INDIAN ASTRONOMY

A Source-Book

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BOMBAY

IN nearly 3000 verses extracted from a large number of original texts on Indian astronomy, and presented with English translation, notes and tables, this publication attempts to provide a scientific insight into the main characteristics of Indian astronomy, the methodologies evolved, instruments used, mathematically developed computation procedures and the innovative trends as well as the rationale associated with them. Several of the passages throw light on the importance attached to continued observation and the concern for accuracy of the traditional Indian astronomer, which would set at rest the general view that Indian astronomy has been, by and large, speculative and empirical. The material presented in this *Source-book* would possibly lead to fresh attempts towards a comparative and critical appreciation of Indian astronomy in relation to those of the other culture-areas.

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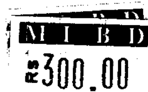
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(Based primarily on Sanskrit Texts)

Compiled By

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And

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FOREWORD

IN the perspective of human history of over five thousand years one of the most fascinating intellectual endeavours of man has been devoted towards a rational understanding and interpretation of the celestial bodies, their nature and structure, their movements and their relation with his own habitat, the Earth. In the past, such an effort had been characterised by prolonged observations, mainly with the naked eye, at times aided by some instruments which may seem to us at present rather crude and primitive. The spirit of man to unravel the mysteries of the so-called heavens has continued unabated and a large number of astronomical observations have come down to us either in the form of recordings or by oral transmission. These observational endeavours, which for long were essentially geo-centric, assumed new dimensions about five hundred years ago when the Copernican helio-centric model provided a new way of understanding and interpretation of the solar system. In the succeeding two centuries, the Newtonian law of gravitation and the telescope not only opened up new vistas of astronomy but also placed them on a truly scientific foundation. Since the beginning of this century, the applications of sophisticated mathematics and of the new physical principles as well as the use of large telescopes have added further dimensions to astronomy and astro-physics.

In India, modern astronomical investigations came to the fore in the beginning of this century and are being fostered specially since Independence. But the Indian astronomical tradition is very ancient, over four thousand years old. It is still a living tradition as an integral part of the social and religious life of the people, in which the traditional calendar called the *Pañcāṅga* continues to play an important role. The origins of the Indian calendrical computation methods can be traced to the Vedic period, over three thousand years ago, and since then astronomy in India has progressed continuously with its own original ideas. At times it has also assimilated the astronomical methods of the other culture-areas in a spirit of open-mindedness, thus displaying a real scientific attitude.

A fact of great importance is that India has produced a vast literature on different aspects of astronomy. According to the American scholar, David Pingree, who has surveyed extensively

the *Jyotiṣa* literature of India, it would appear that more than 1,00,000 manuscripts on *Jyotiṣa* (including astronomy and astrology) still survive in the public and private repositories in India and outside, of which a very substantial part, running to several thousands, relates to astronomy. However, the number of works which have been either studied or which have received general attention is far less when compared with the enormous wealth of source materials, which are mostly in Sanskrit, although a good number of important texts have been studied in some depth. Even so, a comprehensive survey of the more important original sources and their significant passages so as to bring to light the basic concepts and their evolution, the style of presentation, the linguistic terminology, the methods of observation and documentation in which the Indian astronomers were admittedly proficient, has been a long-felt need.

The present publication INDIAN ASTRONOMY: A SOURCE-BOOK is intended to provide a general insight in respect of these aspects. About 3000 verses have been extracted from a large number of original sources, mainly in Sanskrit, and presented with their translations in English and notes, under five major divisions, viz. (i) General Ideas and Concepts; (ii) Astronomical Instruments; (iii) Computation methods; (iv) Occultation and (v) Innovative trends, with appropriate sections, sub-sections and topical headings.

The original passages have brought out authentically the basic characteristics of Indian astronomy, the expertise expected of an astronomer in ancient times, his concern for accuracy, the intricate methodologies adopted by him and, more significantly, the importance given to observation and the mathematically developed documentation. These would, to a great extent, set at rest the prevalent view that the Indian astronomers were generally speculative and empirical.

The Source-book, the first of its type in India on astronomy, does not claim to have presented in it a complete account of Indian astronomy. Its primary object is to present a profile which might lead to further efforts towards examining more of the manuscripts which are still awaiting the attention of scholars.

The scientific manuscript wealth of India is indeed enormous. An authentic discovery of India's scientific heritage demands a critical evaluation of the original sources and a rational presentation of the scientific heritage, through original sources. Guided by this

view, Nehru Centre, as part of its Discovery of India Project, took a step in this direction two years ago leading to the present publication.

I am deeply appreciative of the scholarly work done by Dr. B. V. Subbarayappa and Dr. K. V. Sarma in preparing this volume in a rather short time, for its release on the occasion of the meeting of the General Assembly of the International Astronomical Union, in New Delhi.

Raja Ramanna
General Secretary
Nehru Centre, Bombay
and
Chairman
Atomic Energy Commission

BOMBAY,
November 9, 1985

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PREFACE

THERE has been a considerable number of publications, both general and scholarly, on Indian astronomy. Some of the important texts, mainly in Sanskrit, have also been critically edited and published. These have, in their own way, attempted to bring out several aspects and achievements of some noted Indian astronomers in the ancient and medieval periods.

For quite some time, however, a need for a source-book has been felt not only by those interested in traditional Indian astronomy but, more important, by a number of modern astronomers in order to have a scientific insight into the main characteristics of Indian astronomy, the methodologies developed, instruments used, the manner of documentation of astronomical data and the like, authentically in original source-forms. Most of these scholars have been finding it difficult to get at the desired sources and to understand them, partly because of their inadequate or limited linguistic expertise and partly because of the fact that such sources are not easily available to them in one place, and when desired.

The present compilation, *Indian Astronomy: A Source-book*, is aimed at providing, as far as possible, the important sources, mainly based on Sanskrit works, with their translations in English, and with the necessary notes and references. The source materials on Indian astronomy, admittedly, run into some thousands and it is almost an impossible task for any one to include all of them, or even a substantial part of them, in the form of a single source-book. Nevertheless, a judicious selection could be made, leading to a reasonably representative compilation of important sources. Our present efforts are in the nature of just a first step in this direction.

After going through a large number of sources, we have selected about three thousand verses for inclusion in this volume and presented them under five major divisions: I. General ideas and concepts, II. Astronomical instruments, III. Computation, IV. Occultation, and V. Innovative trends, each divided into several sections, subsections and topics, analysing the numerous traits of Indian astronomy, including the basic views and concepts, the expertise required of an

astronomer, the importance given to observation and accuracy, methods of mathematics-based computation, astronomical constants, the rationale, attempts at making innovations etc.

Generally, the extracts under each section have been presented in a chronological manner and, if there is departure from this, it is largely because of the succinct manner in which a particular idea has been presented in a later text. As to the translations in English, we have utilized such of them as are available in the published works and, when no concerned translation was available, we have given our own translation. It has to be mentioned here that each translator is prone to have his own approach, some giving almost the literal translation and others, translations bordering on an exposition, apart from the individual style of presentation. In any case, the reader has the advantage of having the original passages given above the translations to refer to, when any doubt or ambiguity arises. Further, while short notes and numerical and other types of information have been provided as footnotes, for the rationale and worked out examples, the reader has to consult the references cited in the footnotes.

A Select Bibliography of over 300 important texts on Indian astronomy, in chronological order, has been given in Appendix II for further reference by interested scholars. A Glossary of Technical Terms forms Appendix IV, and an exhaustive list of the terms used to denote numbers according to the *Bhūtasaṅkhyā* notation forms Appendix V.

We are very much thankful to Nehru Centre, Bombay, for giving us an opportunity to work on this *Source-book*. We express our sincere gratitude to Dr. Raja Ramanna, General Secretary, Nehru Centre, who not only provided the financial support for this work from Nehru Centre but also gave us constant encouragement and guidance. He has been good enough also to contribute a Foreword to this publication. We are also thankful to Shri N.V.K. Murthy, Chief Executive, Nehru Centre, for all the help extended by him towards the publication of this volume.

We are greatly beholden to several institutions and scholars whose publications have been of use in this compilation. Among the former, we would like to make a special mention of the Indian National Science Academy, New Delhi, the Vishveshvaranand Institute, Hoshiarpur, the Mathematics Department of the University of Lucknow, the Marthand Bhavan, Kurali (Panjab), the Bombay University Library

and the Adyar Library and Research Centre, Adyar, Madras. Among scholars who are no more with us, we are indebted to the late Prof. T.S. Kuppanna Sastry and Dr. Bina Chatterjee, and among the other scholars, our heartfelt thanks are due to Professor K.S. Shukla, Dr. Arka Somayaji, Dr. S. D. Sharma, Dr. S. R. Sarma, Professor R.C. Gupta and Prof. V.S. Narasimhan for their permission, either oral or in writing, for using their translation in this *Source-book* whenever necessary. By way of acknowledgement, after every such translation, we have indicated the names of these scholars in abbreviation. We are also thankful to Dr. (Miss) Mira Roy for her scholarly help extended to us.

We are deeply appreciative of the devoted cooperation of the Shri B. K. Rao, Manager, Vasanta Press, Adyar, Madras, in undertaking the printing of this work at short notice. But for his organizational efficiency and committed endeavours, this book of intricate composition and execution would not have come out of the press within a period of four months. We are also appreciative of the labours of the technical and other members of staff of Vasanta Press who have worked hard towards this publication.

MADRAS,
November 1, 1985

B. V. SUBBARAYAPPA
K. V. SARMA

INTRODUCTION

THE word 'astronomy' owes its etymology to the Middle English and the Old French term 'astronomie', which, in turn, was derived from the Latin form 'astronomia' through the Greek word 'astronomos' meaning 'star-law'. The stars and the sky, of yore as now, have an irresistible appeal to the human mind. Ever since the ancestral human being was able to stand erect and observe the sky above, he has been alike wondering and speculating on the luminaries of the sky. The Sun, the recurring sequence of day and night, the waxing and waning Moon at periodic intervals, the night sky embedded with countless glimmering stars, the appearance and gradual disappearance of certain bright objects (now called planets) must have not only excited but also stimulated early man to arrive at some sort of understanding about their periodicity. When he settled down as a food-producer, his prolonged observations must have enabled him to relate his agricultural operations principally to the Sun and the Moon, as also to determine, albeit empirically, the cyclic seasons as well as the day and night timings. In course of time, specially when the civilizations began to sprout in the river-valleys, there emerged an abiding interest in him in the heavenly bodies and the need for continuous observations leading to computations and recordings.

The ancient civilizations of the Nile as well as of the Euphrates and Tigris, now designated as the Egyptian and the Mesopotamian civilizations, respectively, have left behind the recordings denoting their astronomical acumen. The Indus Valley civilization (fl. *c* 2350-1700 B.C.), the youngest but by far the largest of the three ancient civilizations, too has developed certain astronomical concepts and their practical applications. Nevertheless, no records of these have been found so far, except for a seal which, according to some scholars, may be suggestive of a lunar motion through an asterism. In any case, of the calendrical computations of the Indus Valley people, we have little information.

As to the earliest sources of Indian Astronomy, then, one has perforce to look to the four Vedas and the Vedic literature. The word *jyotiṣa* in the Vedic literature connotes 'astronomy' (also astrology later) which was recognised as the foremost of the six auxiliaries of the Veda. The earliest Vedic text on astronomy, the *Vedāṅga-Jyotiṣa*,

emphatically states that 'just like the combs of peacocks and the crest jewels of serpents, so does *jyotiṣa* stand at the head of the auxiliaries of the Veda.' The Vedic life was noted for the performance of several sacrifices at prescribed times, thus forging a relationship between the performer of sacrifices (microcosm) with the heavens (macrocosm). There were monthly rituals like the *Darśapūrṇamāsa* and seasonal rites like the *Cāturmāsya*. The sacrificial session called the *Gavāmayana* was specially designed for the daily observation of the movements of the Sun and of the disappearance of the Moon, and this must have given the priests sufficiently precise knowledge about the astronomical elements. We have evidence to show that even knowledge of a special kind, like the *Saros* of the Greeks, for predicting the eclipse, was possessed by the priests of the Atri family. According to the *Vedāṅga-Jyotiṣa* 'one learned in the Vedas who has also learnt the lore of the movement of the Moon, the Sun and the Stars, will enjoy, after death, a life in the world wherein the Moon, the Sun and the Stars move, and he will have also, on the Earth, an unending line of progeny'. In other words, *jyotiṣa* or the science as astronomy, was an integrated part of the life of the Vedic people of whom the Vedic priests were well versed in astronomy. The astronomical knowledge was needed by them, apart from the sacrifices, for festivities, marriages, sowing of seeds and the like—a tradition continuing even to this day.

Though the *Vedāṅga-jyotiṣa* is, as noted before, the earliest text in India exclusively devoted to astronomy, the four Vedic *saṃhitās* (the *Ṛgveda*, the *Yajurveda*, the *Sāmaveda*, and the *Atharvaveda*), the *Brāhmaṇas*, the *Āraṇyakas* and the *Upaniṣads* contain a good deal of astronomical knowledge. As to the *Vedāṅga-jyotiṣa* itself, there are two recensions—the *Ṛgvedic* one containing 35 verses, and the *Yajurvedic* having 43 verses. Like the five other auxiliaries of the Veda, namely, phonetics (*Śikṣā*), ritual (*Kalpa*), grammar (*Vyākaraṇa*), etymology (*Nirukta*) and metrics (*Chandas*), the *Vedāṅga-jyotiṣa* is in an aphoristic form or the *sūtra* style—a style noted for its depth of contents. To quote Winternitz: "there is probably nothing like these *sūtras* of the Indians in the entire literature of the world".

The astronomical knowledge of the Vedic literature may be summarised thus: The universe was conceived as of three distinct parts—the earth (*pṛthvī*), the firmament (*antarikṣa*) and the heavens (*dyaus*). The Sun was regarded as the most important heavenly object and its path, the ecliptic, was considered sacred. The Moon was the next most important and became the obvious choice for time-

reckoning. It was referred to as *māsakṛt* ('maker of the month')—the interval between two consecutive new moons or full moons. There were two systems of month-reckoning, namely, the *amānta* and the *pūrṇimānta*, ending with the new moon and the full moon, respectively. The Moon's path was observed in relation to the 27 or 28 *nakṣatras* or asterisms and the lunar zodiac was well determined. There is no denying the fact that, although there were lunar zodiac presentations in Babylonia, China and Arabia, the method and the manner adopted by the Vedic priests unmistakably point to their originality. The names of the lunar months were given on the basis of the *nakṣatra* in which the full moon occurred. The twelve lunar months were divided into six seasons of two months each. There were also special names for the solar months. (see the text below 7. 1.8-9)

A month was divided into two parts or *pakṣas*, the bright half and the dark half of one lunation, each *pakṣa* having 15 *tithis*, an ingenious devise, which is characteristically Indian, for calendrical purposes and the names of the *pakṣa* following the Sanskrit ordinals. A day was regarded as consisting of 30 *muhūrtas*, (the longest at the summer solstice being 18 and the shortest at the winter solstice 12 *muhūrtas*). Intercalation at regular intervals was known for luni-solar annual calendrical adjustments. The Vedic priests possessed specific knowledge of the solstices and the Svarbhānu legend points to their observation of the solar eclipses. The Atri family of priests seems to have specialized in the observation of both the lunar and solar eclipses.

The *Vedāṅga-jyotiṣa* conceived of a cycle of five years, a luni-solar cycle called the *yuga*, at the beginning of which the Sun and the Moon would lie at the starting point of the *nakṣatra* Dhaniṣṭhā. During this period, there would be 5 revolutions of the Sun, 67 Moon's sidereal and 62 synodic months; 1830 *sāvana* or civil days; 1835 sidereal days, 1800 solar days and 1860 lunar days or *tithis*. The civil day was divided into 30 *muhūrtas*: 1 *muhūrta* into 2 *nāḍikās*: 1 *nāḍikā* into 10 1/20 *kalās*; 1 *kalā* into 124 *kāṣṭhās*; and 1 *kāṣṭhā* into 5 *akṣaras*. The text, in a succinct manner, mentions the 27 *nakṣatras*, 10 *ayanas* and *viṣuvas* and 30 *ṛtus*—all in an archaic, aphoristic language. The object of dividing the day into 124 parts was to have the ending moments of the *tithis* in whole units; likewise of the *nakṣatras*. Each *nakṣatra* was conceived in terms of 124 parts. Since there are 1830 civil days in a five-year period, the year would consist of 366 days each; the *ṛtus* (when divided by 6) would have 61 days, and each *ayana*, 183 days. Two intercalary months were thought of, one at the end of the fifth

ayana and the other at the end of the 10th, for luni-solar adjustments. To make the calendrical computation more accurate, the day-length was considered to change by 6 *muhūrtas*, in one *ayana*, i.e. the daily change would be 6/183 or 2/61 *muhūrtas*.

The later Hindu works like the *Gargasaṃhitā* and the *Paitāmaha* as well as the Jaina works like *Sūryapannatti* and *Jyotiṣakaraṇḍa* follow the same system as that of the *Vedāṅga-jyotiṣa*. However, the *Paitāmaha* thinks of the *Vyatīpāta-yoga* in addition, and the Jaina works indicate the winter solstice to be in *Śrāvaṇa* instead of *Śraviṣṭhā* by including *Abhijit* as the terminal of the zodiac. The Jaina astronomy, in tune with its cosmography, conceived of two Suns, two Moons and two sets of *nakṣatras*. The ecliptic was divided into 28 parts, beginning with the *nakṣatra* Abhijit. The *Sūryapannatti* has some curious statements concerning the movements of planets, the Sun and the Moon as well as of the stars.

The *Vedāṅga-jyotiṣa* is attributed to Lagadha who might have codified the astronomical knowledge which was in vogue for several centuries before him. The classical language employed in the text as now available indicates that Lagadha's work should have been redacted by a later person in about 400 B.C.

While the astronomical computations enunciated in the *Vedāṅga-Jyotiṣa* continued to be in use for a long time, possibly during the few centuries preceding the Christian era, there was emerging a new class of astronomical literature called the *Siddhāntas*. An important development was the gradual replacement of the *nakṣatra* system by the 12 Signs of the zodiac, *Meṣa*, *Vṛṣabha*, *Mithuna*....*Mīna*, similar to the animistic notions of the Babylonians and from them of the Greeks. With the invasion of India by Alexander the Great in the fourth century B.C. and the subsequent Hellenic and the Hellenistic or Greco-Roman contacts with India, conceivably the astronomical elements of the former should have influenced the Indian culture-area. Such elements included the length of the year, planetary motions, calculation of solar and lunar eclipses, ideas of parallax, determination of mean longitudes etc. Another notable aspect was that, during this period and a few centuries after the Christian era, the Indian culture-area developed new mathematical methods, many of them for promoting astronomical calculations. The result was that the Indian astronomers were able mathematicians too and the mathematicization of astronomy added a veneer of accuracy to the study of several astronomical phenomena.

The newly emerging *siddhāntas* were in the nature of rules or the enunciation of methods for arriving at solutions of the concerned astronomical problems. According to the Indian traditional belief, there were principally eighteen such *siddhāntas*, namely, the *Sūrya*, *Paitāmaha*, *Vyāsa*, *Vāsiṣṭha*, *Atri*, *Parāśara*, *Kāśyapa*, *Nārada*, *Gārgya*, *Marīci*, *Manu*, *Aṅgira*, *Lomaśa* (or Romaka), *Pauliśa*, *Cyavana*, *Yavana* (or the Greek), *Bhṛgu* and *Śaunaka*. Five of these *Siddhāntas*, viz., the *Saura*, the *Paitāmaha*, the *Vāsiṣṭha*, the *Romaka* and the *Pauliśa* were ably codified by Varāhamihira in his *Pañcasiddhāntikā* (c. 505 A.D.) who has emphasized that the *Saura* was the most accurate of them all.

The *Saura*, also called the *Sūrya Siddhānta*, has no human authorship associated with it. It is possible that it represents the contributions of more than one author and over a period of time. It has the *Sāṅkhya* principles in its cosmogony and its own conception of the *Yuga*. Varāhamihira might have incorporated certain aspects of Āryabhaṭa's *Ārdharātrika* system into it. The Modern *Sūrya Siddhānta* now current has also certain elements of Brahmagupta and others, and possibly it might have taken its present form during the sixth to the twelfth century.

In Varāhamihira's *Saura*, a period of 180,000 years has been stated in which there would be 66,389 intercalary months and 1,045,095 omitted lunar days, thus giving 65,746,575 days in that period or the *yuga*. The Modern *Sūrya Siddhānta* elaborates upon the *kalpa* or *mahāyuga* (or *Yuga*) of 4,320,000 years (i.e., 24 times 180,000), subdividing the latter into *Kṛta*, *Tretā*, *Dvāpara* and *Kali* ages in the descending order of 4:3:2:1 ('000 divine years) with periods (*sandhyas*) of both twilights for transition from one age to another in tune with the postulates of the *Purāṇas* and the *Smṛtis*. Thus, *Kṛta* (4000+400+400), *Tretā* (3000 + 300 + 300), *Dvāpara* (2000 + 200 + 200), and *Kali* (1000 + 100 + 100) constitute a total 12,000 divine years. A divine year was regarded as being equal to 360 solar years and hence the *mahāyuga* would be equivalent to 4,320,000 (12000×360) solar years. The concept of *kalpa* or *mahāyuga* in Indian astronomy will be briefly dealt with later.

The Modern *Sūrya Siddhānta*, in its 14 chapters, deals with, among others, the mean motion of the planets, their true positions, solar and lunar eclipses, planetary conjunctions, heliacal rising and setting of planets, cosmogony, time-reckoning, and astronomical instruments. As to the cause of planetary motions, the text continues to expound

that they are impelled by a wind called *Pravaha* uniformly, and that an invisible divine force pulls or pushes them at their perigees so as to make their motions become variable. The planets were believed to have been attached to invisible cords of air at the perigees and the nodes. As to the notion of equinoxes, the text adopted the theory of libration of equinoxes and expounds that, in a *mahāyuga*, the number of oscillations of the equinoxes about a fixed point, like the swing of a pendulum eastward and westward, would be 600 and that the maximum eastward or westward deviation would be 27° which works out to a precessional rate of 54″ (modern value is 50.25″). To this aspect we shall return later. It may be emphasized that the *Sūrya Siddhānta* has been an important textual source for the traditional Indian astronomers, mainly in north India especially for their calendrical computations. There have been a large number of commentaries on this text composed even up to the eighteenth century A.D.

In respect of the other four *siddhāntas*, the *Paitāmaha* is the least accurate. Its astronomical elements are more or less similar to those of the *Vedāṅga-jyotiṣa*. The *Vāsiṣṭha* is somewhat better inasmuch as that in addition to the *nakṣatra* system, it deals with a zodiac and its subdivisions in considerable detail. It also contains certain rules for determining day-lengths, anomalistic months etc., besides presenting the true motions of the five planets (Venus, Jupiter, Saturn, Mars and Mercury), including their direct and retrograde motions. According to this text, the length of the solar year works out to 365.36 days approximately and the sidereal year-length is 365 1/4 days.

The *Pauliśa*, according to Varāhamihira, is accurate. It has astronomical calculations for *ahargaṇa* or the number of civil days elapsed from a particular time to a chosen date, derivation of the mean positions of the Sun and the Moon from *ahargaṇa*, and on to their true places, thereby revealing a knowledge of the anomoly and equations of the centre. The text as also sine tables, the radius adopted being 120′ instead of 3438′ generally used in other astronomical works. The calculations concerning the eclipses are far from being accurate. The length of the solar year, according to this text, works out to 365.2583 (43,831 civil days in 120 years). The *Pauliśa-siddhānta* seems to have undergone some modifications in course of time and was one of the sources for an account of Indian astronomy by al-Bīrūnī who regarded Pauliśa as a Greek form the city of Alexandria (Sachau, I, p. 153).

The *Romaka* (possibly pointing to its Hellenistic source) postulates a luni-solar cycle of 2,850 years consisting 1,050 intercalary months, and 16,547 omitted lunar days. The lengths of the year (365 days 5h 55′ 12″) and of the synodic month (29 days 12h 44′ 2.2″) are more or less similar to those found in the works of Hipparchus and Ptolemy. From the number of years of a luni-solar cycle and of the intercalary months, it would seem that the text was well aware of the Meton's cycle of 19 years (*c.* 430 B.C.). Another aspect of the text is that it deals with the equations of the centre of the Sun and the Moon. While the values for the Sun are practically in agreement with those given by Ptolemy, there is some variation concerning the Moon.

The three or two centuries before and after the Christian era were an important period in the Indian social as well as religio-philosophical developments. Buddhism, which had a substantial hold on the masses, was prone to undergo certain changes leading to the *Mahāyāna*, which, in turn, was incorporating into itself certain esoteric elements. Jainism too was streamlining itself with its new epistemological methods concerning matter, motion, space, time and soul. Both had fostered rigorous religious rites and practices. The orthodox Brahmanical hold on the society was equally rigorous with many stratified injunctions for the caste-ridden society. New sub-castes were emerging and they were required to follow the prescribed ways of living, both social and religious. In this socio-religious mileu, the priestly endeavours in determining the astronomical elements for a wide variety of socio-religious activities like marriages, birth and death ceremonies, festivals or worshipping times, occupied an important position. In addition, the native astrology began to establish itself during this period. The concepts of week, its names in terms of planets and their influence on the human beings—concepts which probably owed their inspiration to those of the Hellenistic areas—began to wield influence practically on all segments of the society. A meticulous observation of planets, their movements and their computation as accurately as possible, became a felt necessity.

David Pingree who has carried out an extensive survey of the literature on Indian astronomy states: "At present there exist in India and outside of it some 1,00,000 manuscripts on the various aspects of *jyotiḥśāstra*. The great majority of these were copied within the seventeenth, eighteenth and nineteenth centuries; for manuscripts cannot long survive in India except under exceptional circumstances. We have, therefore, essentially those texts selected for study or

composed by the scholars of the Mughal and British *rājyas*. Since the copying of the manuscripts is virtually dead in modern India, many of these estimated 1,00,000 manuscripts will soon disappear, and the possibility of our achieving a reasonably accurate assessment of the continuity, development, and transformation of the astral and mathematical sciences in India, will be correspondingly diminished. But even without this appalling prospect, we must be constantly aware of the arbitrary way in which was made the selection of texts and commentaries preserved in today's libraries." (*A History of Indian literature*: *Jyotiḥśāstra*: *Astral and Mathematical literature*, p. 118). While one may not agree with Pingree in respect of what he calls 'the arbitrary way of selection', his admirable survey reveals the enormous wealth of the source-materials.

Pingree divides the history of Indian astronomy into five periods, namely (i) Vedic (1000—400 B.C.); (ii) Babylonian (*ca.* 400 B.C.—200 A.D.); (iii) Greco-Babylonian (*ca.* 200—400 A.D.); (iv) Greek (*ca.* 400—1600 A.D.); and (v) Islamic (*ca.* 1600—1800 A.D)., 'depending', as he puts it, 'in most cases on the foreign origin'. He states further, "though the fundamental approach and many of the models and parameters of each period were determined by the foreign sources, the *basic traditions of Indian astronomy imposed on these external systems its peculiar stamp, and transformed the Science of Mesopotamia, Greece, or Iran into something unique to India.*" (*op.cit.* pp 8-9). Inherent contradictions apart in the aforesaid statements, an objective examination of the major Sanskrit sources of Indian astronomy doubtless points out that the Indian astronomers had their own originality, developed their observational and computation methods along with the necessary mathematical aspects in which also they displayed originality. They possessed an open-mindedness, a characteristic of scientific attitude, and a receptivity to the astronomical ideas of other culture-areas. But at the same time, as Pingree concedes, they had their own basic tradition which enabled them to develop the science of astronomy on mathematical grounds. In any case, of the most important intellectual endeavours in India for a long time, was indeed astronomy, as evid-denced by the surviving wealth of manuscripts numbering about 100,000, as stated before.

The richness, flexibility and the rationale of the Sanskrit language were such that they nurtured the scientific terminology in general and that of astronomy and mathematics in particular. Apart from a wide range of technical terms for denoting various astronomical

phenomena as well as the concepts and their explanation, the Indian astronomers who, by and large, were also mathematicians, displayed their ingenuity in devising terms for numbers and mathematical operations. Such a device was needed for capsuling the scientific aspects, by and large, in metrical form. This called for an expertise in Sanskrit and an innovation in communication. Precision and brevity had a great appeal to the authors. In fact, Āryabhaṭa I (born A.D. 476) produced his work, the *Āryabhaṭīya*, in the form of 121 verses enunciating his own alphabetical system for numbers. This is the earliest extant mathematics based astronomical work of great importance in the history of Indian mathematics and astronomy.

In addition to a high level of linguistic attainment, the Indian astronomer had to be proficient in the different systems of measurement of time like the civil and the solar, the sidereal and the lunar, planetary motions, *ahargaṇa* calculation, deductions and reductions from the gnomonic shadow, parallax, eclipses and a host of other aspects as detailed in the *Bṛhatsaṃhitā* of Varāhamihira (see below, pp. 9-10). He had to possess the ability to forecast, by calculation, the times of the commencement and ending, direction, magnitude, duration, intensity and colour of the eclipses of the Sun and the Moon as well as the conjunctions of the Moon with the five *Tārāgrahas* or non-luminous planets and the planetary conjunctions. Various qualifications for an expert astronomer were also stipulated in terms of his proficiency. He was expected to draw and demonstrate for the understanding of his students the diagrams for the computation of astronomical phenomena. His concern for accuracy was indeed great, as evidenced by the corrections which he had to determine by experimentation from time to time. He was to be a keen observer of the planets over long periods of time. The recordings by Parameśvara, an astronomer of Kerala, of the eclipses over a period of 55 years reveal the importance attached to observations as well as the methodology associated with them. (See below, pp. 13 ff.)

Astronomers

All these assiduously nourished a viable tradition, which, from about the fifth century A.D. onwards, produced a galaxy of astronomers. It is well nigh impossible to list even the more important of them whose number is legion and whose works run into thousands. A special mention, might, however, be made of the following astronomers, their major works being indicated within brackets.

5th-6th century: Āryabhaṭa I (*Āryabhaṭīya* and *Āryabhaṭasiddhānta*, the latter available only in quotations and redactions).

6th century: Prabhākara, pupil of Āryabhaṭa; Varāhamihira (*Pañcasiddhāntikā* and *Bṛhatsaṃhitā*).

6th-7th century: Bhāskara I (*Mahābhāskarīya*, *Laghubhāskarīya* and *Āryabhaṭīya-bhāṣya*); Brahmagupta (*Brahmasphuṭasiddhānta* and *Khaṇḍa-khādyaka*); Haridatta (*Grahacāranibandhana*); Devācārya (*Karaṇaratna*).

8th-9th century: Lalla (*Śiṣyadhīvṛddhidatantra*); Govindasvāmin (*Mahābhāskarīya-bhāṣya*); Śaṅkaranārāyaṇa (*Laghubhāskarīya-vivaraṇa*); Pṛdhūdakasvāmin (*Brahmasiddhānta-Vāsanābhāṣya* and *Khaṇḍakhādyaka-vivaraṇa*).

10th century: Vaṭeśvara (*Vaṭeśvarasiddhānta*); Muñjāla (*Laghumānasa*); Śrīpati (*Siddhāntaśekhara*); Āryabhaṭa II (*Mahāsiddhānta*); Bhaṭṭotpala (*Khaṇḍakhādyaka-vyākhyā* and *Bṛhatsaṃhitāvyākhyā*); Vijayanandin (*Karaṇatilaka*).

11th century: Someśvara (*Āryabhaṭīya-vyākhyā*); Śatānanda (*Bhāsvatī*)

12th century: Bhāskara II (*Siddhāntaśiromaṇi* with *Vāsanābhāṣya*, *Karaṇakutūhala*); Mallikārjuna Sūri (*Sūryasiddhānta-vyākhyā*); Sūrya-devayajvan (*Āryabhaṭīya-Prakāśikā* and *Laghumānasa-vyākhyā*); Caṇḍeśvara (*Sūryasiddhānta-bhāṣya*).

13th century: Āmarāja (*Khaṇḍakhādyaka-Vāsanābhāṣya*).

14th century: Makkibhaṭṭa (*Gaṇtiabhūṣaṇa*); Mādhava of Saṅgama-grāma (*Sphuṭacandrāpti*, *Agaṇitagrahacāra*, *Veṇvāroha*); Madanapāla (*Vāsanārṇava* on the *Sūryasiddhānta*); Viddaṇa (*Vārṣikatantra*).

15th century: Parameśvara (*Dṛggaṇita*, *Goladīpikā*, *Grahaṇamaṇḍana*, *Grahaṇanyāyadīpikā*, *Candracchāyāgaṇita*, *Āryabhaṭīya-vyākhyā* *Bhaṭadīpikā*, *Mahābhāskarīya-vyākhya*, *Laghubhāskarīya-vyākhyā*, *Sūryasiddhānta-vyākhyā* and *Mahābhāskarīya-bhāṣya-vyākhyā*); Yallaya (*Āryabhaṭīya-vyākhyā*, *Jyotiṣadarpaṇa*, *Laghumānasa-kalpataru* and *Kalpavallī* on the *Sūrya-siddhānta*); Rāmakṛṣṇa Ārādhya (*Sūryasiddhānta-subodhinī*); Cakradhara (*Yantracintāmaṇi*); Nīlakaṇṭha Somayāji (*Jyotirmīmāṃsā*, *Golasāra*, *Candracchāyāgaṇita*, *Siddhāntadarpaṇa*, *Tantrasaṅgraha* and *Āryabhāṭīya-bhāṣya*).

16th century: Jyeṣṭhadeva (*Yuktibhāṣā*, *Dṛkkaraṇa*); Śaṅkara Vāriyar (*Karaṇasāra*, *Tantrasaṅgraha-Yuktidīpikā*); Bhūdhara (*Sūryasiddhānta-vivaraṇa*); Tammayajvan (*Grahaṇādhikāra*, *Sūryasiddhānta-Kāmadogdhrī*); Gaṇeśa Daivajña (*Grahalāghava*, *Tithi-cintāmaṇi*, *Pratodayantra* and *Siddhāntaśiromaṇi-vyākhyā*); Acyuta Piṣāraṭi (*Karaṇottama*, *Sphuṭanirṇaya* with *Vivaraṇa*, *Uparāgakriyākrama*, *Rāśigolasphuṭānīti*); Rāma (*Rāma-vinoda*).

17th century: Viśvanātha (*Grahaṇārthaprakāśikā*, *Grahalāghavaṭīkā*, *Karaṇakutūhala-Udāharaṇa*); Caṇḍīdāsa (*Karaṇakutūhala-ṭīkā*); Putumana Somayāji (*Karaṇapaddhati*, *Pañcabodha*, *Nyāyaratna*); Nityānanda (*Siddhāntarāja* and *Siddhāndasindhu*).

18th century: Maharājā Sawai Jayasiṃha (*Yantrarājaracanā*, *Jayavinodasāraṇī*); Jagannātha Samrāṭ (*Samrāṭsiddhānta*).

19th century: Śaṅkaravarman (*Sadratnamālā*).

The Indian astronomical literature in Sanskrit and allied languages can be broadly classified into *Siddhāntas*, *Karaṇas* and *Koṣṭhakas*, besides those which deal specially with the astronomical instruments (*yantras*). The *Siddhāntas*, some of which are large and many others small, are in the nature of composite texts concerning several astronomical aspects including the fundamental principles on which they are based and commencing their computations from the beginning of the *Kalpa* or *Yuga*. The *Karaṇas* are noted for their practical rules specially for computational purposes. They do not attempt to compute the motion of the planets from the beginning of the *Kalpa* or the *Yuga*, but take a contemporary epoch as the starting point for calculation. Besides, except for the Moon's, the longitudes of the apogees and nodes of the planets are regarded by them as fixed. The *Grahacāranibandhana* of Haridatta (*c*. 650-700), *Karaṇaratna* of Devācārya (fl. 689), the *Karaṇatilaka* of Vijayanandin (10th century), *Karaṇaprakāśa* by Brahmadeva (11th century), *Bhāsvatī* by Śatānanda (11th century), *Karaṇakutūhala* by Bhāskara II (12th century), *Grahalāghava* of Gaṇeśa Daivajña (16th century), *Rāma-vinoda* by Rāma (16th century); *Sūryaprakāśa-karaṇa* by Viṣṇu (17th century) and the *Karaṇa-vaiṣṇava* by Śaṅkara are among the noted works of the *Karaṇa* literature which is indeed vast.

Certain types of astronomical tables which are computed with a view to aiding the computation of planetary positions are called the *Koṣṭhakas* or *Sāraṇis*. These are helpful specially to those practising astrology as well as those preparing *pañcāṅgas* or almanacs. Such planetary tables have been prepared and presented in ingenious ways. The early examples of the planetary tables are *Grahajñāna* by Āśādhara (epoch: 20th March 1132); *Laghukhecarasiddhi* by Śrīdhara (epoch: 20th March 1316); *Makaranda* by Makaranda (epoch: 27 March 1478); and *Kheṭamuktāvalī* by Nṛsiṃha (epoch: 31 March 1566). Pingree has noticed 19 such planetary tables in his work cited above.

He has also given a good account of the second category of *Koṣṭhakas* which present the beginnings and ends of *tithis*, *nakṣatras* and *yogas* which are useful for religious observations. In addition, there are astronomical tables intended for computing lunar and solar eclipses (*op. cit.*, 42-46).

As Pingree has pointed out after ably analysing a large number of astronomical texts, there developed gradually in India five main *pakṣas*—the Brāhma, the Ārya, the Ārdharātrika, the Saurya and the Gāṇeṣa (daivajña), some texts of course, not falling under any of these *pakṣas* (*op.cit.*, pp. 13-16).

Main conceptual aspects of Indian astronomy

As already noted, the Indian astronomers conceived of a huge period of 43,20,000 years called a *mahāyuga* or *caturyuga* and even a period 1000 times longer, i.e., 43,20,000,000 years called a *Kalpa*. In the Saura and the Brāhma schools, the *kalpa* is regarded as being equivalent to 14 *manvantaras*, a *manvantara* being equal to 71 *mahāyugas* or 306,720,000 years to which must be added 15 *sandhis*, each *sandhi* being 17,28,000 years, which again is equal to the period of *Kṛtayuga*. According to the *Ārdharātrika* system of Āryabhaṭa, the *kalpa* consists of 4,354,560,000 years but equivalent to 14 *manvantaras*, each *manvantara* being composed of 72 *mahāyugas*. Thus the *kalpa* consists of (14×72) 1008 *mahāyugas*. In this system also, a *mahāyuga* is equal to 43,20,000 years. However, here, the four ages, namely, *Kṛta*, *Tretā*, *Dvāpara* and *Kali*, are regarded as being of equal duration (each having 1,080,000 years), unlike in the *Saura* and *Brāhma*, where they are in the ratio of 4:3:2:1 with *sandhis*.

Rotations of the planets in a Mahāyuga

	Brāhma	Ārya	Ārdharā-trika	Saura	Adjusted Saura
Saturn	146,567.298	146,564	146,564	146,568	146,580
Jupiter	364,226.455	364,224	364,220	364,220	364,122
Mars	2,296,828.522	2,296,824	2,296,824	2,296,832	2,296,832
Venus' śīghra	7,022,389.492	7,022,388	7,022,388	7,022,376	7,022,364
Mercury's śīghra	17,936,998.984	17,937,020	17,937,000	17,937,060	17,937,076
Moon	57,753,300.000	57,753,336	57,753,336	57,753,336	57,753,336
Lunar node	—232,311.168	—232,226	—232,226	—232,238	—232,246

Source: David Pingree, *Op. cit.*, p. 15

Such a huge period for computing the mean motion of planets is peculiar to Indian astronomy and it was the ingenuity of the astronomers to develop methods of calculation in relation to this huge period, evidently to avoid decimal fractions concerning intercalary months, omitted lunar days, civil days and the like.

The Indian traditional astronomy is essentially geo-centric and geo-static inasmuch as the Earth is considered to be a stationary sphere at the centre of the solar system. The Sun, the Moon and the planets have a motion of their own from west to east while the asterisms or the stellar sphere is considered to have their motion from east to west, as a result of which the former are supposed to fall behind the latter. This geocentric and geo-stationary view was for the first time modified by Āryabhaṭa I who, while maintaining the geo-centric idea, conceived of a direct rotation of the Earth about its axis, and even gave a precise rate of rotation stating that the Earth rotates through an angle of one second in one *prāṇa* of time (see below p. 26). He explained the apparently retrograde or westward motion of the stationary asterisms by giving an example: "Just as a man in a boat moving forward sees the stationary objects on either side of the river as moving backward, so are the stationary stars seen by people at Laṅkā, as moving exactly towards the west". This concept of Āryabhaṭa I was opposed by the other succeeding leading astronomers like Varāhamihira and Brahmagupta, while his own commentator, Bhāskara I tried to explain it by giving a different interpretation so as to be in tune with the geo-centric and geo-stationary idea.

The Indian astronomers conceived of the celestial sphere in all its details—the zenith, the nadir, the horizon, the prime vertical, the hour circle, the meridian, the ecliptic, the celestial equator and the inclination of the ecliptic to it, the celestial poles etc. Brahmagupta explains the concerned circles as follows: One circle called the *Samamaṇḍala* (prime vertical) has its plane stretching east and west; another lying north and south is known as *Yāmyottara-vṛtta* (the meridian); the *Kṣitija* (horizon) encircles these two like a girdle. The observer on the Earth is situated at the Centre common to these circles (*Br. Sp. Si.*, Gola. 48). Likewise, Āryabhaṭa I, Brahmagupta, Varāhamihira, Lalla and other leading astronomers have vividly explained the different parts of the celestial sphere and in relation to the observer. In general, the Indian astronomers, through calculations of their own, have arrived at the obliquity of the ecliptic with the celestial equator as 24° which forms the basis of their other computations.

The Celestial Sphere has been divided into 12 *rāśis* or Signs (Meṣa, Vṛṣabha,........Mīna). The motions of the planets, the reasons for their very slow, slow, fast and very fast motions, and their retrograde or transverse movements as observed, have been dealt with in a lucid manner, all expectedly within the geo-centric framework.

In this connection, it may be desirable to reflect in brief on one conceptual aspect of astronomy in the Hellenic and the Hellenistic culture-areas. Aristorachus of Samos, known as 'Copernicus of Antiquity' had expounded the idea of the Sun being the stationary centre of the system (heliocentric), a view which, to some extent had originated among the Pythagoreans earlier. Nevertheless, the geo-centric and geo-static concept prevailed and, in the third century B.C., probably Apollonius of Perga developed the eccentric and epicyclic models for explaining the diverse planetary motions. Later, Hipparchus (2nd century B.C.) used these geometric models to explain the solar and lunar inequalities. But, the most important was the development of planetary theories, using these models, by Ptolemy (2nd century A.D.) and the Ptolemiac system ruled the minds of astronomers for centuries to come. In view of the commercial and other contacts which the Indian culture-area had with the Hellenistic or the Greco-Roman world then, as stated before, it is not unlikely that the eccentric and epicycle concepts found favour among the receptive Indian astronomers. In any case, the application of such a concept is found in the *Āryabhaṭīya*. Varāhamihira appears to have used it in recasting the older *Saurasiddhānta* in his *Pañcasiddhāntikā*. Later, Brahmagupta also adopted it, but used the hypotenuse for the correction of conjunctions and the radius as an approximation for finding out the equation of the centre, while Āryabhaṭa I used the hypotenuse for determining both the equation of the centre and for correcting conjunctions. Such methods of adaptation not only reveal the ingenuity of the Indian astronomers but also their open-mindedness towards assimilation of new concepts from outside.

However, Kuppanna Sastry has a different view on this problem. Posing the question: 'Did the Hindu astronomers borrow these ideas (i.e. eccentric-epicyclic) from the Greeks or did these occur to the Hindus naturally, as it had occurred to the Greeks', he writes: "We cannot answer this question with certainty. But an independent origin in India seems more probable, when we see that the clearly western and earlier *Pauliśa* and *Romaka* (*siddhāntas*) did not have either of these theories, that the Hindu constants were different and better in general, that there was already the anology of the representation of the equation of the conjunction, which must occur to the astronomer naturally to explain the motion of the star-planets as seen from the Earth, and that the Hindus, having already a theory of the variant motion in the form of the pull or repulsion of the apogee on the planet,

required the epicyclic representation as a geometrical model, for which there would certainly be an urge." He has presented several more arguments relating to this subject (*IJHS*, 9 (1974) 37-41).

In Indian astronomy, the first point of the *nakṣatra* Aśvinī (near the star zeta Piscium of Revatī) is the fixed point from which the longitudes are measured. According to the *Sūryasiddhānta*, the longitudes of the planets were zero simultaneously 1,95,58,52,179 years before the commencement of the Śaka era (which is reckoned from 78 A.D.) and the apogees and ascending nodes of planets were all supposed to be at the first point of the *nakṣatra* Aśvinī.

The mean motions of the planets computed by Indian astronomers in a period of 4,320,000 years in terms of complete revolutions has been referred to already. They also calculated the number of civil days in this period, thus forging a relation between the revolutions of each planet and the number of civil days. All the planets are supposed to have zero longitude at the beginning of the *mahāyuga*. The beginning of the *Kaliyuga* is an epoch which is presumed to have commenced on the midnight between February 17 and 18, 3102 B.C. The number of civil days elapsed from a given epoch is known as *ahargaṇa*. The Indian astronomers had developed the methods of computing the mean longitude of a planet by multiplying the *ahargaṇa* by the related revolutions of the planet and dividing the product by the concerned civil days. The Indian astronomers have also developed methods for determining the true longitudes of the Sun and the Moon as well as those of the other planets. As and when the necessity arose they had also devised elegant and reasonably accurate methods for the corrections like the deficit of the Moon's equation of the centre and the evection as also for the equation of time due to the obliquity of the ecliptic for all the planets.

According to Kuppanna Sastry, it would be wrong to arrive at a conclusion that the longitudes given in the texts are polar. In his view, Indian astronomers "give the polar longitude for a specific purpose. For the sake of astrological predictions, the Moon, the Sun and the planets' conjunction in polar longitude is given; also to check their correctness in their positions by comparing them with the polar longitudes of the Star. For this purpose, most of the *siddhāntas* give the coordinates of the star in polar longitude readymade, and this has misled the people." (*IJHS*, 9 (1974) 31-33)

Precession of equinoxes

We have noted already that the *Sūryasiddhānta* has dealt with the libration of equinoxes, i.e. an oscillation about the fixed point of *Aśvinī*. The text states that there are 600 such to and fro oscillations in a *mahāyuga* of 4,320,000 years, the period of one such oscillation being 7200 years. The position, according to this text, would be that in the first 1800 years of *Kali*, it moved forward uniformly by 27° (to a point 20′ beyond in respect of the star Bharaṇī and, then in the next 1800 years, i.e. till (3600—3102=about 498 or 499 A.D.), it would oscillate backwards to the first point of Aśvinī, and this regression would continue till 2299 A.D. till it reached 27° behind Aśvinī. Thereafter, it would start moving forwards.

Astronomers like Āryabhaṭa I, Brahmagupta and Lalla do not make any mention about the precession of equinoxes. Bhāskara I was even averse to such a concept, as he dismisses the idea of the precession of equinoxes held by the Romakas (*Romakasiddhānta*) as not correct. Though Varāhamihira was aware of the phenomenon, he had no inkling of the rate of precession.

Devacāryā, author of *Karaṇaratna*, who flourished in A.D. 689, appears to be the earliest astronomer to give a method for computing precession. Later, Vaṭeśvara and Āryabhaṭa II presented methods of their own in terms of solsticial points. (See below, pp. 179-80). In the tenth century A.D., Muñjāla in his work *Laghumānasa* (commented by Munīśvara) recognized the precessional motion, again in terms of '*ayanacalana*', (or movement of solisticial points) and gave the number of revolutions in a *kalpa* as 1,99,669 (which would give an annual precessional rate of 59″.9). About the same time, Pṛthūdakasvāmin (commentator of *Khaṇḍakhādyaka*) gave the revolution of the *ayana* in one *kalpa* as 189,411, calling it the *Ayanayuga*; the rate of precessional motion according to this works out to 56″.82 per year.

The position of Indian astronomy in relation to the precession of equinoxes has been succinctly stated by Kuppanna Sastry as follows: "Even the shifting of the Vernal Equinox from *Mṛgaśiras* to *Rohiṇī*, and from *Rohiṇī* to *Kṛttikā* had been observed in the Vedic peirod. But this had not resulted in the idea of the tropical year as such, as distinguished from the sidereal year, so strong was the hold of the sidereal year upon the astronomical and calendric system of the age. This must be the reason why, even after knowing the continuous motion of the equinoxes, they considered the phenomenon oscillatory, i.e.

something temporary which would rectify itself by an equal motion in the opposite direction. Even if *Muñjāla* gives it as continuously regressing, it can have no value unless he had adequate reason to know it for certain (and he had no reason, it could only be a guess), for the real cause of the phenomenon, viz. the behaviour of the Earth rotating like a spinning top in resisting the pull of the Sun, Moon and planets on the extra matter on its equatorial bulge, was not known at that period.

"Another fact must be mentioned here. We have seen that the Hindu sidereal year, being more than 8 *vināḍīs* longer than the correct sidereal year, the point of *Aśvinī* itself has a progressive motion of more than 8″ per annum. Since the correct precession is about fifty and a quarter seconds, the rate of precession with respect to the Hindu First Point must be more than 58 1/4″, since the *siddhāntas* advocate getting the precession by the observation of the Sun's shadow. Accordingly, *Muñjāla* and the later works give a rate of precession nearly equal to 1′ per annum, which is quite proper. It would be a mistake to suppose, as far as ancient Hindu works are concerned, that the nearer their rate of precession is to 50 1/4″ the more correct it is, e.g. it would not be proper to commend the *Sūryasiddhānta* for its rate of precession of 54″ per annum, on the ground that it is so near 501/4″. (*IJHS*, 9 (1974) 35-37).

Eclipses, Rāhu and Ketu

The mythological view of Rāhu devouring the Sun or the Moon, thus causing the eclipse, was current among the superstitious laymen and even among some astrologers cum astronomers. However, in the fifth century A.D. Āryabhaṭa I gave an explanation for the eclipses in terms of the Sun being obscured by the Moon, and the shadow of the Earth obscuring the Moon. His junior contemporary, Varāhamihira categorically stated that a lunar eclipse is caused by the entry of the Moon into the shadow of the Earth. (See below, pp. 196-97). Even so, he maintained the nomenclature of *Rāhu,* stating that the ascending node is Rāhu's head, and the descending node, Rāhu's tail. Later the word *pāta* began to be used for connoting the nodes. Yet, the word *Rāhu* and *Ketu* are used in the technical sense of ascending and descending nodes in a number of astronomical texts. In some of the religious texts, the word *Ketu* also means either a comet or an earthly activity caused by the Sun.

The *Vāsiṣṭhasiddhānta* of the *Pañcasiddhāntikā* which gives instructions concerning a lunar eclipse in terms of mathematical

calculation as to how to determine the points of the first and the last contacts etc., does not deal with the solar eclipse which, however, is discussed in the *Pauliśa*, the *Romaka*, and the *Saura*. The word '*Rāhu*' is maintained for the node. As to the parallax, while the Indian astronomers have developed methods for the parallax correction, such methods are generally used in connection with the calculations of eclipses (and not for any other astronomical purpose), even so in terms of the relative differences.

Diameters of the Sun, the Moon and the Earth, distances of the Sun and the Moon from the Earth

	SUN			MOON			Earth's diameter (in *yohana*)
	Diameter (in *yojana*)	Distance (in *yojana*)	Diameter / Distance	Diameter (in *yojana*)	Distance (in *yojana*)	Diameter / Distance	
Āryabhaṭa I,[1]							
Bhāskara I	4,410	459,585	0.009596	315	34,377	0.009163	1,050
Bhāskara II	6,522	689,377	0.009461	480	51,566	0.009308	1,581
Sūryasiddhānta	6,500	689,378	0.009429	480	51,566	0.009309	1,600
Modern values (in miles)	86,400	92,900,000	0.0093	2,160	238,900	0.009	7,926.70 (equatorial) 7,900.02 (polar)

Source: *A Concise History of Science in India*, p. 118.

[1] The *yojana* measure used here is about 1.5 times as that used by Bh. II and *SūSi*.

The length and the diameter of the Earth's shadow, of necessity, were to be determined and the Indian astronomers had developed equations for finding them.

The Indian astronomers attached great importance not only to the detailed calculations concerning the occurrence and the duration of the solar and lunar eclipses but also to observe them carefully. For instance, Parameśvara has observed and recorded the lunar and solar eclipses that occurred during nearly fifty years. (See pp. 13-15). The rites and rituals to be performed during the eclipses, were, however, in the realm of the religious texts. The inscriptional records of the eclipses generally mention religious acts, including gifts made by rulers and chieftains, following the religious commandments. (See below, pp. 239-40). These records, however, have not been studied in detail from the astronomical point of view.

Trigonometrical aspects

The Indian astronomers-cum-mathematicians have adopted the half-chord (*jyārdha*, terming it generaly as *jyā*), unlike the Greco-Roman astronomers who used the full chord and used certain assumed units for the radius, computing the half-chords and the arcs in the same units. Thus the Indian half-chords or *jyā* are R sines. Āryabhaṭa I has taken the circumference to be 21,600 units, and the radius becomes equal to 3438 units. While this system has been followed by many astronomers, others have adopted some different units. For example, Varāhamihira has adopted 120 units. The half-chord values in the former case are $3438' \sin \theta$ and those in the case of the latter are $120' \sin \theta$. (See below, p. 71). In general, the R sines are tabulated for intervals of 3° 45′, i.e. twentyfour of them for a quadrant (90°) and the methods or the rules for $R \sin \theta$, R versine θ etc, are given. Such trignometrical computations are required for computing the equation of the centre as well as the equation of conjunction, by way of solutions of plane and spherical triangles with a view to determining the right ascensions, declination, polar latitude, longitudes of planets, length of day light, azimuth, zenith distance etc.

Instruments

Indian astronomers, realising the importance of observation, had developed several types of instruments for measuring time and for astronomical observations. These included different types of gnomon, spherical and circular instruments, water instruments etc. Some of the instruments, like the *Phalaka* and the *Yaṣṭi*, were noted for their versatility and ingenuity. (See below, sections 9 and 10 on Observatories and Instruments, pp. 81-99).

The Eras

In addition to what may be called the natural periodicity of the day, the month and the year, a continuously running era is required for the recording of dates. Till the eighth century B.C. or so, there does not seem to have been any agreed attempt to fix an era; instead, the day, the month and the regnal years of a king were in use for calendrical recordings. The Nabu Nazar (King of Babylon) era, though not used either by the Babylonians nor found even in the later recordings of astronomers like Hipparchus and Ptolemy, is said to date from February 26, 747 B.C., when there was a unique conjunction of planets. Following this era were other eras like the *Greek Olympiads* (776 B.C.) and *Foundation of Rome* (757 B.C.).

Of interest to us are the Seleucidean and the Parthian eras. The former denoted the occupation of Babylon by Seleucus (312 B.C.) and the latter was in commemoration of the liberation of the Parthians from the Seleucidian rule in 248 B.C. Though the north-western part of the Indian sub-continent came under the Parthian suzerainty, there has been no tangible evidence yet regarding the use of the Parthian era in India. Even Aśoka used his regnal years for recording. However, by the first century B.C. or so, the concept of an era seems to have entered India, probably deriving its inspiration from the Parthians. On the basis of the inscriptional recordings, there are more than 30 eras either used or in use in India. Of them, the *Vikrama* (57 B.C.), the *Śaka* (78 A.D.), the Hijri (622 A.D.) and Kollam (825 A.D.) are still being used. (Cf. the table of 'Indian Eras', with the indication of their zero-year, the current year in 1954, the year-beginnings and their provenance, given as App. I, below.)

Transmission

Indian astronomical texts and ideas were not without influence on the other culture-areas. In fact, not a few of them travelled beyond the frontiers of India, into China, Thailand, Indonesia and other south-east Asian regions as well as into the Islamic West Asia. Though the influx of astronomical texts into China started from the 6th century A.D. itself, it was during the Sui dynasty (A.D. 581-618) and the 'Glorious period' of the Thang dynasty (618-907) that the process reached its height. R.C. Gupta has listed a large number of Indian texts which were translated into Chinese during this period and several Indian scholars who went to China and engaged themselves in this work. (See 'Indian astronomy in China', *Vishveshvaranand Ind.*

Jl., 19 (1981) 266-72). The Indian calendrical system was popular also in Nepal, Tibet, Thailand, Java and other places.

During the reign of Caliph al-Mansūr, an Indian astronomer visited Baghdad, carrying with him the textual material concerning planetary tables, calculation of eclipses and the like, as evidenced by the account of Ibn al-Adamī in his astronomical tables *Nazm al-igd*. The interest in Indian astronomical texts evinced by the Caliph was so great that he ordered the *Brāhmasphuṭa-siddhānta* and *Khaṇḍakhādyaka* of Brahmagupta to be translated into Arabic. Muḥammed ibn Ibrāhīm al-Fazārī rendered the former, and Ya'qūb ibn Ṭāriq, the latter into Arabic under the titles *Sindhind* and *Arkand*, respectively, understandably with the help of Indian *pandits* who had already established their reputation in the royal court. Āryabhaṭa I was known to the Arab astronomers as 'Arjabahr'. The well-known astronomers like al-Khwārizmī, al-Ḥasan, al-Nairizi, ibn aṣ-Saffar, ibn Yunis and al-Battāni were quite familiar with the Indian astronomical computations and used them even in their works. Some of the Islamic *zījes* included the zero meridian of Ujjain, under the name 'Arin', the *Kaliyuga* era, spherical trignometrical calculations etc. It may be noted that the Sanskrit word '*jyā*' or '*jīvā*' meaning half-chord, had got mutilated in its Arabic form, which in its Latin translation came to be called '*sinus*', and later 'sine'.

The most outstanding scientific transmitter and synthesiser was al-Bīrūnī who came to India in the eleventh century A.D., stayed here for a considerable time and acquired intimate knowledge of Indian astronomy. Before he came to India, he had already an insight into the Indian astronomical endeavours, through the Arabic translations. He wrote many books in Arabic and his *Ta'rikh al-Hind*, is a classic and a veritable source for Indian astronomy of the times.

The Islamic culture-area played an important role not only in the transmission of astronomical and other scientific ideas but also in engendering its scientific development. In astronomy, the Marāgha school led by Nasir al-Din aṭ-Ṭūsī had established itself by the thirteenth century A.D. Later, an offshoot of this school flourished in Samarkand, of which Ulugh Bek was a well-known astronomer. He built there an observatory and compiled diligently certain sets of astronomical tables. During the Mughal period in India. Islamic astronomical elements became rather widely current. Earlier, in the fourteenth century, Mahendra Sūri had incorporated Islamic elements into his *Yantrarāja* and, during the Mughal period, Malayendu Sūri produced a commentary on it. In the court of Abkar was

Nīlakaṇṭha Jyotirvid who compiled a popular work called *Tājika-Nīlakaṇṭhī*, introducing into it a large number of Persian technical terms of astronomical import.

The most important development during this period, however, was the erection of huge masonry observatories by Maharaja Sawai Jai Singh II in the second quarter of the eighteenth century, deriving inspiration from those in Sarmakand. He built observatories in Delhi, Jaipur, Ujjain, Mathura and Banaras. (See plates between pp. 88-89). Bāpudeva Śāstri has written a detailed description of the instruments installed by Jai Singh in the observatory at Banaras. (See below pp. 81-85). In his endeavours, Jai Singh had the expert assistance of the then well-known astronomer Jagannātha Samrāṭ. Jagannātha ably rendered Ptolemy's *Almajest* into Sanskrit under the title *Samrāṭsiddhānta*. Certain Jesuit missionaries were also associated with the astronomical endeavours of Jai Singh.

As to the telescope, there is no doubt that Jai Singh used it for observing the Moon, the sun-spots and the moons of Jupiter and the like. Nevertheless, he appears to have relied more on the instruments erected by him, for his recordings, which wereme ticulously documented under the title *Zīj Muhammad-shāhī*, a text which still needs to be studied and evaluated in depth. There is also a Sanskrit version of this work.

In the closing decades of the eighteenth century, came into being what was known as the Madras Observatory (1782), marking the beginning, in India, of systematic modern astronomical investigations. It made important contributions for well over a century and paved the way for the establishment of the Solar Physics Observatory in Kodaikanal (1900). As modern astronomical studies were in progress before Independence, astro-physics too emerged as an important scientific discipline. M.N. Saha was a pioneer in this field and his 'Ionisation Formula' for explaining the orderd sequence of the spectra of stars, has been recognised as one of the ten fundamental contributions since the use of the telescope by Galileo in 1608.

Since Independence, not only have the earlier observatories been developed, but also new observatories have been set up. The investigations of Indian astronomers and astro-physicists have also received international acclaim. Side by side, the traditional Indian astronomy continues to be an integral part of the socio-religious life of the people.

B.V. SUBBARAYAPPA
K.V. SARMA

ABBREVIATIONS

ĀBh.	*Āryabhaṭīya*
Ait.	*Aitareya*
AS	Arka Somayaji
AV	*Atharvaveda*
BC	Bina Chatterjee
Br.	*Brāhmaṇa*
Bṛ. Saṃ.	*Bṛhat-Saṃhitā*
BrSpSi.	*Brāhmasphuṭasiddhānta*
C, Com.	Commentary
Cr.	Critical
*Dh.*G	*Dhyānagrahādhyāya*
Gola.D	*Goladīpikā*
IJHS	Indian Journal of History of Science
KK	*Khaṇḍakhādyaka*
KKau.	*Karaṇakaumudī*
KP	*Karaṇapaddhati*
KPr.	*Karaṇaprakāśa*
KR	*Karaṇaratna*
KSS	Kripa Shankar Shukla
KVS	K.V. Sarma
LBh.	*Laghubhāskarīya*
LMā.	*Laghumānasa*
MBh.	*Mahābhāskarīya*
Mahā.	*Mahāsiddhānta*
Par.	Parameśvara
PS	*Pañcasiddhāntikā*
Q	Quoted by, in
ṚV	*Ṛgveda*
ṚV-VJ	*Ṛg-Vedāṅga Jyotiṣa*
SDS	S.D. Sharma
SD, Si.Dar.	*Siddhāntadarpaṇa*
ŚiDhVṛ.	Śiṣyadhīvṛddhida
SiSā.	*Siddhāntasārvabhauma*
SiŚe.	*Siddhāntaśekhara*
SiŚi.	*Siddhāntaśiromaṇi*
SiTvi.	*Siddhāntatattvaviveka*
SRS	S.R. Sarma
SūSi.	*Sūryasiddhānta*
Tait.	*Taittirīya*
TS	*Taittirīyasaṃhitā*
TSK	T.S. Kuppanna Sastry
VJ	*Vedāṅga Jyotiṣa*
VK	*Vākyakaraṇa*
VSN	V S. Narasimhan
Varāha VM	Varāhamihira
VSi.	*Vaṭeśvarasiddhānta*
VVSi.	*Vṛddha-Vāsiṣṭha-saṃhitā*
YV	*Yajurveda*
YV-VJ	*Yajur-Vedāṅga Jyotiṣa*
YV-VS	*Yajurveda Vājasaneyisaṃhitā*

TRANSLITERATION

VOWELS

Short:	अ	इ	उ	ऋ	ऌ		
	a	i	u	ṛ	ḷ		
Long:	आ	ई	ऊ	ए	ओ	ऐ	औ
	ā	ī	ū	e	o	ai	au

Anusvāra: ं = ṁ, ṃ

Visarga: : = ḥ

CONSONANTS

क्	ख्	ग्	घ्	ङ्				
k	kh	*g*	gh	ṅ				
च्	छ्	ज्	झ्	ञ्				
c	ch	j	jh	ñ				
ट्	ठ्	ड्	ढ्	ण्				
ṭ	ṭh	ḍ	ḍh	ṇ				
त्	थ्	द्	ध्	न्				
t	th	d	dh	n				
प्	फ्	ब्	भ्	म्				
p	ph	b	bh	m				
य्	र्	ल्	व्	श्	ष्	स्	ह्	ळ्
y	r	l	v	ś	ṣ	s	h	!

INDIAN ASTRONOMY

A Source-Book

MAJOR DIVISIONS

1. गणितज्योतिषम् – ASTRONOMY

ज्योतिषं नाम प्रमुखं वेदाङ्गम्

1. 1. 1. यथा शिखा मयूराणां नागानां मणयो यथा ।
तद्वद् वेदाङ्गशास्त्राणां ज्योतिषं मूर्धनि स्थितम् ।।

(*RV-VJ* 35, *YV-VJ* 4)

Astronomy—The foremost auxiliary of the Veda

Just like the combs of peacocks and the crest jewels of serpents, so does *Jyotiṣa* (astronomy) stand at the head of the auxiliaries of the Veda. (35). (KVS)

1. 1. 2. वेदास्तावद् यज्ञकर्मप्रवृत्ता
यज्ञाः प्रोक्तास्ते तु कालाश्रयेण ।
शास्त्रादस्मात् कालबोधो यतः स्याद्
वेदाङ्गत्वं ज्यौतिषस्योक्तमस्मात् ।। ६ ।।

(*Bhāskara* II, *SiŚi.*, 1.1.9)

The Vedic lore prescribes sacrifices to be performed; these sacrifices are based upon a knowledge of appropriate times to perform them. This science of astronomy gives the knowledge of time; hence it has been reckoned as one of the six Vedāṅgas or auxiliaries of the Veda. (9). (AS)

1. 1. 3. शब्दशास्त्रं मुखं ज्यौतिषं चक्षुषी
श्रोत्रमुक्तं निरुक्तं च कल्पः करौ ।
या तु शिक्षाऽस्य वेदस्य सा नासिका
पादपद्मद्वयं छन्द आद्यैर्बुधैः ।। १० ।।

(*Bhāskara* II, *SiŚi.*, 1.1.1.10)

(Out of the six Vedāṅgas), the Science of Grammar is the face of the person of the Veda; the Science of Jyotiṣa, the eyes; the Nirukta, the ears; the Kalpa, the hands; the Śikṣā, the nose; and the Chandas, the feet. (10). (KVS)

ज्योतिश्शास्त्रस्य प्रयोजनम्

1. 2. 1. वेदा हि यज्ञार्थमभिप्रवृत्ताः
कालानुपूर्व्या विहिताश्च यज्ञाः ।
तस्मादिदं कालविधानशास्त्रं
यो ज्योतिषं वेद स वेद यज्ञान् ।।

(*RV-VJ* 36; *YV-VJ* 3)

Purpose of astronomy

The Vedas have indeed been revealed for the sake of the performance of sacrifices; but these sacrifices have been set out (to be performed) according to the sequences of time. Therefore, only he who knows astronomy, the science of time, understands the sacrifices. (36). (KVS)

1. 2. 2. श्रुत्युत्तमाङ्गमिदमेव यतो नियोगः
कालेऽयनर्तुतिथिपर्वदिनादिपूर्वे ।
वेदीककुब्भवनकुण्डतदन्तरादि
ज्ञेयं स्फुटं श्रुतिविदां बहुमान्यमस्मात् ।। ४ ।।

(Vaṭeśvara, *VSi.*, 1. 1. 4)

It is this science (of astronomy) that has been regarded as the crown of the Veda, for the reason that the Vedic sacrifices are performed at the specified times defined by *ayana* ('northward or southward course of the Sun'), season, *tithi*, *parva* ('full moon or new moon') and day etc., and the sacrificial altars, the cardinal points, the fire-pits (meant for offering oblation into fire) and the distances involved therein, etc., are to be correctly known (by the Vedic priests); and so this science stands highly honoured amongst the Vedic scholars. (4). (KSS)

1. 2. 3. सोमसूर्यस्तृचरितं विद्वान् वेदविदश्नुते ।
सोमसूर्यस्तृचरितं लोकं लोके च सन्ततिम् ।। ४३ ।।

(*YV-VJ* 43)

One learned in the Vedas, who has also learnt the lore of the movement of the Moon, the Sun and the stars, will enjoy, after death, a life in the world wherein the Moon, the Sun and the stars move, and he will have also, on the Earth, an unending line of progeny. (43). (TSK)

ज्योतिश्शास्त्रस्य विभागः

1. 3. 1. सिद्धान्त-संहिता-होरारूपं स्कन्धत्रयात्मकम् ।
वेदस्य निर्मलं चक्षुर्ज्योतिश्शास्त्रमनुत्तमम् ।।

(*Nāradīya-Saṃhitā*, 1. 4)

Division of Jyotiśśāstra

The Science of Jyotiṣa, with the three divisions, *Siddhānta*, *Saṃhitā* and *Horā*, is the unparalleled and clear eye of the Veda. (KVS)

1. 3. 2. स्कन्धत्रयात्मकं ज्योतिश्शास्त्रमेतत् षडङ्गवत् ।
गणितं संहिता होरा चेति स्कन्धत्रयं मतम् ।। ५ ।।
जातक-गोल-निमित्त-प्रश्न-मुहूर्ताख्य-गणितनामानि ।
अभिदधतीह षडङ्गान्याचार्यो ज्योतिषे महाशास्त्रे ।।
गोलो गणितं चेति द्वितयं खलु गणितसंज्ञिते ग्रन्थे ।
होरासंहितयोरपि निमित्तम् अन्यत्त्रयं च होराख्ये ।।
जनपुष्टिक्षयवृष्टिद्विरदतुरङ्गादिसकलवस्तूनाम् ।
केतूल्कादीनां वा लक्षणमुदितं हि संहितास्कन्धे ।। ८ ।।
प्रमाणफलभेदेन द्विविधं च भवेदिदम् ।
प्रमाणं गणितस्कन्धः स्कन्धावन्यौ फलात्मकौ ।। ९ ।।

(*Praśnamārga*, 1. 1. 5-9)

Jyotiṣa (in its fuller sense) comprises three sections (*skandhas*) and is divided into six branches. *Gaṇita* (Astronomy), *Saṃhitā* (Electional astrology) and *Horā* (Horary astrology) are the three sections. The teachers speak of the following as branches of the great science of Jyotiṣa: *Jātaka* (Horoscopy), *Gola* (Spherics), *Nimitta* (Omenology), *Praśna* (Astrological query), *Muhūrta* (Auspicious times) and *Gaṇita* (Astronomical computations). (5-6)

In a text on astronomy would be found the two branches *Gola* and *Gaṇita*. *Nimitta* would occur both in *Horā* and *Saṃhitā* texts. The other three (viz. *Jātaka*, *Praśna* and *Muhūrta*) would occur only in *Horā* texts. (7)

In *Saṃhitā* texts would be explained the progress and decay of people, rainfall, the nature of elephants, horses and all other animals and things, and of the comets, meteors and allied matters. (8)

This (Science of Jyotiṣa) is further classified in two other ways, according as they pertain to computations or to (the prognostication of) fruits (of actions). Of these, the section on Astronomy concerns itself with computations while the other two are concerned with results. (9). (KVS)

1. 3. 3. ग्रहगणित-पाटीगणित-बीजगणित-रूपसुनिश्चलमूलस्य बहु-विधवितत- होरातन्त्रशाखस्य ज्योतिश्शास्त्रवनस्पतेः संहितार्था एव फलानीत्यवधार्य जातकर्म-नामकरण-मौञ्जीबन्धन-विवाहयात्रादौ निखिलार्थमल्पग्रन्थेनाभिधातुमिच्छुः . . . आह ।

(Mahādeva's Com. on Śrīpati's *Ratnamālā*, Intro.)

Knowing well that Gaṇita comprising planetary computations, arithmetic and algebra, forms the deep roots of the Tree of the Science of Jyotiṣa, that Horary astrology of divers aspects forms the branches, and that *Saṃhitā* (Natural astrology) forms the fruit, here is being set out, in brief, post-natal sacrement, naming the child, girdle cermony, marriage, travel etc., which form the full intent of the *Saṃhitā* section. (KVS)

ज्योतिषम्—सिद्धान्तग्रन्थाः करणग्रन्थाश्च

1. 4. 1. समयमितिरशेषा साधनं खेचराणां
गणितमखिलमुक्तं यत्र कुट्टाद्युपेतम् ।
ग्रहभगणमहीनां संस्थितिर्यत्र सम्यक्
स खलु मुनिवरिष्ठैः स्पष्टराद्धान्त उक्तः ।। ५ ।।

(Vaṭeśvara, *VSi.*, 1. 1. 5.)

Astronomy: Siddhānta and Karaṇa texts

An astronomical work which describes all measures of time as well as the determination of longitudes of the planets, which treats all mathematics including the theory of the pulveriser, etc., and which correctly states the configurations and positions of the planets, the asterisms, and the Earth, is verily called a true Rāddhānta (or Siddhānta) by the distinguished sages. (5). (KSS)

1. 4. 2. त्रुट्यादिप्रलयान्तकालकलना मानप्रभेदः क्रमा-
च्चारश्च द्युसदां द्विधा च गणितं प्रश्नास्तथा सोत्तराः ।
भूधिष्ण्यग्रहसंस्थितेश्च कथनं यन्त्रादि यत्रोच्यते
सिद्धान्तः स उदाहृतोऽत्र गणितस्कन्धप्रबन्धे बुधैः ।।

(Bhāskara II, *SiŚi.*, 1. 1. 1. 6)

A *Siddhānta* work is an astronomical treatise which deals with the various measures of time ranging from a *truṭi* up to of a *Kalpa* which culminates in a deluge, planetary theory, arithmetical computations as well as algebraical processes, questions relating to intricate ideas and their answers, location of the Earth, stars and planets, and the description and use of instruments. (6). (AS)

करणग्रन्थानां वैशिष्ट्यम्

1. 4. 3. शिष्याणां ग्रहगतिपरीक्षासामर्थ्यपादनमेव शास्त्रप्रयोजनम् । करणानामेव हि व्यावहारिकत्वं सूक्ष्मत्वं च स्यात् । . . . अत उक्तम् —"दृष्टानुरूपं करणं" . . . "बहुशो यत् परीक्षितं करणम्" इति ।

(Nīlakaṇṭha, *Jyotirmīmāṃsā*, pp. 8-9)

The significance of Karaṇa texts[1]

The aim of the Siddhānta texts is primarily to create in students the capacity for testing the motion of planets.

[1] The Karaṇa texts are astronomical manuals designed to make computations simpler, concise and accurate. The working herein is made facile by adopting easier methods. They adopt a contemporary date as the zero point for calculation and direct the deletion of the big lump of days up to the chosen date and use as zero corrections the longitudes of the planets as computed accurately for the zero point.

But, it is the Karaṇa texts that conduce to practical utility and accuracy.... Hence it is said, 'Karaṇas accord with observation.'... 'Karaṇa is the result of repeated observation.' (KVS)

1. 4. 4. शिष्यप्रशिष्यपरम्परया सर्वैरपि परीक्षणं कार्यम् ।

जातकेऽप्याह—

योगे ग्रहाणां, ग्रहणेऽर्कसोमयो-
मौढ्ये तथा वक्रगतौ तु पञ्चसु ।
दृष्टानुरूपं करणं तदन्वहं
तेन ग्रहेन्द्रान् गणयेत् त्रिवारकम् ॥

अन्यश्चाह—

ग्रहणग्रहयोगादौ बहुशो यत् परीक्षितम् ।
करणं तेन संगण्य ज्ञेयाः सूर्यादयो ग्रहाः ॥

(Nīlakaṇṭha, *Jyotirmīmāṁsā*, p. 4)

Through the teacher-disciple tradition, all (astronomers) should conduct observation and verification. It is also stated in the *Jātaka*:

'In the undermentioned five, viz. the conjunction of planets, the eclipses of the Sun and the Moon, occultation of planets, and retrograde motion of planets, (the results of computation according to) Karaṇa texts accord with observation. Hence the planets should be computed according to the Karaṇa texts daily at three times (of the day).'

Another authority has said:

'Karaṇa texts are the results of tests conducted during the eclipses and the conjunctions of planets. Hence the Sun and other planets shall be computed (only) through Karaṇas.' (KVS)

ज्योतिषग्रन्थगतविषयाः

1. 5. 1. द्युगणानयनं खेटमध्यमस्फुटयोरपि ।
ग्रहणद्वितयं खेटकलहस्तत्समागमः ॥ १९ ॥
अस्तोदयौ च खेटानां नक्षत्राणां च सङ्गमः ।
इति भेदास्तु विज्ञेया ग्रहाणां गणिते दश ॥ २० ॥

(*Praśnamārga*, 1. 1. 19-20)

General contents of astronomical texts

Ten are the different topics known to be dealt with in (astronomical) texts of planetary computations, these being: (i) Calculation of the days from epoch, (ii) Mean planets, (iii) True planets, (iv-v) the two Eclipses (Lunar and Solar), (vi-vii) Planetary war and Approach (i.e. Conjunction of planets), (viii-ix) Heliacal rising and setting of planets, and (x) Conjunction of (planets and) stars. (19-20). (KVS)

स्फुटगणने निर्बन्धः

1. 6. 1. तिथिनक्षत्रच्छेदप्रतिपत्तिर्यदि तथा ततः साधुः ।
न तथा च भद्रविष्णोस्तथापि (न)विनिवर्तते लोकः ॥३२॥
न युगपदुदयो भानोरस्तमयो वापि भवति सर्वत्र ।
कस्मिन् देशेऽस्तमये पादादित्येन नोक्तमिदम् ॥ ३३ ॥
मार्गादपेतमेतत् काले लघुता न तावदतिदूरे ।
'खविषयभूताष्टरसै'रब्दैः पश्यास्य विनिपातम् ॥ ३४ ॥
रौमकमहर्गणं वा तदर्कमिन्दुं च गणयतां ग्राह्यम् ।
चैत्रस्य पौर्णमास्यां नवमीनक्षत्रमादित्यम् ॥ ३५ ॥
कालापेक्षा विधयः श्रौताः स्मार्ताश्च तदपचारेण ।
प्रायश्चित्ती भवति द्विजो यतोऽतोऽधिगम्येदम् ॥ ३६ ॥
स्फुटगणितविदिह लब्ध्वा धर्मार्थयशांसि दिनकरादीनाम् ।
कुकरणकारः सत्यं सहते नरके कृताऽऽवासाः ॥ ३७ ॥

(Varāha, *PS*, 3. 32-37)

Insistance on accuracy

If the *tithis* and *nakṣatras* as found from observation agree with those computed according to the Śāstra, then the Śāstra is correct and is fit to be accepted. It is not so in the case of Bhadraviṣṇu's work;[1] still people do not turn away from that and follow the correct Śāstra. (32)

Sunrise and sunset are not at the same moments in all places on the earth. (So, in order to find the days and making computation, the place must be mentioned at which the sunrise and sunset are taken as the epoch). But Pādāditya, who has placed the epoch at sunset, has not mentioned the place he refers to. (So, his work is faulty.). (33)

The *gaṇita* text (of Pādāditya) has deviated from the right path handed down by a hierarchy of good teachers and the day of its exposure is not far distant. Witness its downfall in 68,550 years. (34)

If we adopt the days of epoch resulting from the tropical years as adopted by the Romaka, and (the positions of) the Sun or the Moon resulting therefrom, we must accept Punarvasu as the *nakṣatra* of the full moon of the month of Caitra, it being the *nakṣatra* of Caitra-Śukla-navamī, instead of the expected Hasta or Citrā. (35)

All the injunctions of the Vedas and Smṛtis are based on the proper times, and by not performing the rites at those times, the performer, especially the twice-born, acquires sin which is to be expiated. Therefore, a study of the Romaka itself is to be expiated. (36)

[1] Bhadraviṣṇu and Pādāditya, mentioned below are earlier writers on astronomy.

The person having a correct knowledge of the Sun, the Moon etc. acquires *Dharma* (merit), which will take care of his future world, *Artha* (wealth) which will ensure his prosperity in this world, and *Yaśas* (fame) which will perpetuate his memory. But the bad astronomer who misleads people by his writings will certainly have to go to hell and dwell therein. (37). (TSK)

ग्रन्थोद्देशः—संग्रहणम्

1. 7. 1. पूर्वाचार्यमतेभ्यो यद्यत् श्रेष्ठं लघु स्फुटं बीजम् ।
तत्तदिहाविकलमहं रहस्यमभ्युद्यतो वक्तुम् ।। २ ।।
पौलिश-रोमक-वासिष्ठ-सौर-पैतामहास्तु सिद्धान्ताः ।
पञ्चभ्यो द्वावाद्यौ व्याव्यातौ लाटदेवेन ।। ३ ।।
पौलिशतिथिः स्फुटोऽसौ तस्यासन्नस्तु रोमकः प्रोक्तः ।
स्पष्टतरः सावित्रः परिशेषौ दूरविभ्रष्टौ ।। ४ ।।
यत्तत्परं रहस्यं भ्रमति मतिर्यत्र तन्त्रकाराणाम् ।
तदहमपहाय मत्सरमस्मिन् वक्ष्ये ग्रहं भानोः ।। ५ ।।
दिक्स्थितिविमर्दकर्णप्रमाणवेलाग्रहाग्रहाविन्दोः ।
ताराग्रहसंयोगं देशान्तरसाधनं चास्मिन् ।। ६ ।।
सममण्डलचन्द्रोदययन्त्रच्छेद्यानि शाङ्कवच्छायाः ।
उपकरणाद्यक्षज्यावलम्बकापक्रमाद्यानि ।। ७ ।।

(Varāha, *PS*, 1. 2-7)

Aim: Consolidation

I shall state in full the best of the secret lore of astronomy extracted from the different schools of the ancient teachers so as to make it easy and clear. (2)

The five Siddhāntas, of which this work is a compendium, are the Pauliśa, the Romaka, the Vāsiṣṭha, the Saura and the Paitāmaha. Of these five, the first two, *viz.*, the Pauliśa and the Romaka, have been commented upon by Lāṭadeva. (3)

The *tithi* resulting from the Pauliśa is tolerably accurate and that of the Romaka approximates to that. The *tithi* of the Sura is very accurate but that of the remaining two (*viz.*, the Vāsiṣṭha and the Paitāmaha) is far from correct. (4)

I shall tell in this work, avoiding all jealously, the computation of the solar eclipse, which is guarded as a great secret and in which the mind of the astronomer revels. I shall also tell the occurrence and non-occurrence of the lunar eclipse, the directions of the first and last contacts, the duration, the total phase, the 'hypotenuse' at any moment with related quantity of obscuration and time, and also the mutual conjunctions of the stars and the planets, and the computations of differences in longitude as also the prime vertical, moonrise, astronomical instruments and other requirements, graphical representations, the gnomonic shadow, the sines of latitude, co-latitude and declinations and such other matters. (5-7). (TSK)

ग्रन्थोद्देशः—परिष्कारः

1. 8. 1. कृता यद्यप्याद्यैश्चतुरवचना ग्रन्थरचना
तथाऽप्यारब्धेयं तदुदितविशेषान् निगदितुम् ।
मया मध्ये मध्ये त इह हि यथास्थाननिहिता
विलोक्याऽतः कृत्स्ना सुजनगणकैर्मत्कृतिरपि ।।४।।

(Bhāskara II, *SiŚi.*, 1. 1. 1. 4)

Aim: Refinement

Ancient astronomers did write, of course, treatises abounding in intelligent expression; none the less, this work is composed to fill some lacunae in their works. I am going to make up for the deficiencies of the older works and these improvements will be found here and there in their respective places. So I beseech the good-minded mathematicians to go through this entire work of mine (for, otherwise, they may not locate my contribution). (4). (AS)

ग्रन्थोद्देशः—लाघवम्

1. 9. 1. प्रणिपत्य महादेवं जगदुत्पत्तिस्थितिप्रलयहेतुम् ।
वक्ष्यामि खण्डखाद्यकमाचार्यार्यभटतुल्यफलम् ।। १ ।।
प्रायेणार्यभटेन व्यवहारः प्रतिदिनं यतोऽशक्यः ।
उद्वाहजातकादिषु तत् समफललधुतरोक्तिरतः ।। २ ।।

(Brahmagupta, *KK*, 1. 1. 1-2)

Aim: Simplification

Having made obeisance to Mahādeva, the cause of creation, existence and destruction of the world, I compose the astronomical treatise *Khaṇḍakhādyaka*, which, in the first part, gives the same results as those arrived at by Ācārya Āryabhaṭa (in his *Āryabhaṭa-Siddhānta*). (1)

The methods given by Āryabhaṭa are generally impracticable for everyday calculation (of the longitudes of the planets etc.), in connection with marriage, nativity and the like. My statements in this work are more concise, yet give the same results. (2). (BC)

ग्रन्थोद्देशः—शोधः

1. 10. 1. शिष्यस्य बुद्धिमान्द्याद् आचार्यस्योपदेशसंवरणात् ।
गुणभागयोश्च शेषात् पुराणकरणानि न घटन्ते ।। ३ ।।
नष्टानि स्यापयितुं नवानि करणानि च प्रकाशयितुम् ।
तन्त्रज्ञानस्य फलं वदन्ति, तदयं ममोत्साहः ।। ४ ।।

(Deva, *KR*, 1.3-4)

Aim: Revision

The *Karaṇa* texts of ancient times do not yield accurate results either because of the dullness of the pupil's intellect, or because of the cryptic teaching of the preceptor; or else, because of the inexactitude of the multipliers and divisors. (3)

They say that the aim of acquiring the knowledge of astronomy is to rectify and re-establish the lost methods or to discover and highlight new methods. Hence this attempt of mine. (4). (KSS)

ग्रन्थोद्देशः—दृगैक्यम्

1. 11. 1. यात्राविवाहोत्सवजातकादौ
खेटैः स्फुटैरेव फलस्फुटत्वम् ।
स्यात् प्रोच्यते तेन नभश्चराणां
स्फुटक्रिया दृग्गणितैक्यकृद्या ।। १ ।।

(Bhāskara II, *SiŚi.*, 1.2.1)

Aim: Accord with observation

Inasmuch as true positions of the planets alone are required to decide auspicious moments for journeys, marriages, celebrations in temples, astrology and the like, we shall now give the methods of rectifying the mean positions of the planets so as to accord with their observed positions. (1). (AS)

1. 11. 2. दृश्यन्ते विहगा दृष्टया भिन्नाः परहितोदिताः ।
प्रत्यक्षसिद्धाः स्पष्टाः स्युर्ग्रहाः शास्त्रेष्वितीरितम् ।। २ ।।
सत्कर्मोदितकालस्य ग्रहा हि ज्ञानसाधनम् ।
अस्पष्टविहगैः सिद्धः कालः शुद्धो न कर्मणि ।। ३ ।।
ये तु शास्त्रविदस्तद्वद् गोलयुक्तिविदश्च तैः ।
स्फुटखेचरविज्ञाने यत्नः कार्यो द्विजैरतः ।। ४ ।।
सञ्चिन्त्येति समालोच्य पूर्वतन्त्राणि यत्नतः ।
स्फुटयुक्ति खेचराणां गोलदृष्टया समीक्ष्य च ।। ५ ।।
स्फुटखेचरविज्ञानं शिष्यैर्यैः प्रार्थितं द्विजैः ।
तेभ्यो दृग्गणितं नाम गणितं क्रियते मया ।। ६ ।।

(Parameśvara, *Dṛgganita*, 1. 1. 2-6)

Planets computed according to the Parahita system are actually observed at places which are different. And the sciences affirm that the correct planetary positions are only those where they are observed. (2)

Through the instrumentality of the planets alone can the timings for auspicious rituals be known. Times computed from incorrect planetary positions are impure for rituals. (3)

Hence, the twice-born who are knowers of the sciences and also have been initiated into the rationale of the spherics should take pains towards ascertaining the correct positions of planets. (4)

Cogitating thus and having comprehended well the earlier texts and also having observed planetary conjunctions (for long)[1] by means of instruments of spherics, (the present work), *Dṛgganita*, is being composed by me for the sake of students who have prayed for instruction on the true places of planets. (5-6) (KVS)

1. 11. 3. पण्टुळ्ळ गणितज्ञन्मार् चोन्नतोट्टु परञ्ञटाम् ।
कल्यब्दं त्रिसहस्रत्तिल् परं चेन्नुळ्ळ नाळिलु ।। ५ ।।
ओत्तु वन्नीटुमारिल्ल ग्रहणादिकळ् ओन्नुमे ।
करणङ्ङ्ळुमोत्तीटा सिद्धान्तङ्ङ्ळुमोत्तिटा ।। ६ ।।
तदा ह्यार्यभटो नाम गणकस्त्वभवद् भुवि ।
'ज्ञानतुङ्गे'ति कल्यब्दे जातनायवनीतले ।। ७ ।।
'गिरितुङ्गे'ति कल्यब्दे गणितं निर्मितं परम् ।
शास्त्रम् आर्यभटीयाख्यं तस्मिन् पर्ययमुक्तवान् ।। ८ ।।

. . . .

इव अन्नेय्क्कुपायेन कुरच्चिटट्टुं करेट्टियुम् ।। १२ ।।
कल्यादिध्रुवमिल्लाते ओप्पिच्चानन्नु पर्ययम् ।

. . . .

पिन्ने इग्गणितत्तिन्नु नीक्कं कण्टितु भूतले ।
'मन्दस्थले'ति कल्यब्दे 'तनुते'ति शकाब्दके ।। १८ ।।
पलरुं गणितज्ञन्मार् कूटे नोक्कीट्टु वेच्चतु ।
कल्यब्दाद् 'गिरितुङ्गो'नात् शेषं वेच्चु पेरुक्कणम् ।। १९ ।।

. . . .

पेरुं परिहितमेन्नु गणितं सूक्ष्ममेन्नितु ।
इति निश्चित्य पलरुमाचरिच्चवनीतले ।। २५ ।।
चिरकालं कळिञ्ञप्पोळ् नीक्कं वळरे वन्नितु ।
'रङ्गशोभा नु' कल्यब्दे कश्चिद् विप्रवरस्तदा ।। २६ ।।
पश्चिमाम्भोधितीरत्तु निन्नु नोक्कीट्टु वेच्चतु ।
द्वादशाब्दं कळिञ्ञिट्टु तन्त्रसंग्रहमेन्नतु ।। २७ ।।
तस्मिन् पर्ययवुं पिन्ने स्थूलमायिट्टु चोल्कयाल् ।
'वसुस्मरे'ति कल्यब्दे तुटन्नारवनीतले ।। २८ ।।
पश्चिमाम्बोधितीरत्तु निन्नु मुप्पतु वर्षवुम् ।
नोक्कीतु गणकन्मारुं 'जनसेवा नु' वत्सरे ।। २९ ।।
सूक्ष्मं वरुत्ति निर्म्मिच्चार् गणितागममन्नवर् ।
इनियुं कण्टुकोळ्ळेणं चिरकालाद् वरुन्नतु ।। ३० ।।

(*Dṛkkarana*, 1. 5-30)

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[1] The fact that Parameśvara had been observing planetary movements and working on the same has been referred to by his pupil Nīlakaṇṭha Somayāji, who says: *Parameśvarācāryeṇa punaḥ grahaṇa-grahayogādikam yantraiḥ pañcapañcāśad-varṣakālam samyak parīkṣitam* ('Parameśvara had examined closely, by means of instruments, the eclipses and planetary conjunctions for 55 years)'.
Cf. N's *Āryabhaṭīya-Bhāṣya* under 4. 48.

Parameśvara himself corroborates this in the words:

grahendrāḥ pañcapañcāśadvarṣakālam nirīkṣitāḥ ।
mayā tadā dṛśā bhinnā dṛṣṭā Parahitoditāḥ ॥

'The planets have been observed by me for fiftyfive years. Their positions have been noted by me as different from the positions computed by the Parahita (constants).'
Q by Nīlakaṇṭha in *Āryabhaṭīya-Bhāṣya* under 4.48.

Now, I shall set out in brief what the early astronomers enunciated. Before Kali 3000, the eclipses and other observed phenomena did not tally with the astronomical manuals or the *Siddhāntas*. (5-6)

Then, in the Kali year *jñānatuṅga* (3600 = A.D. 499) an astronomer by name, Āryabhaṭa was born in this world. (7)

In the Kali year *girituṅga* (3623 = A.D. 522) was his work *Āryabhaṭīya* composed and therein he enunciated the revolutions (of the planets). (8)

He had adjusted these revolutions by reduction and addition in such a way that there was no zero-correction at the beginning of Kali. (12b-13a)

In course of time, deviations were observed in (the results arrived at by) this computation. Then, in the Kali year *mandasthala* (3785 = A.D. 684) equivalent to Śaka *tanutā* (606), several astronomers gathered together and devised, through observation, (a system), wherein (the correct mean longitudes were to be found) by multiplying the current Kali year-*minus-girituṅga* (Kali 3623, viz., the Āryabhaṭan epoch) (as direeted by the *vāgbhāva*, i.e. *bhaṭābada* or *śakābda*, correction enunciated by Haridatta and applying the correction). (18-19)

. . . .

This system was termed *Parahita* and many followed it, assuring themselves of its accuracy. (25)

When a long time had elapsed, there occurred substantial deviations. Then, (Parameśvarà), a noble brāhmaṇa, residing on the coast of the western ocean, revised it (i.e., the Parahita system) by means of (astronomical) observations, in the Kali year *raṅgaśobhānu* (4532 = A.D. 1431). (26-27a)

The work *Tantrasaṅgraha* (by Nīlakaṇṭha), (with revised constants) is for twelve years later. (27b)

The revolutions given therein (i.e., in *Tantrasaṅgraha*) too, becoming imperfect (in course of time), observations were continued by the astronomers on the west coast for thirty years, from the Kali year *jasustava* (4678 = A.D. 1577) to the Kali year *janasevā nu* (4708 = A.D. 1607) and, by observation, the astronomical tradition was revised accurately. (28-30a)

Hencefore, too, (the deviations) that would occur should be carefully observed (and revisions effected). (30b)

आर्यभटीयोपरि ब्रह्मगुप्तकृतः शोधः

1. 12. 1. न स्फुटमार्यभटोक्तं स्पष्टीकरणं यतस्ततो वक्ष्ये ।

(Brahmagupta, *KK*, 2.1.1a)

Emendations to Āryabhaṭīya by Brahmagupta

I now give emendations as Āryabhaṭa's formulae do not give accurate results. (1a). (B.C.)

ब्रह्मगुप्तमते शोधः भास्करकृतः

1. 13. 1. यौ ब्रह्मगुप्तकथितौ किल कोटिकर्णौ
ताभ्यां कृते तु परिलेखविधौ यथोक्ते ।
नास्तीव भाति मम दृग्गणितैक्यमत्र
शृङ्गोन्नतौ सुगणकैर्निपुणं विलोक्यम् ।। १० ।।
यत्राक्षो'ऽङ्गरसा' लवाः क्षितिजवत् तत्रापवृत्ते स्थिते
मेषादावुदयं प्रयाति तपने नक्रादिगेन्दोर्दलम् ।
याम्योदग्वलयेन खण्डितमिव प्राच्यां सितं स्यात्तदा
नैतद् ब्रह्ममतेऽस्य हि त्रिभगुणो बाहुश्च कोटिस्तदा ।।
शृङ्गे समे स्तो यदि बाह्वभाव उर्ध्वाधरे ते यदि कोटयभावः।
त्रिज्यासमौ तस्य च कोटिबाहू . . . ।। १२ ।।

(Bhāskara II, *SiŚi.*, 1.9.10-12)

Correction of Brahmagupta by Bhāskara II

The *koṭi* and *karṇa* defined by Brahmagupta do not accord between computation and observation in locating the cusps. I request expert mathematicians to verify this carefully. (10)

In a place where the latitude is $(90\text{-}\omega) = (90-24) = 66^\circ$, when the ecliptic coincides with the horizon, and when the Sun is in the beginning of Meṣa which is then rising in the east, and the Moon in the beginning of Makara, then the Moon is dichotomized by the meridian and the illuminated part of the Moon's disc is towards the east. This does not hold good according to Brahmagupta's definition of *koṭi*, because then the *bhujā* as well as *koṭi* according to his definition is equal to R. (11)

When the *bhujā* is zero, the cusps will be horizontal, and when the *koṭi* is zero, they will be vertical. Brahmagupta's *bhujā* and *koṭi*, both being equal to R, the cusps cannot be vertical, which is against truth as stated above.[1] (12). (AS)

1. 13. 2. शशिदृक्क्षेपार्थं यद्वित्रिभलग्नेषुणात्र संस्करणम् ।
जिष्णुजमतं तदुक्तं, न मन्मतं, वच्मि युक्तिमिह ।। १ ।।
यत्राक्षो जिनभागास्तत्रार्केन्दू तुलादिगावुदये ।
पातः किल गृहषट्कं सममण्डलवत् तदापवृत्तं स्यात् ।।२।।
अर्काल्लम्बितचन्द्रो न जहात्यपमण्डलं ह्यविक्षिप्तः ।
वित्रिभशरसंस्कारान्नतिरत्रायाति सा व्यर्था ।। ३ ।।

(Bhāskara II, *Vāsanā-bhāṣya on SiŚi.*, 1. 6. 18-19)

[1] For a note, see *SiŚi:AS*, pp. 498-500.

Bhāskara's correction to Brahmagupta's statement

The statement of Brahmagupta, namely that the arc of the Moon's *dṛk-kṣepa* will be obtained by the sum or difference of that of the Sun with the latitude of the *vitribha*, I (Bhāskara) do not accept. I shall give the reason why. (1)

At a place where the latitude is 24°, when the longitudes of the Sun, the Moon and the Node are all 180° at the time of sun-rise, the ecliptic occupies the position of the prime vertical. (2)

The Moon will not leave the ecliptic even though depressed by parallax in longitude. Thus there is no parallax in latitude. The *vitribha* then being in the zenith, and its latitude being $4\frac{1}{2}°$, the *dṛk-kṣepa* of the Moon according to Brahmagupta's formula will be $4\frac{1}{2}°$; and therefore the parallax in latitude obtained by the *dṛk-kṣepa* will be

$$\frac{790' - 35}{15} \times \frac{R \sin 4\frac{1}{2}°}{3438} = \frac{52' - 42''}{3438} \times 270 = 4'\ 8'',$$

which is actually not the case.[1] (3). (AS)

ज्योतिश्शास्त्रस्य शोधनात्मकत्वम्

1. 14. 1. मानसव्याख्यातापि कश्चिदाह—"ननु पैतामहादिभेदेन परस्परविरुद्धाश्च सिद्धान्ता भवन्ति । सिद्धान्तभेदे सति कालभेदः । कालभेदे सति कालाङ्गानि श्रौतस्मार्तलौकिकानि कर्माणि विकलानि स्युः । कर्मवैकल्ये सति लोकयात्रोच्छेदः । हा धिक्! सङ्कटे महति पतिताः स्मः ।"

अत्रोच्यते—ऋजुमते! स न शोचितव्यः । गुरुचरणपरिचरणपरैः किमिव न ज्ञायते । पञ्चसिद्धान्तास्तावत् क्वचित्काले प्रमाणमेव इत्यवगन्तव्यम् । अपि च—यः सिद्धान्तः दर्शनाविसंवादी भवति सोऽन्वेषणीयः । दर्शनसंवादश्च तदानीन्तनैः परीक्षकैर्ग्रहणादौ विज्ञातव्यः । ये पुनरन्यथा, प्राक्तनसिद्धान्तस्य भेदे सति, यन्त्रैः परीक्ष्य ग्रहाणां भगणादिसंख्यां ज्ञात्वा अभिनवसिद्धान्तः प्रणेय इत्यर्थात् ।

(Nīlakaṇṭha, *Jyotirmīmāṁsā*, p. 6)

Testing, verification and revision

A commentator on the *Mānasa* (viz. *Laghumānasa* of Muñjāla) has lamented: 'Indeed, the Siddhāntas, like *Paitāmaha*, differ from one another (in giving the astronomical constants). Timings are different as the Siddhāntas differ (i.e. the measures of time at a particular moment differs as computed by the different Siddhāntas). When the computed timings differ, Vedic and domestic rituals, which have (correct) timings as a component (of their performance) go astray. When rituals go astray, worldly life gets disrupted. Alas! we have been precipitated into a big calamity.'

Here, it needs to be stated: 'O faint-hearted, there is nothing to be despaired of. Wherefore does anything remain beyond the ken of those intent on serving at the feet of the teachers (and thus gain knowledge)? One has to realise that the five Siddhāntas had been correct (only) at a particular time. Therefore, one should search for a Siddhānta that does not show discord with actual observation (at the present time). Such accordance with observation has to be ascertained by (astronomical) observers during times of eclipses etc. When Siddhāntas show discord, i.e. when an early Siddhānta is in discord, observations should be made with the use of instruments and the correct number of revolutions etc. (which would give results which accord with actual observation) found, and a *new* Siddhānta enunciated. (KVS)

यन्त्रेण ग्रहस्फुटशोधनम्

1. 14. 2. अनावृतधरातले व्यासार्धाङ्गुलप्रमाणविस्तारं दृगुच्छ्रितं समवृत्तं पीठं पूर्वापरदक्षिणोत्तररेखान्वितं 'खखषड्घन'विभक्तपरिधिं कारयेत् । तत्र सवितुरुदयकालेऽस्तकाले च पीठापरपूर्वदिशोः स्थित्वा तत्पीठपरिधौ अर्धोदितं विवस्वन्तं शङ्कुच्छन्नं दृष्ट्वा पीठपरिधौ चिह्नं कुर्यात् । शङ्कुच्छाया च तद्व्यतिरिक्तदिक्परिधौ यत्र स्पृशपि तत्रापि चिह्नं कुर्यात् । मध्याह्नशङ्कुच्छायाग्रे चिह्नं कुर्यात् ।

उत्तरगोलस्थेऽर्के समपूर्वापरसूत्रे शङ्क्वन्तरं स्थापयित्वा तच्छायायास्तद्रेखाप्रवेशकालं प्रतिदिनं उपलक्षयेत् । एवं भगणद्वित्रिभोगकालं यावत् पश्येत् । एवंदृष्टोऽर्कः शास्त्रप्रतिपादितदेशेषु दृष्टश्चेत् गणितं सम्यगिति ज्ञेयम् । मध्यप्रवेशकालश्च शास्त्रोक्तसमश्चेत् तेनापि गणितशुद्धिः । एवं दक्षिणोत्तरमध्यच्छायापरशून्यतावशादपि गणितसम्यक्त्वं ज्ञेयम् । एवं अर्कदृक्साधनम् ।

एवं दृक्समेनार्केण शशिसंयोगं शास्त्रीयगणितसिद्धं ग्रहणगतदृक्साम्येन दृष्टं चेत् चन्द्रोऽपि स्फुटः । तेनेन्दुना गणितसिद्धसमागमो भौमादीनां दृक्समश्चैत् तेऽपि स्फुटाः ।

(Sūryadeva Yajvan, Com. on *ABh.*, 4. 48)

Verification of computed planets

In an open spot construct a perfectly circular platform of desired radius in *aṅgulas,* raised to the height of the eye, with the east-west and north-south lines drawn through the centre and 21,600 equal divisions (of seconds) marked on its circumference. Then, at sunrise and sunset, stand (in turn) at the west and east sides of the platform and observe the half-risen and half-set Sun, respectively, and mark their positions on the circumference as indicated by the gnomon (fixed at the centre of the circle).

[1] For an exposition, see *SiŚi:AS*, pp. 446-48.

Mark also the opposite points on the circumference as indicated by the shadow of the gnomon. Mark also the point at the tip of the mid-day shadow. When the Sun is in the northern hemisphere, place another gnomon on the east-west line and observe daily the entrance of the shadow on that line. These observations should be continued for two or three years. If the Sun, observed thus, is seen exactly in the positions arrived at by computation, then that computation is to be understood as correct. If the time of the entrance of the midday shadow on the north-south line corresponds to that derived by computation, then also the correctness of the computation is verified. In the same way, the correctness of the computation may be verified also by the observation of the north-south midday shadow, when it is either maximum or totally absent. The verification of the computed Sun with observed Sun is done in this manner.

The Moon will also be correct if the computed conjunction of the Moon with the observed Sun is found to be identical with the observed one at the time of eclipses. And, the other planets too will be correct if the computed conjunctions of the Moon (as verified) above with the computed planets are in conformity with the observed one. (If observations do not accord with the computations, necessary changes shall have to be effected in the *bhagaṇas* of the planets, as instructed elsewhere.) (KVS)

2. गणकः – ASTRONOMER

वेदकालिको गणकः

2. 1. 1. प्रज्ञानाय नक्षत्रदर्शम्...(*YV-VS*, 30-10)

यादसे...गणकम्...(*YV-VS*, 30-20)

Vedic astronomer

(In the Puruṣamedha sacrifice, offer) an observer/indicator of the stars to the deity Prajñāna . . . and an astronomer to the deity Yādas. (KVS)

2. 1. 2. स्वर्भानुर्वा आसुरः सूर्यं तमसाऽविध्यत् । तदत्रिरपनुनोद । तदत्रिरन्वपश्यत् । यदात्रेयाय हिरण्यं ददाति तम एव तेनापहते । अथो ज्योतिरुपरिष्टाद्धारयति, स्वर्गस्य लोकस्य समष्ट्यै ।।

(*Gopatha Brāhmaṇa*, 2.3.19)

Svarbhānu the demon, hid the Sun with darkness. Atri removed it. That was identified by Atri. When any gift of gold is given to a descendent of Atri, through that act darkness is removed. Then light from above holds one towards the attainment of the heavenly world. (KVS)

वेदकालिकी गणकपरम्परा

2. 2. 1. जनको ह वैदेहः अहोरात्रैः समाजगाम । तं होचुः—यो वा अस्मान् वेद विजहत् पाप्मानमेति । सर्वमायुरेति । अभि स्वर्गं लोकं जयति । नास्याऽमुष्मिन् लोकेऽन्नं क्षीयते ।...अहीना ह आश्वत्थः सावित्रं विदांचकार । सह हंसो हिरण्मयो भूत्वा स्वर्गं लोकमियाय । आदित्यस्य सायुज्यम्...। देवभागो ह श्रौतर्षः सावित्रं विदांचकार । तं ह वाग् अदृश्यमानाऽभ्युवाच—सर्वं बत गौतमो वेद यः सावित्रं वेदेति । ...तस्माद् ये के च सावित्रं विदुः सर्वे ते जितलोकाः ।...शूषो ह वै वार्ष्णेयः आदित्येन समाजगाम । तं होवाच—एहि सावित्रं विद्धि ।

(*Taitt. Brāhmaṇa*, 3.10.9)

A line of Vedic astronomers

The Vaideha (king) Janaka went with 'days and nights' (being the units of time, which he studied). (About the benefit of such study) they told him: "He who understands us becomes sinless; he lives a full life; he attains to the heavens; for him food will not be scarce in this world."

Ahīna, son of Aśvattha, learnt the science of the Sun. He became a swan and ascended the heavens and was merged with the Sun.

The *śrauta* priest Devabhāga learnt the science of the Sun. The invisible Goddess of Speech told him: "Gautama who knew the science of the Sun knows everything . . . . Hence those who know the science of the Sun, they overcome the world . . ."

Śūṣa, the son of Vārṣṇi, went with the Sun. The Sun told him: "Come and learn the Science of the Sun . . ." (KVS)

ज्यौतिषिको वेदवित्

2. 3. 1. वेदा हि यज्ञार्थमभिप्रवृत्ताः
कालानुपूर्व्या विहिताश्च यज्ञाः ।
तस्मादिदं कालविधानशास्त्रं
यो ज्योतिषं वेद स वेद यज्ञान् ।। ३६ ।।

(*RV-VJ*, 36)

An Astronomer is a Vedist

The Vedas have, indeed, been revealed for the sake of the performance of sacrifices. But the sacrifices are laid down on the sequence of appropriate times. Hence, he who knows astronomy, which is the science specifying time, knows the sacrifices (and, so, is a Vedist). (36). (KVS)

गणकस्य योग्यता

2. 4. 1. अथातः सांवत्सरसूत्रं व्याख्यास्यामः ।। १ ।।

. . . .

तत्र ग्रहगणिते पौलिशरोमकवासिष्ठसौरपैतामहेषु पञ्चस्वेतेषु सिद्धान्तेषु युगवर्षायनर्तुमासपक्षाहोरात्रयाममुहूर्तनाडीप्राणत्रुटित्रुट्याद्यवयवादिकस्य कालस्य क्षेत्रस्य च वेत्ता ।। ४ ।।

चतुर्णां च मानानां सौरसावननाक्षत्रचान्द्राणामधिमासकावमसम्भवस्य च कारणाभिज्ञः ।। ५ ।।

षष्ट्यब्दयुगवर्षमासदिनहोराधिपतीनां प्रतिपत्तिच्छेदवित् ।। ६ ।।

सौरादीनां च मानानामसदृशसदृशयोग्यायोग्यत्वप्रतिपादनपटुः ।।

सिद्धान्तभेदेऽप्ययननिवृत्तौ प्रत्यक्षसममण्डललेखासम्प्रयोगाभ्युदितांशकानां छायाजलयन्त्रदृग्गणितसाम्येन प्रतिपादनकुशलः ।। ८ ।।

सूर्यादीनां च ग्रहाणां शीघ्रमन्दयाम्योत्तरनीचोच्चगतिकारणाभिज्ञः ।। ९ ।।

सूर्याचन्द्रमसोश्च ग्रहणे ग्रहणादिमोक्षकालदिक्प्रमाणस्थितिविमर्दवर्णादेशानामनागतग्रहसमागमयुद्धानामादेष्टा ।। १० ।।

प्रत्येकग्रहभ्रमणयोजनकक्ष्याप्रमाणप्रतिविषययोजनपरिच्छेदकुशलः ।। ११ ।।

भूभगणभ्रमणसंस्थानाद्यक्षावलम्बकाहर्व्यासचरदलकालराश्युदयच्छायानाडीकरणप्रभृतिषु क्षेत्रकालकरणेष्वभिज्ञः ।। १२ ।।

(Varāha, *Bṛ. Saṃ.*, 2. 1-12)

Qualifications of an astronomer

We shall now explain the aphorisms, i.e. rules or qualifications, for an astronomer. (1)

Among the astronomical calculations, the astronomer should be conversant with the various sub-divisions of time such as the *yuga*, year, solstice, season, month, fortnight, day and night, *yāma* (a period of an hour and a half), *muhūrta* (fortyeight minutes or two *ghaṭīs*), *nāḍī* (equal to 24 minutes) *prāṇa* (time required for one inhalation), *truṭi* (a small unit of time) and its further subdivisions, as well as with the ecliptic (or with geometry) that are treated of in the five *Siddhāntas* entitled *Pauliśa*, *Romaka*, *Vāsiṣṭha*, *Saura* and *Paitāmaha*. (4)

He should also be thoroughly acquainted with the reasons for the existence of the four systems of measurement of time, viz. *Saura* or the solar system, *Sāvana*, or the terrestrial time, i.e. the time intervening between the first rising of any given planet or star and its next rising, *Nākṣatra* or sidereal, and *Cāndra* or lunar, as well as for the occurrence of intercalary months and increasing and decreasing lunar days. (5)

He should also be well-versed with the calculation of the beginning and ending times of the cycle of sixty years, a *yuga* (a five-year period), a year, a month, a day, a *horā* (hour), as well as of their respective lords. (6)

He should also be capable of explaining, by means of arguments, the similarities and dissimilarities as well as the appropriateness or otherwise of the different systems of measurement of time according to the solar and allied systems. (7)

Despite differences of opinion among the *Siddhāntas* regarding the expiry or ending time of an *ayana* (solstice), he should be capable of reconciling them by showing the agreement between correct calculation and what has been actually observed in the circle drawn on the ground by means of the shadow of the gnomon as well as water-instruments. (8)

He should also be well acquainted with the causes that are responsible for the different kinds of motions of the planets headed by the Sun, viz. fast, slow, southerly, northerly, towards perigee and apogee. (9)

He must be able to forecast, by calculation, the times of commencement and ending, direction, magnitude, duration, intensity and colour at the eclipses of the Sun and the Moon, as well as the conjunctions of the Moon with the five *Tārāgrahas* or non-luminous planets and the planetary conjunctions. (10)

He should also be an expert in determining accurately for each planet, its motion in *yojanas*, its orbit, other allied dimensions etc., all in terms of *yojanas*. (11)

He must be thoroughly acquainted with the Earth's rotation (on its own axis round the Sun) and its revolution along the circle of constellations, its shape and such other details, the latitude of a place and its complement, the difference in the lengths of the day and night (*lit.* diameter of the day-circle), the *carakhaṇḍas* of a place, rising periods of the different Signs of the zodiac at a given place, the methods of converting the length of shadow into time (in *ghaṭīs*) and time into the length of shadow and such other things, as well as those to find out the exact time in *ghaṭīs* that has elapsed since sunrise or sunset at any required time from the position of the Sun or from the Ascendant, as the case may be. (12)
(M.R. Bhat)

छेद्यकविद् गणकः

2. 4. 2. पूर्वापरायतां तद्भित्तावुत्तरपार्श्वके ।
दर्शयेच्छिष्यबोधार्थं लिखित्वा छेद्यकं सुधीः ।। ८ ।।
(Bhāskara II, *SiŚi.*, 2.4.8)

Expert in astronomical diagrams

An intelligent astronomer should draw (and demonstrate), for the understanding of his students, diagrams on the computation of astronomical phenomena, on the northern side of a wall extending from east to west. (KVS)

चतुरो गणकः

2. 5. 1. सममण्डललेखासंप्रवेशवेलाः करोति योऽर्कस्य ।
तत्प्रत्ययं च जनयति जानाति स भास्करं सम्यक् ।। ३६ ।।
(Varāha, *PS*, 4.36)

Expert astronomer

(Only he is fit to be called an expert astronomer) who knows the problems dealing with the Sun, who can compute the time of the Sun crossing the prime-vertical, and prove his method (mathematically or graphically). (36). (TSK)

2. 5. 2. आनयति यो द्युराशिं विनाधिमासैस्तथा तिथिप्रलयैः ।
रविदिवसेभ्योऽस्माद् वा द्युचरार्धं यः स तन्त्रज्ञः ।। २ ।।
अधिमासैः शशिमासैरवमैः कुदिनैर्विना च य आनयति ।
द्युगणं रविदिवसेभ्यो वेत्ति प्रकटं स मध्यगतिम् ।। ३ ।।
कुदिनैः शशिदिवसान् तैः खरांशुदिवसान् करोति तैर्भाहान् ।
अधिकैरवमानवमैरधिकान् वा यः स तन्त्रज्ञः ।। ४ ।।
द्युगणादृते रवीन्दू ताभ्यामिष्टं ग्रहं चान्यम् बहुधा यः ।
शशिन इनं तत इन्दुं करोति गणकः स तन्त्रज्ञः ।। ५ ।।
अश्विन्यौदयिकानथवाऽभीष्टदिवौकसाभ्युदयकाले ।
साधयति दिविचरान् यो गणको मुख्यः स तन्त्रविदाम् ।।
वारं विलोमविधिना स्वसप्तमाद् यः करोति संक्षेपात् ।
द्युसदां च विलोमगतिं मध्यगतिं वेत्ति विमलां सः ।। ७ ।।
महदल्पगती द्युचरावन्योन्यं यः प्रसाधयेद् बहुधा ।
ग्रहमर्कमर्कमथवा करोति खचरं स तन्त्रज्ञः ।। ८ ।।

प्रत्युदयं प्रतिपाद्य ग्रहभुक्तिं वेत्ति यो ग्रहाभ्युदयात् ।
बहुधा करोति तेभ्यो भावर्त्तान् यः स तन्त्रज्ञः ।। ९ ।।
अन्यग्रहभगणगुणाद् द्युगणात् प्रश्नाहताक्षरादथवा ।
कुरुते यो ग्रहमिष्टं सच्छेदगुणापवर्तज्ञः ।। १० ।।
इष्टग्रहावमेभ्यो मध्यतिथिं तद्दिवौकसाभ्युदयात् ।
रविशीतगू च बहुधा यो वेत्ति स वेत्ति मध्यगतिम् ।। ११ ।।
अपवर्तितगुणहारैर्यो द्युगणादीन् करोति संक्षेपात् ।
कल्पात्कजन्मनो वा कृतात्कलेर्वा स तन्त्रज्ञः ।। १२ ।।
द्वित्रिगुणयो रवीन्द्वोर्योगादष्टोद्धृताज् ज्ञहीनाद् यः ।
आनयतीष्टद्युचरं करामलकवत्स वेत्ति मध्यगतिम् ।। १३ ।।
नव-धी-गो-हृत-भूमिज-गुरु-शनियोगाद् दिगीशगुणिताभ्याम् ।
ज्ञसिताभ्यां युक्ताद् यो वेत्तीष्टखगं स तन्त्रज्ञः ।। १४ ।।
रविशशिकुजबुधयोगः पृथक् पृथक् त्रिगुणितैश्च तैर्हीनः ।
युक्तो वा तद्योगात् स्वधनगुरुं वेत्ति यः स तन्त्रज्ञः ।।१५।।
(Vaṭeśvara, *VSi.*, 1.9.2-15)

Only he who finds the *ahargaṇa* from the solar days (elapsed) without making use of the intercalary months and the omitted days and the mean longitude of a planet from that (*ahargaṇa*), is an (expert) astronomer. (2)

Only he who finds the *ahargaṇa* from the solar days (elapsed) without making use of the intercalary months, the lunar months, the omitted days, and the civil days, knows the mean motion clearly. (3)

Only he who finds the lunar days from the civil days, therefrom the solar days, and therefrom the sidereal days, and also finds the omitted days from the intercalary days, and the intercalary days from the omitted days, is an astronomer. (4)

Only he who finds the longitudes of the Sun and Moon without making use of the *ahargaṇa* and therefrom obtains the longitude of a different desired planet, and also derives the Sun's longitude from the Moon's longitude and vice versa in a variety of ways, is an astronomer. (5)

Only he who finds the longitudes of the planets for the time for the rising of the asterism Aśvinī or for the time of rising of the desired planet is the foremost amongst the astronomers. (6)

Only he who finds easily (the lord of) the current day from the seventh day of the succession of weekdays by the inverse process, and calculates the retrograde (or westward) longitudes of the planets knows the mean motion clearly. (7)

Only he who determines the longitudes of the faster and slower planets from each other in a variety of ways and also derives the Sun's longitude from the planet's longitude and vice versa, is an astronomer. (8)

Only he who, having obtained the times of rising of a planet (in a *yuga*), finds the daily motion of a planet from those risings of the planet, and from them derives the revolutions of the asterisms (in a *yuga*), in a variety of ways, is an astronomer. (9)

Only he who determines the longitude of the desired planet from the *ahargaṇa* multiplied by the revolution-number of another planet or multiplied by the multiplier given in the problem, is proficient in the reduction of fractions. (10)

Only he who determines the longitude of the desired planet from the omitted days, the mean *tithi* from the risings of that planet, and the longitude of the Sun and the Moon in a variety of ways, knows the mean motion. (11)

Only he who, by using the abraded multipliers and divisors, briefly obtains the *ahargaṇa* since the commencement of the current *kalpa*, since the birth of Brahmā, since the beginning of Kṛtayuga, or since the beginning of Kaliyuga, is an astronomer. (12)

Only he who computes the longitude of the desired planet from

$$\frac{2 \times \text{Sun's long.} + 3 \times \text{Moon's long.}}{8} - \text{Mercury's long.}$$

knows the mean motion (of the planets) like an emblic myrobalan placed on his palm. (13)

Only he who obtains the longitude of the desired planet from 9 × (Mars' long.) + 8 × (Jupiter's long). + 9 × (Saturn's long.) + 10 × (Mercury's long.) + 11 × (Venus' long.) is an astronomer. (14)

'The sum of the longitudes of the Sun, Moon, Mars and Mercury is severally diminished or increased by three times the individual longitudes of those planets.' One who, from those sums (and differences), obtains the individual longitudes of those planets is an astronomer. (15)

2.5.3. यः कोटिभागैः कुरुते भुजांशान्
भुजांशकैर्वेत्ति च कोटिभागान् ।
भुजाच्च केन्द्रं द्युचरस्य केन्द्रा-
न्मध्यं स वेद स्फुटखेटचेष्टाम् ।। २ ।।
कोटचंशकैर्यः कुरुते भुजज्यां
बाह्वंशकैर्वेत्ति च कोटिजीवाम् ।
बाहुज्ययाऽग्रामनया च दोर्ज्यां
जानात्यसौ स्पष्टगतिं ग्रहाणाम् ।। ३ ।।
क्रमज्यया स्वोत्क्रममौर्वीकां तथा
निजक्रमज्यां श्रवणं विनाग्रकम् ।
भुजज्यया च श्रवणाच्च कोटिकां
तया च दोर्ज्यां कुरुते सुधीवरः ।। ४ ।।

स्पष्टीकरोति बहुधा स्वफलैर्ग्रहान् यो
भुक्तीश्च केन्द्रमपि मन्दचलोच्चसंज्ञम् ।
कुर्यात् स्फुटं दिविचरं वशगं च मध्यं
वेत्ति स्फुटां ग्रहगतिं खलु दैववित् सः ॥ ५ ॥

तुङ्गमेव कुरुते स्फुटग्रहं तद्गतिं च स्फुटभुक्तिमैन्दवीम् ।
यातयेयदिनयोस्तथाद्यजां वेत्त्यसौ स्फुटगतिं दिवौकसाम् ॥

यातकालमवमावशेषकं भूदिनौघखगपर्ययादिना ।
पातकालमवमस्य वेत्ति यः स्यादसाववमपातविद् बुधः ॥

स्पष्टमेव खचरं द्युराशितो
वेत्ति वाऽभिहितखेचरोदये ।
अश्विनस्य खलु वा प्रसाधयेद्
यः स वेत्ति विमलां स्फुटां गतिम् ॥ ९ ॥

ज्याभिर्विनैव कुरुते भुजकोटिजीवे
चापं च यः स्फुटखगं च करोति मध्यम् ।
तुङ्गात्तथोच्चगतिमध्यगती स्फुटे य-
श्चेष्टां करामलकवद् द्युसदां स वेत्ति ॥ १० ॥

केन्द्रमिष्टफलतस्ततोऽथवा तद्ग्रहस्य दृगदृश्यकेन्द्रके ।
वक्रकेन्द्रमनुवक्रकेन्द्रकं तद्दिनानि गणकः स उच्यते ॥ १२ ॥

स्फुटर्क्षभोगं बहुधाऽभिजिद्गतिं
विनिघ्निनः स्पष्टगतिं च वेत्ति यः ।
दिवौकसः सङ्क्रमकालनाडिकाः
स वेत्ति सम्यग्गणितं स्फुटां गतिम् ॥ १३ ॥

आद्यन्तौ व्यतिपातवैधृतिकयोः पर्वान्तयोश्च स्फुटं
तिथ्यन्तं करणान्तमेव हि तथा योगान्तमार्क्षं तथा ।
यो जानाति समौ खरांशुशशिनौ लिप्तांशराश्यादिकौ
त्र्यहःस्पृग्दिवसाधिपं स गणको नान्योऽस्ति तस्यापरः
॥ १४ ॥

अत्यन्तशीघ्रमथ शीघ्रसंज्ञां निसर्गजातां मृदुसंज्ञितां च ।
सुमन्दवेगां खलु वक्रनाम्नीमतीव वक्रां कुटिलां तथैव ॥१५॥
अष्टप्रकारां द्युचरस्य भुक्तिं
यः केन्द्रभेदैर्गणकस्तु सम्यक् ॥ १६ ॥

(Vaṭeśvara, *VSi.*, 2.7.2-16)

One who finds the degrees of the *bhuja* from the degrees of the *koṭi*, the degrees of the *koṭi* from the degrees of the *bhuja*, from the *bhuja* the *kendra*, and mean planet from the planet's *kendra*, knows the true motion of the planets. (2)

One who finds the R sine of the *bhuja* from the degrees of the *koṭi*, the R sine of the *koṭi* from the degrees of the *bhuja*, the *koṭijyā* from the *bāhujyā*, and the *bāhujyā* from the *koṭijyā*, knows the true motion of the planets. (3)

One who finds the corresponding R versed sine from the R sine, the R sine from the R versed sine, the *karṇa* without using the *koṭijyā*, the *koṭijyā* from the *bhujajyā* and the *karṇa*, and the *bhujajyā* from the *koṭijyā* (and the *karṇa*), is endowed with flawless intellect. (4)

One who corrects, in many ways, the (mean) planets, the (mean) motions, as well as the *manda* and *śīghra-kendras* with the help of their own corrections, converts the true planet into the corresponding mean planet, knows the true motion of the planets is indeed an astronomer. (5)

One who derives the true planet from its *ucca*, and the true motion of the Moon for the preceding, succeeding and current days from that of its *ucca*, knows the true motion of the planets. (6)

One who finds the time elapsed corresponding to the *avamaśeṣa* (i.e., residue of the omitted days) and the period of occurrence of the omitted days with the help of civil days and planetary revolutions in a *yuga*, etc., is a learned astronomer, proficient in the subject of *avamapāta*. (8)

One who finds the true planet from the *ahargaṇa*, or for the time of rising of a given heavenly body, or for the time of rising of the *nakṣatra* Aśvinī (ζ Piscium), knows the flawless true motion (of the planets). (9)

One who, without making use of the (tabular) R sines, calculates the *bhujajyā* and the *koṭijyā*, and the arc (corresponding to the *bhujajyā* or *koṭijyā*); who, with the help of the longitude of the *ucca*, converts the true planet into the mean planet; and who finds the true motion (of a planet) with the help of the motion of the *ucca* and the mean motion (of the planet), knows the motion of the heavenly bodies (as if submitted to the eye) like an emblic myrobalan placed on his palm. (10)

One who finds the *kendra* from the given *phala* (i.e., *manda-phala* or *śīghra-phala*); obtains the (*śīghra*)*kendra* for the heliacal rising or setting of a planet; or one who finds the (*śīghra*)*kendra* for the beginning of retrograde or re-retrograde motion (of a planet) or the corresponding days (i.e., the days of retrograde or direct motion) is designated as gaṇaka (astronomer). (12)

One who knows the true *bhogas* of the *nakṣatras*, the *bhoga* of Abhijit as well as the true location of that malignant (*nakṣatra*), and the true *nāḍīs* of *saṅkrāntikāla* (i.e., the time in *nāḍīs* at which the Sun crosses the end of a sign), is an astronomer well-versed with *gaṇita* and the true motions (of the heavenly bodies). (13)

One who knows (how to find) the times when *Vyatipāta* and *Vaidhṛta* begin and end; the times when the new moon and full moon days, *tithi*, *karaṇa*, *yoga* and *nakṣatra* end; the longitudes of the Sun and Moon corresponding to minutes, degrees, signs, etc., as well as the lord of the

day which touches three *tithis* is a *gaṇaka* having none to match him. (14)

One who correctly knows the eight varieties of planetary motion, viz. very fast, fast, natural (or mean), slow, very slow, retrograde, very retrograde, and re-retrograde, along with the corresponding (*śighra*)-*kendras*, is a good astronomer. (15-16). (KSS)

स्फुटगणिते गणकस्य व्यग्रता

2. 6. 1. कार्यो ग्रहेषु ग्रहणमण्डनोक्तेष्वतः परम् ।
संस्कारस्तं च वक्ष्यामि, तत्र नोक्तं यतो मया ॥ ४७ ॥

शतद्वयाब्दे ग्रहणमण्डनोदितभास्करे ।
एकैका लिप्तिका शोध्या तत्सिद्धौ; शीतगौ पुनः ॥ ४८ ॥

योज्यैका वत्सरे सैकचत्वारिंशन्मिते कला ।
पञ्चत्रिंशद्युतशते वर्षे तुङ्गेंऽशकस्तथा ॥ ४९ ॥

योज्य एका; राहुमध्ये शोध्यैकाब्दत्रये कला ।
एतत्संस्कारसंयुक्तास्तत्रार्काद्याः स्फुटाः स्मृताः ॥ ५० ॥

(Parameśvara, *Dr̥gganita*, 2. 47-50)

Anxiety for accuracy

A further correction has to be applied to (the mean positions of) the planets (as computed by the methods) enunciated (in my) *Grahaṇamaṇḍana* (verses 5-8). That correction too, I shall state (here) since that has not been specified by me there.[1] (47)

One second should be subtracted for every 200 years from (the mean position of) the Sun derived according to the *Grahaṇamaṇḍana* to get its (correct mean position).

In the case of the Moon, however, one second should be added (to its mean position) for every 41 years.

In the case of the Node, one second should be added to (12—Node) for every 135 years.

From the mean of the Higher Apsis should be subtracted one minute for every three years. (48-50a)

With the application of this correction (the mean positions of) the Sun and other (planets) will become accurate. (50b). (KVS)

ग्रहणदर्शको गणकः

परमेश्वरपरीक्षितानि ग्रहणानि

2. 7. 1. इति विस्तरतः प्रोक्तं ग्रहणं सूर्यचन्द्रयोः ।
दृश्यते तत्र दृग्भेदः काले बिम्बे कदाचन ॥ १ ॥

तिथिविश्व (1315) समे शाके प्रारभ्य ग्रहणं मया ।
अनेकमीक्षितं, तेषु भिन्नः कालो दृशा सदा ॥ २ ॥

प्रत्यक्षकालस्तेषु प्राग्गणितानीतकालतः ।
अतः कार्योऽत्र संस्कारो यः कश्चिद् गणकोत्तमैः ॥ ३ ॥

आचार्योदितखेटेषु संस्कारः क्रियते बुधैः ।
शकाब्दाख्यः स चान्यत्र 'वाग्भावे'त्यादिनोदितः ॥ ४ ॥

तत्रेन्दोः शाकजा लिप्ताः स्वपञ्चांशेन वर्जिताः ।
ग्राह्या, राहोर्द्वादशांशहीनास्तुङ्गस्य केवलाः ॥ ५ ॥

विशेषोऽयं दृष्टिसाम्यसिद्धये क्रियतेऽधुना ॥ ६a ॥

रविग्रहणानि

द्युगणे सप्तनागाग्निगुणेषुरसभू (1653387) मिते ।
गोकर्णे ग्रहणं भानोर्दृष्टं नात्र निलातटे ॥ ६९ ॥

शून्याग्निभूशरेष्वङ्गभू (1655130) तुल्ये द्युगणे रवेः ।
गोकर्णे ग्रहणं दृष्टं निलाब्ध्योः सङ्गमेऽत्र न ॥ ७० ॥

प्रोक्ते दिनेऽपि बिम्बस्य पार्श्वे वर्णस्य भेदनम् ।
कैश्चित् कुमारैरत्रापि कल्पितं वा निलातटे ॥ ७१ ॥

द्युगणे खखशून्याक्षिबाणाङ्गशशि (1652000) सम्मिते ।
सुव्यक्तं ग्रहणं दृष्टं नावाक्षेत्रेऽत्र तीक्ष्णगोः ॥ ७२ ॥

स्पर्शोपलब्धौ पदभा चत्वारिंशन्मितात्र तु ।
पञ्चत्रिंशन्मितेत्येके वदन्ति व्यक्तिभेदतः ॥ ७३ ॥

द्विषड्रसेषुपञ्चाङ्गविधुभि (1655662) र्द्युगणे मिते ।
ईषद्ग्रस्तो रविर्दृष्टो निलायां तु सुदृष्टिभिः ॥ ७४ ॥

स्पर्शकाले तु पदभा तत्र पञ्चदशोन्मिता ।
प्रायशो मोक्षकाले तु दशभिर्वा दलोनितैः ॥ ७५ ॥

अस्मिन् दिने किलार्कस्य मण्डलं पश्यतां नृणाम् ।
नातितप्ते दृशौ मान्द्यं तैक्ष्ण्यस्यातोऽत्र कल्पितम् ॥ ७६ ॥

कृतद्विबाणरामाब्धिषट्चन्द्रैः (1643524) द्युगणैः समे ।
पदभैकादशारम्भे मोक्षो दृष्टोऽपराह्नजः ॥ ७७ ॥

स्थितिकालोऽधिको ह्यत्र नाडिकानवकादतः ।
कार्यमेवाविशेषादि धीमता द्युमणेरपि ॥ ७८ ॥

द्युगणे रसतिथ्यद्रिवेदषट्भूमि (1647156) सम्मिते ।
मुक्तेऽर्केऽस्तमयो दृष्टो नाडीपादोऽस्ति वान्तरे ॥ ७९ ॥

द्व्यक्ष्यद्र्यष्टाब्धिषट्चन्द्र (1648722) सम्मिते द्युगणे पुनः ।
स्पर्शे तु पदभार्कस्य चतुर्विंशतिसम्मिता ॥ ८० ॥

अह्नां गणेऽब्धिनागाब्धिपञ्चेष्वङ्गैक (1655484) सम्मिते ।
मोक्षकाले रवेः सार्धैः पञ्चभिः पदभा मिता ॥ ८१ ॥

चन्द्रग्रहणानि

दिनौघेऽद्र्यब्धिषट्पञ्चबाणाङ्गैक (1655647) मिते विधोः ।
संस्पर्शे तिथिभि (15) र्द्वाभ्यां मोक्षे च पदभा मिता ॥८२॥

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[1] The correction enunciated here is to be applied from the date of the *khaṇḍa* of this work, given in verese 5, viz. Kali day 16,48,157, corresponding to Kali year 4512, Kaṭaka 17 or A.D. July 15, 1410.

रामरन्ध्रयमप्राणबाणषट्शशिभि(1655293)मिते ।
द्युगणे नैव दृष्टं खे ग्रहणं शीतदीधितेः ।। ८३ ।।
वेदरन्ध्ररसाक्षीषुरसशीतांशुभि(1652694)मिते ।
द्युगणे शीतगुर्दृष्ट ईषद्ग्रस्तोऽम्बरे नृभिः।। ८४ ।।
मन्वङ्गाब्धीषुषट्चन्द्रैः(1654614)सम्मिते द्युगणे विधोः ।
दृष्टो विमर्दस्तद्वत् त्रिखाब्धित्रीषुरसेन्दुभिः(1653403)।।
उक्तेभ्योऽन्ये चोपरागा मया दृष्टा विवस्वतः ।
इन्दोश्च बहवो दृष्टास्ते तु नोदाहृता इह ।। ८६ ।।
एतानतीतोपरागान् सञ्चिन्त्य परिकल्पिताः ।
विलिख्यन्ते मया भानुचन्द्रचन्द्रोच्चराहवः ।। ८७ ।।
द्युगणे व्योमशून्याद्रिचन्द्रेषुरसभू(1651700)मिते ।
सूर्यस्य मेषसंस्थस्य तिथिभिः (15) सम्मिता कलाः ।।८८।।
धटस्थस्य विधोर्भागा वेदा (4) लिप्ता रसै (6)मिताः ।
तुङ्गस्य कर्कटे भागा नव (9)लिप्ता स्वरेषवः (57) ।।
पातस्य सिंहे त्रियमा(23) भागाः प्राणशराः(55)कलाः ।
अर्धाधिकं गृहीतं वै प्रोक्तेष्वर्केन्दुराहुषु ।। ९० ।।
अस्मिन् काले रवीन्दूच्चपातानां स्थितिरीदृशी ।
एतत्सिद्ध्यर्थमस्माभिः संस्कारान्तरमादृतम् ।। ९१ ।।
यदा परहितप्रोक्ता गृह्यन्ते विहगास्तदा ।
कार्यः पूर्वोक्तसंस्कारो यतोऽस्मिन् स्थितिरीदृशी ।। ९२ ।।
कालान्तरे तु संस्कारश्चिन्त्यतां गणकोत्तमैः ।। ९३a ।।

(Parameśvara, *Siddhāntadīpikā: MBh. Bhāṣya-vyākhyā*, under 5.77)

Eclipses examined by Parameśvara[1]

Thus far, I have elucidated in detail (the computation of) the eclipses of the Sun and the Moon. But at times there is found differences both in the time (of the eclipses) and in (the extent of) the orbs (eclipsed). (1)

Beginning from Śaka (1315, i.e. A.D. 1393), I have computed and observed a large number of eclipses. However, there had uniformly been difference in the time (of the eclipses) as observed (and as computed). (2)

In those cases, the times when (the eclipses were) observed occurred before the computed times. Hence it was patent that an appropriate correction was required to be effected by expert astronomers. (3)

Additional correction to the Śakābda correction

A correction by name *Śakābda-saṃskāra*, otherwise called *Vāgbāva-saṃskāra*, is applied by astronomers to the (mean positions) of the planets as computed according to the system of Ācārya (Āryabhaṭa). (4)

There, the minutes of the Śakābda correction of the Moon should be taken one-fifth less, and that of Rāhu's correction one-twelfth less. The correction for the Higher Apsis can be taken as it is. (5)

This special correction is done for (making computed positions) accord with observed positions. (6a)

Solar eclipses

A solar eclipse was observed in Gokarṇa on Kali day 16,53,387; it was not, however, seen on the banks of the Nilā river. (69)

Again on Kali 16,55,130 a solar eclipse was seen in Gokarṇa which was not observed at the confluence of river Nilā and the sea. (70)

However a discoloration at the fringes of the solar orb was suspected by some students (of astronomy) even here on the banks of Nilā on the above date. (71)

On Kali 16,52,000 a clear solar eclipse was observed in the region of Nāvā. (72)

The gnomonic shadow at the moment of contact at this eclipse has been stated by some to be forty, while others stated it as thirtyfive, on individual (reckonings). (73)

On Kali 16,55,662 on the banks of the Nilā river the Sun was seen by keen viewers slightly eclipsed. (74)

The gnomonic shadow at the first contact of this eclipse was fifteen and at the last contact it was nine and a half. (75)

On this day, the eyes of those observing the Sun's orb were not hurt; it is therefore to be presumed that the heat then was much subdued. (76)

On Kali 16,43,524, the gnomonic shadow at the commencement was eleven and the end (of the eclipse) occurred in the afternoon. (77)

In this case, the duration (of the eclipse) was more than nine *nāḍikās* and hence the Sun also had to be computed by experts by the method of successive approximation. (78)

On Kali day 16,47,156, as the eclipse ended, the Sun was setting with a quarter *nāḍī* to go. (79)

On Kali 16,48,722 at the first contact, the Sun's gnomonical shadow was twentyfour. (80)

On Kali 16,55,484, at the time of last contact, the gnomonic shadow of the Sun was five and a half. (81)

[1] The set of verses extracted and translated here form a disquisition by the author Parameśvara at the close of his commentary on *Mahābhāskarīya-Bhāṣya*, ch. V, verse 77, dealing with the eclipses (see the edition of the work, Madras 1957, pp. 321-32.)

Lunar eclipses

On Kali 16,55,647 the Moon's shadow at the moment of first contact was fifteen and at the moment of last contact it was two. (82)

On Kali 16,55,293, the lunar eclipse due was not visible in the sky. (83)

On Kali 16,52,694, (at lunar eclipse), the Moon was seen by people as slightly eclipsed. (84)

On Kali 16,54,614 a total lunar eclipse was witnessed; so also on Kali 16,53,403. (85)

In fact several more eclipses, both solar and lunar, have been observed by me though not documented here. (86)

On considering the differences between the observed and (computed times of) the said past eclipses (and identifying the corrections which would obviate the said differences), I am setting down below the (true positions of) the Sun, the Moon, the Higher Apsis and the Node (at sunrise for a contemporary date, for the first point of Aries, so as to serve as zero-corrections for calculations beginning from that date). (87)

On Kali 16,51,700, the positions of the Sun at sunrise at the first point of Aries is 15′, the Moon at Libra is 4° 6′, the Moon's Higher Apsis in Capricorn is at 9° 57′ and the Node in Leo is 23° 55′. In the case of the Sun, Moon and Node, half and more than half of a second has been taken as a full second. (88-90)

The above are the positions of the Sun, the Moon, Higher Apsis and Node at the above specified time. For arriving at these results the special correction has been postulated by me as above. (91)

If the planets are taken as computed according to the Parahita system, then the correction enunciated above (in verse 5) has to be applied, for that is the situation. (92)

In the times to come also, similar corrections may have to be postulated by expert astronomers. (93a). (KVS)

गणकादृतो शोधप्रकारः

2. 8. 1. 'वाग्भावोनाद्' इत्यादिनोक्तो भटाब्दसंस्कारः इह शकाब्दसंस्कारो विवक्षितः। सिद्धान्तशेखराद्युक्तमध्यमेभ्यः परमेश्वरोक्तानां नूतनत्वाद् अस्य दृष्टिसाम्यं स्यात् । तस्मात् सिद्धान्तदीपिकोदाहृतानि अस्माभिः (नीलकण्ठेन) दृष्टानि च तत्तदवसरे वक्ष्यमाणानि परमेश्वरोक्तप्रकारेण अर्कादिमध्यमान्यानीय श्रीपत्युक्तप्रकारेण स्फुटीकृत्य कालक्रिया-गोलपादोक्ताभिः अस्माभिर्व्याख्याताभिर्युक्तिभिस्सिद्धैः क्रियाविशेषैश्च गण्यन्ताम् । तत्र प्रथमम्—

'कृतद्विबाणरामाब्धिषट्चन्द्रे' द्युगणे समे ।
पदभैकादशारम्भे मोक्षो दृष्टोऽपराह्नतः ।।[1]

इत्येतद् गणयेत् ।

ग्रहणात् पूर्वोदयकालजोऽयमहर्गणः । तत्र रविमध्यं 'मध्यं सूक्ष्मं ह्यर्कः' (0ʳ 11° 57′ 15″) इति विकलादिः । 'मन्ये वनस्थः' (7ʳ 4° 15′) इति कलादिश्चन्द्रः। 'तन्त्रधी राजा' (8ʳ 29° 26′) इति तत्तुङ्गः। 'वाक्यार्थज्ञः सः' (7ʳ 7° 14′) इति तत्पातः। स्पर्शो वा प्रथमं परीक्ष्यः, उक्तत्वात् तस्य 'पदभैकादशारम्भे' इति। तथाप्यविशेषः कार्यः। तत्र छायावाक्यैरेव प्रथमं प्रायिकं द्युगतं ज्ञेयम् । तच्च 'अवम' (5^{n} 40^{vin}) संख्यम् । तत्र निलातीरे अश्वत्थग्रामे देशान्तरकालो 'नख' (20) विनाड्यः। तच्च धनम्। विषुवच्छाया च 'दुष्करा। (2^{ang} 18^{vyang}) । देशान्तरद्युगतयोगो नाडीषट्कम्। तद्गतीरुदयकालमध्यमे क्षेप्याः। तदार्कमध्यमं 'प्रयोगो न स्फुटार्थम्' (7ʳ 12° 3′ 12″) इति । तच्च न स्वदेशार्कोदयकालादूर्ध्वं नाड़ीषट्कान्तरजम्; किन्तु स्वदेशदक्षिणोत्तरभूवृत्तनिरक्षपूर्वापरवृत्तयोः संयोगे लङ्कातः प्रतीच्यां भूवृत्तस्य अशीत्यधिकशतांशे घटिकामण्डलस्य। एतदर्कमध्यमतुल्यप्रदेशोदयकालादेव। तत्रापि नार्कोदयकालात् प्रभृति। ततोऽर्कमध्यमस्य तत्स्फुटप्रतिचयस्य च विवरप्राणगतिः स्वा स्वा सर्वं मध्यमे कार्या। मध्यमात् स्फुटप्राणेऽधिके धनम्, न्युने ऋणम्। स्फुटप्राणनिश्चयश्च प्रथमपदे 'इष्टज्यागुणितम्' इत्यादिना आनीत एव षष्टया-(?डा)रोपितः। द्वितीये तु राशिषट्काद् विशोधयेत्। सः स्फुटप्राणराशिः। तृतीये राशिषट्के क्षिपेत्। चतुर्थे मण्डलाच्छोधयेत्। साक्षदेशे पुनः स्वदेशचरदलप्राणतुल्याः कलाः मेषादौ स्फुटप्राणेभ्यः शोध्याः। तुलादौ क्षेप्याः। एतदेव रव्युदये घटिकामण्डललग्नम्। तस्य चार्कमध्यमस्य च विवरकलाः प्राणा एव। तानेव सर्वेषां स्वस्वगत्या निहत्य चक्रकलाहृताः कलाः स्वस्वमध्यमे संस्कार्याः। स्फुटे चेत् स्फुटगत्या हन्तव्याः। ततस्तदर्थमर्कस्फुटीकरणं कार्यम्। प्रथमं तद्भुजाफलं 'हंसः' (78′) इति ऋणम्। 'प्रियो मघोनः पार्थः' (7ʳ 10° 45′ 12″) इति स्फुटरविः। तत्र अयनचलनं क्षिप्त्वा चरप्राणादिकमानयेत् ।।[2]

(Nīlakaṇṭha, *Jyotirmīmāṁsā*:*KVS*, pp. 35-36)

Methodology of Revision adopted by astronomers

The Bhaṭa-correction enunciated through the verse commencing with the expression *vāgbhāvona* is referred to here as Śakābda-saṃskāra. Since the system of Parameśvara (A.D. 1360-1455) is posterior to that of *Siddhāntaśekhara* (of Śrīpati, A.D. *c*.1000), the mean positions of planets computed according to the former would accord (better) with observation. Hence, for (the eclipses) enumerated (by Parameśvara) in his

[1] The dates of the several eclipses noticed above occur on dates beginning from A.D. 1393.

[1] This solar eclipse observed by Parameśvara has been depicted in verse 77 of the series of eclipses cited by him in the extract given above, 2.7.1.

[2] For further steps, see *Jyotirmīmaṃsā*:*KVS*, pp. 36 ff.

Siddhāntadīpikā and of those observed and enumerated by me (Nīlakaṇṭha) in various contexts, compute: (i) the mean Sun etc., as directed by Parameśvara, (ii) their true positions as directed by Śrīpati and (iii) by the special process explained by me in my (*Bhāṣya*) on the *Kālakriyā* and *Gola pādas* of the *Āryabhaṭīya*.

Take as the first case the following instance:

'On Kali day 16,43,524, (at a solar eclipse), the gnomonic shadow at commencement was eleven and the end (of the eclipse) occurred in the afternoon.'

Here the *ahargaṇa* referred to is at sunrise preceding the eclipse. The mean Sun then is $0^r\ 11^\circ\ 57'\ 15''$, correct to seconds. The mean Moon correct to minutes is $7^r\ 4^\circ\ 15'$. The Moon's Higher Apsis is $7^r\ 7^\circ\ 14'$.

The commencement of the eclipse is to be tested first, since it has been stated that 'the gnomonic shadow is eleven at commencement.' There too the process of successive approximation has to be done. The rough time elapsed after sunrise might be understood from the measure of the shadow. That comes to 5 *nāḍikās*, 40 *vināḍikās*. Now, at Aśvatthagrāma, on the bank of river Nilā, the *deśāntara* (longitudinal time difference) is 20 *vināḍīs*, positive. The Equinoctial shadow is 2 *aṅgulas* and 18 *vyaṅgulas*. The sum of *deśāntara* and time elapsed after sunrise is 6 *nāḍīs*. (The longitude for) the motion thereof should be added to the longitude at sunrise. The mean Sun would then be $7^r\ 12^\circ\ 3'\ 12''$. This is not for six *nāḍīs* after the local sunrise but for that of the *ghaṭikāmaṇḍala* at the western horizon at 180° from Laṅkā at the junction at the local N-S line, the horizon and the Equatorial E-W line. This is from the rise at a place having the longitude of mean Sun, but not from the time of sunrise. The difference in motion in terms of *prāṇas* of mean Sun and the corresponding true Sun has now to be applied in toto to the mean, it being positive if the true *prāṇas* are greater, and negative if less. The true *prāṇas* are calculated for the first quadrant using the formula enunciated in the verse *iṣṭajyāguṇita* etc. and added to six. In the second quadrant it has to be subtracted from six *rāśis*. In the third, it is to be added to six *rāśis*. In the fourth, it is to be subtracted from twelve *rāśis*. In places having latitude (i.e. north or south of the equator), *prāṇas* equal to half local ascensional difference have to be subtracted from the true *prāṇas* if (the Sun is in the six *rāśis*) beginning from Aries and added if beginning from Libra. The result would indicate the rising point of the *ghaṭikāmaṇḍala* at sunrise. The difference in minutes between this point and the mean Sun would be in *prāṇas*. In all cases these have to be multiplied by their respective rates of motion and divided by 21,600 and the resultant minutes applied to the respective means. In the case of true longitudes, the multiplication is to be by the rates of true motion. For this purpose the true Sun has to be computed. Now, the *bhujāphala* (of the mean Sun found above, viz., $7^r\ 12^\circ\ 3'\ 12''$) is 78′ (1° 18′). (Applying this to the mean), the true Sun is $7^r\ 10^\circ\ 45'\ 12''$. To this the precession of the equinoxes has to be applied and the *prāṇas* of ascensional differences are to be derived.[1] (KVS)

केरलीय-गोविन्दशिष्यपरम्परा

2. 9. 1. 1. 'गोविन्दन्' तलक्कुळत्तूर् भट्टतिरि । 'रक्षेत् गोविन्दमर्कः' एन्नु अद्देहत्तिन्टे जननकलि । केरलेश्वरसमीपत्तु इल्लं आकुन्नु ॥ . . . .

2. 'परमेश्वरन्' वटश्शेरि नम्पूरि । निलायाः सौम्यतीरस्थः परमेश्वरः । . . .

3. अस्य तनयो 'दामोदरः' ।

4. अस्य शिष्यो 'नीलकण्ठसोमयाजी' । इद्देहं तन्त्रसंग्रहं आर्यभटीयभाष्यम् मुतलाय ग्रन्थङ्ङळ्क्कु कर्तावाकुन्नु । 'लक्ष्मीशनिहितध्यानैः' इत्यस्य कलिना कालनिर्णयः ।

5. पूर्वोक्तस्य दामोदरस्य शिष्यः 'ज्येष्ठदेवः' । इद्देहं परङ्ङोट्टु नम्पूरियाकुन्नु । युक्तिभाषाग्रन्थत्ते उण्टाक्कियतुं इद्देहं तन्ने ।

6. ज्येष्ठदेवन्टे शिष्यन् तृक्कण्टियूर् 'अच्युतपिषारटि । इद्देहं स्फुटनिर्णयं, गोलदीपिक मुतलाय ग्रन्थकर्तावाकुन्नु । . . . .

(ओरु ज्योतिषग्रन्थवरि, Ms. Baroda 9886)

'राम'नेन्नेल्लाटवुं विश्रुतनायिट्टभि-
रामनामाशासितावेन्नुळ्ळ कीर्तियोटुम् ।
गुरुदैवज्ञन्माकुं गुरुभूतनामेन्टे
गुरुवां पिताविन्टे चरणाम्बुजं वन्दे ॥ १ ॥

गुरुविन् गुरु 'व्याघ्रमुखमन्दिरवासि'
गुरुकारुण्यशालि तन्नेयुं वणङ्ङुन्नेन् ॥ २ ॥

तद्गुरुभूतनायिट्टेत्रयुं मनीषियाय्
हृद्गतभावज्ञनाय् गणिततत्त्वज्ञनाय् ।
ताषात कीर्तियोटुं 'नावायिक्कुळत्तुळ्ळो-
राषातिप्रवरनां गुरुवे वन्दिक्कुन्नेन् ॥ ३ ॥

आयवन् तन्टे गुरुभूतनायुळ्ळ देह-
मायतमतिकळाल् पूजितनायुळ्ळवन् ।
कोलत्तुनाट्टु 'तृप्पाणिक्करप्पोतुवाळ'-
क्कालत्तेग्गुरुवरन्मारिल् वच्चग्रेसरन् ॥ ४ ॥

एन्नुटे गुरुविन्टे गुरुविन् गुरुभूतन् ।
तन्नुटे गुरुवाकुं तत्पदं वणङ्ङुन्नेन् ॥ ५ ॥

पोतुवाळिन्टे गुरु'वच्युतप्पिषारटि'-
यतिमानुषनवन् सकलविद्यात्मकन् ।

[1] For various and other allied revisions, see *Jyotirmīmāṃsā*: *KVS*, pp. 36 ff.

अन्पत्तिमून्नुवयस्सिरट्टियायिरुन्नुळ्ळ
मेल्पुत्तूर् पट्टेरिक्कुं गुरुवायिरुन्नवन् ।
तन्नुटे पादपद्मयुगलं विशेषिच्चु-
मेन्नुटे मनक्काम्पिल् सन्ततं निनय्क्कुन्नेन् ।। ६ ।।

कोच्चु-'कृष्ण'नाशान्टे शिष्यन् आरन्मुळ मङ्गलश्शेरि 'दक्षिणामूर्ति मूत्ततु', मूत्ततिन्टे शिष्यन् मान्नार् नालेक्काट्टिल् 'बालरामन् पिळ्ळ', सम्प्रतिप्पिळ्ळ, अद्देहत्तिन्टे शिष्यन् किळिमानूर् विद्वान् 'चेरुण्णि कोयित्तम्पुरान्' एन्निङ्ङने आ छात्नपरम्पर पिन्नेयुं तुटर्न्नु पोकुन्नु ।।

(Ulloor, *Kerala Sāhitya-Caritram*, II. pp. 321-22)

Academic genealogy of Govinda of Kerala

Govinda is a Bhaṭṭatiri of the Talakkuḷattūr family (of nampūtiri brahmins). His date of birth is Kali 15,84,362 (in A.D. 1237). His ancestral house is near Keraḷeśvara (in north Kerala). (1)

Parameśvara, (the grandson of Govinda's pupil)[1] is a nampūtiri of the Vaṭaśśeri family. Parameśvara resided on the northern bank of the Nilā river (in north Kerala). (2)

His son is Dāmodara. (3)

His pupil is Nīlakaṇṭha Somayāji. He is the author of *Tantrasaṅgraha*, *Āryabhaṭīyabhāṣya* and other works. His date is known from the Kali day 16,80,553 (in A.D. 1500). (4)

Of the aforesaid Dāmodara, Jyeṣṭhadeva is the pupil. He is a nampūtiri of the Paraṅkoṭṭu (family). It was he who wrote the work *Yuktibhāṣā*. (5)

The pupil of Jyeṣṭhadeva is Acyuta Piṣāraṭi belonging to Tṛkkaṇṭiyūr (in Central Kerala). He is the author of *Sphuṭanirṇaya*, *Goladīpikā* and other works.[2] (6)

I, (Kṛṣṇadāsa or Kṛṣṇan Āśān), worship the feet of my father and teacher widely known everywhere as Rāman (Āśān), famed as a promulgator of the name of god Rāma and teacher of even teachers of Jyotiṣa. (1)

I adore the teacher of my teacher, most compassionate and resident of Vyāghra-mukha-mandira (viz. Pulimukhattu Potti, in Malayalam). (2)

I adore also his (viz. Potti's) teacher, the great intellectual, thought-reader, noted astronomer and of unimpaired reputation, Nāvāyikkuḷattu Āzhāti. (3)

His teacher was Tṛppāṇikkara Potuvāḷ of Kolattunāḍu (in North Kerala), the foremost of all contemporary teachers, respected by the learned. I bow at the feet of this scholar who is my teacher's teacher. (4-5)

Potuvāḷ's teacher was Acyuta Piṣāraṭi who was a superman, a repository of all sciences, and who was the teacher of Melputtūr (Nārāyaṇa) Bhaṭṭatiri who lived for twice fiftythree years. I always keep in mind, in a very special manner, the twin lotus feet of this teacher.[3] (6)

The pupil of Kṛṣṇan Āśān was Dakṣiṇāmūrti Mūttatu of the Maṅgalaśśeri family at Āranmuḷa (in central Kerala). The pupil of Mūttatu was Bālarāman Piḷḷa, a holder of the office of *Samprati*. His pupil was Vidvān Ceruṇṇi Koyittampurān of Kiḷimānūr (south Kerala). The pupil-lineage extends still further.[4] (KVS)

कमलाकरस्य परम्परा

2. 10. 1. गोदावरीसौम्यतटोपकण्ठगोलाख्यसद्ग्रामसुसिद्धभूमौ ।
विप्रो महाराष्ट्र इति प्रसिद्धो रामो भरद्वाजकुलावतंसः ।।
बभूव तज्जोऽखिलमान्यभट्टाचार्योऽतिशास्त्रे निपुणः पवित्रः ।
सदा मुदासेवितभर्गसूनुर्दिवाकरस्तत्तनयो बभूव ।। ८ ।।
वेदान्तशास्त्राभ्यसनेन काश्यां यः पुण्यराश्यां तनुमुत्ससर्ज ।
अस्यार्यवर्यस्य दिवाकरस्य श्रीकृष्णदैवज्ञ इति प्रसिद्धः ।।
बभूव पुत्रः सुतरां पवित्रः सत्तीर्थकर्ताखिलशास्त्रवेत्ता ।
तज्जस्तु सद्गोलविदां वरिष्ठो नृसिंहनामा गणकार्यवन्द्यः ।।
बभूव येनात्र च सौरभाष्यं शिरोमणेर्वार्त्तिकमुत्तमं हि ।
स्वार्थं परार्थं च कृतं त्वपूर्वसद्युक्तियुक्तं ग्रहगोलतत्त्वम् ।।
तज्जस्तु तस्यैव कृपालवेन स्वज्येष्ठसद्बन्धुदिवाकराख्यात् ।
सांवत्सरार्याद् गुरुतः प्रलब्धशास्त्रावबोधो गणकार्यतुष्ट्यै ।।
दृग्गोलजक्षेत्रनवीनयुक्त्या पूर्वोक्तितः श्रीकमलाकराख्यः ।
समस्तसिद्धान्तसुगोलतत्त्वविवेकसंज्ञां किल सौरतत्त्वम् ।।
'खनागपञ्चेन्दु'शके व्यतीते सिद्धान्तमार्याभिमतं समग्रम् ।
भागीरथीसौम्यतटोपकण्ठवाराणसीस्थो रचायांबभूव ।।

(Kamalākara, *Si. Tattvaviveka*, *Upasaṃhāra Sn.*, 7-14)

Kamalākara's academic genealogy

There lived in the prosperous village of Gola, lying on the northern bank of river Godāvarī, a Maharashtra

[1] Cf. Parameśvara's own statement towards the beginning of his commentary on Govinda's *Muhūrtaratna*: "By revered Govinda was raised (or 'composed') the *Muhūrtaratna* from (churning or ransacking) the ocean of the science of *muhūrta*, with a compassionate mind (towards learners of the subject). And, on that work a short commentary is being composed by me, Parameśvara, who is the grandson of the pupil of Govinda."

[2] The above genealogy furnishes a lineage as noted below, with dates determined from other sources: Govinda (born A.D. 1237)→pupil: grandfather of Parameśvara (15th cent. A.D.)→ grandson Parameśvara (1360-1455)→son: Dāmodara (15th cent.) →pupil: Nīlakaṇṭha Somayāji (1443-1560)→pupil: Jyeṣṭhadeva (1500-1610)→pupil: Acyuta Piṣāraṭi (1550-1621).

[3] The above lineage might be set down as follows, with dates furnished from other sources: Acyuta Piṣāraṭi (1550-1621)→pupil: Tṛppāṇikkara Potuvāḷ (17th cent.)→pupil: Nāvāyikkuḷattu Āzhāti (17th cent.)→pupil: Pulimukhattu Potti (1686-1758)→Rāman Āśān (18th cent.)→son, pupil: Kṛṣṇan Āśān (Kṛṣṇadāsa) (1756-1812)

[4] The lineage with dates: Kṛṣṇan Āśān (1756-1812)→pupil: Dakṣiṇāmūrti Mūttatu (18th cent.)→pupil: Bālarāman Piḷḷa (19th cent.)→pupil: Vidvān (Karīndran) Ceruṇṇi Koyittampurān (1812-1846).

The lineage depicted above thus extends to more than 600 years, from 1237 to 1846.

brahmin named Rāma (c. A.D. 1450), an ornament to the Bhāradvāja gotra. (7)

Rāma had a son Akhilamānya Bhaṭṭācārya (?), saintly and learned in the śāstras. (8-a)

He had a son named Divākara, who constantly worshipped with pleasure (god Gaṇeśa) son of Śiva and who passed away in the holy city of Kāśi (Varanasi) studying the Vedānta philosophy. (8b-9a)

The great teacher Divākara had a renowned son named Kṛṣṇa Daivajña,[1] a great saint, teacher of brilliant pupils and a master of all sciences. (9b-10a)

His son was Nṛsiṁha, the greatest of the masters of the sphere and respected by all astronomers. By him were composed a *Bhāṣya*-commentary on the *Sūrya-siddhānta*, an excellent *Vārttika*-commentary on the *Siddhāntaśiromaṇi* (of Bhāskara II) and the twin works on astronomical rationale *Graha-* and *Gola-tattva*, for his own use and for the benefit of others. (10b-11)

His son was Kamalākara, who, at his father's benign instance, imbibed the science (of astronomy) from his elder astronomer-brother Divākara.[2] For the delectation of astronomers, he expounded astronomy fully according to the Ārya system through his work *Siddhānta-tattvaviveka*, correlating earlier theories with the new theories of spherics and geometry. This he (Kamalākara) did at the elapse of the Śaka year 1580 (A.D. 1658), while residing at Varanasi on the northern banks of river Ganges.[3] (12-14). (KVS)

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[1] Kṛṣṇa Daivajña had three brothers, all astronomers: Viṣṇu (fl. 1608), author of *Bṛhaccintāmaṇi-ṭīkā* and *Saurapakṣagaṇita*; Mallāri, author of *Grahalāghava-ṭīkā*; and Viśvanātha, author of several commentaries, including *Siddhāntarahasyodāharaṇa*, *Brahmatulyodāharaṇa*, *Keśavapaddhatyudāharaṇa* and *Karaṇaprakāśodāharaṇa*. The last had an astronomer son, Tryambaka, who has at least two works to his credit, *Viṣṇukaraṇa-ṭīkā* (1663) and *Paddhatikalpavallī* (1673).

[2] Divākara (b. 1606) was as eminent scholar, author of *Jātakamārga* with commentary, *Prauḍhamanoramā*, *Makaranda-vivaraṇa*, *Pātasāraṇī-ṭīkā* and *Varṣagaṇitabhūṣaṇa* with commentary.

[3] Kamalākara had also an younger brother Raṅganātha, author of several astronomical works like *Lohagolakhaṇḍana* and *Bhaṅgivibhaṅgīkaraṇa*.

It might be noted that the lineage of this family of astronomers has been traced in the above verses for about 250 years, from 1450 to 1700.

3. विश्वसृष्टिः – COSMOGONY

विश्वसृष्टेर्गहनता

3. 1. 1. नासदासीन्नो सदासीत् तदानीं
नासीद्रजो नो व्योमा परो यत् ।
किमावरीवः कुह कस्य शर्मन्
अम्भः किमासीद् गहनं गभीरम् ॥ १ ॥

न मृत्युरासीदमृतं न तर्हि
न रात्र्या अह्न आसीत् प्रकेतः ।
आनीदवातं स्वधया तदेकं
तस्माद्धान्यं न परः किंच नास ॥ २ ॥

तम आसीत्तमसा गूळ्ह-
मग्रेऽप्रकेतं सलिलं सर्वमा इदम् ।
तुच्छ्येनाभ्वपिहितं यदासीत्
तमसस्तन्महिनाजायतैकम् ॥ ३ ॥

कामस्तदग्रे समवर्तताधि
मनसो रेतः प्रथमं यदासीत् ।
सतो बन्धुमसति निरविन्दन्
हृदि प्रतीष्या कवयो मनीषा ॥ ४ ॥

तिरश्चीनो विततो रश्मिरेषाम्
अधः स्विदासीदुपरि स्विदासीत् ।
रेतोधा आसन् महिमान आसन्
स्वधा अवस्तात् प्रयतिः परस्तात् ॥ ५ ॥

को अद्धा वेद क इह प्रवोचत्
कुत आजाता कुत इयं विसृष्टिः ।
अर्वाग् देवा अस्य विसर्जनेना-
ऽथा को वेद यत आबभूव ॥ ६ ॥

इयं विसृष्टिर्यत आबभूव
यदि वा दधे यदि वा न ।
यो अस्याध्यक्षः परमे व्योमन्
सो अङ्ग वेद यदि वा न वेद ॥ ७ ॥

(*ṚV*, 10.129. 1-7)

The Universe: Mystery of its origin

Then was not non-existent nor existent: there was no realm of air, no sky beyond it.

What covered in, and where? and what gave shelter? Was water there, unfathomed depth of water? (1)

Death was not then, nor was there aught immortal: no sign was there, the day's and night's divider.

That One Thing, breathless, breathed by its own nature: apart from it was nothing whatsoever. (2)

Darkness there was: at first concealed in darkness this All was indiscriminated chaos.

All that existed then was void and formless: by the great power of Warmth was born that Unit. (3)

Thereafter rose Desire in the beginning, Desire, the primal seed and germ of Spirit.

Sages who searched with their heart's thought discovered the existent's kinship in the non-existent. (4)

Transversely was their severing line extended: what was above it then, and what below it?

There were begetters, there were mighty forces, free action here and energy up yonder. (5)

Who verily knows and who can here declare it, whence it was born and whence comes this creation?

The Gods are later than this world's production. Who knows then whence it first came into being? (6)

He, the first origin of this creation, whether he formed it all or did not form it.

Whose eye controls this world in highest heaven, he verily knows it, or perhaps he knows not. (7) (R.H.T. Griffith)

3. 2. 1. आसीत्तमः किलेदं तत्रापां तैजसेऽभवद् हैमे ।
स्वर्भूशकले ब्रह्मा विश्वकृदण्डेऽर्कशशिनयनः ॥ ६ ॥

(Varāha, *Bṛ Saṃ.*, 1.6)

Evolutionary ideas

It appears that originally, i.e. before creation, there was nothing but darkness everywhere. Then water came into existence. From that sprang a fiery golden egg consisting of the two parts of the shell, viz. heaven and earth. Out of this arose Brahman, Creator of the universe, with the luminaries (the Sun and the Moon) as his eyes. (6). (M.R. Bhat)

3. 2. 2. ऋतं च सत्यं चाभीद्धात् तपसोऽध्यजायत ।
ततो रात्र्यजायत ततः समुद्रो अर्णवः ॥ १ ॥
समुद्रादर्णवादधि संवत्सरो अजायत ।
अहोरात्राणि विदधद् विश्वस्य मिषतो वशी ॥ २ ॥
सूर्याचन्द्रमसौ धाता यथापूर्वमकल्पयत् ।
दिवं च पृथ्वीं चान्तरिक्षमथो स्वः ॥ ३ ॥

(*ṚV*, 10.190. 1-3)

Truth and truthfulness were born from intense penance. Hence was darkness born and thence the watery ocean. (1)

From the watery ocean was born the year, ordaining days and nights, the controller of every living moment. (2)

The Creator then created, in due order, the Sun, the Moon, the sky, the earth and the regions of the air and light. (3)

3. 2. 3. तस्माद्वा एतस्मादात्मन आकाशः सम्भूतः । आकाशाद्वायुः । वायोरग्निः । अग्नेरापः । अद्भ्यः पृथिवी । पृथिव्या ओषधयः । ओषधीभ्योऽन्नम् । अन्नात् पुरुषः ।।

(*Taitt. Upd., Brahmavallī*, 1)

Verily from this Soul space arose; from space, arose air; from air, fire; from fire, the waters; from the waters, the Earth; from the Earth, vegetation; from vegetation, food; and from food, man. (KVS)

3. 2. 4. देवानां नु वयं जाना प्र वोचाम विपन्यया ।
उक्थेषु शस्यमानेषु यः पश्यादुत्तरे युगे ।। १ ।।
ब्रह्मणस्पतिरेता सं कर्मार इवाधमत् ।
देवानां पूर्व्ये युगेऽसतः सदजायत ।। २ ।।
देवानां युगे प्रथमेऽसतः सदजायत ।
तदाशा अन्वजायन्त तदुत्तानपदस्परि ।। ३ ।।
भूर्जज्ञ उत्तानपदो भुव आशा अजायन्त ।। ४a ।।
यद्देवा यतयो यथा भुवनान्यपिन्वत ।
अत्रा समुद्र आ गूळ्हमा सूर्यमजभर्तन ।। ७ ।।
अष्टौ पुत्रासो अदितेर्ये जातास्तन्वस्परि ।
देवाँ उप प्रैत् सप्तभिः परा मार्ताण्डमास्यत् ।। ८ ।।
सप्तभिः पुत्रैरदितिरुप प्रैत् पूर्व्यं युगम् ।
प्रजायै मृत्यवे त्वत् पुनर्मार्ताण्डमाभरत् ।। ९ ।।

(*ṚV* 10.72 1-4, 7-9)

Let us with tuneful skill proclaim these generations of the Gods, that one may see them when these hymns are chanted in a future age. (1)

These Brahmaṇaspati produced with blast and smelting, like a smith. Existence in an earlier age of Gods, from Non-existence sprang. (2)

Existence, in the earliest age of Gods, from Non-existence sprang. Thereafter were the regions born. This sprang from the Productive Power. (3)

Earth sprang from the Productive Power. The regions from the Earth were born. (4a)

When, O ye Gods, like Yatis, ye caused all existing things to grow, then ye brought Sūrya forward who was lying hidden in the sea. (7)

Eight are the Sons of Aditi who from her body sprang to life. With seven she went to meet the Gods: she cast Mārtāṇḍa far away. (8)

So with her Seven Sons Aditi went forth to meet the earlier age. She brought Martāṇḍa (Sun) thitherward to spring to life and die again. (9). (R.H.T. Griffith)

3. 2. 5. चन्द्रमा मनसो जातश्चक्षोः सूर्यो अजायत ।
मुखादिन्द्रश्चाग्निश्च प्राणाद् वायुरजायत ।। १३ ।।

नाभ्या आसीदन्तरिक्षं शीर्ष्णो द्यौः समवर्तत ।
पद्भ्यां भूमिर्दिशः श्रोत्रात् तथा लोकानकल्पयन् ।। १४ ।।

(*ṚV*, 1.90 13-14)

The Moon was gendered from his mind, and from his eye the Sun had birth; Indra and Agni from his mouth were born, and Vāyu from his breath. (13)

Forth from his navel came mid-air; the sky was fashioned from his head; Earth from his feet, and from his ear the regions. Thus they formed the worlds. (14). (R.H.T. Griffith)

4. भावनाः सिद्धान्ताश्च – VIEWS AND CONCEPTS

भूसंस्थानम्

4. 1. 1. पञ्चमहाभूतमयस्तारागणपञ्जरे महीगोलः ।
खेऽयस्कान्तस्थो लोह इवावस्थितो वृत्तः ।। १ ।।
तरुनगनगरारामसरित्समुद्रादिभिश्चितः सर्वः ।
विबुधनिलयस्तु मेरुः तन्मध्येऽधःस्थिता दैत्याः ।। २ ।।
सलिलतटासन्नानामवाङमुखी दृश्यते यथा छाया ।
तद्वद् गतिरसुराणां मन्यन्ते तेऽप्यधो विबुधान् ।। ३ ।।
गगनमुपैति शिखिशिखाक्षिप्त-
मपि क्षितिमुपैति गुरु किञ्चित् ।
यद्वदिह मानवाना-
मसुराणां तद्वदेवाधः ।। ४ ।।

(Varāha, *PS*, 13. 1-4)

Nature and situation of the Earth

The spherical Earth, which is constituted of the five elements, stands poised in the region of space as if it is an iron ball held in position in a cage of magnets. (1)

The whole earth-surface is spotted by trees, mountains, cities, rivers, oceans, etc. The Meru mountain, (forming the North pole), is the abode of the devas (gods). The Asuras (demons) are down below (i.e. at the South pole.) (2)

Just as the reflection of the objects on the bund of a water source is upside down, so the asuras are, (with respect to the devas.) The asuras too consider the devas to be upside down. (3)

Just as the flame of the fire, observed by men here, flares upwards, and anything thrown up falls down towards the earth, the same upward flaring of the flame, and the downward falling of a heavy object is experienced by the asuras (at the anti-podal region.) (4). (TSK)

4. 1. 2. गगनमरुदग्निजलमृण्मयो महाभूतगुणयुतः स्वस्थः ।
कक्षाभिरावृतोऽयं भपञ्जरान्तश्च भूगोलः ।। १ ।।
मध्येऽयस्कान्तानां यथा स्थितोऽयोगुडः खमध्यस्थः ।
तद्वदनाधारोऽपि हि सर्वाधारो महीगोलः ।। २ ।।
लङ्काप्राग् यमकोटिः पश्चाद्रोमकमधश्च सिद्धपुरम् ।
मेरुः स्थित उत्तरतो दक्षिणतो दैत्यनिलयः स्यात् ।। ३ ।।
एते जलस्थलस्था मेरुः स्थलगोऽसुरालयो जलगः ।
कुपरिधिपादान्तरगा मन्यन्ते तिर्यगन्योऽन्यम् ।। ४ ।।
भूपरिधिदलान्तरगाः के नृच्छाये यथा तथोर्ध्वमधः ।
सर्वेषामुपरि नभस्तदधो भूरुपरि ते भूमेः ।। ५ ।।
तरुनगनगरसुरनरैरयं केसरैरिव समन्तात् ।
गोलः कादम्बो मधुकरीभिरिव सर्वतः प्रचितः ।। ६ ।।

(Lalla, *SiDhVr.*, 17.1-6)

This sphere of Earth, made of ākāśa, air, fire, water and clay and thus having all the properties of the five elements, surrounded by the orbits (of the Moon, etc.), and extending up to the sphere of stars, remains in (the centre of) space. (1)

Just as a ball of iron, when placed amidst pieces of magnets, remains suspended in space, in the same manner this globe of Earth remains in space unsupported, while itself remaining the abode of all. (2)

Yamakoṭi is to the east of Laṅkā (which is in the middle of the Earth), and Romaka is to the west. Siddhapura is beneath Laṅkā (just opposite); the Meru (mountain) is to the north and the abode of the demons is to the south. (3)

These (four cities) are on islands. Meru is on the land and the abode of the demons (in the south) is surrounded by water. These six places are believed to be situated transversely at a distance of one fourth of the Earth's circumference, (that is 90°), each from the next one. (4)

Those who are at a distance of half the Earth's circumference from each other are antipodes, just as a man (standing on the bank of a river) and his reflection in the water. The sky is above all. This (globe of Earth) is beneath it. The inhabitants are on the surface of the Earth. (5)

This globe of the Earth is covered on all sides by trees, mountains, cities, gods, demons and human beings, just as a (ball-like) *kadamba* flower is beset with anthers and bees (all around). (6). (BC)

भूतलं भूयशः जलान्तर्गतम्

4. 1. 3. भूगोलो जलमग्नोऽस्ति जलाद् बहिरपि स्थितः ।
तत्राधिको जलान्तःस्थो बहिः स्वल्पोऽस्ति . . . ।।

(Kamalākara, *Si. Tattvaviveka*, Madhyama, 124)

Earth's crust mostly under water

The surface of the Earth sphere is covered with water, with portions also above water. However, the greater part of it is under water, and only a smaller part of it is above. (124). (KVS)

भूमेः समतलभ्रमः

4. 1. 4. समो यतः स्यात् परिधेः शतांशः
पृथ्वी च पृथ्वी नितरां तनीयान् ।
नरश्च तत्पृष्ठगतस्य कृत्स्ना
समेव तस्य प्रतिभात्यतः सा ।। १३ ।।
(Bhāskara II, *SiŚi.*, 2.ii. Bhuvana. 13)

Illusion of the flatness of the Earth

An infinitesimal part of the surface of the huge Earth would, indeed, appear flat. And, to a man standing on that (infinitesimal part) the whole Earth would give the illusion of flatness. (KVS)

भूकम्पहेतुः

4. 1. 5. पाषाणैः कठिना भूमिः यत्र तत्र कुतो बलात् ।
बाष्पनिस्सरणात् कम्पः शब्दोऽपि सततं भुवि ।।
(Kamalākara, *Si. Tattvaviveka*, Madhyama, 206b-7a)

Cause of Earthquakes

The Earth's crust is hard and rocky. Where, however, a fissure occurs due to lack of strength, gases emerge forcibly causing the Earth to quake, when there would also be constant terrific noise. (206b-207a). (KVS)

भूभ्रमणम्

4. 2. 1. सदैव नित्यं प्रवहेण वायुना
निरक्षदेशोपरिगो भपञ्जरः ।
स्वपश्चिमाशाभिमुखेऽपि नीयते-
ऽसुरामराणामपसव्यसव्यगः ।। ३ ।।
(Lalla, *SiDhVr.*, 1.8.3)

Motion of the Earth

The celestial sphere, at the Earth's equator, is constantly carried towards the west by the *Pravaha* wind. To the gods (at the north pole) it appears to move (from the left) to the right and to the demons (at the south pole from the right) to the left. (3). (BC)

4. 2. 2. आवहः प्रवह उद्वहस्तथा संवहः सुपरिपूर्वकौ वहौ ।
सप्तमस्तु पवनः परावहः कीर्तितः कुमरुदावहोऽपरैः ।।१।।
'शराद्रिरामानल'योजनानि
कुवायुकक्षा परितः पृथिव्याम् ।
'समुद्रशैलाम्बरशीतभास'-
स्तदीयविष्कम्भमुशन्ति सन्तः ।। २ ।।
(Lalla, *ŚiDhVr.*, 18.1-2)

The seven winds

The seven winds (surrounding the Earth) are Āvaha, Pravaha, Udvaha, Saṁvaha, Suvaha, Parivaha and Parāvaha. Āvaha is known as *Ku-marut* (atmosphere) according to others. (1)

The circumference of the orbit of the Āvaha surrounding the Earth is 3375 *yojanas*. The wise maintain that its diameter is 1074 *yojanas*. (2)

भुवः ज्यौतिषिको स्थितिः

4. 3. 1. निरक्षदेशोपरिगौ ध्रुवावुभौ
स्वदक्षिणोदक्क्षितिजप्रसक्तौ ।
सदात्र पश्यन्ति तदुत्तरं सुराः
स्वमूर्ध्वगं दक्षिणमिन्द्रशत्रवः ।। ४ ।।
सदैव दैत्यैस्तनयैस्तथादिते-
र्भचक्रमुर्वीजगतं प्रदृश्यते ।
न दृश्यतेऽपक्रमवृत्तदक्षिणं
दलं सुरैर्नान्यदपीतरैः क्वचित् ।। ५ ।।
सुरासुराणां विषयाद् यतो यतो
यथा यथा गच्छति यश्च कश्चन ।
स तत्र तत्रोन्नतमृक्षमण्डलं
तथा तथा पश्यति खान्नतं ध्रुवम् ।। ६ ।।
भवेत् खमध्याद् ध्रुवकस्य या नति-
र्भपञ्जरस्योन्नतिरेव साथवा ।
स्वलम्बभागाः 'स्वषडंशगो'हता
नरापसारः फलयोजनैर्भवेत् ।। ७ ।।
समुन्नतिर्या क्षितिजाद् ध्रुवस्य सा
भपञ्जरस्य प्रणतिर्नभस्तलात् ।
फलांशका वा गुणिताश्च पूर्वव-
न्निरक्षदेशात् कथयन्ति योजनैः ।। ८ ।।
इनस्य कक्षा गुणिता नतांशकै-
र्नतिः खमध्यात् 'खषडग्नि'हृद् भवेत् ।
यदा स्वकक्षा चरणेन सम्मिता
वि'तत्त्वबाणेन' कुजे तदा रविः ।। ९ ।।
तुलाद्यजाद्योः स्थितमर्कमण्डलं
समं समीक्षन्ति परिभ्रमं कुजे ।
त्रिभागयुक्तेऽह्निगतेऽसुरामराः
क्रमाद्धि मुक्तं क्षितिजं ततस्ततः ।। १० ।।
(Lalla, *ŚiDhVr.*, 18.4-10)

Astronomical position of the Earth

At the equator men always see the north and south celestial poles coinciding with the north and south points respectively of the horizon. The gods see the north celestial pole on their zenith while the demons see the south celestial pole on their zenith. (4)

To the gods and the demons the equinoctial appears to coincide with their horizon. The gods never see the southern half of the ecliptic and the demons the northern half. (5)

Whosoever proceeds from the abode of the gods or of the demons towards the equator, would observe the celestial sphere gradually rising and the celestial pole more and more depressed from his zenith, the depression of the pole from the zenith being the same as the elevation of the celestial sphere.

The number of degrees in the colatitude of a place, multiplied by $9\frac{1}{8}$, gives the corresponding length in *yojanas*. (6-7)

The elevation of the pole above the horizon is the same as the depression of the celestial sphere from the zenith. This latitude in degrees (elevation of the pole) multiplied as before by $9\frac{1}{8}$, gives the latitude of the place in *yojanas*, measured from the equator. (8)

The zenith distance of the Sun multiplied by the (number of *yojanas*) in its orbit and divided by 360, gives the zenith distance (in *yojanas*).

(When the zenith distance is) $\frac{1}{4}$ of the orbit *minus* 525, the Sun is on the horizon. (9)

When the Sun is at the first points of Aries and Libra, the gods and the demons see it simultaneously on their horizons. Then after $1\frac{3}{4}$ days they gradually see it leaving the horizon. (10).[1] (BC)

4. 3. 2. दृश्ये चक्रस्यार्धे त्रयः खमध्यात्तु राशयस्तेषाम् ।
नवतिस्तानि च खण्डान्युदयात् परिकल्पनीयानि ।। १४ ।।
एकैकोंऽशे नवभिर्नवभागोनैश्च योजनैर्याति ।
समदक्षिणोत्तराणां प्रत्यक्षं खेऽप्ययं मध्यात् ।। १५ ।।
एवं च नवत्यंशैरष्टौ दृष्टानि योजनशतानि ।
स हि तत्प्रमाणदेशे मध्याह्ने द्रष्टुरुदयो यः ।। १६ ।।
उज्जयिनी लङ्कायाः सन्निहिता योत्तरेण समसूत्रे ।
तन्मध्याह्नो युगपत् विषमो दिवसो विषुवतोऽन्यः ।। १७ ।।
(Varāha, *PS*, 13. 14-17)

(For the devas, i.e. at the North pole, the zenith is the north celestial pole, any visible semi-great-circle passing through the zenith being a meridian on the earth.) It is 3 Signs' distance, i.e. 90°, from the zenith to the horizon. This is to be divided into 90 parts, so that each division is a degree. (14)

These degree-divisions in the sky are zeniths of corresponding degrees of latitudes on the Earth along any meridian, and the distance between each degree of latitude is 8 8/9 *yojanas*. (15)

Thus for 90°, there are 800 *yojanas*, as calculated. When an observer on any meridian sees the Sun rising, it is midday on the meridian 800 *yojanas* (east) of him. (16)

The midday of Laṅkā is the same as that at Ujjain, which is north of Laṅkā on the same longitude. But their day-time durations are different, except when the Sun is on the equator. (17). (TSK)

4. 3. 3. प्रतिविषयमुदक् तुङ्गो हरिजाद् यावद् ध्रुवः खमध्यात्तु ।
दिनकृदपि नमति विषुवति दक्षिणतस्तावदेवांशैः ।। २० ।।

[1] For notes see *ŚiDhVṛ*: *BC*, II, pp. 252-53.

त्रिशतीं त्रिसप्ततियुतो यात्वोदक् योजनत्रिभागं च ।
उज्जयिनीतो विघटति पर्याप्तोऽयं भगणगोलः ।। २१ ।।
षष्टिर्नाड्यस्तस्मिन् सकृदुदितो दृश्यते दिवसनाथः ।
परतः परतो बहुतरमाषण्मासादिति सुमेरौ ।। २२ ।।
योजनपञ्चनवांशांस्त्र्यधिकांश्च चतुःशतीमुदगवन्त्याः ।
गत्वा न धनुर्मकरौ कदाचिदपि दर्शनं व्रजतः ।। २३ ।।
तस्मादेव स्थानाद् द्व्यशीतियुक्तां चतुःशतीं साग्राम् ।
नोदयमुपयान्त्यलिमृगघटचापधराः कदाचिदपि ।। २४ ।।
षडशीतिं पञ्चशतीं त्र्यंशोनं योजनं च तत एव ।
गत्वान्त्यं चक्रार्धं नोदेत्याद्यं न यात्यस्तम् ।। २५ ।।
लङ्कास्था भूलग्नां नभसो मध्यस्थितां च मेरुगताः ।
ध्रुवतारामीक्षन्ते तदन्तरालेऽन्तरालोपगताः ।। २६ ।।
सकृदुदितः षण्मासान् दृश्योऽर्को मेरुपृष्ठसंस्थानाम् ।
मेषादिषु षट्सु चरन् परतो दृश्यः स दैत्यानाम् ।। २७ ।।
मेषस्तेषां नित्यं लग्नं त्र्यंशश्च भूमिपुत्रस्य ।
त्रिंशद्भागनवांशद्वादशभागाश्च तस्यैव ।। २८ ।।
विषुवल्लेखाऽधस्ताल्लङ्का तस्यां समो भगणगोलः ।
त्रिंशन्नाड्यो दिवसः त्रिंशच्च तस्यां सदा च निशा ।।
(Varāha, *PS*, 13.20-29)

At any latitude, the equatorial Sun is bent so many degrees south at midday, as the north pole is raised from the north point of the horizon. (20)

Going north from Ujjain, 373 1/3 *yojanas*, the stellar sphere, (marked by the 27 asterisms of the ecliptic rising in order), becomes discontinuous, (i.e. the order in the rising is disrupted.)[1] (21)

At that latitude, the Sun can be visible even throughout a day. North of this place, the Sun may not set more than one day, until at the North pole it will not set for six months at a stretch. (22)

At a distance greater than 403 5/9 *yojanas* north of Ujjain, the signs Dhanus and Makara can never be visible.[2] (23)

At latitudes north of Ujjain greater than 482 *yojanas* and a fraction, the signs, Vṛścika, Dhanus, Makara, and Kumbha will never be visible.[3] (24)

[1] At 66° North latitude, which is 42° = (66° − 24°) north of Ujjain, peculiarities occur in the rising of the signs of the ecliptic, duration of day-time etc. 42°=$373\frac{1}{3}$ *yojanas*.

[2] The Dhanus and Makara segments of the ecliptic have a south declination greater then 20° 36′. In the north latitudes 90° − 20° 36′ (= 69° 24′) and beyond, the zenith distance of these signs becomes greater than 90°, and so they are not visible in those latitudes. 69°24′ is 45°24′ north of Ujjain, i.e. 45°24′ × 8 8/9=403 8/9 *yojanas* north.)

[3] These four signs have a declination greater than 11°44′ south. Therefore, at latitudes 90°−11°44′=78°16′ North and more, these signs cannot be seen, since their zenith distance is greater then 90°. 78°16′ is 54°16′ north of Ujjain=54°16′ × 8 8/9=482 10/27 *yojanas*.)

586 2/3 *yojanas* north of Ujjain, i.e. at the North pole, the second half of the ecliptic, i.e. the signs Tulā to Mīna, cannot be seen.[1] (25)

People on the equator see the north polar star on the horizon. At the north pole they observe it at the zenith. In between, people observe it at latitudes 0° to 90°. (26)

For the people on Meru, i.e. at the North pole, the Sun is visible at a stretch when it is in the six Signs, Meṣa to Kanyā. When it is in the next six Signs, it is visible to the demons at the south pole, at a stretch. (27)

For them, the first point of Meṣa is the *Lagna* or Orient ecliptic point, permanently. Mars is the Lord of the *Drekkāṇa*, *Navāṁśa*, *Dvādaśāṁśa* and *Triṁśāṁśa lagnas* permanently. (28)

Laṅkā is beneath the celestial equator, i.e. the celestial equator itself is the prime vertical at Laṅkā. There the stellar sphere is equally divided (into the northern half with the North pole at its centre, and the southern half with the South pole at its centre). There the day and night are always 30 *nāḍīs* each. (29). (TSK)

मेरुः

4. 4. 1. मेरुर्योजनमात्रः प्रभाकरो हिमवता परिक्षिप्तः ।
नन्दनवनस्य मध्ये रत्नमयः सर्वतो वृत्तः ।। ११ ।।
स्वर्मेरू स्थलमध्ये नरको बडवामुखं च जलमध्ये ।
अमरमरा मन्यन्ते परस्परमधःस्थितान् नियतम् ।। १२ ।।
(Āryabhaṭa I, *ABh.*, 4.11-12)

Meru mountain

The Meru (mountain) is exactly one *yojana* (in height). It is light-producing, surrounded by the Himavat mountain, situated in the middle of the Nandana forest, made of jewels, and cylindrical in shape. (11)

The heaven and the Meru mountain are at the centre of the land (i.e., at the North pole); the hell and the Baḍavāmukha are at the centre of the water (*i.e.*, at the South pole). The gods (residing at the Meru mountain) and the demons (residing at the Baḍavāmukha) consider themselves positively and permanently below each other. (12). (KSS)

लङ्का, उज्जयिनी, मध्यरेखा च

4. 5. 1. उदयो यो लङ्कायां सोऽस्तमयः सवितुरेव सिद्धपुरे ।
मध्याह्नो यवकोटयां रोमकविषयेऽर्धरात्रं स्यात् ।।
(Āryabhaṭa I, *ABh.*, 4.13)

[1] Being situated south of the celestial equator, the zenith distance of these signs from the North pole is greater than 90°, and therefore they are not visible at the North pole. The distance of the North pole from Ujjain is 90°—24°=66°=586 2/3 *yojanas*.

Laṅkā, Ujjayini and Prime meridian

When it is sunrise at Laṅkā, it is sunset at Siddhapura, midday at Yavakoṭi, and midnight at Romaka.[1] (13). (KSS)

मध्यरेखा

4. 5. 2. लङ्कावात्स्यपुरावन्तीस्थानेश्वरसुरालयान् ।
अवगाह्य स्थिता रेखा देशान्तरविधायिनी ।। २३ ।।
(Bhāskara I, *LBh.*, 1.23)

Indian prime meridian

The line which passes through Laṅkā, Vātsyapura, Avantī, Sthāneśvara, and Meru (the abode of the gods) is the prime meridian. (23). (KSS)

4. 5. 3. स्थलजलमध्याल्लङ्का भूकक्ष्याया भवेच्चतुर्भागे ।
उज्जयिनी लङ्कायाः तच्चतुरंशे समोत्तरतः ।।
(Āryabhaṭa I, *ABh.*, 4.14)

From the centres of the land and the water, at a distance of one-quarter of the Earth's circumference, lies Laṅkā; and from Laṅkā, at a distance of one-fourth thereof, exactly northwards, lies Ujjayini. (14). (KSS)

4. 5. 4. रेखा विषयेषु खमध्यमेति युगपद् ग्रहः समस्तेषु ।
भूपरिधेरष्टयंशेऽवन्ती स्यात् सौम्यदिग्भागे ।। ४० ।।
(Lalla, *ŚiDhVṛ.*, 19.40)

A planet reaches simultaneoulsy the zenith at all the regions on the prime-meridian.

(The city of) *Avanti* (or Ujjain) is at a distance of 1/16th part of the earth's circumference (from Laṅkā) to its north. (40). (BC)

4. 5. 5. लङ्कातः खरनगरं सितोरुगेहं
पाणाटो मिसितपुरी तथा तपर्णी ।
उत्तुङ्गस्सितवरनामधेयशैलो
लक्ष्मीवत्पुरमपि वात्स्यगुल्मसंज्ञम् ।। १ ।।

विख्याता वननगरी तथा ह्यवन्ती
स्थानेशो मुदितजनस्तथा च मेरुः ।
अध्वाख्यः करणविधिस्तु मध्यमाना-
मेतेषु प्रतिवसतां न विद्यते सः ।। २ ।।
(Bhāskara I, *MBh.*, 2.1-2)

From Laṅkā (towards the north, we have the following places on the prime meridian): Kharanagara, Sitorugeha, Pāṇāṭa, Misitapurī, Taparṇī, the lofty mountain called Sitavara, the wealthy town called Vātsyagulma, the well

[1] Laṅkā is supposed to be at the place where the meridian of Ujjayinī (long. 75°.43 E., lat. 23°.09 N) intersects the equator, Yavakoṭi 90° to the east of Laṅkā, Romaka 90° to the west of Laṅkā, and Siddhapura diametrically opposite to Laṅkā.

known Vananagarī, Avantī, Sthāneśa, and then Meru, which is inhabited by happy people. For those who reside in these places, the correction for the longitude (of the local place) does not exist. (1-2). (KSS)

4. 5. 6. लङ्का कुमारी तु ततस्तु काञ्ची
मानाटमश्वेतपुरी त्वथोदक् ।
श्वेतोऽचलोऽस्मादपि वात्स्यगुल्मं
पूः स्यादवन्ती त्वनु गर्गराटम् ॥ १ ॥
आश्रमपत्तनमालवनगरे पट्टशिवमेव रोहितकम् ।
स्थाण्वीश्वरस्तु हिमवान् मेरुर्लेखाध्वकर्म नास्त्येषाम् ॥ २ ॥
(Vaṭeśvara: *VSi*, 1.8.1-2)

Laṅkā, (then northwards), Kumārī, then Kāñcī Mānāṭa, Aśvetapuri, then northwards, the Śveta mountain, thereafter Vātsyagulma, the city of Avantī, then Gargarāṭa, Āśramapattana, Mālavanagara, Paṭṭa-śiva, Rohitaka, Sthāṇvīśvara, the Himalaya mountain and lastly Meru—(these are situated on the prime meridian). For these places correction for longitude is not needed. (1-2). (KSS)

स्वदेशस्थानम्

4. 5. 7. अतीत्य गणितानीतं यदा स्यातामुपप्लुती ।
पूर्वेण समरेखाया द्रष्टा स्यात् पश्चिमेऽन्यथा ॥ २९ ॥
(Deva, *KR*, 1.29)

Local place relative to prime meridian

If the (lunar and solar) eclipses occur after the calculated time, then the observer is to the east of the prime meridian; otherwise, to the west.[1] (29). (KSS)

दिशः

4. 6. 1. नुः पदमधस्तादाशामूर्ध्वोपरि तस्य चोर्ध्वकाष्ठा स्यात् ।
यत्रोदेति प्राची यत्रास्तमितोऽपरा सा दिक् ॥ ३३ ॥
सौम्यो ध्रुवश्च यस्मिन् शेषा दिग् दक्षिणा समुद्दिष्टा ।
द्रष्टा यस्मिन् देशे तत्राशानां भवति सम्पातः ॥ ३४ ॥
(Lalla, *ŚiDhVṛ*., 19. 33-34)

Cardinal directions

The feet of a man point downwards and his head upwards. These directions will, therefore, be up and down. The direction in which the Sun rises is the east and that in which it sets is the west. (33)

The celestial pole points to the north; and the direction opposite to it is known as the south. *All these directions meet at the point where the observer is.* (34). (BC)

भुवः केन्द्राकर्षणशक्तिः

4. 7. 1. भवनभावतुलोपतुलास्वलं
स्थितवती च यथा गृहगोधिका ।
समभिधावति नूनमनाकुला
कुवलयस्य तथैव जनोऽप्यधः ॥ ७ ॥
शिखिशिखा गगनं गुरुमेदिनीं
व्रजति यद्वदिहास्मदवस्थितौ ।
तलगतामितरेष्वपि तत्तथा
न तलमस्ति भुवः क्व पतत्वसौ ॥ ८ ॥
(Lalla, *ŚiDhVṛ*., 17.7-8)

Gravitational pull of the Earth

Just as a house-lizard, resting underneath the cross beams of the ceiling of a dwelling, runs forward without hesitation, so do people on the bottom side of the sphere of the earth. (7)

In our daily life (we see) that the flame of fire goes towards the sky and a heavy weight falls towards the earth. In the same manner, everything that has a surface to reach, makes for it. The Earth, however, has no such surface. Where can it fall? (8). (BC)

निराधारो महीगोलः

4. 8. 1. वृत्तभपञ्जरमध्ये कक्ष्यापरिवेष्टितः खमध्यगतः ।
मृज्जलशिखिवायुमयो भगोलः सर्वतो वृत्तः ॥ ६ ॥
यद्वत् कदम्बपुष्पग्रन्थिः प्रचितः समन्ततः कुसुमैः ।
तद्वद्धि सर्वसत्त्वैर्जलजैः स्थलजैश्च भूगोलः ॥ ७ ॥
ब्रह्मदिवसेन भूमेरुपरिष्टाद् योजनं भवति वृद्धिः ।
दिनतुल्ययैकरात्र्या मृदुपचितायास्तदिह हानिः ॥ ८ ॥
(Āryabhaṭa I, *ABh*., 4.6-8)

Supportless Earth

The globe of the Earth stands (supportless) in space at the centre of the circular frame of the asterisms (*i.e.*, at the centre of the *Bhagola*) surrounded by the orbits (of the planets); it is made of water, earth, fire and air and is spherical (*lit.* circular on all sides). (6)

Just as the bulb of a *kadamba* flower is covered all around by blossoms, so is the globe of Earth surrounded by all creatures, terrestrial as well as aquatic. (7)

During a day of Brahmā, the size of the Earth increases externally by one *yojana*; and during a night of Brahmā,[1] which is as long as a day, this growth of the earth is destroyed. (8). (KSS)

4. 8. 2. पृच्छामि त्वा परमन्तं पृथिव्याः ।
पृच्छामि यत्र भुवनस्य नाभिः ॥ ३४ ॥
इयं वेदिः परो अन्तः पृथिव्या ।
अयं यज्ञो भुवनस्य नाभिः ॥ ३५ ॥
(*RV*, 1.164.34-35)

I ask of thee, where the ultimate end of the Earth is.
I ask of thee, where the centre of the Earth is.

[1] The calculated time (as stated above) is the local time for the place lying at the intersection of the prime meridian and the local circle of latitude.

[1] For the measures of the day and night of Brahmā, see below 7.7.1-2.

This altar (and, for that matter, any point on the surface of the Earth) is the ultimate end of the Earth; this sacrifice (performed on the altar) is (again) the centre of the Earth (since the Earth is spherical). (4-35) (KVS)

4. 8. 3. आप्रा रजांसि दिव्यानि पार्थिवा
श्लोकं देव: कृणुते स्वाय धर्मणे ।
प्र बाहू अस्राक् सविता सवीमनि
निवेशयन् प्रसुवन् नक्तुभिर्जगत् ।।

(*RV*, 4.53.3)

The brilliant Sun has filled the regions of the heaven with light. He enlivens the hymn for his own stregthening. He has stretched out his rays, putting the world to sleep and awakening it by turns. (3)

भुव: स्वयंभ्रमणम्

4. 9. 1. भपञ्जर: स्थिर: । भूरेवावृत्यावृत्य प्रतिदैवसिकौ उदयास्तमयौ सम्पादयति नक्षत्रग्रहाणाम् ।

(Pṛthūdaka-svāmi, Com. on *Brāhmasphuṭa Si.*)

Rotation of the Earth

The sphere of the stars is fixed. It is only the Earth that is regularly rotating once a day, causing the rising and setting of the stars and the planets. (KVS)

4. 9. 2. भूमि: प्राङ्मुखी भ्रमति ।

(*Makkibhaṭṭa*, Com. on *SiŚe.*, 1.39)

The Earth rotates (from west) to east. (KVS)

4. 9. 3. आचार्यार्यभटेनापि भूभ्रमणमभ्युपगतम् । 'प्राणेनैति कलां भू:' (*ABh.* 1. 6) . . . लोकभयाद् भास्करादिभिरन्यथा मत्वेयमार्या व्याख्याता ।

(Pṛthūdaka-svāmi, Com on *Brāhmasphuṭa-Si.*, Gola., 30)

The Earth's rotation had been accepted by Āryabhaṭa also, vide his words 'The Earth rotates through (an angle of) one second per one *prāṇa* (of time)' (ĀBh. 1.6). On account of (possible) adverse criticism by people, Bhāskara I and others explained the verse to give it a different meaning. (KVS)

4. 9. 4. भूभ्रमणमप्याचार्येणाभ्युपगतम्, नक्षत्राणां स्थिरता च । विद्यते हि पाठान्तरं 'प्राणेनैति कलां भू:' (आर्यभटीयम्, 1.6) इति ।

(Udayadivākara, Com. on *LBh.*, 1. 3-33)

Rotation of the Earth also has been accepted by Ācārya (Āryabhaṭa). Note that there is a variant reading (for *ĀBh.* 1. 6) as 'The Earth rotates through (an angle of) one second per one *prāṇa* (of time).' (KVS)

4. 9. 5. अनुलोमगतिर्नौस्थ: पश्यत्यचलं विलोमगं यद्वत् ।
अचलानि भानि तद्वत् समपश्चिमगानि लङ्कायाम् ।। ९ ।।
उदयास्तमयनिमित्तं नित्यं प्रवहेण वायुना क्षिप्त: ।
लङ्कासमपश्चिमगो भपञ्जर: सग्रहो भ्रमति ।। १० ।।

(Āryabhaṭa I, *ABh.*, 4.9-10)

Just as a man in a boat moving forward sees the stationary objects (on either side of the river) as moving backward, so are the stationary stars seen by people at Laṅkā (on the equator), as moving exactly towards the west. (9)

(It so appears as if) the entire structure of the asterisms together with the planets were moving exactly towards the west of Laṅkā, being constantly driven by the provector wind, to cause their rising and setting.[1] (10) (KSS)

4. 9. 6. 'पृथिवी प्रतिष्ठा' इति श्रुत्यन्तरात् आर्यभटाद्यभिमतभूभ्रमणादिवादानां श्रुतिन्यायविरोधेन हेयत्वात् ।

(Appaya Dīkṣita, *Vedāntakalpataru-parimala*, p. 201)

(On the authority of) the Vedic statement, 'The Earth is stable', the views of Āryabhaṭa and others to the effect that the Earth is rotating has to be discarded as being against scripture and logic.[2] (KVS)

भूमानादि:

4. 10. 1. यदिन्विन्द्र पृथिवी दशभुजि-
रहानि विश्वा ततनन्त कृष्टय: ।
अत्राह ते मघवन् विश्रुतं महो
द्यामनु शवसा बर्हणा भुवम् ।।

(*RV*, 1.52.11)

Dimensions of the Earth etc.

Oh (God) Indra, were this Earth to magnify itself tenfold (i.e., infinitely) and men who live on it multiplied day by day, then and then alone will the lauded might and glory of yours be as vast as the heavens. (11). (KVS)

4. 10. 2. योजनशतानि भूमे: परिमाणं षोडश द्विगुणितानि ।
तापयति मेरुमध्यात् विषुवस्थेऽर्क: क्षितिं चैवम् ।। १८ ।।
षडशीतिं पञ्चशतीं त्रिभागहीनं च योजनं गत्वा ।
क्षितिमध्यमुदगवन्त्या: लङ्काया योजनाष्टशतीम् ।।१९।।

(Varāha, *PS*, 13.18-19)

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[1] The theory of the Earth's rotation underlying the above passage was against the view generally held by the people and was severely criticised by Varāhamihira (*d.* A.D. 587) and Brahmagupta (628 A.D.) The followers of Āryabhaṭa I, who were unable to refute the criticism against the theory, fell in line with Varāhamihira and others of his ilk and have misinterpreted the above verses as conveying the contrary sense.

Pṛthūdaka (860 A.D.) in his commentary on the *Brāhma-sphuṭa-siddhānta*, supports Āryabhaṭa I's theory of the Earth's rotation. According to him, the followers of Āryabhaṭa I, who misinterpreted Āryabhaṭa I, were afraid of the public opinion which was against the motion of the Earth.

[2] For a disquisition on the point, see P.K. Gode, 'Appaya Dīkṣita's criticism of Āryabhaṭa's theory of the diurnal motion of the Earth (Bhūbhramaṇavāda), *Annals of the BORI*, 19 (1938) 93-95.

The circumference of the earth is 3200 *yojanas*. When situated on the equator the Sun is visible from pole to pole at all latitudes, (making the day and night equal). (18)

The middle of the Earth, (the North pole is meant here,) is north of Ujjain by 586 2/3 *yojanas*. It is north of Laṅkā by 800 *yojanas*.[1] (19). (TSK)

भूव्यासपरिधी

4. 10. 3. योजनानि शतान्यष्टौ भूकर्णो द्विगुणानि तु ।
तद्वर्गतो दशगुणात्पदं भूपरिधिर्भवेत् ।। ५८ ।।
(*SūSi.*, 1.58)

Earth: Diameter and Circumference

Twice eight hundred *yojanas* is the diameter of the Earth: the square root of ten times the square of that is the Earth's circumference. (58). (E. Burgess)

4. 10. 4. 'खखखशरा' लम्बहता व्यासार्धहृताः स्फुटकुपरिणाहः ।
(Brahmagupta, *KK*, 2.1.6a)

Multiply 5000 by the *jyā* of the colatitude of the place and divide the product by the *trijyā*. The result is the correct circumference of the Earth at that place. (6a) (BC)

लङ्कातः उज्जयिनी

4. 10. 5: 'नवनिधिदन्ता' लङ्कावन्त्यो-
र्मध्ये कुपरिधिरष्टचाप्ता ।। ३३ab ।।
(Deva, *KR*, 1.33 a-b)

Ujjayinī from Laṅkā

3299 (*yojanas*) is the Earth's circumference; this divided by 16 gives the distance between *Laṅkā* and *Avantī* (or *Ujjayinī*.) (33. a-b) (KSS)

4. 10. 6. प्रोक्तो योजनसंख्यया कुपरिधिः 'सप्ताङ्गनन्दाब्धयस्'
तद्व्यासः 'कुभुजङ्गसायकभुवः' 'सिद्धांशकेनाधिकाः' ।
पृष्ठक्षेत्रफलं तथा 'युगगुणत्रिंशच्छराष्टाद्रयो'
भूमेः कन्दुकजालवत् कुपरिधिव्यासाहतेः प्रस्फुटम् ।।
(Bhāskara II, *SiŚi.*, 2.2.52)

The circumference of the Earth sphere is said to be 4967 *yojanas*. Its diameter is 1581 1/24 *yojanas*. The surface area thereof is 7,85,034 square *yojanas*, for it is obvious that, as in the case of a spherical ball, the product of the Earth's circumference and its diameter gives its surface area. (52). (KVS)

4. 10. 7. प्रोक्तो योजनसंख्यया कुपरिधिः 'सप्ताङ्गनन्दाब्धय'-
स्तद्व्यासः 'कुभुजङ्गसायकभुवो'ऽथ प्रोच्यते योजनम् ।
याम्योदक्पुरयोः पलान्तरहतं भूवेष्टनं 'भांश'हृत्
तद्भक्तस्य पुरान्तराध्वन इह ज्ञेयं समं योजनम् ।।१।।
लम्बज्यागुणितो भवेत् कुपरिधिः स्पष्टस्त्रिभज्याहृतो
यद्वा द्वादशसंगुणः स विषुवत्कर्णेन भक्तः स्फुटः ।। २ ।।
(Bhāskara II, *SiŚi.*, 1.1.7.1)

The circumference of the Earth's globe is 4967 *yojanas*; its diameter 1581. (A *yojana* is equal to $\frac{d \times 360}{c \times \delta\phi}$, where $\delta\phi$ is the difference in the latitudes of two places on the same terrestrial meridian in degrees, c the circumference of the Earth's globe given above and d the distance between the two places.) (1)

The equatorial circumference of the earth multiplied by cos θ and divided by R, or multiplied by 12 and divided by the hypotenuse of the right angled triangle formed by the gnomon and the equinoctial midday shadow thereof, (hereafter called equinoctial hypotenuse), gives the circumference of the Earth parallel to the equator and passing through the locality (hereafter called the rectified circumference).[1] (2). (AS)

4. 10. 8. भूगोलः 'खेषुदिग्'व्यासो
भमध्ये व्योम्न्यधः स्थितः ।। १४a ।।
(Nīlakaṇṭha, *SiDar.*, 14 a)

The terrestrial sphere is 1050 *yojanas* in diameter and it stands in the sky in the centre of the celestial sphere, as the lowest point. (14a). (KVS)

भूपरिध्यानयनम्

4. 10. 9. समयाम्योदक्स्थौ देशौ ज्ञात्वा तयोः पलज्ये च ।
योजनसंख्यां च तयोर्मध्ये त्रैराशिकं ततः कार्यम् ।।१२।।
अक्षद्वयान्तरांशैर्देशद्वयमध्ययोजनानि यदि ।
लभ्यन्ते चक्रांशैः कानीति स्याद् भुवः परिधिमानम् ।।
अथवाक्षद्वयविवरजभागैस्तन्मध्ययोजनानि यदि ।
त्रिभसममेरुपलांशैस्तदा कियन्तीति भूपरिधिपादः ।।१४।।
(Par., *Gola. D*, 3.12-14)

Earth's circumference

Having fixed upon two places situated exactly north and south, determine their latitudes and the number of *yojana-s* between them. Then apply the rule of three: If the distance between the two places is caused by their difference in latitudes, how much (will the distance be) for the degrees in a circle (*i.e.*, 360°)? The result will be the circumference of the Earth. (12-13)

Or, if the difference in degrees of the two latitudes are the *yojanas* between them, how many will the *yojanas* be for 90° which is the latitude of Meru? This will give a quarter of the circumference of the Earth. (14). (KVS)

[1] The distance in latitude from Laṅkā to Ujjain is 24′ and from *Ujjain* to the North pole is 66°. Thus 66 × 8 8/9 = 586 2/3, and 90 × 8 8/9 = 800).

[1] For explanation and rationale, see *SiŚi*: *AS*, pp. 83-89.

स्वदेशपरिधिः

4. 10. 10. लम्बकेनाहतं भूमे'र्नवरन्ध्राश्विवह्नयः' ।
व्यासार्धापहृतं वृत्तं स्वदेशे तत्प्रकीर्त्यते ॥

(Bhāskara I, *LBh.*, 1.24)

Circumference of the local circle of latitude

3299 (*yojanas*), (the circumference) of the Earth, multiplied by the R sine of the colatitude (of the local place), and divided by the radius (i.e., 3438′) is known as the (Earth's) circumference at the local place. (24). (KSS)

सूर्यः—वैदिकी भावनाः

4. 11. 1. एकः सूर्यो विश्वमनु प्रभूतः ।
एकैवोषाः सर्वमिदं विभाति ॥

(*Ṛ.V*, 8.59.2)

Sun—Vedic views

The One Sun is the lord of the universe.

One Dawn, it lights up all this. (KVS)

4. 11. 2. मित्रो दाधार पृथिवीमुत द्याम् ।

(*YV:TS*, 3.4.11)

The Sun supports the heaven and the Earth. (KVS)

4. 11. 3. सूर्यस्य चक्षू रजसैत्यावृतं
तस्मिन्नर्पिता भुवनानि विश्वा ॥

(*Ṛ.V*, 1.164.14)

The Sun's eye keeps revolving;

On it depends all the worlds. (KVS)

4. 11. 4. इदं श्रेष्ठं ज्योतिषां ज्योतिरुत्तमं
विश्वजिद्धनजिदुच्यते बृहत् ।
विश्वभ्राड् भ्राजो महि सूर्यो दृश
उरु पप्रथे सह ओजो अच्युतम् ॥ ३ ॥
विभ्राजन् ज्योतिषा
स्वरागच्छो रोचनं दिवः ।
येनेमा विश्वा भुवनान्याभृता
विश्वकर्मणा विश्वदेव्यावता ॥ ४ ॥

(*Ṛ.V*, 10. 170. 3-4)

This light, the best of lights, supreme, all-conquering, winner of riches, is exalted with high laud.

All-lighting, radiant, mighty as the Sun to see, he spreadeth wide unfailing victory and strength. (3)

Beaming forth splendour with thy light, thou hast attained heaven's lustrous realm. By thee were brought together all existing things, possesser of all Godhood, All-effecting God. (4). (R.H.T. Griffith)

4. 11. 5. उरुं हि राजा वरुणश्चकार सूर्याय पन्थानमन्वेत वा उ ।

(*Ṛ.V*, 1.24.8)

King Varuṇa (the Vedic god) has made a spacious pathway for the Sun to traverse. (8). (KVS)

सूर्यो नोदेति, नास्तमेति

4. 11. 6. स वा एष न कदाचनास्तमेति नोदेति । तं यदस्तमेतीति मन्यन्ते अह्न एव तदन्तमित्वाथात्मानं विपर्यस्यते, रात्रिमेवावस्तात् कुरुतेऽहः परस्तात् । अथ यदेनं प्रातरुदेतीति मन्यन्ते रात्रेरेव तदन्तमित्वाथात्मानं विपर्यस्यतेऽहरेवावस्तात् कुरुते रात्रिं परस्तात् । स वा एष न कदाचन निम्लोचति ।

(*Aitareya Br.*, 14.6)

He (the Sun) never sets or rises. When (men) think that he is setting, he is only turning round, after reaching the end of the day, and makes night here and day below. Then, when (men) think he is rising in the morning, he is only turning round after reaching the end of the night, and makes day here and night below. Thus, he (the Sun) never sets at all. (6)

4. 11. 7. अथ तत ऊर्ध्वं उदेत्य नैवोदेता नास्तमेतैकल एव मध्ये स्थाता । तदेष श्लोकः ॥ १ ॥

न वै तत्र न निम्लोच नोदियाय कदाचन ।
देवास्तेनाहँ सत्येन मा विराधिषि ब्रह्मणा ॥ २ ॥ इति ॥

न ह वा अस्मा उदेति न निम्लोचति सकृद् दिवा हैवास्मै भवति य एतामेवं ब्रह्मोपनिषदं वेद ॥ ३ ॥

(*Chāndogyopaniṣad*, 3.11.1-3)

Then rising from these upward, he (the Sun) will neither rise nor set. He will remain alone in the middle. There is this verse about it. (1)

'Never does this happen here. Never did the Sun set there nor did it rise. O Gods! by this my assertion of the truth, may I not fall from Brahman.' (2)

Verily, for him, the Sun neither rises nor sets. He who thus knows this secret of the Vedas, for him, there is perpetual day. (3). (Swami Swahananda)

रवेः कृष्णरूपम्

4. 11. 8. तद् व्यक्षरत् । तदादित्यमभितोऽश्रयत् । तद् वा एतद् यदेतदादित्यस्य कृष्णं रूपम् ।

(*Chāndogyopaniṣad*, 3.3.3.)

तद् व्यक्षरत् । तदादित्यमभितोऽश्रयत् । तद् वा एतद् । एतदादित्यस्य मध्ये क्षोभत इव ।

(*Chāndogyopaniṣad*, 3.5.3)

Black spots in the Sun

It flowed forth; it settled by the side of the Sun. Verily, it is that appears as the black form (spot) in the Sun. (3.3.3.)

It flowed forth; it settled by the side of the Sun. Verily this is that appears as the quivering in the middle of the Sun. (3.5.3.). (Swami Swahananda)

4. 11. 9. तस्मादादित्यः षण्मासो दक्षिणेनैति षडुत्तरेण ।

(*Taitt. Saṁhitā*, 6.5.3.4)

Southward and northward passage of the Sun

Therefore the Sun goes south for six months and north for six months. (4). (KVS)

सूर्यापेक्षया शशिनः स्थानशौक्ल्यादि

4. 12. 1. अर्कादुपरि शशी स्यादिति कवयो ये वदन्ति तत्पक्षे ।
नेन्दुरयं प्रत्यक्षस्तत्रान्यच्चन्द्रदैवतं कल्प्यम् ।। २९ ।।
भानि खगानामूर्ध्वं तेषां मुनयस्तथोर्ध्वगाः सप्त ।
उपरि ध्रुवश्च तेषामिति च प्रोक्तं पुराणविद्भिस्तैः ।।
मेरावेव ह्येवं भवति ततो मेरुगाः पुराणविदः ।
मेरौ तु यमान्तेऽर्के सौम्यक्षेपो विधुस्तदासन्नः ।। ३१ ।।
उपरिगत एव सूर्यात्प्रदृश्यते तत्स्मृतिस्ततो वा स्यात् ।
अर्कोऽमावास्यान्ते चन्द्रमसा क्षेपहीनेन ।। ३२ ।।
संछाद्यते च, तस्मात्
निश्चितमर्काच्छशी ह्यधःस्थ इति ।
यद्यर्कादुपरि विधुस्तथा
ह्यधःस्थार्करश्मिसंपातात् ।। ३३ ।।
दृश्येत पूर्णसदृशः कृष्णे पक्षे त्रयोदश्याम् ।। ३४a ।।

(Par., *Gola. D.*, 2. 29-34a)

Moon: Its position in relation to the Sun

In the statement of the sages that the Moon is above the Sun, it is not the visible Moon that is envisaged; there a distinct Moon deity is to be presumed.[1] (29)

It is stated by those sage-authors of the Purāṇa-s that the stars are above the planets; above them are the Seven Sages (the Great Bear), and above them the Pole. (30)

This is so only on Meru and so the composers of the Purāṇa-s were on Meru. At Meru when the Sun is at the (northern) extremity of the southern (hemisphere, i.e., on the horizon), the Moon near it, having a north-latitude (*uttara-vikṣepa*), is seen above the Sun. The above statement may perhaps be due to this (phenomenon). (31-32a)

(In fact) the Sun is hidden by the Moon at the end of the new moon when it has no latitudinal deflection (*vikṣepa*); and hence it is definite that the Moon is below the Sun. (32b-33a)

If the Moon is above the Sun it would be seen almost full on account of the Sun's rays falling on it from below even on the thirteenth day of the dark fortnight.[2] (33b-34ab). (KVS)

[1] The reference is to Upaniṣadic statements like the description of the 'Path of the Gods' (*Devayāna*) in the *Chāndogya Upaniṣad*: *Ādityāc candramasam, candramaso vidyutam... Brahma gamayati. Eṣa devayānaḥ panthāḥ*, etc. (4. 15.5). Śaṅkara in his *Brahmasūtrabhāṣya* 4. 3.4 says that distinct deities are to be presumed in cases like these.

[2] The significance of taking the thirteenth day and not the fourteenth or the new moon day itself is that on those days the Moon may not be visible due to the bright light of the Sun, and sometimes even be hidden by the Sun.

4-*

शशिनः शौक्ल्यम्

4. 12. 2. शौक्ल्यं स्वाभाविकमिति मन्यन्ते ये पुनस्तु चन्द्रमसः ।।
शौक्ल्यस्य क्षयवृद्धयोरपि हेतुस्तैश्च वक्तव्यः ।
अर्काधःस्थस्येन्दोरूर्ध्वार्धे ह्यर्करश्मयोऽमान्ते ।। ३५ ।।
दृश्यार्धमतः कृष्णं तदार्कतः पूर्वगे तु चन्द्रमसि ।
पश्चिमपार्श्वेऽर्ककरा लम्ब्यन्ते तत्र शौक्ल्यमत इन्दोः ।।
एवं क्रमवृद्धिः स्यात् शौक्ल्यस्य हि पूर्णता च पूर्णान्ते ।
पश्चिमभागे कार्ष्ण्यं ततः क्रमात् सूर्यरश्मिराहित्यात् ।।

(Par., *Gola D.*, 2.34-37)

Moon's brightness

If brightness is presumed by anybody as inherent in the Moon, then, he will also have to state the reason for its increase and decrease. (34b-35a)

At the end of the new moon (i.e., conjunction), the rays of the Sun fall upon the upper half of the Moon which is below it. The visible (lower) half (of the Moon) is therefore dark at that time (and hence not seen). (35b-36a)

When the Moon is to the east of the Sun, the rays of the Sun fall on its western side, and therefore the Moon is bright there. Thus occurs the gradual increase of the Moon's brightness, and fullness at the end of the full moon day. From then onwards (as the Moon moves to the west of the Sun) the darkness (of its western side) gradually increases due to the absence of the Sun's rays. (36b-37). (KVS)

4. 12. 3. चन्द्रमा अमावास्यायामादित्यमनुप्रविशति । . . . आदित्याद् वै चन्द्रमा जायते ।

(*Ait. Brāhmaṇa*, 8.28)

The Moon enters the Sun on the new moon day... The Moon is (again) born of the Sun.[1] (28). (KVS)

ग्रहाः राशयः नक्षत्राश्च

4. 13. 1. ग्रहाः सूर्येन्दुभौमज्ञगुर्वच्छार्क्यहिकेतवः ।
मेषादयो राशय स्युरश्विन्याद्याश्च तारकाः ।। ६ ।।

(Śaṁkara Varman: *Sadratnamālā*, 2.6)

Planets

The (nine) planets are: Sun, Moon, Mars, Mercury, Jupiter, Venus, Saturn (and the two invisible *grahas*) Rāhu and Ketu.

The Signs

The (twelve) signs are: (1) Meṣa (Aries) and others (viz. (2) Vṛṣabha or Taurus, (3) Mithuna or Gemini, (4) Karka or Karkaṭaka, Cancer, (5) Siṃha or Leo, (6) Kanyā or Virgo, (7) Tulā or Libra, (8) Vṛścika or Scorpio, (9) Dhanus or Sagittarius, (10) Makara or Capricorn, (11) Kumbha or Aquarius and (12) Mīna or Pisces). (6). (KVS)

[1] Being born again refers to the reappearance of Moon in the evening of the day next to the new moon.

The (twentyseven) asterisms are Aśvini and others.[1]

नक्षत्रापेक्षया ग्रहाणां गतिः

4. 13. 2. ननु सर्वाणि ग्रहनक्षत्राणि पूर्वं पूर्वस्यां दिश्युदयं कृत्वा क्रमेणाम्बरमध्यमतीत्य पश्चिमस्यां दिश्यस्तं यान्तीति दृश्यन्ते । अतो नक्षत्राणामपि प्रत्यङ्मुखमेव गमनं, न तु प्राङ्मुखमिति ।

उच्यते—नक्षत्राणि तावद् भचक्रे पूर्वापरस्थितानि, अश्विन्याः पूर्वतो भरणी, तत्पूर्वतः कृत्तिका, तत्पूर्वतो रोहिणीत्यादि । एतानि भचक्रे प्रतिबद्धानि, तद्भ्रमणवशान्नित्यं पश्चिमाभिमुखं गच्छन्ति । भचक्रस्थानां ग्रहाणामपि नक्षत्राणामिव यदि चक्राधीना गतिरेव स्यात्, तर्ह्येकस्मिन् काले अश्विन्यां दृष्टास्तत्परतो बहुतिथेऽपि काले गते भरण्यादिभिस्सह नोपलभ्येरन् । उपलभ्यते च ग्रहा अश्विन्यादिभिस्सह पूर्वस्मिन् काले दृष्टाः परस्मिन् काले भरण्यादिभिस्सह पूर्वपूर्वस्थितैः । तस्माच्चक्रगतिव्यतिरिक्तं स्वकीयं गमनं प्राङ्मुखमस्तीति निश्चीयते । तेन प्राङ्मुखेन गमनेन य एकश्चक्रपरिवर्तः स एको भगणः ॥

(Sūryadeva-yajvan, Com. on *A.Bh.* 1.3.)

Relative motion of stars and planets

Now, it is seen that all celestial bodies, inclusive of both stars and planets, initially rise in the east, gradually cross the middle of the heavens, and set in the west. Hence it has to be conceded that the motion of the stars too is westward and not towards the east.

(No). It is explained (thus): The stars have been distributed on the ecliptic east-west, in such a manner that to the east of Aśvinī is Bharaṇī, to the east of that Kṛttikā, to its east Rohiṇī and so on. These are firmly fixed to the ecliptic and move continuously westwards. due to the (westward) motion of the ecliptic as a whole. If the planets on the ecliptic too had their motion with the ecliptic as a whole, then a planet seen alongside Aśvinī will not be seen with Bharaṇī etc. even after the lapse of considerable time. Actually, however, it is observed that planets which were earlier seen alongside Aśvinī etc. are later seen with Bharaṇī etc. which are situated more and more to the east. Therefore it is conclusively proved that there is, for the planets, an independent eastward motion different from that of the ecliptic. When a planet completes one circle in its eastward motion, it would have done one revolution. (KVS)

राहूच्चयोः वास्तविकं रूपम्

4. 13. 3. ननु सप्तैव ग्रहा व्योम्नि भ्रमन्तो दृश्यन्ते । तेषां च भगणाः पूर्वमेवोपदिष्टाः । क एते उच्चपाताः येषामत्र भगणोपदेशः क्रियते?

उच्यते । तेषामेव ग्रहाणां स्फुटगत्यादिपरिज्ञानोपायभूताः केचन संख्याविशेषाः । नैषां व्योम्नि दर्शनमस्ति । तथा च ब्रह्मगुप्तः (ब्राह्मस्फुटसिद्धान्तः 21.30)—

प्रतिपादनार्थमुच्चं प्रकल्पितं ग्रहगतेस्तथा पाताः ।

(Sūryadeva-yajvan, Com. on *ABh.* 1.4)

Real nature of Nodes and Apogees

Now, only seven planets are to be seen revolving in the sky. And, their revolutions have already been enumerated. What are these (new bodies), the nodes and apogees, whose revolutions are being enumerated here?

(The answer) is stated thus: These (nodes and apogees) are (not material objects but) just some special numerical figures which have been conceived (by astronomers) as a means to compute the true motion etc. of the self-same planets (mentioned earlier). These do not have visual appearance in the sky. Thus Brahmagupta observes: 'The apogee has been improvised in order to compute the motion of planets; so also the nodes.' (*Br.Sp.Si.*, 21.30).[1] (KVS)

ताराणां दीप्तिः

4. 13. 4. तारारूपाणि यानीह दृश्यन्ते द्युतिमन्ति वै ।
दीपवद् विप्रकृष्टत्वादणूनि सुमहान्त्यपि ॥

(*Mahābhārata*, Cr. edn., 3.43.30)

[1]

No.	Nakṣatra	Junction star (*Yogatārā*)	Latitude		Longitude	
1	Aśvinī	β Arietis	+ 8°	20′	33°	22′
2	Bharaṇī	41 Arietis	+10	27	37	36
3	Kṛttikā	η Tauri	+ 4	3	59	23
4	Rohiṇī	α Tauri	− 5	28	69	11
5	Mṛgaśiras	λ Orionis	−13	23	83	6
6	Ārdrā	α Orionis	−16	2	88	9
7	Punarvasu	β Geminorum	+ 6	41	112	37
8	Puṣya	σ Caneeri	+ 0	5	128	7
9	Āśleṣā	α Canceri	— 5	5	133	2
10	Maghā	α Leonis	+ 0	28	149	13
11	Pārva-Phalgunī	δ Leonis	+14	20	160	42
12	Uttara-Phalgunī	β Leonis	+12	16	171	1
13	Hasta	δ Corvi	−12	12	192	51
14	Citrā	α Virginis	− 2	3	203	14
15	Svātī	α Bootis	+30	46	203	38
16	Viśakhā	α Libra	+ 0	20	224	28
17	Anurādhā	δ Scorpii	− 1	59	241	58
18	Jyeṣṭhā	α Scorpii	− 4	34	249	9
19	Mūla	λ Scorpii	−13	47	263	59
20	Pūrvāṣāḍhā	δ Sagittarii	− 6	28	273	58
21	Uttarāṣādhā	α Sagittarii	− 3	27	281	47
22	Śravaṇa	α Aquilae	+29	18	301	10
23	Dhaniṣṭhā	β Delphini	+31	55	315	44
24	Śatabhiṣaj	λ Aquarii	− 0	23	340	58
25	Pūrva Bhādrapada	α Pegasi	+19	24	352	53
26	Uttara-Bhādrapada	γ Pegasi	+12	36	8	33
27	Revatī	δ Piscium	− 0	13	19	16

[1] It is well known that the node and apogee of a planet are not material objects like the planet itself, the former being just the point where the planet, in its northward course, crosses the ecliptic and the latter the remotest point of the planet's orbit from the Earth. However, these points keep on changing their positions, the said motion being capable of being counted in terms of revolutions during long periods of time. This point is well brought out here.

The luminous stars, though really very large, appear small and, twinkle like lamps on account of their great distance. (30). (KVS)

ध्रुवतारं स्थिरं न वर्तते

4. 13. 5. ध्रुवतारं स्थिरं ग्रन्थे मन्यन्ते ते कुबुद्धयः ।

(Kamalākara, *Si. Tattvaviveka*, Madhyamādhikāra, 78)

The Pole star is not fixed

It is the unintelligent who accept in their works that the Pole star is in a fixed position. (KVS)

ग्रहाणां गोलाकारत्वम्

4. 14. 1. गोलाकारं मण्डलमर्कादीनां स्मृतं गणकमुख्यैः ।
दर्पणवृत्ताकारं दूरगतत्वात् प्रदृश्यतेऽस्माभिः ।। ४६ ।।
शशिबिम्बे दर्पणवद् वृत्ते तस्यैकदेशसितलब्धिः ।
भानोः करसम्पाते व्यवधानाभावतो न संभवति ।।४७।।

(Para., *Gola D*, 2.46-47)

Spherical shape of the Planets

The shape of the orbs of the Sun, etc., is said to be spherical by great astronomers. (46a)

They appear to us in the shape of plane circular mirrors on account of their being at a (great) distance. If the orb of the Moon is like a plane circular mirror, when the Sun's rays fall on it, the phenomenon of a part alone being lighted cannot happen, since there is no obstruction.[1] (46b-47). (KVS)

ग्रहाणां स्थितिः

4. 15. 1. भानामधः शनैश्चरसुरगुरुभौमार्कशुक्रबुधचन्द्राः ।
एषामधश्च भूमिर्मेधीभूता खमध्यस्था ।।

(Āryabhaṭa I, *ABh.* 3.15)

Order of the Planets

(The asterisms are the outermost). Beneath the asterisms lie (the planets) Saturn, Jupiter, Mars, the Sun, Venus, Mercury, and the Moon (one below the other); and beneath them all lies the Earth like the hitching peg in the midst of space. (15). (KSS)

4. 15. 2. चन्द्रादूर्ध्वं बुधसितरविकुजजीवार्कजास्ततो भानि ।
प्राग्गतयस्तुल्यजवा ग्रहास्तु सर्वे स्वमण्डलगाः ।। ३९ ।।
तौलिकचक्रस्य यथा विवरमराणां घनं भवति नाभ्याम् ।
नेम्यां स्यान्महदेवं स्थितानि राश्यन्तराण्यूर्ध्वम् ।। ४० ।।
पर्येति शशी शीघ्रं स्वल्पं नक्षत्रमण्डलमधस्थः ।
ऊर्ध्वस्थस्तुल्यजवो विचरति तथा न महदर्कसुतः ।।४१।।

(Varāha, *PS*, 13.39-41)

Beyond the Moon, are orbiting higher and higher, Mercury, Venus, the Sun, Mars, Jupiter and Saturn, and beyond that there are fixed stars. All the planets (from Mercury to Saturn) move in their own individual orbits at a constant speed. (39)

Just as the spokes of the oil-press wheel are close to one another near the navel, and the space between one another increases as the rim is approached, so the linear extension of the *rāśi* increases as the orbits are situated higher and higher. (40)

Situated near-most, the Moon goes round in the shortest time, its orbit being the shortest. But Saturn situated farther-most, in its longest orbit, cannot move so fast, i.e. moves slowest. (41). (TSK)

गोलपरिस्थितिः

4. 16. 1. समघनवृत्ता भूमिः स्वयैव शक्त्या धृता मृदादिमयी ।
ज्योतिर्गोलकमध्ये बिभर्ति विश्वं समन्ततो वस्तु ।। १ ।।
द्रव्याणि गुरूणि यतः पतन्ति भूमौ समन्ततो नभसः ।
अध एव सर्वतो भूस् तस्मात् प्रतियोगिनी दिगूर्ध्वाख्या ।।
कतिपययोजनपरिमितम् अतः समन्ताद्विहायसि विहायः ।
भ्रमति ह्यूर्ध्वं वायुः प्रवहाख्यो भ्रामयन् विहगान् ।। ३ ।।
तत्र भ्रमन्ति यत्र क्रमेण दृश्यानि भानि सर्वाणि ।
पार्श्वस्थे ध्रुवतारे निरक्षसंज्ञो भुवि प्रदेशः सः ।। ४ ।।
घटिकामण्डलमाहुस्तत्र यदधऊर्ध्वगं भ्रमद् वृत्तम् ।
अभितोऽपि च तद् भ्रमतां भवन्ति नाना ध्रुवा द्युवृत्तानि ।।
प्रत्यक् भ्रमति भचक्रं मेधीकृत्य ध्रुवौ नियतम् ।
चक्रकलासमसंख्यैः प्रवहेण भ्राम्यते च तत्प्राणैः ।। ६ ।।
द्वादशराशिविभक्तो भगोल इह, तस्य मध्यवलयं यत् ।
अपयातं घटिकाख्यात् तदर्धशस् सौम्ययाम्यदिशोः ।। ७ ।।
तत्समतिर्यग्वलयैः प्रविभज्यन्तेऽत्र राशिभागकलाः ।
यत्र क्वापि च दृष्टं ज्योतिर्मेषादिराशिगं तस्मात् ।। ८ ।।
लग्नव्यवहारस्तु भ्रमद्भगोलस्थमध्यवृत्तवशात् ।
स्वाधिष्ठितात्तथोद्यति भिन्ने राशावुडूदयोऽपि भवेत् ।।
निजमन्दपरिधिगोच्चं केन्द्रीकृत्य भ्रमन्ति कक्ष्यासु ।
विहगाः, रविचन्द्रमसोर्भगोलमध्यं स्वमन्दवृतिमध्यम् ।।
अपमण्डलमध्यस्थस्वशीघ्रवृतिसंगतोच्चमन्येषाम् ।
पाताद् विक्षिप्तमुदङमृदुवृत्तार्धं, ततोऽन्यार्धम् ।। ११ ।।
चन्द्रादीनां मन्दानुसारतः स्वस्वकक्ष्याः स्युः ।
क्षयवृद्धी सर्वेषां परिधेर्मन्दस्य तु स्वकर्णवशात् ।। १२ ।।
प्राचीं भ्रमतां स्वे स्वे कक्ष्यावलये तु योजनैस्तुल्या ।
लिप्ताभोगाद् भिन्ना गतिर्ग्रहाणां मिथो वापि ।। १३ ।।
प्रत्यग् भ्रमतां तेषां दिनार्धक्लृप्त्यै प्रकल्प्यते स्थायि ।
वलयमुदग्दक्षिणतः समपार्श्वस्थं च दिवसनिशोः ।।
एतन्निरक्षदेशजम् उन्मण्डलमवनिजं ततोऽन्यत्र ।
सममण्डलमपि भिन्नं घटिकावृत्तात् स्वदेशवशात् ।।

(Nīlakaṇṭha, *Golasāra*, 2.1-15)

[1] The idea is that if light falls on a plane mirror, the whole surface will be lighted, ill or well, whatever be its slope. Hence phases of the Moon which we actually see cannot occur if its surface is plane.

The Celestial sphere and the planets

The Earth, a regular sphere, composed of mud etc. and sustaining itself by its own power and situated at the middle of the celestial globe supports all things around it. (1)

Since all weighty things fall on the Earth from the sky all around, the Earth is 'down' from everywhere and any direction opposite to it (i.e., pointing away from the Earth) is 'up'. (2)

The atmosphere extends some *yojanas* into the sky, all round the Earth. Above that, the wind known as *Pravaha* blows, causing the celestial bodies to revolve. (3)

That region upon the Earth where *all* the stars revolving in that (*Pravaha*) region can be seen rising one after another, and where the Pole stars are (exactly) on the two sides, is called the region of Zero latitude (i.e., the Equatorial region). (4)

The revolving vertical circle there (i.e., the Prime vertical at that region) is called the *Ghaṭikā-maṇḍala*[1] (Hour circle). On both sides of that are the different fixed Diurnal circles of the heavenly bodies.[2] (5)

The stellar sphere is constantly revolving westwards with the two Celestial Poles as the apices and is rotated by the *Pravaha* wind (completely once) in (a period containing) *prāṇa-s* equal in number to the minutes of arc in a circle, (viz., 21,600), (i.e., in one sidereal day).[3] (6)

This celestial globe is divided into twelve *rāśis*; the central great circle, (called *Apakrama-vṛtta* or the Ecliptic), is inclined to the Celestial Equator[4] so that one half of it lies to the north of it and the other to the south. (7)

This (circle, *viz*., the Ecliptic) is divided into *rāśis* (segments of 30°), *bhāgas* (degrees) and *kalās* (minutes) by circles perpendicular to it, so that, a body, wherever it may be seen (on the Celestial sphere) lies in one of the *rāśis, Meṣa* etc.[5] (8)

The reckoning of *Lagna* (Rising point of the Ecliptic) is in relation to the central circle of the revolving celestial sphere, (*viz*., the Ecliptic). Hence, at the moment of the rising of the *Lagna* in a particular *rāśi*, a star in a different *rāśi*, may also be rising.[6] (9)

The planets move in their orbits with centres at the Higher Apses of their Epicycles of the Equation of the centre. For the Sun and the Moon, the centre of the celestial sphere is the centre of the above epicycles. (10)

For the other (planets), the centres (of their *Manda* circles) are on (the circumference of) their *Śighra* circles concentric with the Ecliptic. One half of their *Manda* circles is deflected northwards from the Ascending Node, and the subsequent half southwards. (11)

The Moon and other (planets) have their respective orbits increasing according to (the changes in their *manda* (circles).[7] And for all (planets), the increase and decrease of the circumference of the *manda* circles depend upon their hypotenuse.[8] (12)

Of the planets which move, each in its own orbit, eastwards (in relation to the stars fixed on the celestial globe which, as a whole, is moving westwards), the motion in *yojanas* is equal. But their (angular) motion in minutes is different (for the same planet at different positions) as also from one another's (owing to variance in the hypotenuse and the magnitude of the orbits, respectively.) (13)

To fix the midday for these (planets) which move (apparently) westwards, a fixed north-south-lying great circle situated equally on both sides of (mid)day and night is presumed. (14)

This (circle) is the horizon at the equator, and is called *Unmaṇḍala* elsewhere. In regions other than the equator, the *Samamaṇḍdala* (which coincides with the Hour circle at the equator), too, varies from place to place, (i.e., in different terrestrial longitudes). (15). (KVS)

4. 17. 1. विषुवत्संस्थो भानुर्लङ्कायां नभसि मध्यरेखायाम् ।
भ्रमति ह्यपराभिमुखं निरक्षदेशे तथैव सर्वत्र ।। १८ ।।
अर्कः स एव मेरौ भ्रमति समन्तात् क्षितेस्तु पार्श्वगतः ।
अन्यत्र स्वाक्षवशात् खमध्यभूपार्श्वयोर्भ्रमति मध्ये ।।१९।।
अचलानि भानि तेषामधः क्रमान्मन्दजीवकुजदिनपाः ।
सितबुधशशिनश्चैते गच्छन्तः प्राङ्मुखं स्वगत्या तु ।।२०।।

[1] It is so called because time, in units like the *ghaṭikā*, are measured on it. The term *Viṣuvan-maṇḍala* (Equinoctial) is also applied to this great circle.

[2] These are known by the name *dyu-vṛtta* or *ahorātra-vṛtta*.

[3] A sidereal day is equal to 60 *nāḍikās*, a *nāḍikā* equal to 60 *vināḍikās* and a *vināḍikā* equal to 6 *prāṇas*; hence, one day is equal to $60 \times 60 \times 6 = 21,600$ *prāṇas*.

[4] This obliquity is equal to 24°.

[5] The Ecliptic is divided into 12 *rāśis* called *Meṣa* etc. commencing from a point situated near the junction-star called Zeta Piscium in the asterism of Revatī.

[6] This may happen in the case of stars which are removed from the Ecliptic.

[7] That is, the hypotenuse got in the *Manda*-operation is the radius of the orbit on which the planet is measured after this operation. This is a peculiarity of the school of Āryabhaṭa.

[8] This is another peculiarity of the Āryabhaṭan school. The mutual dependence is resolved by resorting to what is called *aviśeṣakriyā* or successive approximation.

अपराभिमुखं गोलभ्रमणात् सर्वे भ्रमन्ति लङ्कायाम् ।
घटकारकस्य चक्रे विलोमगाः कीटका यथा तद्वत् ।। २१ ।।

अधउपरिगतखगानां प्राग्गत्या योजनानि तुल्यानि ।
सर्वेषां दिवसे स्युस्तथापि तेषां कलागतिर्भिन्ना ।। २२ ।।

कक्ष्याल्पाधःस्थस्य ग्रहस्य, महती तथोर्ध्वगस्य भवेत् ।
स्वल्पायां च महत्यां लिप्ता भागाश्च राशयस्तुल्याः ।।

पूरयति चाल्पवृत्ते चरन्निजं वृत्तमल्पकालेन ।
महता कालेन चरन् महति दिने गतिकलास्ततो भिन्नाः ।।

पूर्वपक्षः

दृश्यन्ते सर्वखगा अपराभिमुखं भ्रमन्त एवेह ।
एवं स्थिते ग्रहाणां पूर्वगतिः कल्प्यते कया युक्त्या ।।

समाधानम्

दस्रयुतः शीतकरो ह्यपरदिने दृश्यते यमर्क्षस्थः ।
भौमादयश्च तद्वत् पूर्वगतिर्निश्चिता ततस्तेषाम् ।। २६ ।।

पूर्वपक्षः

मन्दगतिरिन्दुरार्किः शीघ्रगतिस्तारकास्तु शीघ्रतराः ।
गच्छन्त्यपराभिमुखं सर्वेऽप्येवं वदन्ति किल केचित् ।।

समाधानम्

मन्ये तदपि न युक्तं यस्माद्वक्रिग्रहोऽनलर्क्षस्थः ।
तत्पश्चिमगभरण्यां दिनान्तरे दृश्यते न पूर्वदिशि ।।२८।।

(Par., *Gola.D.*, 2.18-28)

The true eastward motion of planets

The Sun on the Celestial equator at Laṅkā moves in the heavens westwards along the Prime vertical, and so also at all places on zero latitude. The same (Equinoctial) Sun moves around along the horizon at Meru. At other places it moves between the zenith and the horizon according to the latitude (of the place). (18-19)

The stars are stationary. Below them are, in order, Saturn, Jupiter, Mars, the Sun, Venus, Mercury and the Moon, all moving eastwards by their individual motion, and westwards with reference to (places on zero latitude, like) Laṅkā, due to the rotation of the entire Starry sphere, in the same manner as insects on a potter's wheel moving in a direction opposite (to that of the wheel). (20-21)

The (distance in) *yojanas* covered per day by all planets, whether high or low, (*i.e.*, distant or near) in their easterly motion is the same. But their motion in terms of minutes of arc is diverse. (22)

The orbit of a planet which is low down is small and that of one high up is great. The number of minutes, degrees and signs is the same in both small and big circles. Moving along a smaller circle, a planet completes its circle in a shorter time, and moving along a bigger circle (completes it only) in a longer period. Hence the number of minutes covered in a day varies (from one planet to another). (23-24)

(*Objection*). All the planets are seen moving only westwards. This being the case, by what reasoning is their easterly motion presumed? (25)

(*Answer*). The Moon (seen) in conjunction with the *Aśvinī* constellation, is seen the next day in (conjunction with) *Bharaṇī*.[1] Mars and other planets also behave in the same way. Their easterly motion is determined from this. (26)

(*Objection*). The Moon is slow; Saturn moves fast; and the stars are faster still. All move westwards. (If we suppose thus, then also will they behave as stated in verse 26): So argue some. (27)

(*Answer*). We assert, however, that this (explanation), too, is wrong, because a retrograde planet in *Kṛttikā*[2] is seen a few days later in Bharaṇī which is to its west, and not in an easterly position. (28). (KVS)

ग्रहचारः

4. 17. 2. सर्वेषां कक्षार्धं सममभितः कुदलवर्जितं दृश्यम् ।
भूगोलान्तरितं यत् कुदलेन संयुतमदृश्यम् ।। ३५ ।।

कक्षा मध्यगतिघ्न्यः स्फुटगत्याप्ताः परिस्फुटा भवन्ति ।
स्वमृदुकर्णकलाहता विभाजितास्त्रिगृहमौर्व्या वा ।। ३६ ।।

मध्यमकक्षावृत्ते मध्यमया गच्छति ग्रहो गत्या ।
उपरिष्टात् तल्लब्ध्या तदधिकगत्या त्वधस्थः स्यात् ।।

वक्री यात्यपराशां निसर्गतो गच्छति ग्रहः प्राचीम् ।
क्रान्त्या याम्योदीच्योर्ग्रहगतिरेवं भवेत् षोढा ।। ३८ ।।

स्फुटकक्षया यदाप्तं नाड्यादि क्ष्मार्धषष्टिसंवर्गात् ।
तेनोदितोऽप्यनुदितो नास्तमितोऽप्यस्तमुपयाति ।। ३९ ।।

(Lalla, *SiDhVṛ.*, 19. 35-39)

Planetary Motion

Whatever is obtained by subtracting the Earth's radius from the radius of the orbit of a planet, is the portion visible (to man). And whatever is obtained by the semi-diameter of a planet's orbit increased by the semi-diameter of the Earth, remains hidden from the Earth. (35)

The (mean) orbit of each planet multiplied by its mean motion and divided by its true motion, gives the corrected measure of the orbit.

Or, the (mean) orbit multiplied by the *manda* hypotenuse in minutes of the planet, and divided by the radius, gives the corrected result. (36)

[1] The *Aśvinī* constellation is called *Dasra* because it is presided over by the *Aśvins*, also called *Dasrau*. *Bharaṇī* is called *Yama* because its presiding deity is Yama, the God of death.

[2] The God *Anala* (Fire) is the presiding deity of *Kṛttikā*.

A planet moves along its mean circular orbit at the rate of its mean motion. When it is above (its mean orbit), it moves at a slower rate, and when below, at a higher rate. (37)

A planet naturally moves to the east, but, when retrograde, to the west. It is drawn towards north or south by its declination. Thus the motion of a planet is of six kinds. (38)

Whatever *nāḍīs* are obtained from the division of the product of the Earth's semidiameter and 60, by the true circumference of a planet's orbit, give the horizontal parallax of the planet (in terms of *nāḍīs*). If the planet has theoretically risen so many *nāḍīs* before, it has not risen actually; and if theoretically it is to set so many *nāḍīs* afterwards, it actually appears to set even now. (39). (BC)

दैनंदिनं ग्रहभ्रमणम्

4. 17. 3. अपमण्डले तु सूर्यः पूर्वाभिमुखः सदा चरति ।
भवति ह्यपराभिमुखं वेगाधिक्यात्तथापि गोलस्य ।। १ ।।
प्रवहमरुत्प्रक्षिप्तो भगोल उर्वी प्रदक्षिणीकृत्य ।
अपराभिमुखं षष्टचा घटिकाभिर्भ्रमति भूयोऽपि ।। २ ।।
भूपृष्ठादुपरि मरुद् 'रवि'योजनसम्मितान्तरे प्रवहः ।
नियतगतिरपरगः स्याद् भूवायुरधश्च तस्य भिन्नगतिः ।।
घटिकाख्यषष्टिभागभ्रमणे कालोऽत्र नाडिकेत्युदिता ।
न तु दिवसषष्टिभागो गोलभ्रमणाद्यतोऽधिको दिवसः ।।
भूपार्श्वगतं क्षितिजं तदूर्ध्वगास्तूदिता ग्रहा दृश्याः ।
भूमितनुव्यवधानात् तस्याधःस्था ग्रहा न दृश्यन्ते ।।
इति गदितं शास्त्रेषु क्षितिपृष्ठादूर्ध्वगा ग्रहा ह्येव ।
अस्माभिर्दृश्यन्ते कुपृष्ठतोऽधोगता न दृश्यन्ते ।। ६ ।।
अपमगतत्वाद् भानुस्तस्य स्थानानुसारतः सौम्ये ।
याम्ये वा यात्युदयं मध्यदिशस्तद्वदेव चास्तमपि ।। ७ ।।
चन्द्राद्या विक्षेपादपमादपि सौम्ययाम्यदिक्स्थाः स्युः ।
साक्षे देशे ह्युदयाद् याम्ये यान्ति ग्रहा नभोमध्यम् ।।
प्रागुन्मण्डलगोऽर्कस्त्रिशद्घटिकाभिरेति पश्चिमगम् ।
उन्मण्डलं, ततोऽपि च तावत्कालेन पूर्वगतम् ।। ९ ।।
अत्र तु घटिकाशब्दो दिनषष्टयंशस्य वाचकः प्रोक्तः ।
व्यवहारो ह्यनयैव स्याद् गोलभ्रमणतोऽन्यत्र ।। १० ।।
क्षितिजे ह्युदयास्तमयौ सौम्ये तून्मण्डलादयः क्षितिजम् ।
तस्मात्सौम्ये गोले त्रिंशद्घटिकाधिकं दिनं भवति ।।११।।
याम्ये तूर्ध्वं क्षितिजं त्रिंशद्घटिकात ऊनमत्र दिनम् ।
दिनवृद्धौ तु निशोना निशाधिका तत्क्षये तु सर्वत्र ।।१२।।
त्रिंशद्घटिका दिवसो निरक्षदेशे सदा निशा च तथा ।
उन्मण्डलमेव यतः क्षितिजाख्यं मण्डलं भवेत्तत्र ।। १३ ।।

(Para., *GolaD*, 2.1-13)

Diurnal motion of planets

The Sun is constantly moving eastwards along the Ecliptic; still, on account of the more rapid (westward) rotation of the (Starry) sphere (of which it also forms a part), it (appears) to move westwards. (1)

The Starry sphere, impelled by the Pravaha wind, keeps on rotating westwards round the Earth in the *pradakṣiṇa* (clockwise) direction once in 60 *ghaṭikās*. (2)

Above the surface of the Earth, at a height equal to 12 *yojanas*, the Pravaha wind blows with a constant westward motion. Below it (i.e., up to a height of 12 *yojanas*) is the terrestrial wind; (the direction of) its motion is diverse. (3)

By *ghaṭikā* (or *nāḍikā*) is meant the time required for the rotation (of the Celestial Equator) through its one-sixtieth part, and not one-sixtieth of the duration of a (*sāvana*) day,[1] for the day is (a little) longer[2] than the time of rotation of the (Starry) sphere.[3] (4)

The horizon is the horizontal circle around the Earth. Planets become visible when they rise above it. Planets below it are not seen, being hidden by the body of the Earth. (5)

So is it said in the ancient texts. (6a)

(However,) only those planets which rise above (the level of) the Earth's surface (*viz.*, the visual or sensible horizon) are seen by us; and those below (the level of) the Earth's surface are not visible.[4] (6a-b)

Since the Sun is on the Ecliptic (which is inclined to the Celestial Equator), it rises (a little) to the north or south (of the central point, *viz.*, due east) according to its position; it also sets accordingly.[5] The Moon and other (planets) move further north or south than the Ecliptic due to their Celestial latitude (*viksepa*). (7-8a)

In places having (north) latitude (i.e., north of the terrestrial equator), the planets reach the centre of the heavens (*viz.*, the Celestial Meridian) at a more southerly position than at their rising.[6] (8b)

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[1] The *sāvana* day is from sunrise to sunrise.

[2] The difference between a *sāvana* day and a *nākṣatra* day is about 10 *vināḍikā-s* or 4 minutes.

[3] That is called the Sidereal (or *nākṣatra*) day.

[4] The difference in the position of planets viewed from the surface of the Earth and from the centre of the Earth is termed *lambana* or parallax.

[5] The angle subtended by the east or west point and the point of rising or setting, respectively, is callled the amplitude.

[6] This is generally known as 'Southing'. For places south of the equator, however, the deflection will be to the north. This phenomenon for the southern hemisphere is not envisaged by Indian astronomers and hence 'Northing' is not separately described here.

The Sun on the eastern *Unmaṇḍala* reaches the western *Unmaṇḍala* in 30 *nāḍikās,* and from there in an equal length of time, it reaches the eastern (*Unmaṇḍala*). (9)

Here by *ghaṭikā* (*nāḍikā*) is meant one-sixtieth of the (*sāvana*) day. This will be its connotation elsewhere except in the case of the rotation of the Starry sphere. (10)

Rising and setting take place on the horizon. In the northern (hemisphere) the horizon is below the *Unmaṇḍala.* Hence (when the Sun is) in the northern hemisphere the day will be more than 30 *ghaṭikās.* In the southern (hemisphere) the horizon is above the *Unmaṇḍala*; and so, here, the day is less than 30 *ghaṭikās.* At all places, as the day increases, night becomes shorter, and as it shortens, the night grows longer. (11-12)

At places of zero-latitude (i.e., on the equator), the day is (always) 30 *ghaṭikās*, and so also the night, since there the *Unmaṇḍala* itself is the horizon (13). (KVS)

मन्दशीघ्रवृत्तानि

4. 18. 1. ग्रहभ्रमणवृत्तानि गच्छन्त्युच्चगतीन्यपि ।
मन्दवृत्ते तदर्केन्द्वोर्घनभूमध्यनाभिकम् ।। १९ ।।
मध्यार्कगति चान्येषां तन्मध्यं शीघ्रवृत्तगम् ।
तेषां शैघ्र्यं भचक्रान्न विक्षिप्तं गोलमध्यगम् ।। २० ।।
शैघ्रत्वेन तदंशैः स्वं प्रमायोक्तं ज्ञशुक्रयोः ।
मन्दवृत्तस्य चैवात्र क्षयवृद्धी स्वकर्णवत् ।। २१ ।।
(Nīlakaṇṭha, *Si.Dar.*, 19-21)

Epicycles of the eq. of the centre and conj.

(The centres of) the circles along which (the mean motion of) the planets take place, move on (the circumference of) the epicycle of the equation of the centre (i.e. *manda* epicycles) with the velocity of the Higher apses. (19 a-b)

In the case of the Sun and the Moon it (i.e., the *manda* epicycle) has its centre at the centre of the Earth-sphere. (19 c-d)

For the other (five planets) the centre (of the *manda* or 'slow' epicycle) moves on (the circumference of) the epicycle of the equation of conjunction, (i.e., the *śīghra* or 'fast' epicycle) with the mean velocity of the Sun. (20 a-b)

The centre of their *śīghra* epicycle is the centre of the (celestial) globe itself and (the planes of) these (epicycles) are not oblique to (that of) the ecliptic; (i.e., they are in the same plane as the ecliptic.)[1] (20 c-d)

[1] The idea is that the 'fast' (*śīghra*) epicycles are concentric with the ecliptic and lie in its plane and that the celestial latitude (*vikṣepa*) of the planets is caused by the deflection of the 'slow' (*manda*) epicycles which move on the circumference of the 'fast' (*śīghra*) ones. The maximum deflection of each planet is given in verses 8c-9a.

In the case of Mercury and Venus, their own orbits are stated to be the same as their *śīghra* epicycles, measuring them with respect to their *śīghra* epicycles (which are taken as 360 degrees).

In the case of these, the increase and decrease of the *manda* epicycles alone are according to the hypotenuse (extended from the centre of the *manda* epicycle to its circumference where the planet is). (21). (KVS)

प्रतिमण्डले कक्ष्यामण्डले च ग्रहगत्यादेशः

4. 19. 1. ज्ञातभोगग्रहं वृत्तं सर्वत्र प्रतिमण्डलम् ।
कक्ष्यावृत्तं च तत्तुल्यं ज्ञेयभोगप्रदेशगम् ।। २२ ।।
त एवाप्युच्चनीचाख्ये बहिश्चेदितरेतरम् ।
तद्व्यासार्धान्तरे चान्ये तन्मध्यान्तरनिर्मिते ।। २३ ।।
ग्रहोच्चयोर्विपर्यस्तौ भोगावप्यत्र कल्पयेत् ।
तिर्यक्त्वे ज्ञेयभोगात्तत् कोटीव्यासदले तयोः ।। २४ ।।
दोर्भेदो वा युतिः क्षेपस्तन्मध्यग्रहगोलगः ।

भुजादिलक्षणम्

दो:कोटिभुजयोर्योगः कर्णबाहुचतुर्भुजः ।। २५ ।।
कक्ष्यामध्योच्चनीचस्पृक्सूत्रखेटान्तरं भुजा ।
कोटिस्तदुच्छ्रितिः कर्णः कक्ष्यामध्याद् ग्रहान्तरम् ।। २६ ।।
(Nīlakaṇṭha, *Si. Dar.*, 22-26)

Projection of planetary motion on Eccentric and Orbital Circles

In all cases, the circle on which the velocity of a planet is measured (from fundamentals) is termed the 'eccentric circle' (*pratimaṇḍala*). And, that circle on which the motion of the planet is to be (projected and) understood (i.e. measured) is termed the 'orbital circle' (*kakṣyā-maṇḍala*). (22)

When they are one outside the other, (the outer) one is called the 'upper' (*ucca*) (circle) and (the inner) one 'lower' (*nīca*) (circle). (23 a-b)

Here, the two circles of radii equal to the difference between the centres of the orbital and eccentric circles are drawn at the extremities of the line joining the two centres; these two will then be the *ucca* (upper) and *nīca* (lower) circles. (23 c-d)

The motions of the planets and the Higher apses are to be taken here in opposite directions. (24 a-b)

When the eccentric circle (*jñātabhoga-vṛtta*) is oblique in relation to the orbital circle (*jñeyabhoga-vṛtta*), their cosines (*koṭi*) are to be taken as their radii. And the difference or sum, (as the case may be), of the sines of the two circles is to be taken as the latitude of the eccentric circle. (24c-25b)

Definition of bhuja etc.

The sum of the squares of the sine (*doḥ-bhuja*) and cosine (*koṭi-bhuja*) is a square with the hypotenuse as a side. (25-c-d)

The sine is the (perpendicular) distance from the planet to the line joining the upper (*ucca*) and lower (*nīca*) points through the centre of the orbital circle. The cosine is the height of the perpendicular to the planet (from the centre of the orbital circle). And the hypotenuse is the distance from the centre of the orbital circle to the planet. (26). (KVS)

4. 19. 2. कक्ष्याप्रतिमण्डलगा भवन्ति सर्वे ग्रहाः स्वचारेण ।
मन्दोच्चादनुलोमं प्रतिलोमं चैव शीघ्रोच्चात् ॥ १७ ॥
कक्ष्यामण्डलतुल्यं स्वं स्वं प्रतिमण्डलं भवत्येषाम् ।
प्रतिमण्डलस्य मध्यं घनभूमध्यादतिक्रान्तम् ॥ १८ ॥
प्रतिमण्डलभूविवरं व्यासार्धं स्वोच्चनीचवृत्तस्य ।
वृत्तपरिधौ ग्रहास्ते मध्यमचाराद् भ्रमन्त्येव ॥ १९ ॥
यः शीघ्रगतिः स्वोच्चात् प्रतिलोमगतिः स्ववृत्तकक्ष्यायाम् ।
अनुलोमगतिर्वृत्ते मन्दगतिर्यो ग्रहो भवति ॥ २० ॥
अनुलोमगानि मन्दात् शीघ्रात् प्रतिलोमगानि वृत्तानि ।
कक्ष्यामण्डललग्नस्ववृत्तमध्ये ग्रहो मध्यः ॥ २१ ॥
(Āryabhaṭa I, *ABh.*, 3. 17-21)

(The mean planets move on their orbits and the true planets on their eccentric circles). All the planets, whether moving on their orbits (*kakṣyā-maṇḍala*) or on the eccentric circles (*prati-maṇḍala*), move with their own (mean), motion, anticlockwise from their apogees and clockwise from their *śīghroccas*. (17)

The eccentric circle of each of these planets is equal to its own orbit, but the centre of the eccentric circle lies at a distance from the centre of the solid Earth. (18)

The distance between the centre of the Earth and the centre of the eccentric circle is (equal to) the semi-diameter of the epicycle (of the planet). (19a)

All the planets undoubtedly move with mean motion on the circumference of the epicycles. (19b)

A planet when faster than its *ucca*, moves clockwise on the circumference of its epicycle, and when slower than its *ucca*, moves anticlockwise on its epicycle. (20)

The epicycles move anticlockwise from the apogees and clockwise from the *śīghroccas*. The mean planet lies at the centre of its epicycle, which is situated on the (planet's) orbit.[1] (21). (KSS)

[1] For an analytical elucidation and figurative representation of the epicyclic and eccentric theories of Indian astronomy, see Bose, Sen, Subbarayappa, *A Concise History of Science in India*, pp. 111-16; *SiDhi Vṛ.:BC*, pp. 311-24.

छेद्यकविधिः

4. 19. 3. निजकक्षावृत्तं यत्तस्माद् व्यासार्धमण्डलं विलिखेत् ।
तद् द्वादशराश्यङ्क्यं तस्योर्वी मध्यतः कल्प्या ॥ ७ ॥
द्युचरस्यान्त्यफलज्यासमानमपि वर्धितं नयेत् सूत्रम् ।
भूमध्यात् स्वाभिमुखं त्रिभज्यया केन्द्रवृत्तमत्र लिखेत् ॥८॥
परमफलजीवया स्वोच्चनीचवृत्तं लिखेत् स्वकक्षाग्रे ।
स्वोच्चसमानो मध्यो यदा तदोच्चे ग्रहस्थानम् ॥ ९ ॥
स्वोच्चात् षड्भाभ्यधिको यदा तदा भवति स्वनीचस्थः ।
दूरेणोच्चग उर्व्याः कर्णवशान्नोच्चगो निकटे ॥ १० ॥
अत एव द्युचराणामणुता भवति क्रमान्महत्त्वं वा ।
सवितुश्चासन्नानां दूरगतानां क्रमेणैव ॥ ११ ॥
अनुलोमं निजमन्दात् प्रतिलोमं गच्छति स्वशीघ्रोच्चात् ।
कक्षावृत्ते मध्यः स्वकेन्द्रवृत्ते ग्रहः स्पष्टः ॥ १२ ॥
वेगः स्वकेन्द्रवृत्ते यः कक्षामण्डलेऽपि स एव ।
मध्यैव गतिः स्पष्टा वृत्तद्वययोगगे द्युचरे ॥ १३ ॥
मध्यमतुल्यं स्पष्टं भान्तगते भार्धगेऽपि वा केन्द्रे ।
द्रष्टा पश्यति यस्मात् मध्यस्यातः फलाभावः ॥ १४ ॥
स्पष्टं पश्यति यस्मात् मध्यादूनाधिकं नरस्तस्मात् ।
विवरं तयोः फलमृणं धनं च मध्यग्रहे क्रियते ॥ १५ ॥
अन्त्यफलज्याग्रात् कोटिरुपरि केन्द्रे मृगादिगे यस्मात् ।
कीटादिगते चाधस्तत्र तदैक्यान्तरं कोटिः ॥ १६ ॥
कोटिफलमुपरि यस्माद् वान्त्ये प्रथमे पदे भवेत् त्रिगुणात् ।
मध्यमयोश्च यतोऽधस्ततो युतिर्वान्तरं कोटिः ॥ १७ ॥
भूग्रहमध्ये कर्णः स्वकर्णमध्यग्रहान्तरे च फलम् ।
भवति सदा बाहुज्या ग्रहनीचोच्चरेखान्तः ॥ १८ ॥
मध्यैव योक्ता हि गतिर्ग्रहाणां
स्पष्टा गतिः सैव च पर्ययास्ते ।
वक्रोदये दृष्टिवशाद् विकारा-
स्तेभ्यो हि नान्ये भगणा भवन्ति ॥ १९ ॥
(Lalla, *SiDhVr.*, 14. 7-19)

Diagrammatic representation

(With any point as centre) and the given radius, describe a circle which is the circular orbit of the planet or *kakṣāvṛtta*. Divide its circumference into 12 Signs (following the 12 Signs of the zodiac). Assume that the Earth is at the centre of this circle. (7)

From the centre of the Earth draw a vertical line upwards equal in length to the radius of the epicycle of the planet or *antyaphalajyā* or R sine of the maximum correction. With the extremity of this line as centre and the given radius (equal to that of the *kakṣāvṛtta*), describe a circle. It is the eccentric circle or *kendra-vṛtta*. (8)

Or, (draw a vertical diameter in the circular orbit). With the upper end of that diameter as centre and radius

equal to the radius of the epicycle, describe a circle. This is known as the epicycle or *nīcoccavṛtta* of the planet. If the mean place of the planet is at the apogee or *ucca* of the circular orbit, its true place is at the apogee or *ucca* of the epicycle. (9)

When a planet is at a distance of 6 Signs from its apogee, it is said to be at the perigee or *nīca*. When a planet is at the apogee, it is farthest from the Earth; when at the perigee, it is nearest to the Earth. This is so because of the (difference in length of the) hypotenuse in each case. (10)

Thus, the planets appear small or large accordingly. In the same manner, they appear small or large, according as they are near the Sun or removed from it. (11)

A planet moves in the direction of the Signs of the zodiac from its *manda* apogee or *mandocca* (apex of slow motion) and in the opposite direction from its conjunction or *śīghrocca* (apex of quick motion).

The mean planet moves along the circular orbit or concentric and the true planet along the eccentric. (12)

The rate of motion of the true planet in the eccentric is the same as that of the mean planet in the concentric. When the planet is at the points of intersection of these two circles, its mean motion is the same as its true motion. (13)

When the anomaly is either 6 Signs or 12 Signs, the observer sees the mean planet and the true planet in the same straight line. Therefore, at these places the mean planet has no correction. (14)

Since, (for other values of the anomaly), an observer sees the true planet either ahead or behind its mean place, the difference of these positions, that is, the correction, should either be added to or subtracted from the mean place (to obtain the true planet). (15)

When the anomaly is within 6 Signs beginning with Capricorn, its R cosine is above the R sine *antyaphala*. So their sum is the correct R cosine. Again, when the anomaly is within 6 Signs beginning with Cancer, its R cosine is below the R sine *antyaphala*. So, their difference is the correct R cosine. (16)

When the anomaly is either in the first or fourth quadrant, the *koṭiphala* is above the radius. So, their sum is the correct *koṭi*. When the anomaly is either in the second or third quadrant, the *koṭiphala* is below the radius. So, their difference is the correct *koṭi*. (17)

The distance between the centre of the earth and the true position of the planet is known as the hypotenuse or *karṇa*.

The correction or *phala* is the (arc of the concentric intercepted between the point) where the hypotenuse cuts it and the position of the mean planet.

The R sine of the anomaly, *bāhujyā*, is always the perpendicular distance between the place of the planet and the diameter passing through the apogee and perigee. (18-19).[1] (BC)

ग्रहाणां बिम्बकक्षादिमानम्

4. 20. 1. शशिराशयष्ठ चक्रं तेंऽशकलायोजनानि य-व-ञ-गुणाः ।
प्राणेनैति कलां भूः, खयुगांशे ग्रहजवो, भवांशेऽर्कः ॥
(Āryabhaṭa I, *ABh.*, 1.6)

Measures of Planetary orbs, orbits etc.

Reduce the Moon's revolutions (in a *yuga*) to Signs (*rāśis*) multiplying them by 12 (*lit.* using the fact that there are 12 Signs in a circle or revolution). Those Signs multiplied successively by 30, 60 and 10 yield degrees, minutes and *yojanas*, respectively. (These *yojanas* give the length of the circumference of the sky). The Earth rotates through (an angle of) one minute of arc in one respiration (=4 sidereal seconds). The circumference of the sky divided by the revolutions of a planet in a *yuga* gives (the length of) the orbit on which the planet moves. The orbit of the asterisms divided by 60 gives the orbit of Sun.[2] (6). (KSS)

4. 20. 2. नृ-षि योजनं, ञिला भूव्यासो-,
ऽर्कोन्द्वोर्घ्रिञा गिण, क मेरोः ।
भृगु-गुरु-बुध-शनि-भौमाः
शशि-ङ-ञ-ण-न-मांशकाः, समार्कसमाः ॥
(Āryabhaṭa I, *ABh.*, 1.7)

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[1] For observations, see *ŚiDhVṛ.*: *BC*, II. 228-30.

[2] Thus we have:

Orbit of the sky=5,77,53,336 × 12 × 30 × 60 × 10 *yojanas*
= 1,24,74,72,05,76,000 *yojanas*

Orbit of the asterisms=17,32,60,008 *yojanas*

Orbit of the Sun=28,87,666 $\frac{4}{5}$ *yojanas*

Orbit of the Moon=2,16,000 *yojanas*

Orbit of Mars 54,31,291 $\frac{1,32,027}{2,87,103}$ *yojanas*

Orbit of (*Śīghrocca* of) Mercury=6,95,473 $\frac{3,73,277}{8,96,851}$ *yojanas*

Orbit of Jupiter=3,42,50,133 $\frac{699}{1897}$ *yojanas*

Orbit of (*Śīghrocca* of) Venus=17,76,421 $\frac{2,55,221}{5,85,199}$ *yojanas*

Orbit of Saturn=8,51,14,493 $\frac{5,987}{36,641}$ *yojanas*

These orbits are hypothetical and are based on the following two assumptions :

(*a*) That all the planets have equal linear motion in their respective orbits.

(*b*) That one minute of arc (1′) of the Moon's orbit is equal to 10 *yojanas* in length.

8000 *nṛ*[1] makes a *yojana*. The diameter of the Earth is 1050 *yojanas*; of the Sun and the Moon, 4410 and 315 *yojanas*, (respectively); of Meru, 1 *yojana*; of Venus, Jupiter, Mercury, Saturn and Mars (at the Moon's mean distance), one-fifth, one-tenth, one-fifteenth, one-twentieth, and one-twentyfifth, (respectively), of the Moon's diameter. The years (used in this work) are solar years. (7). (KSS)

4. 20. 3. द्वात्रिंशत्पञ्चभिर्हृत्वा भूयो भूयस्तदुत्तरैः ।
शुक्रज्योज्ञार्किभौमानां व्यासलिप्ताक्रमं विदुः ॥ ५६ ॥
(Bhāskara I, *MBh.*, 6.56)

Having (first) divided 32 by 5, divide the same number (i.e., 32) again and again by the same (5) as increased by itself in succession (i.e., by 10, 15, 20 and 25): the results thus obtained are known as the minutes of the diameters of Venus, Jupiter, Mercury, Saturn, and Mars, respectively. (56). (KSS)

ग्रहणसिद्धान्तः

4. 21. 1. छादक इन्दुर्भानोः, शशिनो महती च भूच्छाया ।
अर्कात् सप्तमराशौ भूच्छाया वर्ततेऽर्कसमभुक्तिः ॥
छाद्यस्य शीघ्रगत्वात् प्राक् स्पर्शः पश्चिमे विधोर्मोक्षः ।
छाद्यस्याल्पगतित्वाद् विपरीतं तद्द्वयं रवेर्भवति ॥ ४२ ॥
छादकवृत्तं भानोः स्वल्पमतस्तीक्ष्णशृङ्गता भवति ।
छादकवृत्तं शशिनो महदेवातोऽस्य कुण्ठशृङ्गत्वम् ॥४३॥
(Para, *Gola D*, 2. 41-43)

Theory of Eclipses

(Really) the Moon is the hiding object of the Sun, and (the hiding object) of the Moon is the huge shadow of the Earth. This shadow of the Earth will (always) be at the seventh Sign from the Sun, moving with a velocity equal to the Sun's. (41)

In the case of the Moon, since the eclipsed object (*viz.*, the Moon) is faster, the beginning of the eclipse is at the east, and the end at the west. In the csae of the Sun's (eclipse), due to the slower motion of the eclipsed body (*viz.*, the Sun), these two, (the beginning and the end), are the other way. (42)

The orb hiding the Sun, (*viz.*, the projection of the Moon), is small and hence sharp horns are formed. The orb hiding the Moon, (*viz.*, the Earth's shadow), is big and hence it is blunt-horned. (43). (KVS)

भगोलः

4. 22. 1. भूव्यासार्धेनोनं दृश्यं देशात् समाद् भगोलार्धम् ।
अर्धं भूमिच्छन्नं भूव्यासार्धाधिकं चैव ॥ १५ ॥
देवाः पश्यन्ति भगोलार्धमुदङ्मेरुसंस्थिताः सव्यम् ।
अर्धं त्वपसव्यगतं दक्षिणबडवामुखे प्रेताः ॥ १६ ॥
रविवर्षार्धं देवाः पश्यन्त्युदितं रविं तथा प्रेताः ।
शशिमासार्धं पितरः शशिगाः कुदिनार्धमिह मनुजाः ॥
(Āryabhaṭa I, *ABh.*, 4.15-17)

Sphere of the asterisms

One half of the Bhagola as diminished by the Earth's semi-diameter is visible from a level place (free from any obstructions). The other one-half as increased by the Earth's semi-diameter remains hidden by the Earth. (15)

The gods living in the north at the Meru mountain (*i.e.*, at the north pole) see one half of the Bhagola as revolving from left to right (or clockwise); the demons living in the south at the Baḍavāmukha (*i.e.*, at the south pole), on the other hand, see the other half as revolving from right to left (or anticlockwise). (16)

The gods see the Sun, after it has risen, for half a solar year; so is done by the demons too. The manes living on (the other side of) the Moon see the Sun for half a lunar month; the men here see it for half a civil day. (17). (KSS)

खगोलः

4. 23. 1. पूर्वापरमध ऊर्ध्वं मण्डलमथ दक्षिणोत्तरं चैव ।
क्षितिजं समपार्श्वस्थं भानां यत्रोदयास्तमयौ ॥ १८ ॥
पूर्वापरदिग्लग्नं क्षितिजादक्षाग्रयोश्च लग्नं यत् ।
उन्मण्डलं भवेत्तत् क्षयवृद्धी यत्र दिवसनिशोः ॥ १९ ॥
पूर्वापरदिग्रेखाधश्चोर्ध्वा दक्षिणोत्तरस्था च ।
एतासां सम्पातो द्रष्टा यस्मिन् भवेद् देशे ॥ २० ॥
ऊर्ध्वमधस्ताद् द्रष्टुर्ज्ञेयं दृङ्मण्डलं ग्रहाभिमुखम् ।
दृक्क्षेपमण्डलमपि प्राग्लग्नं स्यात् त्रिराश्यूनम् ॥ २१ ॥
(Āryabhaṭa I, *ABh.*, 4.18-21)

Sphere of the Sky, Celestial Sphere

The vertical circle which passes through the east and west points is the Prime vertical, and the vertical circle passing through the north and south points is the meridian. The circle which goes by the side of the above circles (like a girdle) and on which the stars rise and set, is the horizon. (18)

The circle which passes through the east and west points and meets (the meridian above the north point and below the south point) at distances equal to the latitude (of the place) from the horizon is the equatorial horizon (or six o'clock) circle on which the decrease and increase of the day and night are measured. (19)

[1] *Nṛ* is a unit of length whose measure is equal to the height of a man, assumed to be 96 aṅgulas, or 4 cubits.

The east-west line, the nadir-zenith line, and the north-south line intersect where the observer is. (20)

The great circle which is vertical in relation to the observer and passes through the planet is the *dr̥ṅmaṇḍala* (*i.e.*, the vertical circle through the planet). The vertical circle which passes through that point of the ecliptic which is three Signs behind the rising point of the ecliptic is the *dr̥kkṣepavr̥tta*. (21). (KSS)

अपक्रमवृत्तम्

4. 24. 1. मेषादेः कन्यान्तं सममुदगपमण्डलार्धमपयातम् ।
तौल्यादेर्मीनान्तं शेषार्धं दक्षिणेनैव ॥ १ ॥
ताराग्रहेन्दुपाता भ्रमन्त्यजस्रमपमण्डलेऽर्कश्च ।
अर्काच्च मण्डलार्धे भ्रमति हि तस्मिन् क्षितिच्छाया ॥२॥
अपमण्डलस्य चन्द्रः पाताद् यात्युत्तरेण दक्षिणतः ।
कुजगुरुकोणाश्चैवं शीघ्रोच्चेनापि बुधशुक्रौ ॥ ३ ॥

(Āryabhaṭa I, *ABh.*, 4. 1-3)

Ecliptic

One half of the ecliptic, running from the beginning of the sign Aries to the end of the sign Virgo, lies obliquely inclined (to the equator) northwards. The remaining half (of the ecliptic) running from the beginning of the sign Libra to the end of the sign Pisces, lies (equally inclined to the equator) southwards. (1)

The Nodes of the star-planets (Mars, Mercury, Jupiter, Venus and Saturn) and of the Moon incessantly move on the ecliptic. So also does the Sun. From the Sun, at a distance of half a circle, moves thereon the Shadow of the Earth. (2)

The Moon moves to the north and to the south of the ecliptic (respectively) from its (ascending and descending) Nodes. So also do the planets Mars, Jupiter and Saturn. Similar is also the motion of the *śighroccas* of Mercury and Venus. (3). (KSS)

4. 24. 2. भगोलमध्यवृत्तार्धे वायुगोलस्य मध्यतः ।
अपक्रान्ते चतुर्विंशत्यंशैः सौम्येतरराशयोः ॥ १६ ॥

(Nīlakaṇṭha, *SiDar.*, 16)

The two halves of the central circle of the celestial globe (i.e. the ecliptic) move away, respectively, towards the north and the south from the central (circle) of the *Vāyugola* (viz. the *Viṣuvan-maṇḍala* or celestial equator) by 24 degrees. (16). (KVS)

अर्काग्रादीनां स्वरूपम्

4. 25. 1. क्षितिजे यत्रार्कयुतिस्तस्मात्पूर्वापराख्यसूत्रान्ता ।
जीवार्काग्राऽस्तोदयसूत्रं पूर्वापरं तदग्राच्च ॥ १४ ॥
उन्मण्डलार्कयोगप्रागपरस्वस्तिकान्तरालज्या ।
क्रान्तिज्या, सात्र भुजा, सूर्यस्थितभागगं द्युवृत्तं यत् ॥१५॥
तद्व्यासार्धं द्युदलं कोटिः सा स्यात्, त्रिजीवका कर्णः ।
यस्मिन्वृत्ते सूर्यस्थितभागो भ्रमति तद् द्युवृत्ताख्यम् ॥
क्षितिजोन्मण्डलविवरे द्युवृत्तजीवाभुजा क्षितिज्याख्या ।
क्रान्तिज्यात्र तु कोटिः स्यादर्काग्रा श्रुतिस्त्रिभिस्त्र्यश्रम् ॥

(Para., *GolaD.*, 2.14-17)

Sun's Amplitude, etc.

Sine Amplitude (*arkāgrā*) is the distance in the shape of the sine of the point from where the Sun touches the horizon (i.e., points of rising and setting) to the east-west line.[1] (14a)

The *Astodaya-sūtra*, or the line of rising and setting, is the east-west line joining the two points (of rising and setting, respectively). (14b)

Sine Declination (*Krāntijyā*) is the sine of the (angular) distance between the point where the Sun touches the *Unmaṇḍala* and the east or west point (*Svastika*). Let this be the *bhujā* (altitude). Let the radius of the diurnal circle on which the Sun is, *viz.*, cos. declination (*dyu-dala*) be the *koṭi* (base); the radius of the sphere will (then) be the hypotenuse (*karṇa*) of the right-angled triangle. (15-16a)

The circle along which the position of the Sun moves is called the Diurnal circle (*dyuvr̥tta*). (16b)

The sine of the (arc of the) diurnal circle between the horizon and the *Unmaṇḍala* is called *Kṣitijyā*. Let it be the *bhujā*. Let sine declination be the *koṭi* here. Then sine amplitude will be hypotenuse; these three make a (right-angled) triangle. (17). (KVS)

नक्षत्रपरिधिः

4. 26. 1. रवेः षष्टिगुणो भानि प्रत्यगीरयति ह्यतः ॥ १५ ॥

(Nīlakaṇṭha, *Si.Dar.*, 15b)

Orbit of the stars

(The Pravaha wind) is driving the stars in a westerly direction (on an orbit equal to) sixty times that of the Sun. (15). (KVS)

आकाशकक्ष्या ग्रहकक्ष्याश्च

4. 27. 1. 'वेदाश्विराम 'गुणितान्ययुताहतानि
चन्द्रस्य शून्यरहितान्यथ मण्डलानि ।
स्वैः स्वैर्हृतानि भगणैः क्रमशो ग्रहाणां
कक्ष्या भवन्ति खलु योजनमानदृष्ट्या ॥ ३५ ॥

(Bhāskara I, *M.Bh.*, 7.35)

[1] This angle of amplitude is subtended at the centre of the Earth (or the Earth's surface, which makes little difference) by the point of rising or setting and the east or west point respectively. At the equator, it is equal to the declination and at other places it is a function of the declination and the latitude of the place; sin amplitude=sin decl. sec. lat.

Circle of the sky and the orbits of planets

Multiply the revolutions of the Moon (in a *yuga*) by 32,40,000 and then discard the zero at the unit's place; (this is the length of the circle of the sky in terms of *yojanas*).

(Severally) divide that by the revolutions of the planets (in a *yuga*); thus are obtained the lengths of the orbits of the respective planets in terms of *yojanaṣ*. (35).[1] (KSS)

ब्रह्माण्डमानम्

4. 28. 1. कोटिघ्नै'र्नखनन्दषट्कनखभूभूभृद्भुजङ्गेन्दुभिर्'
ज्योतिश्शास्त्रविदो वदन्ति नभसः कक्षमिमां योजनैः ।

(Bhāskara II, *SiŚi*, 2.2.67)

Circumference of the sky

Astronomers state that the circumference of the sky is 18,71,20,69,20 multiplied by one crore (i.e. 18,71,20,69,20,00,00,000) *yojanas*. (67). (KVS)

उल्का धूमकेतुश्च

4. 29. 1. शं नो भूमिर्वेप्यमाना शम् उल्कानिर्हतं च यत् ।
शं नो ग्रहाश्चन्द्रमसा शम् आदित्यश्च राहुणा ।
शं नो मृत्युर् धूमकेतुः शं रुद्रास्तिग्मतेजसः ।।

(*AV* 19.9.8-10)

[1] It may be noted that one *yojana* according to the Midnight System is two-thirds of a *yojana* according to the Sunrise system.

Meteors and Comets

Weal for the quaking Earth, and weal for the meteor-smitten. Weal for us be the planets belonging to the Moon and weal the Sun with Rāhu. Weal for us the deadly comet, weal the Rudras of keen brightness. (8-10)

4. 29. 2. दक्षिणां दिशमास्थाय धूमकेतुः स्थितोऽभवत् ।
अनिशं चाप्यविच्छिन्ना ववुर्वाताः सुदारुणाः ।। ४६ ।।

(*Harivaṃśa Purāṇa*, Cr. Edn., 106.46)

Comets, Meteors etc.

A comet stood forth towards the south; strong winds blew for long and incessantly. (46). (KVS)

4. 29. 3. दर्शनमस्तमयो वा न गणितविधिनास्य शक्यते ज्ञातुम् ।
दिव्यान्तरिक्षभौमास्त्रिविधाः स्युः केतवो यस्मात् ।।

(Varāha, *Bṛ.Saṃ.*, 11.2)

It is not possible to determine by calculation the rising or setting of the comets, since there are three categories of comets, viz. Celestial, Atmospheric and Terrestrial.[1] (2). (M.R. Bhat)

[1] In ch. 11 of his *Bṛhatsaṃhitā*, Varāhamihira takes note and gives the characteristics of one thousand comets, the burden being primarily to indicate the good and evil effects of their appearance.

5. मिथ्याज्ञानम् – ERRONEOUS NOTIONS

मिथ्याज्ञानं तन्निराकरणप्रयोजनं च

5. 1. 1. संस्थानमशेषमीरितं ब्रह्माण्डोदरवर्तिनामिदम् ।
प्रतिवादिवचांसि शृण्वतो मनसि भ्रान्तिरिवावतिष्ठते ॥

यत एवमतः कुहेतुमद्वचनानि प्रथमं ब्रवीम्यहम् ।
उपपत्तिमदागमं ततो मनसः स्थैर्यकरं परिस्फुटम् ॥ २ ॥

(Lalla, *SiDhVṛ.*, 20. 1-2)

Need for removing erroneous notions

Different situations have been ascribed (by different people) on matters relating to the universe. When one listens to the different statements made by opposing schools, doubts, as it were, will assail one's mind. (1)

For this reason, I shall first enumerate the irrational views; and then I shall put forward the truths together with proofs. That will make things clear and thus steady the mind. (2). (BC)

मिथ्याज्ञानानि

5. 2. 1. असुरामरवासरं क्रमादयनं दक्षिणमुत्तरं जगुः ।
हिमदीधितितिग्मतेजसोर्ग्रहणं राहुकृतं तथापरे ॥ ३ ॥

उपरीन्दुमधो दिवाकरं तमसा मेरुभुवा विभावरी ।
प्रतिवासरमिन्दुमण्डलं विबुधैः पीयत इत्यतः कृशम् ॥ ४ ॥

ककुभश्च सुमेरुभूभृतो युगलं चन्द्रमसोस्तथार्कयोः ।
पितृवासरमादितोऽसितं सितपक्षं च वदन्ति शर्वरीम् ॥ ५ ॥

अमितामवनीं प्रचक्षते सुसमां केचन दर्पणोपमाम् ।
अपरे बहुयोजनामिमां सलिलस्थामथ यानपात्रवत् ॥ ६ ॥

कमठाहिवराहदिग्गजैः कुलशैलैर्विधृतामथापरे ।
जगुरूर्ध्वमथ प्रयात्यधो रथचक्रभ्रमवद् भ्रमत्यपि ॥ ७ ॥

(Lalla, *Si Dh Vṛ.*, 20. 3-7)

The several erroneous notions

Some opine that the day of the demons (at the South pole) starts when the Sun goes (from the northern to) the southern solstice; and, that of the gods (at the North pole) starts, when the Sun goes from there towards the northern solstice.

Again, others say that the solar eclipse and the lunar eclipse are caused by Rāhu. (3)

Some say that the Moon is above and the Sun is below. Some others say that the night is caused by the darkness due to the mountain Meru. Yet, others say that the disc of the Moon decreases daily because it is being drunk by the gods. (4)

Some people state that the mountain Meru has directions. Others think that there are two Suns and two Moons. Again, some say that the day of the manes commences with the dark half of the lunar month, and the night, with the bright half. (5)

Some think that the earth is infinite; others, that it is plane as a mirror. Again, others say that it extends to many *yojanas* and floats on water like a boat. (6)

Some opine that the Earth is supported by a tortoise, a serpent, a boar, an elephant or by mountain ranges. Again, others say that the Earth goes up and down; and some say that it goes round and round like the wheel of a chariot.[1] (7). (BC)

अयनोपरि मिथ्याज्ञाननिराकरणम्

5. 3. 1. मिथुनान्तसमीपसंस्थितो यदि यज्ञांशभुजां दिवाकरः ।
सततं समुपैति दृक्पथं न कुलीरे वद केन हेतुना ॥ ८ ॥

अपमस्य वशात् समुन्नमन् क्रमशो यत्र स यत्र वीक्षितः ।
विनमन्नपि तद्वशात् तथा विपरीतं च कथं न दृश्यते ॥ ९ ॥

(Lalla, *ŚiDh Vṛ.*, 20. 8-9)

Doubts on Solstice removed

If the Sun is always visible to the gods when it is near the end of Gemini, then, for what reason, pray, it is not visible when in Cancer? (8)

If the Sun is visible to the gods while ascending due to (the increase) in the declination, why should it not be

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[1] In these verses Lalla mentions some of the beliefs prevailing then with regard to astronomical phenomena. Most of these beliefs occur in the Purāṇas.

The views are as follows :

1. The day of the observers at the North pole begins when the Sun starts towards the summer solstitial point and that of the observers at the South pole begins, when it starts towards the winter solstitial point.
2. Night falls when the mountain Meru covers the Sun.
3. Directions can be determined at Meru.
4. The dark half of the lunar month is the day of the manes and the light half is their night.
5. Rāhu, a mythical demon or snake, is the cause of eclipses.
6. The Moon's orbit is above that of the Sun.
7. The illuminated portion of the Moon decreases because it is being sucked by the gods.
8. The Earth is infinite.
9. The Earth is plane like a mirror.
10. The Earth is supported in various ways.
11. The Earth moves.
12. There are two Suns and two Moons.

visible to them while descending through the same declination. (9)[1]. (BC).

निशासम्बन्धि मिथ्याज्ञाननिराकरणम्

5. 4. 1. रज्ज्वा सहितो यथा हयो भ्रममाणो निकटेऽथ दूरगः ।
तद्वद् रविमीक्षते सुरा रजनीचक्रपदे समं कथम् ।। १० ।।
पिहिते यदि मेरुणा रवौ मनुजानामिह शर्वरी भवेत् ।
कथयस्व कथं सुधाभुजां रजनी जायत इत्ययुक्तिमत् ।।
दिनभर्तरि याति दक्षिणां वणिजो मेरुसदां सदा निशा ।
सौम्यां सुरविद्विषामजादपरां भूपिहिते नृणामपि ।। १२ ।।
विपरीतदिशा दिवाकराच्छाया दीर्घतरा तरोर्यथा ।
अवनेरपि जायते तथा सुरदैतेयनृणां तथा निशा ।। १३ ।।

(Lalla, *ŚiDh Vṛ.*, 20. 10-13)

Doubts on nightfall removed

How is it that the gods see the same Sun, even when the wheel of darkness (approaches the world), just as one sees a horse attached to a rope going round and round, sometimes near and sometimes at a distance. (10)

If the night of the men in this world is caused by the Meru covering the Sun, tell me how the night of the gods, (who live on the Meru) and feed on nectar, is caused? (Therefore the statement) is irrational. (11)

When the Sun goes to the southern side (of the ecliptic beginning with) Libra, it is always night for the dwellers of Meru. When it moves along the north (of the ecliptic, beginning with) Aries, (it is continuous night) for the demons. When the Sun is hidden by a part of the Earth, it is night also for men. (12)

Just as the shadow of a tree is longer when it is directly opposite to the Sun than at other times, so is the shadow of the part of the Earth (directly) opposite to the Sun. (That shadow causes night in the part which is away from the Sun). (I have thus explained how) the nights of the gods, demons and men come about.[2] (13). (BC.)

मेरुसम्बन्धि मिथ्याज्ञाननिराकरणम्

5. 5. 1. यदुदेति रविः समन्ततः कथमाशास्तदमर्त्यभूभृतः ।
प्राची प्रथमोदयेन चेत् तत्रैवास्तमयेन का भवेत् ।। १४ ।।
केषांचिदुदग् दिवाकरः पुरतः पृष्ठगतः परस्य च ।
इतरस्य तदैव दक्षिणः परमार्थात् ककुभो न कुत्रचित् ।।

(Lalla, *ŚiDhVṛ.*, 20. 14-15)

Doubts on Meru cleared

How can Meru, the dwelling place of the gods, have directions when the Sun rises all round it? If the point where the Sun rises first is called the east point, what will it be called, when the Sun sets there? (14)

The Sun is simultaneously to the north of some and to the south of others. It is again in front of some and behind others. Thus, in reality, there are no directions (for the Meru). (15).[1] (BC)

पितॄणां चन्द्रदर्शनम्

5. 6. 1. सवितारमवेक्ष्य शीतगोरुपरिष्टात् पितरः स्वमूर्ध्वगम् ।
असितान्ततिथौ ततः परं न च पश्यन्ति कथं सिते दले ।।

(Lalla, *ŚiDhVṛ.*, 20.16)

Visibility of the Moon to the manes

The manes, living in the upper part of the Moon, see the Sun on their zenith on the day of the new moon. That being so, how is it that they do not see it from the middle of the light half of the lunar month?[2] (16). (BC)

रवेरधश्चन्द्र इति निरूपणम्

5. 7. 1. यदि शीतगुरूर्ध्वमुष्णरश्मे-
रुडुवत् किन्नु भवेत् सदा सितोऽसौ ।
ग्रहणं स भवेद्रवेर्न चेन्दो-
र्भूच्छायाग्रमपास्य दूरगस्य ।। २८ ।।

(Lalla, *ŚiDhVṛ.*, 20.28)

Assertion that the Moon is below the Sun

If the Mooon is above the Sun, will it not always appear illuminated like a star? (If the Moon were above the Sun) there would be no solar eclipse; there would not even be a lunar eclipse, as the Moon would then be (further) away from the extremity of the earth's shadow.[3] (28). (BC)

चन्द्रसम्बन्धि मिथ्याज्ञाननिराकरणम्

5. 8. 1. अथ शापवशात् परिक्षयः स्याद्
विबुधैर्वा शशिनो निपीतमूर्तिः ।
गणितेन चयक्षयौ कथं स्तो
यदि पीतश्च समीक्ष्यते स कृष्णः ।। २९ ।।

(Lalla, *ŚiDhVṛ.*, 20. 29)

[1] These verses refute the first belief. According to this belief, observers at the North pole can see the Sun travelling from Capricorn to Cancer and not from Aries to Libra. Lalla's argument is that if the observers at the North pole can see the Sun descending from Aries to Gemini, they should also see it as descending from Cancer to Virgo when the Sun travels exactly along the same diurnal circles. So the current belief cannot be correct.

[2] These refute the second belief. Lalla maintains that the night is not caused by Meru but by the shadow of the Earth.

[1] This refutes the third belief. Lalla says that no direction can be determined at Meru, because there the observer's horizon coincides with the celestial equator and hence there is no east point.

[2] This refutes the fourth belief. Lalla states that the whole of the light half of the lunar month cannot be the day of the manes, because they do not see the Sun after the eighth day.

[3] This refutes the sixth belief. Lalla says that if the Moon were above the Sun, it would always be illuminated like a star. Moreover, it could neither cause a solar eclipse nor could it be obscured by the Earth's shadow.

Views about the Moon corrected

If the Moon decreases because of some curse or because it is being sucked by the gods, how can the increase and decrease (in the illuminated portion) be determined by calculation? (And, moreover, if it is being sucked), it would appear completely dark when fully sucked.[1] (29). (BC)

भूरनन्तेति मिथ्याज्ञाननिराकरणम्

5. 9. 1. अमिता यदि भूरियोजना वा
क्षितिरह्ना परिवर्त्यते कथं भैः ।
परिधेः खलु षोडशे स्थितांशे
न च लङ्काविषयाद् भवत्यवन्ती ।। ३० ।।
ग्रहणं ग्रहसङ्गमोदयौ
शशिशृङ्गोन्नतिरिष्टभाविधिः ।
स्यात् प्रत्ययपञ्चकं स्फुटं
क्षितिमानेन भवेत् महत् कथम् ।। ३१ ।।
'खखषट्कुयमै'र्मिताः कला
ग्रहचक्रे वलये व्यवस्थिताः ।
स्वफलस्य कुवृत्तयोजनै-
रनुपातादमिता कथं मही ।। ३२ ।।
यदि वृत्तवशेन गच्छता-
ममिता भात्यथ भूरियोजना ।
परितस्तु तदा तथाविधा
परिमाणं त्विदमेव नापरम् ।। ३३ ।।

(Lalla, *ŚiDhVṛ.*, 20.30-33)

Doubts on the size and shape of the Earth clarified

If the Earth is infinite (in size) or if it extends up to innumerable *yojanas*, how can the celestial sphere go round it once a day?

Morever, Avanti could not then be at a distance of 1/16 of the circumference (of the Earth) from the meridian of Laṅkā. (30)

The eclipse, the conjunction and rising (and setting) of planets, the cusps of the Moon and the length of the shadow (of the gnomon) at any time—the calculation of all these five (phenomena) depends upon the measurement of the Earth. (And the calculated results) agree with the observed results. So how can the Earth be (infinitely) large? (31)

By using the *grahaphala* (*grahagati*) and the above circumference of the Earth in *yojanas*, it is found by means of simple proportion that the circumference of the circle of one revolution of a planet consists of 21,600 minutes. Then how can the Earth be of infinite size? (32)

[1] This refutes the seventh belief. If the Moon's daily decrease were due to its being sucked by the gods, mathematics would be of no use in computing its light and dark portions.

The Earth may appear to be of infinite size or extending up to innumerable *yojanas* because of its being spherical. But its circumference and diameter are the same as given before and in no way different.[1] (33).

भूः समतलेति मिथ्याज्ञाननिराकरणम्

5. 9. 2. यदि दर्पणवत् समा मही
कथमम्भो गगनात् परिच्युतम् ।
स्थिरतामपहाय यात्यहो
जवमाश्रित्य दिशैकया महत् ।। ३४ ।।
प्रगुणः परिधेः शतांशको
गणितज्ञाः कथयन्ति दृश्यते ।
प्रतिभाति तदा समा मही
विषये यत्र तथैव गम्यते ।। ३५ ।।
समता यदि विद्यते भुव-
स्तरवस्तालनिभा बहूच्छ्रयाः ।
कथमेव न दृष्टिगोचरं
नुरहो यन्ति सुदूरसंस्थिताः ।। ३६ ।।
परितः क्षितिजे प्रदृश्यते
गगनं सङ्गमुपागतं नृभिः ।
तस्यावनिरन्तरे स्थिता
सुसमा दर्पणवद् विभाव्यते ।। ३७ ।।

(Lalla, *ŚiDhVṛ.*, 20. 30-37)

If the Earth is plane as a mirror why does not the water falling from the sky remain static, instead of, alas, flowing with a great speed in one direction? (34)

Mathematicians say that one hundredth of the circumference of the Earth appears to be plane, So, that portion of Earth *appears* to be plane to an observer. (35)

If the Earth is level, why cannot tall trees, like *tāla* (date palm), alas, be seen by man, though at a very great distance from the observer? (36)

To men the sky appears to meet (the Earth) all round horizon. So, the Earth thus surrounded appears to be plane like a mirror. (37).[2] (BC)

भुव आधारसम्बन्धि मिथ्याज्ञाननिराकरणम्

5. 10. 1. सलिले विलयो मृदो भवे-
दिति गोरप्सु न युज्यते स्थितिः ।

[1] These refute the eighth belief. Lalla says that the Earth could not be infinitely large, because the sphere of the fixed stars could not go round it one in day. Moreover, the measurements of the Earth, as given ar. correct because the calculated results relating to the various pnenomena based on these measurements tally with the observed results.

[2] These refute the ninthe belief. Lalla maintains that the Earth is spherical and not a plane. But as only a small portion of it is visible at a time, that may be the reason for its appearing as level as a mirror. If it were level, the tops of high trees could be seen even from a great distance.

अथ पात्रगतेति तत् कथं
न भवेद्यावदिलैव पार्थिवम् ।। ३९ ।।

यदि वाम्भसि संस्थिता मही
सलिलं तद् द्युवदप्रतिष्ठितम् ।
गुरुणोऽम्भसि चेत् स्थितिर्भवेत्
क्षितिगोलस्य न किं विहायसि ।। ४० ।।

कमठादिभिरुद्धृता मही
यदि ते केन धृता नभःस्थिताः ।
अत एषां वियति स्थितिर्यदि
क्षितिगोलस्य तु केन वार्यते ।। ४१ ।।

(Lalla, *ŚiDhVṛ*., 20. 39-41)

Doubts on the Earth's situation cleared

Clay is destroyed by water. So it is not possible for the Earth to remain in water. Again, (the statement) that the Earth (floats on water) like a boat, cannot be correct, as the Earth is itself made of clay. (39)

Supposing that the Earth is on water, the water, like the sky, is also unsupported. If the heavy sphere of Earth can remain on water, (which is unsupported), why can it not remain in space? (40)

If the Earth is supported by tortoise and other things, by whom are they supported in space? If these can remain in space (unsupported), what prevents the Earth from remaining thus (i.e. unsupported)? (41).

भुवश्चलननिराकरणम्

5. 11. 1. यदि च भ्रमति क्षमा तदा
स्वकुलायं कथमाप्नुयुः खगाः ।
इषवोऽभिनभः समुज्झिता
निपतन्तः स्युरपांपतेर्दिशि ।। ४२ ।।
पूर्वाभिमुखे भ्रमे भुवो वरुणाशाभिमुखो व्रजेद् घनः ।
अथ मन्दगमात्तदा भवेत् कथमेकेन दिवा परिभ्रमः ।।

(Lalla, *ŚiDhVṛ*., 20. 42-43)

If the Earth rotates, how could birds come back to their nests. Moreover, arrows shot towards the sky, would fall towards the west. (42)

If the Earth rotates to the east, the clouds would move to the west. If (it is said) that the Earth moves slowly, then how can it go round (the universe) in one day?[1] (43)

भुवश्चलनोपरि मिथ्याज्ञानम्

5. 11. 2. यदि गच्छति भूरधोमुखी
गगने क्षिप्तमुपैति नो महीम् ।
यदि चोर्ध्वमुपैति सा तदा
निकटं किं न भवेद् भपञ्जरः ।। ३८ ।।

(Lalla, *ŚiDhVṛ*., 20. 38)

Doubts on the Earth's motion clarified

If the Earth goes downwards, (anything) thrown towards the sky will not fall on the Earth. If the Earth is (continually) moving upwards, why does not the sphere of constellation come nearer?[1] (38). (BC).

रविचन्द्रद्वितयमिथ्याज्ञाननिराकरणम्

5. 12. 1. द्वितयं दिनरात्रिनाथयोः
कथमेकान्तरितं तदुद्व्रजेत् ।
यदि किं ध्रुवतारकातिमे-
दिवसेनैव भवेत् परिभ्रमः ।। ४४ ।।

(Lalla, *ŚiDhVṛ*., 20. 44)

Views on two Suns and Moons corrected

If there are two Suns and two Moons, which rise alternately, how can the circumpolar constellations complete their revolution in one day![2] (44). (BC).

ग्रहणसम्बन्धि मिथ्याज्ञाननिराकरणम्

5. 13. 1. उदयेऽस्तमये यथा नरै-
र्ग्रहणं तिग्मकरस्य दृश्यते ।
निकटोत्तरदिग्गते विधौ
किल तद्वद्विबुधैर्न तत् तथा ।। १७ ।।
अरिणा हरिणा किलामृतं
पिबतो देवरिपोर्दितं शिरः ।
व्यसुतां न गतं ग्रहाधिपो
ग्रसितः प्राहुरिदं च राहुवत् ।। १८ ।।
रविशीतगुमण्डलाकृति द्युगतं कृष्णतया न दृश्यते ।
किल पर्वणि याति दर्शनं वरदानादरविन्दजन्मनः ।।१९।।
मुखपुच्छयुतं भुजङ्गमं जगुरेके तम एव केवलम् ।
अपरेऽसुरमेव यायिनं द्वयमन्येऽथ परे यथेष्टगम् ।। २० ।।
यदि काययुतोऽथ मण्डली भविचारी खचरः शिरोऽथवा ।
नियतां गतिमुद्वहन् कथं भगणार्धान्तरितो ग्रसत्यगुः ।।
असुरो यदि मायया युतो नियतोऽतिग्रसतीति ते मतम् ।
गणितेन कथं स लभ्यते ग्रहकृत्पर्व विना कथञ्चन ।।

[1] These refute the tenth belief that the Earth is supported by an external energy. Lalla says that the Earth remains unsupported, suspended in space. If it were supported by something, the latter, in its turn, would have to be supported by something else and so on. Then there would be no end of supporters, and that is not possible.
Lalla asserts neither could the Earth be moving from east to west nor from west to east. If it did, the birds would not be able to find their nests. Lalla tries to refute here Āryabhaṭa's theory that the Earth rotates from west to east.

[1] These refute the eleventh belief. The Bauddhas maintain that the Earth was falling down in space. So Lalla says that if it were so, how could a thing thrown up come down on the same place of ground. Again, if the Earth were continuously moving up, the constellations would be nearer every moment.

[2] This refutes the twelth belief held by the Jainas.

यदि राहुयुते मिथोऽस्तगे क्षितिजस्थे हिमधाम्नि सग्रहे ।
इतरेण कथं दिवाकरो दितबिम्बः समगेन नेक्ष्यते ।।२३।।
पुच्छेन मुखेन यद्यहिर्ग्रसतीति प्रतिवादिनो जगुः ।
मध्ये मुखपुच्छयोः स्थितं भगणार्धं न समावृणोति किम् ।।
प्रथमं रविमण्डलं यतो न ततः खण्डितमिन्दुमण्डलम् ।
न समाकृतिरीक्ष्यते स्थितिर्यदतो राहुकृतो न स ग्रहः ।।
सवितुश्च यदन्यथाऽन्यथा प्रतिदेशं सकलं समीक्ष्यते ।
न च कुत्रचिदित्यवेत्य कः कुरुते राहुकृते ग्रहे ग्रहम् ।।
(Lalla, *ŚiDhVṛ.*, 20. 17-26)

Eclipses : Refutation of unscientific views

Men see a solar eclipse either at sunrise or at sunset, when the Moon is a little to the north. But the gods cannot see it then. (17)

It is known that when a demon, an enemy of the gods, was drinking the nectar, his head was chopped off by his enemy Hari. But the head did not die. Some say this is Rāhu. The Sun (and the Moon) are devoured by it. (18)

(Some say that) Rāhu is round like the discs of the Sun and Moon, and being dark cannot (always) be seen in the sky. It is seen only on the new and full moon days and that too due to the boon of Brahmā. (19)

Some opine that the cause (of an eclipse) is a snake with head and tail. Others say it is only darkness. Again, some say that it is a moving demon, and others that there is a pair of them. And yet others say that it goes wherever it wishes. (20)

If Rāhu has a body or is a disc, or a head or is a planet in the sky, and if it is always moving, why should it swallow (the Moon) only at a distance of 6 Signs (from the Sun)? (21)

If you are of the opinion that an artful demon is always the cause of eclipses by swallowing (the Sun or Moon), then how is it that an eclipse can be determined by means of calculation? Moreover, why is there not an eclipse on a day other than the day of new or full moon? (22)

If a lunar eclipse takes place on the western horizon and is caused by Rāhu, then why does not the Sun's disc appear to be swallowed by the second Rāhu of the same speed? (23)

If the opponents say that it is a snake which causes eclipses by means of its head and its tail, then why does it not cover half of the circle between the head and the tail. (24)

An eclipse cannot be caused by Rahu, because the sides of the discs of the Sun and Moon, which are first to be eclipsed, are not the same; nor are the portions eclipsed the same; and nor even are the durations the same. (25)

In a solar eclipse, people at different parts (of the Earth) see different portions of the Sun eclipsed. Some do not see (the eclipse) at all. Knowing this, who can maintain that an eclipse is caused by Rāhu?[1] (26). (BC)

5. 13. 2. ग्रसति यदि राहुरर्कं चन्द्रं वा राहुसप्तमर्क्षस्थौ ।
चन्द्रार्कौ दूरगतौ ग्रस्येते दूरगेण कथमहिना ।। ३८ ।।
चक्रार्धान्तरितौ द्वौ राहू इति चेत्तदापि कथमिन्दोः ।
स्पर्शः प्राग्भागे स्यान्मोक्षः पश्चात्ततोऽन्यथा च रवेः ।।
राहोर्वदनगतश्चेत् तीक्ष्णविषाणः कथं रविर्भवति ।
तत्र गतः शीतांशुः कुण्ठविषाणश्च दृश्यते कस्मात् ।।
(Par., *Gola. D.* 2. 38-40)

If it is presumed that Rāhu swallows the Sun or the Moon, how is it that the Sun or the Moon at the seventh Sign from it is not also swallowed by Rāhu which is far from it? (38)

If, however, two Rāhu-s are presumed at a distance of 6 Signs (from one another), how is it then that in the case of the Moon, the first contact (*sparśa*) is at its east and the last contact (*mokṣa*) at its west, while for the Sun it is the reverse (i.e., *sparśa* in the west and *mokṣa* in the east)?[2] (39)

Again, if the Sun is gripped in Rāhu's mouth, how does it appear sharp-horned, while the Moon in the same position appears blunt-horned? (40). (KVS)

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[1] These refute the fifth belief. The Paurāṇika story goes that a demon in disguise was drinking the nectar churned by the gods from the milky ocean. The Sun and Moon pointed him out to Hari who severed his head from his body. But the head did not die. It remained immortal since it had drunk the nectar. Out of spite, this head swallows the Sun and the Moon and thus causes eclipses. Another belief is that of a snake causing the eclipse.

Lalla refutes these views by pointing out that since there are differences with regard to the duration, obscured portion, etc., between a solar and a lunar eclipse, the obscuring body in both the cases cannot be the same. Moreover, why should Rāhu choose only the new and full moon days for eclipsing the Sun and the Moon, respectively. Again, if Rāhu were the cause, calculation would be of no use in determining an eclipse.

[2] The idea is that if Rāhu swallows the Moon and the Sun, since their direction of motion is the same, *viz.*, eastwards, Rāhu should swallow them both in the same manner, i.e., from the east end, which is not actually the case.

6. संख्यानम् – NUMERATION

संख्यानस्य अनिवार्यता

6. 1. 1. लौकिके वैदिके वापि तथा सामयिकेऽपि यः ।
व्यापारस्तत्र सर्वत्र संख्यानमुपयुज्यते ।। ६ ।।
बहुभिर्विप्रलापैः किं त्रैलोक्ये सचराचरे ।
यत्किञ्चिद् वस्तु तत्सर्वं गणितेन विना नहि ।। १६ ।।

(Mahāvīra, *Gaṇitasārasaṅgraha*, 1.9, 16)

Need of numerals for calculation

In worldly life or Vedic matters, or even in religious practices, whatever be the dealings, everywhere enumeration is essential. (9)

Why say much. In the three worlds, living or nonliving, whatever is to be transacted, that cannot be done without calculation. (16). (KVS)

6. 2. 1. स्थानात् स्थानं दशगुणमेकस्माद् गुण्यते द्विज ।

(*Viṣṇu Purāṇa*, Aṁśa 6, ch. 3, Verse 4)

Oh brāhmaṇa! from one place to the next one, the places are multiples of ten. (KVS)

दशगुणाः संख्याः

6. 3. 1. एकं दश च शतं च सहस्रं त्वयुतनियुते तथा प्रयुतम् ।
कोटचर्बुदं च वृन्दं स्थानात् स्थानं दशगुणं स्यात् ।।

(Āryabhaṭa I, *ABh.*, 2.2)

Decimal numbers

Eka (unit's place), Daśa (tens place), Śata (hundreds place), Sahasra (thousands place), Ayuta (ten thousands place), Niyuta (hundred thousands place), Prayuta (millions place), Koṭi (ten millions place), Arbuda (hundred millions place), and Vṛnda (thousand millions place) are, respectively, from place to place, each ten times the preceding. (2) (KSS)

6. 3. 2. एकं दश च शतं चाथ सहस्रमयुतं क्रमात् ।
नियुतं प्रयुतं कोटिरर्बुदं वृन्दमप्यथ ।। ५ ।।
खर्वो निखर्वश्च महापद्मः शङ्कुश्च वारिधिः ।
अन्त्यं मध्यं परार्धं च संख्या दशगुणोत्तराः ।। ६ ।।

(Śaṅkara Varman: *Sadratnamālā*, 1. 5-6)

Eka (1), Daśa (10), Śata (100), Sahasra (1000), Ayuta (10,000), Niyuta (or Lakh, 10^5), Prayuta (10^6), Koṭi (10^7), Arbuda (10^8), Vṛnda (10^9), Kharva (10^{10}), Nikharva (10^{11}), Mahāpadma (10^{12}), Śaṅku (10^{13}), Vāridhi (10^{14}), Antya (10^{15}), Madhya (19^{16}) and Parārdha (10^{17}) are numbers, each tenfold of the previous. (5-6) (KVS)

6. 3. 3. इमा म अग्न इष्टका धेनवः सन्तु––एका च दश च, दश च शतं च, शतं च सहस्रं च, सहस्रं चायुतं चायुतं च नियुतं चँ, नियुतं च प्रयुतं चार्बुदं च न्यर्बुदं च, समुद्रश्च मध्यं चान्तश्च परार्धश्चैता मे अग्न इष्टका धेनवः सन्त्वमुत्रामुष्मिँल्लोके ।

(*YV-VS*, 17. 2)

O Agni, may these (sacrificial) bricks be mine own milch-kine: one and a ten, a ten and a hundred, a hundred and a thousand, a thousand and a ten thousand, a ten thousand and a hundred thousand, a hundred thousand and a million, a ten million, a hundred million, a thousand million, a ten thousand million, a hundred thousand million, a million million or billion. May these bricks be mine milch-kine in yonder world and in this world.[1]

6. 3. 4. एका च मे तिस्रश्च मे, तिस्रश्च मे पञ्च च मे, पञ्च च मे सप्त च मे, सप्त च मे नव च मे, नव च म एकादश च म, एकादश च मे त्रयोदश च मे, त्रयोदश च मे पञ्चदश च मे, पञ्चदश च मे सप्तदश च मे, सप्तदश च मे नवदश च मे, नवदश च म एकर्विशतिश्च म, एकर्विशतिश्च मे त्रयोर्विशतिश्च मे, त्रयोर्विशतिश्च मे पञ्चर्विशतिश्च मे, पञ्चर्विशतिश्च मे सप्तर्विशतिश्च मे, सप्तर्विशतिश्च मे नवर्विशतिश्च मे, नवर्विशतिश्च म एकत्रिंशच्च म, एकत्रिंशच्च मे त्रयस्त्रिंशच्च मे यज्ञेन कल्पन्ताम् ।।

(*YV-VS*, 18.24)

May my one and my three, my 3 and 5, my 5 and 7, my 7 and 9, my 9 and 11, my 11 and 13, my 13 and 15, my 15 and 17, my 17 and 19, my 19 and 21, my 21 and 23, my 23 and 25, my 25 and 27, my 27 and 29, my 29 and 31, and my 31 and my 33 prosper by sacrifice.

6. 3. 5. चतस्रश्च मेऽष्टौ च मे, ऽष्टौ च मे द्वादश च मे, द्वादश च मे षोडश च मे, षोडश च मे विंशतिश्च मे, विंशतिश्च मे चतुर्विंशतिश्च मे, चतुर्विंशतिश्च मेऽष्टाविंशतिश्च मे, ऽष्टाविंशतिश्च मे द्वात्रिंशच्च मे, द्वात्रिंशच्च मे षट्त्रिंशच्च मे, षट्त्रिंशच्च मे चत्वारिंशच्च मे, चत्वारिंशच्च मे चतुश्चत्वरिंशच्च मे, चतुत्वारिंशच्च मेऽष्टाचत्वारिंशच्च मे यज्ञेन कल्पन्ताम् ।।

(*YV-VS*, 18.25)

[1] Similar lists occur also elsewhere in the Veda, e.g., *YV-TS* 4.4.11; *Kāṭhaka Saṃhitā*, 17.10; *Maitrāyaṇīya Saṃhitā*, 2.18.14.

May my four and my eight, my 8 and my 12, my 12 and my 16, my 16 and my 20, my 20 and my 24, my 24 and my 28, my 28 and my 32, my 32 and my 36, my 36 and my 40, my 40 and my 44 and my 44 and my 48 prosper by sacrifice.[1]

6. 3. 6. अथार्जुनो गणको महामात्रो बोधिसत्त्वमेवमाह—'जानीषे त्वं कुमार कोटिशतोत्तरां नाम गणनाविधिम्?'

आह—'जानाम्यहम्'

आह—'कथं पुनः कोटिशतोत्तरा गणनागतिरनुप्रवेष्टव्या?'

बोधिसत्त्व आह—'शतमयुतानां नियुतं नामोच्यते। शतं नियुतानां कङ्कारं नामोच्यते। शतं कङ्काराणां विवरं नामोच्यते। शतं विवराणां अक्षोभ्यं नामोच्यते। शतमक्षोभ्याणां विवाहं नामोच्यते। शतं विवाहानां उत्सङ्गं नामोच्यते। शतमुत्सङ्गानां बहुलं नामोच्यते। शतं बहुलानां नागबलं नामोच्यते। शतं नागबलानां तिटिलम्भं नामोच्यते। शतं तिटिलम्भानां व्यवस्थानप्रज्ञप्तिर्नामोच्यते। शतं व्यवस्थानप्रज्ञप्तीनां हेतुहिलं नामोच्यते। शतं हेतुहिलानां करहूर्नामोच्यते। शतं करहूणां हेत्विन्द्रियं नामोच्यते। शतं हेत्विन्द्रियाणां समाप्तलम्भं नामोच्यते। शतं समाप्तलम्भानां गणनागतिर्नामोच्यते। शतं गणनागतीनां निरवद्यं नामोच्यते। शतं निरवद्यानां मुद्राबलं नामोच्यते। शतं मुद्राबलानां सर्वबलं नामोच्यते। शतं सर्वबलानां विसंज्ञागतिर्नामोच्यते। शतं विसंज्ञागतीनां सर्वसंज्ञा नामोच्यते। शतं सर्वसंज्ञानां विभूतंगमा नामोच्यते। शतं विभूतंगमानां तल्लक्षणं नामोच्यते।।

(*Lalitavistara*, pp. 168-69)

Numbers upto 10^{53}

The mathematician, minister Arjuna asked Bodhisattva, "O prince, do you know the counting which goes beyond hundred *koṭi* (in the centesimal scale)?"

Bodhisattva: "I know."

Arjuna: "How does the counting proceed beyond hundred *koṭi* (in the centesimal scale)?"

Bodhisattva: "Hundred *koṭis* make one *ayuta*; hundred *ayutas* make one *niyuta*; hundred *niyutas* make one *kaṅkāra*; hundred *kaṅkāras* make one *vivara*; hundred *vivaras* make one *akṣobhya*; hundred *akṣobhyas* make one *vivāha*; hundred *vivāhas* make one *utsaṅga*; hundred *utsaṅgas* make one *bahula*; hundred *bahulas* make one *nāgabala*; hundred *nāgabalas* make one *tiṭilambha*; hundred *tiṭilambhas* make one *vyavasthānaprajñapti*; hundred *vyavasthānaprajñaptis* make one *hetuhila*; hundred *hetuhilas* make one *karahū*; hundred *karahūs* make one *hetvindriya*; hundred *hetvindriyas* make one *samāptalambha*; hundred *samāptalambhas* make one *gaṇanāgati*; hundred *gaṇanāgatis* make one *niravadya*; hundred *niravadyas* make one *mudrābala*; hundred *mudrābalas* make one *sarvabala*; hundred *sarvabalas* make one *visaṃjñāgati*; hundred *visaṃjñāgatis* make one *sarvasaṃjñā*; hundred *sarvasaṃjñās* make one *vibhūtaṅgamā*; hundred *vibhūtaṅgamas* make one *tallakṣaṇā* (i.e. 10^{53}).[1] (KVS)

[1] On numerals mentioned in the Vedas, their variant forms and etymologies, see Satya Prakash, *Founders of Sciences in Ancient India*, ch. IX. 'Medhātithi—First to extend numerals to billions', pp. 355-94.

संख्याविन्यासः—आर्यभटानुसारी

6. 4. 1. वर्गाक्षराणि वर्गेऽवर्गेऽवर्गाक्षराणि कात् ङमौ यः।
खद्विनवके स्वरा नव वर्गेऽवर्गे नवान्त्यवर्गे वा।।

(Āryabhaṭa I, *ABh.* 1.2)

Depiction of numbers—Acc. to Āryabhaṭa

The *varga* letters (*k* to *m*) (should be written) in the *varga* places and the *avarga* letters (*y* to *h*) in the *avarga* places. (The *varga* letters take the numerical values, 1, 2, 3, etc.) from *k* onwards; (the numerical value of the initial *avarga* letter) *y* is equal to *ṅ* plus *m* (i.e., 5+25). In the places of the two nines of zeros (which are written to denote the notational places), the nine vowels should be written (one vowel in each pair of the *varga* and *avarga* places). In the *varga* (and *avarga*) places beyond (the places denoted by) the nine vowels too (assumed vowels or other symbols should be written, if necessary).[2] (2) (KSS)

संख्याविन्यासः—कटपयादिक्रमः १

6. 5. 1. नञावचश्च शून्यानि संख्याः कटपयादयः।
मिश्रे तूपान्तहल्संख्या न च चिन्त्यो हलः स्वरः।।

(Śaṅkara Varman: *Sadratnamālā*, 3.4.)

Depiction of numbers—Kaṭapayādi notation (i)

n, *ñ* and the vowels (when standing alone) denote zero. (The consonants) beginning with *ka*, *ṭa*, *pa* and *ya* denote, in order, the digits. In a conjoint consonant,

[1] For the enumeration of large numbers, see Kaccāyana's Pāli grammar under sūtras 51 and 52 where numbers up to 10^{140} are mentioned.

[2] In the Sanskrit alphabet the 25 letters *k* to *m* have been classified into five *vargas* (classes) and are here supposed to bear the numerical values 1 to 25. The letters *y* to *h* are called *avarga* letters, and bear the following numerical values:

y 30, *r* 40, *l* 50, *v* 60, *ś* 70, *ṣ* 80, *s* 90, *h* 100.

The values of the said *avarga* letters are taken to increase by 10; and increase by 1 in the *avarga* place means increase by 10 in the *varga* place. The odd places are called the *varga* places (because 1, 100, 10000, etc. are perfect squares); and the even places are called the *avarga* places (because 10, 1000, etc., are non-square numbers). The *varga* letters should be written down in the *varga* places and the *avarga* letters in the *avarga* places. When a letter is joined with a vowel (for example, in *gṛ* the letter *g* is joined with the vowel *ṛ*), the letter denotes a number and the vowel the place where that number is to be written down. Thus *gṛ* stands for the number *g* (3) written in the *varga* place occupied by the vowel *ṛ* in the *varga*. Thus *gṛ* 30,00,000. *g* has been written in the *varga* place because *g* is a *varga* letter.

only the last consonant counts. The vowel suffixed to a consonant, too, is to be ignored, (the digits being written from right to left to form the number.)[1] (4) (KVS)

6. 5. 2. कटपयवर्गभवैरिह पिण्डान्त्यैरक्षरैरङ्काः ।
नेञे शून्यं ज्ञेयं, तथा स्वरे; केवले कथिते ॥

(Stray verse)

The (ten) digits are denoted by the letters in the groups (of ten each) beginning with *ka, ṭa, pa* and *ya*, the end letters alone being taken in the case of conjunct syllables. *Na* and *ña* are to be understood as zero; so also the vowels when standing alone.[1] (KVS)

[1] This system has been very popular in the southernmost part of India, especially in Kerala. The system was prevalent in this part of the country at least from the 4th cent. A.D. when the Moon-chronograms beginning with *gīr naḥ śreyaḥ* were composed by a Kerala Vararuci who is traditionally ascribed to the said period. Āryabhaṭa's commentator Sūryadeva Yajvan refers to this system as a pre-Āryabhaṭan notation. Thus, while commenting on *ĀBh.* 1.2, which sets out the notation newly introduced by Āryabhaṭa, he says that the values of the consonants *ka* etc. is different from their values in the *kaṭapādi* system, which was already well known then. Cf. *vargākṣarāṇām saṅkhyāpratipādane kaṭapāditvam nañayoś ca śūnyatvam api prasiddham. tannirāsārtham kāt-grahaṇam. kāt-prabhṛty eva vargākṣarāṇām saṅkhyā na ṭakārāt pakārāt ca prabhṛti. kāt-prabhṛti sarvām saṅkhyām pratipūdayanti, na tu ñakāra-nakārayoḥ śūnyatvam ity arthaḥ.* (Edn., K.V. Sarma, New Delhi, 1976, p. 10).

संख्याविन्यासः—कटपयादिः २

6. 6. 1. रूपात् कटपयपूर्वा वर्णा वर्णक्रमाद् भवन्त्यङ्काः ।
ञनौ शून्यं प्रथमार्थे आ छेदे ऐ तृतीयार्थे ॥ २ ॥

(ABh. II, *Mahā.* 1.2)

Depiction of numbers—Kaṭapayādi (ii)

The consonants starting from ka, ṭa, pa, ya represent the numerals from one (*rūpa*) (in succession) in the order of the (respective) consonants; ña and na (denote) zero. (The chronograms, when) separated (from each other) have ā and ai (at their end) in the nominative (plural) and in the instrumental (plural, respectively)[2]. (2). (SRS)

[1] *Alphabetic notation of the numerals*

1	2	3	4	5	6	7	8	9	0
k	kh	g	gh	ṅ	c	ch	j	jh	ñ
ṭ	ṭh	ḍ	ḍh	ṇ	t	th	d	dh	n
p	ph	b	bh	m					
y	r	l	v	ś	ṣ	s	h	ḷ	

[2] According to this system, the vowels, whether standing alone or in conjunction with consonants, have no numerical significance. Each component conjunct of a consonant has a numerical significance. The letters of the chronograms are read from left to right, unlike several other systems. The *visarga* at the end of a chronogam is dropped if the latter is followed by a word or a chronogram.

7. कालादिमानम् – MEASURES OF TIME ETC.

वैदिकः कालविभागः

संवत्सरः, मासाः, दिवसाश्च

7. 1. 1. द्वादशारं नहि तज्जराय
वर्वर्ति चक्रं परि द्यां ऋतस्य ।
आ पुत्रा अग्ने मिथुनासो
अत्र सप्त शतानि विंशतिश्च तस्थुः ।।
(*RV*, 1. 164. 11)

Division of Time in the Veda

Year, months and days

The wheel (of time) formed with twelve spokes revolves round the heavens without wearing out. O Agni! on it are 720 sons (viz. days and nights).[1] (KVS)

7. 1. 2. द्वादश प्रधयश्चक्रमेकं
त्रीणि नभ्यानि क उ तच्चिकेत ।
तस्मिन्त्साकं त्रिशता न शङ्कवो-
ऽर्पिताः षष्टिर्न चलाचलासः ।। (*RV*, 1. 164. 48)

The fellies (or arcs) are twelve; the wheel is one; three-(partitioned) are the axles (or hubs); but who knows it? Within it are collected 360 (spokes), which are, as it were, movable and unmovable.[2]

7. 1. 3. वेद मासो धृतव्रत द्वादश प्रजावतः ।
वेद या उपजायते ।। (*RV*, 1. 25.8)

Dhṛtavrata (the sage) knows the twelve months. He knows also the month that is created (i.e. added as intercalary).

7. 1. 4. संसर्पाय स्वाहा, चन्द्राय स्वाहा, ज्योतिषे स्वाहा, मलिम्लुचाय स्वाहा, दिवांपतये स्वाहा ।। (*YV-VS*, 22. 30)

Oblation to (the intercalary month) Saṁsarpa, oblation to the Moon, oblation to the luminaries, oblation to (the intercalary month) Malimluca, oblation to the Sun. or amhaspati.

त्रयोदशानां मासानां नामानि

7. 1. 5. अरुणोऽरुणरजाः पुण्डरीको विश्वजिदभिजित् ।
आर्द्रः पिन्वमानोऽन्नवान् रसवानिरावान् ।
सर्वौषधः सम्भरो महस्वान् ।।
(*Tait. Brāhmaṇa*, 3. 10. 1)

[1] For this elucidation, see *Aitareya Brāhmaṇa*: *triṇī ca vai śatāni ṣaṣṭiś ca saṁvatsarasyāhāni . . . sapta ca vai śatāni viṃsatiś ca saṃvatsarasyāhorātrayaḥ.*: '360 is the number of days in the year . . . 720 is the number of days and nights.'

[2] The 12 arcs refer to the twelve months, the wheel to the year, the three-partitioned hubs to the four-month or *cāturmāsya* sections and the 360 spokes to the 360 days in the year.

Names of the 13 months

(The thirteen months are): Aruṇa, Aruṇarajas, Puṇḍarīka, Viśvajit, Ābhijit, Ārdra, Pinvamāna, Annavān, Rasavān, Irāvān, Sarvauṣadha, Sambhara and Mahasvān. (KVS)

अर्धमासानां नामानि

7. 1. 6. पवित्रं पविष्यन् पूतो मेध्यः ।
यशो यशस्वानायुरमृतः ।
जीवो जीविष्यन्त्स्वर्गो लोकः ।
सहस्वान्त्सहीयानोजस्वान्त्सहमानः ।
जयन्नभिजयन्त्सुद्रविणो द्रविणोदाः ।
आर्द्रपवित्रो हरिकेशो मोदः प्रमोदः ।।
(*Tait. Brāhmaṇa*, 3.10.1)

The names of the 24 half-months

(The names of the 24 half-months are): Pavitra, Paviṣyan, Pūta, Medhya, Yaśas, Yaśasvān, Āyus, Amṛta, Jīva, Jīviṣyan, Sarga, Loka, Sahasvān, Sahīyān, Ojasvān, Sahamāna, Jayan, Abhijayan, Sudraviṇa, Draviṇodas, Ārdrapavitra, Harikeśa, Moda and Pramoda. (KVS)

चान्द्रवत्सरः दिवसाश्च

7. 1. 7. आजिं वा एते धावन्ति ये दर्शपूर्णमासाभ्यां यजन्ते । स वै पञ्चदश वर्षाणि यजेत । तेषां पञ्चदशानां वर्षाणां त्रीणि च शतानि षष्टिश्च पौर्णमास्यश्चामावास्याश्च त्रीणि च वै शतानि षष्टिश्च संवत्सरस्य रात्रयस्तद् रात्रिराप्नोति ।। १० ।।

अथा पराणि पञ्चदश वर्षाणि यजेत । तेषां पञ्चदशानां वर्षाणां त्रीणि चैव शतानि षष्टिश्च पौर्णमास्यश्चामावास्याश्च त्रीणि चैव शतानि षष्टिश्च संवत्सरस्याहानि । तदहान्याप्नोति । तदेव संवत्सरमाप्नोति ।। ११ ।। (*Śatapatha Brāhmaṇa*, 11.1.2.10-11)

Lunar year of 354 days

Verily, they who perform the Full and New Moon sacrifices run a race. One ought to perform it during fifteen years. In these fifteen years, there are three hundred and sixty full moons and new moons. And, there are, in a year, three hundred and sixty nights; it is the nights he thus gains. (10)

He should then sacrifice for another fifteen years. In these fifteen years, there are three hundred and sixty full moons and new moons, and there are in a year

three hundred and sixty days; it is the days he thus gains, and the year itself he thus gains. (11)[1]

मासाः, अधिमासकौ च

7. 1. 8. मधुश्च माधवश्च, शुक्रश्च शुचिश्च, नभश्च नभस्यश्च, इषश्चोर्जश्च, सहश्च सहस्यश्च, तपश्च तपस्यश्चोपयाम गृहीतोऽसि संसर्पोऽस्यंहस्पत्याय त्वा ॥ (*YV-TS*, 1.4.14)

Months and intercalary months

Thou (Soma) art Madhu and Mādhava; thou art Śukra and Śuci; thou art Nabha and Nabhasya; thou art Iṣa and Ūrja; thou art Saha and Sahasya; thou art Tapa and Tapasya. Thou art taken with a support; thou art Saṃsarpa, to thee Aṃhaspatya. (A.B. Keith)

मासा ऋतवश्च

7. 1. 9. मधुश्च माधवश्च वासन्तिकावृतू । शुक्रश्च शुचिश्च ग्रैष्मावृतू । नभश्च नभस्यश्च वार्षिकावृतू । इषश्चोर्जश्च शारदावृतू । सहश्च सहस्यश्च हैमन्तिकावृतू । तपश्च तपस्यश्च शैशिरावृतू ॥

(*YV-TS*, 4.4.11)

Seasons and months

(The two months) Madhu and Mādhava (constitute) the Vasanta season; Śukra and Śuci (constitute) the Grīṣma season; Nabhas and Nabhasya (constitute) the Varṣa season; Iṣa and Ūrja (constitute) the Śarad season; Sahas and Sahasya (constitute) the Hemanta season; and Tapas and Tapasya (constitute) the Śiśira season. (KVS)

7. 1. 10. द्वादश मासाः पञ्चर्तवो हेमन्तशिशिरयोः समासेन ।

(*Ait. Brāhmaṇa*, 1.1)

The months are twelve, and the seasons five through the union of winter and the cool seasons. (A.B. Keith)

दिन-रात्रीणां नामानि

7. 1. 11. संज्ञानं विज्ञानं प्रज्ञानं जानद् अभिजानत् ।
संकल्पमानं प्रकल्पमानं उपकल्पमानं उपक्लृप्तं क्लृप्तम् ।
श्रेयो वसीय आयत् संभूतं भूतम् ॥
दर्शा दृष्टा दर्शता विश्वरूपा सुदर्शना ।
आप्यायमाना प्यायमाना प्याया सूनृतेरा ।
आपूर्यमाणा पूर्यमाणा पूरयन्ती पूर्णा पौर्णमासी ॥ १ ॥
प्रस्तुतं विष्टुतं संस्तुतं कल्याणं विश्वरूपम् ।
शुक्रम् अमृतं तेजस्वि तेजः समिद्धम् ।
अरुणं भानुमत् मरीचिमद् अभितपत् तपस्वत् ॥ २ ॥
सुता सुन्वती प्रसुता सूयमानाऽभिषूयमाणा ।
पीती प्रपा सम्पा तृप्तिस् तर्पयन्ती ।
कान्ता काम्या कामजाताऽयुष्मती कामदुघा ॥ ३ ॥

(*Tait. Brāhmaṇa*, 3. 10. 1. 1-3)

Names of day-times and night-times

Saṁjñāna, Vijñāna, Prajñāna, Jānad, Abhijānad, Saṅkalpamāna, Prakalpamāna, Upakalpamāna, Upaklptamāna, Klpta, Śreya, Vasīya, Āyat, Sambhūta, and Bhūta (are the names of the fifteen day-times of the bright fortnight).

Darśā, Dṛṣṭā, Darśatā, Viśvarūpā, Sudarśanā, Āpyāyamānā, Pyāyamānā, Pyāyā, Sūnṛtā, Irā, Āpūryamāṇā, Pūryamāṇā, Pūrayantī, Pūrṇā and Paurṇamāsī are the (names of the fifteen night-times of the bright fortnight). (1)

Prastuta, Viṣṭuta, Saṃstuta, Kalyāṇa, Viśvarūpa, Śukra, Amṛta, Tejasvi, Tejas, Samiddha, Aruṇa, Bhānumat, Marīcimat, Abhitapat and Tapasvat (are the names of the fifteen day-times of the dark fortnight).

Sutā, Sunvatī, Prasutā, Sūyamānā, Abhiṣūyamāṇā, Pītī, Prapā, Sampā, Tṛpti, Tarpayantī, Kāntā, Kāmyā, Kāmajātā, Āyuṣmati and Kāmadughā (are the names of the fifteen night-times of the dark fortnight). (KVS)

मुहूर्तानां नामानि

7. 1. 12. चित्रः केतुः प्रभा नाभान् सम्भान् ।
ज्योतिष्मांस् तेजस्वान् आतपंस् तपन्न् अभितपन् ।
रोचनो रोचमानः शोभनः शोभमानः कल्याणः ॥ १ ॥
दाता प्रदाताऽऽनन्दो मोदः प्रमोदः ।
आवेशयन् निवेशयन् संवेशनः संशान्तः शान्तः ।
आभवन् प्रभवन् संभवन् संभूतो भूतः ॥
सविता प्रसविता दीप्तो दीपयन् दीप्यमानः ।
ज्वलन् ज्वलिता तपन् वितपन् सन्तपन् ।
रोचनो रोचमानः शुम्भूः शुंभमानो वामः ॥ २ ॥
अभिशस्ताऽनुमन्ताऽनन्दो मोदः प्रमोदः ।
आसादयन् निषादयन् संसादनः संसन्नः सन्नः ।
आभूर्विभूः प्रभूः शम्भूर्भुवः ॥ ३ ॥
इदानीं तदानीम् एतर्हि क्षिप्रम् अजिरम् ।
आशुर्निमेषः फणो द्रवन्न् अतिद्रवन् ।
त्वरंस्त्वरमाण आशुराशीयान् जवः ॥ ४ ॥

(*Tait. Brāhmaṇa*, 3. 10. 1. 1-4)

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[1] This passage adjusts the lunar year with the sidereal year. In the course of fifteen sidereal years, the sacrificer would gain 180 days of 24 hours each, or 360 nights of 12 hours each, the 180 days of 12 hours each being regarded as nights in the Dakṣiṇāyana. In other words, he would gain six intercalary months in 15 sidereal years, as each sidereal year of 366 days exceeds each lunar year of 354 days by 12 days, and 15 sidereal years would produce $15 \times 12 =$ 180 intercalary days.

It would seem from this passage that in this cycle of 30 sidereal years, no intercalation of any sort was made so as to keep the lunar year in consonance with seasons. The lunar year was allowed to retrograde through all the seasons and begin again with the real season at the close of 30 sidereal years. (Satya Prakash: *Founders of Sciences in Ancient India*, p. 78)

Names of muhūrtas

Citra, Ketu, Prabhā, Nābhān, Sambhān, Jyotiṣmān, Tejasvān, Ātapan, Tapan, Abhitapan, Rocana, Rocamāna, Śobhana, Śobhamāna and Kalyāṇa (are the names of the fifteen *muhūrtas* of the day-time of a day in the bright fortnight). (1)

Dātā, Pradātā, Ānanda, Moda, Pramoda, Āveśayan, Niveśayan, Saṃveśana, Saṃśānta, Śānta, Abhavan, Prabhavan, Sambhavan, Sambhūta and Bhūta (are the names of the fifteen *muhūrtas* of the day-time in a day in the dark fortnight).

Savitā, Prasavitā, Dīpta, Dīpayan, Dīpyamāna, Jvalan, Jvalitā, Tapan, Vitapan, Santapan, Rocana, Rocamāna, Śumbhū, Śumbhamāna and Vāma (are the names of the fifteen *muhūrtas* of the night-time of a day in the bright fortnight). (2)

Abhiśastā, Anumantā, Ānanda, Moda, Pramoda, Āsādayan, Niṣādayan, Saṃsādana, Saṃsanna, Sanna, Ābhū, Vibhū, Prabhū, Śambhū and Bhuva (are the names of the fifteen *muhūrtas* of the night-time of a day in the dark fortnight).[1] (3-4). (KVS)

(**Prati-muhūrtas**)

Idānīm, Tadānīm, Etarhi, Kṣipram, Ajiram, Āśu, Nimeṣa, Phaṇa, Dravan, Atidravan, Tvaran, Tvaramāṇa, Āśu, Āśīyāṅ and Java (are the names of the fifteen Prati-muhūrtas into which each Muhūrta is divided).[2] (4) (KVS)

तिथिः

7. 1. 13. यां पर्यस्तमयादभ्युदियादिति सा तिथिः ।

(*Ait. Brāhmaṇa*, 32. 10)

Tithi, the lunar day

Tithi is that period of time during which the Moon sets and rises (again). (KVS)

युगादिमानम्—वेदाङ्गे पञ्चवत्सरात्मकं युगम्

7. 2. 1. त्रिशत्यह्नां सषट्षष्टिरब्दः षट् चर्तवोऽयने ।
मासा द्वादश सौराः स्युः एतत् पञ्चगुणं युगम् ।

(*YV-VJ*, 28)

Yuga in the Vedāṅga: Five-year yuga

Three hundred and sixtysix days form the solar year. In the year there are six *ṛtus* and two *ayanas*, (i.e. northward and southward courses of the Sun). There are 12 solar months in the year. Five years make a *yuga* (lustrum).[1] (28)

युगध्रुवाः

7. 2. 2. उदया वासवस्य स्युर्दिनराशिः सपञ्चकः ।
ऋषेर्द्विषष्टया हीनः स्याद् विंशत्या सैकया स्तृणाम् ।।२६।।

पञ्चत्रिंशं शतं पौष्णं एकोनमयनान्यृषेः ।
पर्वणां स्याच्चतुष्पादो . . . . . . ।। ३० ।।

सावनेन्दुस्तृमासानां षष्टिः सैकद्विसप्तिका ।
द्युस्त्रिशत् सावनस्यार्धः सौरः स्तृणां च पर्ययः ।। ३१ ।।

(*YV-VJ*, 29-31)

ससप्तैकं भयुक् सोमः, सूर्यो द्यूनि त्रयोदश ।
नवमानि च पञ्चाह्नः, काष्ठा पञ्चाक्षरी भवेत् ।। ३६ ।।

(*YV-VJ*, 39; *RV-VJ*, 18)

Yuga constants

(In a *yuga* of five years), the number of risings of the Vāsava (Indra) asterism (viz. Jyeṣṭhā) (and of all the other asterisms) is the same as the number of days (in the *yuga* plus 5 (i.e. 1830 *plus* 5 = 1835). The number of risings of the Moon is the number of days *minus* 62 (i.e. 1830—32 = 1768). The total number of each of the (Moon's 27) asterisms (coming round 67 times in the *yuga*) is the number of the days *minus* 21 (i.e. 1830—21 = 1809). (29)

The total of the asterisms of the Sun (which comes round five times) is 135. There is one less (i.e. 134) Moon's *ayanas* (i.e. northward and southward courses). There are four *pādas* (i.e. 4 × 31 = 124) *parvas* (or *pakṣas* or *parvāntas*, i.e. light and dark fortnights). (30)

In a *yuga* there are, respectively, 61, 62 and 67 (i.e. 60 + 1, 60 + 2, and 60 + 7) *sāvana* months, lunar (synodic) months and Moon's (cycles). The *sāvana* month contains 30 days. This plus half (i.e. 30½ days) make a solar month. The number mentioned here (viz. 30) is the number of solar sidereal cycles in a *yuga*. (31)

The Moon comes into contact with each asterism 60 + 7, (i.e. 67) times during a year. The Sun stays in each asterism for 13 *plus* 5/9 days. (39 a-c). (TSK)

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[1] The initial words of these sets of names are given elsewhere in the *Brāhmaṇa*, where it is also indicated specifically that these are the *muhūrta* names: *atha yad āha citraḥ ketur dātā pradātā savitā prasavitā 'bhisavitā 'numanteti. eṣa hy eva tat. eṣa hy eva te 'hno muhūrtāḥ.* (*Tait. Brāhmaṇa*, 3.10.9.7).

[2] That these terms denote the divisions of the *muhūrtas* is stated later in the *Brāhmaṇa*: *atha yad āha idānīm tadānīm iti. esa eṣa tat. eṣa hy eva muhūrtānām muhūrtāḥ.* (*Tait. Brāhmaṇa*, 3.10.9.9)

[1] According to the *VJ*, the divisions of time are:

5 *gurvakṣaras* or 10 *mātrās*	=	1 *kāṣṭhā*
124 *kāṣṭhas*	=	1 *kalā*
10 1/20 *kalās*	=	1 *nāḍikā*
2 *nāḍikās*	=	1 *muhūrta*
30 *muhūrtas*	=	1 day (i.e. civil day)
366 days or 12 solar months or six *ṛtus* or 2 *ayanas*	=	1 solar year
5 solar years	=	1 *yuga*

युगारम्भः अयनकालश्च

7. 2. 3. माघशुक्लप्रपन्नस्य पौषकृष्णसमापिनः ।
युगस्य पञ्चवर्षस्य कालज्ञानं प्रचक्षते ॥ ५ ॥

स्वराक्रमेते सोमार्कौ यदा साकं सवासवौ ।
स्यात्तदादि युगं माघः तपः शुक्लोऽयनं ह्युदक् ॥ ६ ॥

प्रपद्येते श्रविष्ठादौ सूर्याचन्द्रमसावुदक् ।
सार्पार्धे दक्षिणार्कस्तु माघश्रावणयोः सदा ॥ ७ ॥
(*YV-VJ* 5-7; *RV-VJ* 32; 5-6)

Yuga-beginning and Solstices

The (Vedic astronomers) expound the knowledge of the time of the cycle (*yuga*) of five years which begins with the bright fortnight of (the lunar) month of Māgha and ends with the dark fortnight of (the lunar) month of Pauṣa (or Puṣya). (5)

(Solstices)

When the Sun and the Moon occupy the same position in the zodiac together with the asterism of Vāsava (i.e. Indra, viz. Śraviṣṭhā), then is the beginning of (i) the yuga, (ii) the synodic month of Māgha, (iii) the solar seasonal month of Tapas, (iv) the bright fortnight (of the synodic month, here Māgha), and (v) the *Uttara-ayana* (Winter solstice, the northward course of the Sun and Moon). (6)

When situated at the beginning of the Śraviṣṭhā segment, the Sun and the Moon begin to move north. When they reach the mid-point of the Āśleṣā segment, they begin moving south. In the case of the Sun, this happens always in the month of Māgha and Śrāvaṇa, respectively. (7). (TSK)

7. 2. 4. एवमर्धतृतीयानामब्दानामधिमासकम् ।
ग्रीष्मे जनयतः पूर्वं पञ्चाब्दान्ते च पश्चिमम् ॥
(Kauṭalya : *Arthaśāstra*, 2.20.66)

Thus, in two years and a half (being half of the five-year *yuga*, the two halves each) beget one intercalary month, the first at (the end of) summer, and the second at the end of the fifth year. (KVS)

वेदाङ्गे कालमानम्

7. 3. 1. पलानि पञ्चाशदपां धृतानि तदाढकं द्रोणमतः प्रमेयम् ।
त्रिभिर्विहीनं कुडवैस्तु कार्यं तन्नाडिकायास्तु भवेत् प्रमाणम् ॥
(*YV-VJ*, 24)

कला दश सविंशा स्यात्, द्वे मुहूर्तस्य नाडिके ।
द्युस्त्रिंशत् तत् कलानां तु षट्छती त्र्यधिका भवेत् ॥
(*YV-VJ*, 38; *RV-VJ*, 16)

पादस्त्रिंशत्तु सैकिका । (*YV-VJ*, 12b)

काष्ठानां चैव ताः कलाः । (*YV-VJ*, 30d)

काष्ठा पञ्चाक्षरा भवेत् । (*YV-VJ*, 39d)

(Measures of time in the Vedāṅga)

A vessel which holds 50 *palas* of water is the measure called *āḍhaka*. From this is derived the measure called *droṇa* (which is four times the *āḍhaka*). This lessened by three *kuḍuvas* (being three-sixteenths of the *āḍhaka*) is the volume (of water) measured (into a clepsydra) for the length of time of one *nāḍikā*. (24)

(The *nāḍikā*) is ten and one twentieth *kalās* (of time). Two *nāḍikās* make a *muhūrta*. Thirty *muhūrtas* make a day, and a day is 603 *kalās*. (38)

A *pāda* ('quarter') is equal to (the number) 31. (12-b)

They (four *pādas*) together (i.e. $4 \times 31 = 124$) in *kāṣṭhās* make one *kalā*. (30-d)

One *kāṣṭhā* is equal to (the time taken to utter) five (long) syllables (*akṣaras*, each equal to two *mātrās*). (39-d). (TSK)

ऋतवः

7. 3. 2. अर्धपञ्चमभस्त्वृतुः ॥ (*RV-VJ*, 9d)

Seasons

Four and a half asterismal segments make one *ṛtu* (season).[1] (9-d). (TSK)

सामान्यः कालविभागः

7. 4. 1. कमलदलनतुल्यः काल उक्तस्त्रुटिस्त-
च्छतमिह लवसंज्ञस्तच्छतं स्यान्निमेषः ।
सदल'जलधि'भिस्तैर्गुर्विहैवाक्षरं तत्-
'कृत'परिमिति काष्ठा त'च्छरार्धे'न चासुः ॥ ७ ॥

आक्षं पलं षडसवो घटिका पलानां
षष्ट्या दिनं च घटिकां खलु षष्टिमह्नाम् ।
मासं 'खवह्नि'भिरथाब्द'मिना'हतं तं
क्षेत्रे च कालसदृशावयवं विनासुम् ॥ ८ ॥

'दन्ताब्धयो'ऽयुतहता युगमर्कवर्षाः
'दस्राद्रयो' युगगणा मनुरेक उक्तः ।
कल्पश्चतुर्दश'मनु'र्द्युनिशं च तौ द्वौ
कस्य स्ववर्षशतमत्र तदायुरुक्तम् ॥ ९ ॥
(Vaṭeśvara, *Vsi.*, 1. 1. 7-9)

General Time-measures

The time taken (by a sharp needle) to pierce (a petal of) a lotus flower is called *truṭi*; one hundred times that is called a *lava*; one hundred times that is a *nimeṣa*; four

[1] The period of time during which the Sun or the Moon traverses through 4½ segments (of a total of 27segments into which the circle is divided) is the *ṛtu* related to the respective segments. In a year there are six *ṛtus*, the months Madhu and Mādhava making up the Vasanta ṛtu, Śukra and Śuci making up the Grīṣma-ṛtu, Nabhas and Nabhasya making up the Varṣa ṛtu, Iṣa and Ūrja making up the Śarad ṛtu, Saha and Sahasya making up the Hemata ṛtu, and Tapas and Tapasya making up the Śiśira ṛtu.

and a half times that is a 'long syllable' (i.e., time required for pronouncing a long syllable by a healthy person with a moderate flow of voice); four times that is a *kāṣṭhā*; and one half of five times that is an *asu*. (7)

Six *asus* make a sidereal *pala*; sixty *palas* make a *ghaṭikā*; sixty *ghaṭikās* make a day; thirty days make a month; and twelve times that is a year. The divisions of the circle too have been defined in the same manner as those of time excepting those up to *asu*. (8)

Solar years amounting to 432 multiplied by 10,000 make a *yuga*; a period of 72 *yugas* is called a *manu*; a period of 14 manus is a *kalpa*; a couple of them is a day-and-night of *Brahmā*; and a century of Brahma's own years is stated to be the duration of his life.[1] (9). (KSS)

7. 4. 2. गुर्वक्षरं विघटिका घटिका दिनं च
पूर्वाणि षष्टिगुणितानि निजोत्तराणि ।
त्रिंशद्गुणं दिवसमत्र च माससंज्ञः
मासो 'दिवाकर'गुणः खलु सावनाब्दः ॥
(Śaṅkara Varman: *Sadratnamālā*, 2.1)

Gurvakṣara		=(Time to utter a long syllable)
(10 *gurvakṣaras*=1 *prāṇa* or respiration)		
(6 *prāṇas* or)	60 *gurvakṣaras*	=1 *vighaṭikā*
	60 *vighaṭikās*	=1 *ghaṭikā*
	60 *ghaṭikās*	=1 day
	30 days	=1 month
	12 months	=1 year

[1] The time-divisions:

lotus-pricking time	=	1 *truṭi*
100 *truṭis*	=	1 *lava*
100 *lavas*	=	1 *nimeṣa* (twinkling of eye)
4½ *nimeṣas*	=	1 long syllable
4 long syllables	=	1 *kāṣṭhā*
2½ *kūṣṭhās*	=	1 *asu* (respiration)= 4 seconds
6 *asus* (*prāṇas*)	=	1 sidereal *pala* (*caṣaka*, *vināḍī* or *vighaṭikū*
60 *palas*	=	1 *ghaṭikā*=24 minutes
60 *ghaṭikās*	=	1 day
30 days	=	1 month
12 months	=	1 year
43,20,000 years	=	1 *yuga*
72 *yugas*	=	1 *manu*
14 *manus*	=	1 *kalpa*
2 *kalpas*	=	1 day of Brahmā (including nig .t)
30 days of Brahmā	=	1 month of Brahmā
12 months of Brahmā	=	1 year of Brahmā
1 year of Brahmā	=	72× 14× 2× 30× 12 *yugas*=7,25,760 *yugas*.
100 years of Brahmā	=	Life of Brahmā *or* Mahākalpa

कालमानम्

7. 5. 1. आर्क्ष-चान्द्रमस-सौर-सावन-ब्राह्म-जैव-पितृ-देव-दैत्यजैः ।
काल एभिरनुमीयतेऽव्ययो येन माननवकव्यवस्थितिः ॥६॥
(Vaṭeśvara, *Vsi.*, 1.2.9)

Modes of reckoning Time

The imperishable time is measured by sidereal, lunar, solar, civil, Brāhma, Jovian, Paternal, divine and demoniacal reckonings. That is how (these) nine varieties of time reckoning have been defined.[1] (9). (KSS)

7. 5. 2. रविभगणा रव्यब्दाः, रविशशियोगा भवन्ति शशिमासाः ।
रविभूयोगा दिवसाः क्वावर्ताश्चापि नाक्षत्राः ॥
(Āryabhaṭa I, *ABh.*, 3.5)

The revolutions of the Sun are solar years. The conjunctions of the Sun and the Moon are lunar months. The conjunctions of the Sun and Earth are (civil) days. The rotations of the Earth are sidereal days. (KSS)

7. 5. 3. वर्षायनर्तुयुगपूर्वकमत्र सौरान्
मासस्तथा च तिथयस्तुहिनांशुमानान् ।
यत् कृच्छ्रसूतकचिकित्सितवासराद्यं
तत्सावनाच्च घटिकादिकमार्क्षमानात् ॥
(Stray verse)

The year, the Sun's courses, seasons and *yugas* are determined on the basis of the solar year; months and *tithis* on the basis of the lunar year; austerities, pollution, and medication on the basis of the *sāvana* year; and *ghaṭī* (*nāḍikā*) and its divisions on the basis of the sidereal year. (KVS)

7. 5. 4. रवेश्चक्रभोगोऽर्कवर्षं प्रदिष्टं,
द्युरात्रं च देवासुराणां तदेव ।
रवीन्द्वोर्युतेः संयुतिर्यावदन्या
विधोर्मास, एतच्च पैत्रं द्युरात्रम् ॥ १६ ॥

[1] Units of the nine varieties of time-reckoning

	Reckoning	Unit used	
i.	Sidereal	Sidereal day	=one star-rise to the next
ii.	Lunar	lunar month	=one new moon to the next
iii.	Solar	solar year	=period of one solar revolution
iv.	Civil	civil day	=one sunrise to the next
v.	Brāhma	day of Brahmā	=period of 2 *kalpas* or 2016 *yugas*
vi.	Jovian	Jovian year	=period of Jupiter's motoin through a sign
vii.	Paternal (Manes)	day of manes	=one lunar month
viii.	Divine	day of gods	=one solar year
ix.	Demoniacal	day of demons	=one solar year

इनोदयद्वयान्तरं तदर्कसावनं दिनम् ।
तदेव मेदिनीदिनं भवासरस्तु भभ्रमः ।। २० ।।

(Bhāskara II, *SiŚi*, 1.Ì.1. 19-20)

The time taken by the Sun to complete one revolution with respect to the stars goes by the name 'sidereal solar year'. This will be a day for the gods and demons. The time that elapses between two consecutive new moons or conjunctions of the Moon with the Sun is called a *Cāndra-māsa* or a lunar month or simply a lunation. This again is the day of the Pitṛs or the manes. (19)

The time that elapses between two consecutive Sunrises at a place is termed the *Sāvana* day or civil day. This is called *Saura-Sāvana* day and it is also the day of the Earth.

The sidereal day is the time taken by the stars to go round the Earth once. (It is called *Nākṣatra-dina*).[1] (20). (AS)

चान्द्रमास-अधिमास-तिथिक्षयाः

7. 5. 5. भवन्ति शशिनो मासाः सूर्येन्दुभगणान्तरम् ।। ३४ ।।
रविमासोनितास्ते तु शेषास्स्युरधिमासकाः ।
सावनाहानि चान्द्रेभ्यो द्युभ्यः प्रोज्झ्य तिथिक्षयाः ।। ३५ ।।

(*Sū.Si.*, 1. 34b-35)

Lunar months etc.

The number of lunar months is the difference between the number of revolutions of the Sun and of the Moon. If from it the number of solar months be subtracted, the remainder is the number of intercalary months. (34b-35a)

Take the civil days from the lunar days; the remainder is the number of omitted lunar days. (*tithikṣaya*). (35b). (Burgess)

प्रभवादि-बार्हस्पत्याब्दाः

7. 5. 6. प्रभवविभवाख्यशुक्लाः प्रमोदनामा प्रजापतिरथाब्दः ।
परतोऽङ्गिरास्ततश्च श्रीमुखभावौ युवाख्योऽन्यः ।। ४ ।।
धातेश्वरबहुधान्यौ प्रमाथिनामाथ विक्रमाख्यातः ।
सवृषोऽथ चित्रभानुरस्मात् सुभानुरथ तारणाख्यश्च ।। ५ ।।
पार्थिवनामाव्यय इति सर्वजिदाख्यः सर्वधारी च ।
तदनु विरोधी विकृतः खरनन्दनविजयजयसंख्याः ।। ६ ।।
मन्मथदुर्मुखसंज्ञावथापरौ हेमलम्बकविलम्बौ ।
तद्वद्विकारिनामाथ शर्वरी प्लव इति शुभकृच्च ।। ७ ।।
शोभनकृदथ परः क्रोधी विश्वावसुरनुपराभवाख्यश्च ।
परतः प्लवङ्गनामा कीलक इति सौम्यसंज्ञकश्च ।। ८ ।।
साधारणो विरोधकृदथ परिधाविप्रमाथिनामानौ ।
आनन्दराक्षसाख्यौ नलपिङ्गलकालयुक्तश्च ।। ९ ।।
सिद्धार्थरौद्रदुर्मतिदुन्दुभयो वत्सराः क्रमादपरे ।
रुधिरोद्गारिरक्ताक्षसंज्ञकः क्रोधनः क्षयकृत् ।। १० ।।
इयं हि षष्टिः परिवत्सराणां बृहस्पतेर्मध्यमराशिभोगात् ।
उदाहृता पूर्वमुनिप्रवर्यैर्नियोजनीया गणनाक्रमेण ।। ११ ।।

(Śrīpati, *Jyotiṣaratnamālā*, 1. 4-11)

The 60 Jovian years beginning with Prabhava

1.	Prabhava	31.	Hemalamba
2.	Vibhava	32.	Vilamba
3.	Śukla	33.	Vikārī
4.	Pramoda	34.	Śarvarī
5.	Prajāpati	35.	Plava
6.	Aṅgiras	36.	Śubhakṛt
7.	Śrīmukha	37.	Śobhakṛt
8.	Bhāva	38.	Krodhi
9.	Yuvā	39.	Viśvāvasu
10.	Dhātā	40.	Parābhava
11.	Īśāna	41.	Plavaṅga
12.	Bahudhānya	42.	Kīlaka
13.	Pramāthi	43.	Saumya
14.	Vikrama	44.	Sādhāraṇa
15.	Vṛṣa	45.	Virodhakṛt
16.	Citrabhānu	46.	Paridhāvi
17.	Subhānu	47.	Pramādi
18.	Tāraṇa	48.	Ānanda
19.	Pārthiva	49.	Rākṣasa
20.	Avyaya	50.	Nala
21.	Sarvajit	51.	Piṅgala
22.	Sarvadhārī	52.	Kalyāṇakṛt
23.	Virodhi	53.	Siddhārtha
24.	Vikṛta	54.	Raudra
25.	Khara	55.	Durmati
26.	Nandana	56.	Dundubhi
27.	Vijaya	57.	Rudhirodgāri
28.	Jaya	58.	Raktākṣi
29.	Manmatha	59.	Krodhana
30.	Durmukha	60.	Kṣayakṛt

These sixty years, as specified by great ancient sages according to the mean motion of Jupietr, shall be used in the serial order. (14-11). (KVS)

अहर्मानम्

7. 6. 1. मकरादौ 'गुण'युक्तो मेषादौ 'तिथि'युतो रविर्दिवसः ।
कर्कटकादिषु सत्सु त्रयस्त्रिकाः शर्वरीमानम् ।। ८ ।।

(Varāha, *PS*, 2.8)

Day-time

When the Sun is in the 3 *rāśis*, Makara etc., the Sun measured in *rāśi* plus three is the duration of day-time

[1] For a detailed note, see *SiŚi*: *AS*, pp. 6-10.

in *muhūrtas*. When in the 3 *rāśis* Meṣa etc., the Sun plus fifteen is the duration of day-time. When in the 6 *rāśis* Karkaṭaka etc. the Sun plus nine is the duration of the night-time. (To get the duration of the day-time, this should be subtracted from 30.) (8). (TSK)

युगव्यवहारः

7. 7. 1. कृतायादिनवदर्शं त्रेतायै कल्पिनं द्वापरायाधिकल्पिनं आस्कन्दाय सभास्थाणुम् । (*YV-VS*, 30.18)

Concept of the Yugas

To Kṛta the offering of Ādinavadarśa is to be offered, to Tretā the offering of Kalpin, to Dvāpara the offering of Adhikalpin and to Āskanda the offering of Sabhāsthāṇu.[1] (18). (KVS)

7. 7. 2. कलिः शयानो भवति संजिहानस्तु द्वापरः ।
उत्तिष्ठंस्त्रेता भवति कृतं सम्पद्यते चरैश्चरैवेति चरैवेति ।।
(*Ait. Brāhmaṇa*, 7.15.4)

One who sleeps becomes Kali; one who sits becomes Dvāpara; one who gets up becomes Tretā; and who moves ahead becomes Kṛta. Therefore keep on moving, keep on moving. (4). (KVS)

7. 7. 3. कृताय सभाविनम् । त्रेताया आदिनवदर्शम् । द्वापराय बहिःसदम् । कलये सभास्थाणुम् ।।
(*Tait. Brāhmaṇa*, 3. 4. 16)

A Sabhāvi offering should be procured for Kṛta, an Ādinavadarśa for Tretā, a Bahiḥsada for Dvāpara and a Sabhāṣthāṇu for Kali. (16). (KVS)

युगव्यवस्था—१

7. 7. 4. सुरासुराणामन्योन्यमहोरात्रं विपर्ययात् ।
षट् षष्टिसङ्गुणं दिव्यं वर्षमासुरमेव च ।। १४ ।।
तद्द्वादशसहस्राणि चतुर्युगमुदाहृतम् ।
सूर्याब्दसंख्यया 'द्वित्रिसागरै'रयुताहतैः ।। १५ ।।
सन्ध्यासन्ध्यांशसहितं विज्ञेयं तच्चतुर्युगम् ।
कृतादीनां व्यवस्थेयं धर्मपादव्यवस्थया ।। १६ ।।
युगस्य दशमो भागः चतुस्त्रिद्वयेकसङ्गुणः ।
क्रमात्कृतयुगादीनां षष्ठोंऽशस्सन्ध्ययोः स्वकः ।। १७ ।।
युगानां सप्ततिस्सैका मन्वन्तरमिहोच्यते ।
कृताब्दसङ्ख्या तस्यान्ते सन्धिः प्रोक्तो जलप्लवः ।। १८ ।।
ससन्धयस्ते मनवः कल्पे ज्ञेयाः चतुर्दश ।
कृतप्रमाणः कल्पादौ सन्धिः पञ्चदश स्मृताः ।। १९ ।।
इत्थं युगसहस्रेण भूतसंहारकारकः ।
कल्पो ब्राह्ममहः प्रोक्तं शर्वरी तस्य तावती ।। २० ।।
परमायुश्शतं तस्य तयाहोरात्रसङ्ख्यया ।
आयुषोऽर्धमितं तस्य शेषात्कल्पोऽयमादिमः ।। २१ ।।
कल्पादस्माच्च मनवः षड् व्यतीतास्ससन्धयः ।
वैवस्वतस्य च मनोः युगानां त्रिघनो गतः ।। २२ ।।
अष्टाविंशाद्युगादस्माद्यातमेकं कृतं युगम् ।
अतः कालं प्रसङ्ख्याय सङ्ख्यामेकत्र पिण्डयेत् ।। २३ ।।
(*Sū.Si.*, 1. 14-23)

Yuga concept—i

The day and night of the gods and of the demons are mutually opposed to one another. Six times sixty of them are a year of the gods, and likewise of the demons. (14)

Twelve thousand of these divine years are denominated a Quadruple Age (*caturyuga*); of ten thousand times four hundred and thirty-two solar years is composed that Quadruple Age, with its dawn and twilight. The difference of the Golden and the other Ages, as measured by the difference in the number of the feet of Virtue in each, is as follows: (15-16)

The tenth part of an Age, multiplied successively by four, three, two and one, gives the length of the Golden and the other Ages, in order: the sixth part of each belongs to its dawn and twilight. (17)

One and seventy Ages are styled here a patriarchate (*manvantara*); at its end is said to be a twilight which has the number of years of a Golden Age, and which is a deluge. (18)

In an aeon (*kalpa*) are reckoned fourteen such Patriarchs (*manu*) with their respective twilights; at the commencement of the aeon is a fifteenth dawn, having the length of a Golden Age.

The aeon is accordingly thus composed. (19)

The aeon, thus composed of a thousand Ages, and which brings about the destruction of all that exists, is styled a day of Brahmā; his night is of the same length. (20)

His extreme age is a hundred, according to this valuation of day and a night. The half of his life is past; of the remainder, this is the first aeon. (21)

And of this aeon, six Patriarchs (*manu*) are past, with their respective twilights; and of the Patriarch Manu son of Vivasvant, twenty-seven Ages are past; (22)

Of the present, the twenty-eighth, Age, this Golden Age is past: from this point, reckoning up the time, one

[1] The terms Kṛta etc. of the Veda, are not used in the definitive sense of time measures, but have, during later ages, been used to connote large units of time called *yugas* (ages, aeons).

should compute together the whole number.[1] (23). (Burgess)

7. 7. 5. युगं कल्पसहस्रांशो मनुस्तान्येकसप्ततिः ।
सन्धयः कृततुल्यास्तदाद्यन्ताभ्यन्तरेष्वपि ।। ११ ।।

युगस्यापि दशांशोऽब्धि-त्रि-द्वये-क-घ्नः कृतादि च ।
अष्टाविंशे युगे तिष्यः सप्तमस्य मनोरयम् ।। १२ ।।

(Nīlakaṇṭha, *Si.Dar.*, 11-12)

One thousandth of a *kalpa* is a *yuga*. Seventyone (of these *yugas*, constitute (the period of) a Manu, (there being, thus, fourteen complete Manu periods in a *kalpa*). Between the periods of (each) Manu and at the beginning and end of the (*kalpa*) there are intercalary periods each equal to (the duration of) a Kṛta age. (11)

The Kṛta and other ages (viz. Tretā, Dvāpara and Kali) are, respectively, four-tenths, three-tenths, two-tenths and one-tenth of a *yuga*. (12). (KVS)

7. 7. 6. यदा चन्द्रश्च सूर्यश्च तथा तिष्यबृहस्पती ।
एकराशौ समेष्यन्ति तदा भवति तत् कृतम् ।।

(*Bhāgavata Purāṇa*, 12. 2. 24)

Beginning of the Kṛta-yuga

When the Moon, the Sun and Jupiter rise together in one zodiacal house and the Puṣya constellation is in the ascendant, then it will be the advent of the Kṛta age. (24)[2] (C.L. Goswami)

कल्पप्रवृत्तिः

7. 7. 7. लङ्कानगर्यामुदयाच्च भानो-
स्तस्यैव वारे प्रथमं बभूव ।
मधोः सितादेर्दिनमासवर्ष-
युगादिकानां युगपत् प्रवृत्तिः ।। १५ ।।

(Bhāskara II, *SiŚi*, 1.1.1.15)

Beginning of Kalpa

The first *mahāyuga*, the first year, the first day of the bright half of the first month named Madhu, all of them began simultaneously at the sunrise at Laṅkā on Sunday, at the beginning of the first *kalpa* which marked the beginning of creation. (15). (AS).

युगव्यवस्था—२

7. 7. 8. काहो मनवो ढ, मनुयुगाः श्ख, तास्ते च, मनुयुगाः छ्ना च ।
कल्पादेर्युगपादा ग च, गुरुदिवसाच्च, भारतात् पूर्वम् ।।

(Āryabhaṭa I, *ABh.*, 1.5)

Yuga concept—ii

A day of (God) Brahmā (or a *kalpa* or Great aeon) is equal to (a period of) 14 manus, and (the period of one) manu is equal to 72 *yugas*. Since Thursday, the beginning of the current *kalpa*, 6 manus, 27 *yugas* and 3 quarter *yugas* had elapsed before the beginning of the current Kaliyuga (*lit.* before Bhārata). (5). (KSS)

युगप्रवृत्तिः

7. 7. 9. युगवर्षमासदिवसाः समं प्रवृत्तास्तु चैत्रशुक्लादेः ।
कालोऽयमनाद्यन्तो ग्रहभैरनुमीयते क्षेत्रे ।।

(Āryabhaṭa I, *ABh.*, 3.11)

Beginning of the yuga

The *yuga*, the year, the month, and the day commenced simultaneously at the beginning of the light half of Caitra. Time, which is without beginning and end, is measured with help the of the planets and the asterisms on the celestial sphere. (11). (KSS)

युगव्यवस्था—३

सौररोमकयोः रविचन्द्रयुगम्

7. 7. 10. वर्षायुते 'धृति'घ्ने 'नववसुगुणरसरसाः' स्युरधिमासाः ।
सावित्रे 'शरनवखेन्द्रियार्णवाशाः' तिथिप्रलयाः ।। १४ ।।

रोमकयुगमर्केन्द्वोर्वर्षा'ण्याकाशपञ्चवसुपक्षाः' ।
'खेन्द्रियदिशो' ऽधिमासाः 'स्वरकृतविषयाष्टयः' प्रलयाः ।।

युगवर्षमासपिण्डं रविमासं साधिमासकं चान्द्रम् ।
अवमविहीनं सावनमैन्दवमब्दान्वितं त्वार्क्षम् ।। १६ ।।

(Varāha, *PS*, 1.14-16)

Yuga concept—iii

Yuga of Sun and Moon—Saura and Romaka

In the *Saura Siddhānta*, a period (actually the minor *yuga*) of 1,80,000 solar years contain 66,389 intercalary months and 10,45,095 elided days. (14)

The luni-solar *yuga* of the *Romaka Siddhānta* consists of 2850 solar years. In this period there are 1050 intercalary months and 16,547 elided days. (15)

The solar years in the *yuga* multiplied by 12 gives the solar months in the *yuga*. The solar months plus the intercalary months are the synodic months in the *yuga*. The *tithis* got by multiplying the synodic months by 30 reduced by the elided days, are the civil days, (i.e., days) in the *yuga*. The civil days plus the solar years are the sidereal days in the *yuga* (or the synodic months plus the solar years are the Moon's revolutions in the *yuga*). (16). (TSK)

युगकल्पनायां मतभेदः

7. 7. 11. कल्पादीनां प्रमाणं तु बहुधा कल्प्यते बुधैः ।
उपेयस्यैव नियमो नोपायस्येति यत् ततः ।। ५.१५ ।।

[1] For a detailed note, see *SūSi* : *Burgess*, pp. 9-13.

[2] This is said of the Kṛta age to come after the dissolution of the current *yuga*.

एवं युगोक्ता भगणादयस्ते
'दिनानय'घ्नास्तु भवन्ति कल्पे ।
चतुर्दश स्युर्मनवोऽत्र तेषां
युगानि 'रास'प्रमितानि यस्मात् ।। १.६ ।।

कृत-त्रेता-द्वापराख्य: कलिश्चैते युगांघ्रय: ।
युगांघ्रयस्तु कल्पेऽस्मिन् 'धिगादित्य' मिता गता: ।।१.७।।

कल्पे युगानि तु सहस्रमुशन्ति केचित्
तत्रैकसप्ततियुगानि पृथङ्‌ मनूनाम् ।
आद्यन्तयोश्च विवरे च तथैव तेषां
स्यु: सन्धयो युगदशांशचतुष्कतुल्या: ।। ५.१६ ।।

मनवोऽथ चतुर्दशैव कल्पे
'पृथु'तुल्यानि युगानि चैव तेषाम् ।
त्रियुगानि गतानि सृष्टित: प्राक्
परत: स्यु: प्रलयात् तथाहुरन्ये ।। १७ ।।

युगस्य दशमो भागो 'भो-ग-प्रि-य' हत: क्रमात् ।
कृतादीनां प्रमाणं स्यात् पक्षयोरनयोर्द्वयो: ।। १८ ।।

कल्पेऽस्मिन् सप्तमस्यास्य वैवस्वतमनोर्युगे ।
अष्टाविंशे कलि: सर्वैर्वर्तमान इह स्मृत: ।। १९ ।।

(Putumana Somayāji, *KP*, 5.15; 1.6-7; 5.16-19)

Different concepts on yuga periods

The measures of *kalpa* etc. are conceived, by scholars, differently, for, ultimately, it is only the result that counts and not the means. (5.15)

Thus, (according to the Āryabhaṭan school), the *yuga* revolutions multiplied by 1008 form *kalpa* revolutions. In a *kalpa* (period) there are 14 *manu* (periods) in each of which there are 72 *yugas*. (1.6)

Kṛta, Tretā, Dvāpara and Kali are the (respective) names of one fourths of the yuga. In the current *kalpa*, 1839 quarter yugas are over.[1] (1.7)

Others (like the *Sūryasiddhānta*, Bhāskara II etc.) take the number of *yugas* in a *kalpa* as 1000. Each of the 14 *manu* periods would have 71 *yugas*; between the beginning and ends of the fourteen *yugas* there are (in all, 15) contact periods, each equal to four tenths of a *yuga*. (5.16)

Still others agree that the number of *manus* in a *kalpa* is only 14, each having 71 *yugas*, but that 3 *yugas* have passed before Creation and 3 *yugas* will occur after Dissolution. (17)

According to the above two views, the measures of the Kṛta, (Tretā, Dvāpara and Kali *yugas*) are 4, 3, 2 and 1 tenths part of the *(Catur-)yuga*. (18)

All agree that today the current *yuga* is Kali, in the 28th *(catur-)yuga* of the 71st *manu* (viz. Vaivasvata Manu) in the present *kalpa*. (19). (KVS)

कल्पभगणसंस्कारोपाय:

7. 7. 12. ग्रहणग्रहयोगाद्यैर्ये ग्रहा: सुपरीक्षिता: ।
दृक्समास्तत्समा: कल्पे कल्प्या वा भगणादय: ।। १ ।।

परीक्षितस्य खेटस्य तन्त्रानीतस्य चान्तरम् ।
लिप्तीकृत्यार्कभगणै: कल्पोक्तैश्च समाहतम् ।। २ ।।

तन्त्रनिर्माणकालस्य परीक्षासमयस्य च ।
अन्तरालगतैरब्दै राशिचक्रकलाहृतै: ।। ३ ।।

हृत्वाऽऽप्तं तन्त्रनीतस्य ग्रहस्याल्पाधिकत्वत: ।
स्वर्णं तत् कल्पभगणे कुर्यान्नैष विधी रवे: ।। ४ ।।

(Putumana Somayāji, *KP*, 5.1-4)

Method of correcting kalpa and yuga revolutions

(The number of) planetary revolutions (taken to constitute) a *kalpa* (according to some Siddhānta or text) has to be revised (periodically) so that eclipses and planetary conjunctions computed using those numbers would accord with the positions actually observed (through instruments, during contemporary times). (1)

(Towards effecting the said revision), take the difference between the true positions of a planet as observed (in the sky) and as calculated using the number of revolutions (as accepted) and reduce the difference to minutes. Multiply this by the Sun's *kalpa* revolutions and divide by the number of years between the time of composition of the said Siddhānta or text (at which time it is to be presumed that the computation accorded with the then observation) and the time of the (current) observation. Reduce the quotient (which would be in full cycles) to minutes by multiplying it by 21,600, being the number of minutes contained in a circle. (2-3)

The result (which is the correction in terms of revolutions) is to be added to or subtracted from the number of *kalpa* revolutions (enunciated by the Siddhānta or text) according as the true planet determined by computation is less or more (than the observed true planet). This mode of correction is not to be applied to revise the Sun's revolutions (since the basis itself of the correction is the Sun's revolutions). (4) (KVS)

भगणादीनां परीक्षणानुमानादिभिर्निर्णय:

7. 7. 13. "ज्योतिश्शास्त्रेऽपि युगपरिवृत्तिपरिमाणद्वारेण चन्द्रादित्यादिगतिविभागेन तिथिनक्षत्रज्ञानमविच्छिन्नसम्प्रदायगणितानुमानमूलम्" इति वार्त्तिककारोऽपि ग्रहगतिज्ञानम् अनुमानेनाह । (कुमारिलभट्ट:, तन्त्रवार्त्तिकम्, 1. 3. 2) ।

[1] 1839 quarter *yugas* would amount to 6 *manus*, 27 full *yugas* and 3 quarter *yugas*, the quarter *yuga* current today being of the Kali *yuga* in the 28th *manu* period.

तत्राविच्छिन्नसम्प्रदायपदमप्येवं व्याचष्टे—'गणितोन्नीतस्य चन्द्रादेः देशविशेषान्वयस्य प्रत्यक्षेण संवादः, ततो निश्चितान्वयस्य परस्य गणितलिङ्गोपदेशः, ततस्तस्याप्तोपदेशवशावगतान्वयस्य अनुमानम्, संवादः, परस्मै चोपदेशः इति सम्प्रदायाविच्छेदात् प्रामाण्यम्' इति । (अजिताव्याख्या) । (Kumārila Bhaṭṭa, *Tantravārttika* and com. *Ajitā*, quoted in Nīlakaṇṭha Somayāji, *Jyotirmīmāṁsā*, p. 3)

Fixation of revolutions by observation and deduction

"Even in the science of astronomy, the computation of *tithis* and *nakṣatras* by means of the motion of the Sun and the Moon, which is determined through the number of their revolutions in a *yuga*, is based on the continuity of tradition and deduction." (Kumārila Bhaṭṭa, *Tantravārttika*, 1.3.2).

Here, the expression 'the continuity of tradition' is explained thus: "The correlation of the Moon etc. as observed in a place and as computed (using extant texts in the science) is done first; this information is then discussed with astronomical rationale with one rooted in the science; there is then deductions by that person rooted in the science; then follows discussions and again instruction to another (on the basis of such deductions). Thus the authority of the science (along with the new deductions and revisions) is due to this continuity of tradition." (Com. *Ajitā*). (KVS)

युगभगणाः मासदिनादि च
—आर्यभटीयानुसारी

7. 8. 1. 'नानाज्ञानप्रगल्भ'-'स्तिलबलमसुसूक्ष्मं' 'धयेद्राजदम्भो'
'भद्रोदन्तो धरेन्द्रो' 'निरनुसृगधिसौख्यं' 'वरिष्ठोऽभिषङ्गः' ।
'दोर्दण्डाग्रेद्रिनाथो' 'विषमितविपिनं' 'चन्द्ररेखाम्बुखिन्ने'-
त्यर्कादेः पर्ययाः स्युः, क्षितिदिन 'मनृशंसः कळार्थी स मर्त्यः' ॥

'रुपा'हतार्कभगणाः खलु सौरमासा
मासा रवीन्दुभगणान्तरमेव चान्द्राः ।
चन्द्रार्कमासविवरं च युगाधिमासा
मासाः पुन-'र्नग'-हता दिवसस्वरूपाः ॥ ४ ॥

चान्द्रमासा 'नगा'भ्यस्ता भूदिनोनास्तिथिक्षयाः ।
भूदिनाढ्यार्कभगणा नाक्षत्रदिवसाः स्मृताः ॥ ५ ॥

एवं युगोक्ता भगणादयस्ते
'दिनानय'घ्नास्तु भवन्ति कल्पे ।
चतुर्दश स्युर्मनवोऽत्र, तेषां
युगानि 'रास'प्रमितानि यस्मात् ॥ ६ ॥

कृत-त्रेता-द्वापराख्यः कलिश्चैते युगांघ्रयः ।
युगांघ्रयस्तु कल्पेऽस्मिन् 'धिगादित्य'मिता गताः ॥ ७ ॥

कल्यब्दतः 'प्रिय'हताद् गतमासयुक्तात्
चान्द्राख्यमासगुणिताद् रविमासलब्धः ।
'नागा'हतस्तिथियुतः क्षितिवासरघ्न-
श्चान्द्रैर्दिनैरपहृतो द्युगणोऽच्छवारात् ॥ ९ ॥

(Putumana Somayāji, *KP*, 1. 3-7, 9)

Yuga revolutions etc.—Āryabhaṭa school

The number of revolutions of the planets in a (*catur-*) *yuga* are:

Sun: 43,20,000; Moon:-5,77,53,336; Moon's apogee: 4,88,219; Mars: 22,96,824; Mercury: 1,79,37,020; Jupiter: 3,64,224; Venus: 70,22,388; Saturn: 1,46,564; and Moon's Node: 2,32,226. The number of civil days are 1,57,79,17,500. (3)

The Sun's revolutions multiplied by 12 give solar months.

The difference between the revolutions of the Sun and the Moon gives the lunar months. The solar months *minus* lunar months are the intercalary months, all in the (*catur-*)*yuga*. Months multiplied by 30 give days. (4)

The lunar months multiplied by 30 and the number of civil days subtracted from it give the 'omitted lunar days'. The number of civil days *plus* revolutions give sidereal days. (5)

In a *kalpa* there are 1008 *yugas* of the type stated above. In it (i.e. the *kalpa*) there are 14 *manus*, for the period of each *manu* is equal to 72 yugas. (6)

The four quarters of the *yuga* are Kṛta, Tretā, Dvāpara and Kali. And, in the current *kalpa*, 1839 quarter yugas have passed by. (7)

The (elapsed) Kali year multiplied by 12, the number of the months (Meṣa etc.) elapsed in the current year added to it, the result multiplied by the lunar months in the *yuga*, and the product divided by the solar months, the result multiplied by 30, the lunar days elapsed in the current lunar month added to the product, and the sum multiplied by the number of days in a *yuga* and divided by the lunar days in the *yuga* will give the number of elapsed Kali days in the *yuga*, as counted from Friday.[1] (9). (KVS)

अधिमासतिथिक्षयौ

7. 8. 2. अधिमासका युगे ते रविमासेभ्योऽधिकास्तु ये चान्द्राः ।
शशिदिवसा विज्ञेया भूदिवसोनास्तिथिप्रलयाः ॥

(Āryabhaṭa I, *ABh.*, 3.6)

Intercalary months and omitted lunar days

The lunar months (in a *yuga*) which are in excess of the solar months (in a *yuga*) are (known as) the inter-

[1] For the rationale, see *KP*: *PKK*.

calary months in a *yuga*; and the lunar days (in a *yuga*) diminished by the civil days (in a *yuga*) are known as the omitted lunar days (in a *yuga*)[1]. (2). (KSS).

—सूर्यसिद्धान्तानुसारि

7. 8. 3. उदयादुदयं भानोः भूमिसावनवासराः ।
'वसुद्वयष्टाद्रिरूपाङ्कसप्ताद्रितिथयो' युगे ।। ३६ ।।
भाना'मष्टद्विवस्वद्रित्रिद्विद्वयष्टशरेन्दवः' ।
चान्द्राः 'खाष्टखखव्योमखाग्निखर्तुनिशाकराः' ।। ३७ ।।
'षड्वह्नित्रिहुताशाङ्कतिथय'श्चाधिमासकाः ।
तिथिक्षया 'यमार्थाक्षिद्वयष्टव्योमशराश्विनः' ।। ३८ ।।
(*Sū.Si.*, 1. 36-38)

—Sūryasiddhānta school

From rising to rising of the Sun are reckoned terrestrial civil days. Of these there are, in an Age, one billion, five hundred and seventy-seven million, nine hundred and seventeen thousand, eight hundred and twenty-eight; of lunar days one billion, six hundred and three million, and eighty. (36-37)

Of intercalary months, one million, five hundred and ninety-three thousand, three hundred and thirty-six; of omitted lunar days, twenty-five million, eighty-two thousand, two hundred and fifty-two. (38) (Burgess)

ग्रहयोगसङ्ख्याः केन्द्रपर्ययाश्च

7. 9. 1. या मण्डलान्तरमितिर्ग्रहयोर्द्वयोः स्यात्
सैवोभयोरपि तयोर्युगयोगसङ्ख्या ।
ये स्वोच्चपर्ययनभश्चरमण्डलानां
विश्लेषजास्त इह केन्द्रजपर्ययाः स्युः ।। ११ ।।
(Lalla, *SiDhVṛ.*, 1.11)

Planetary conjunctions and Revolutions of the apogees

The difference between the revolutions of two planets in a *yuga* is the number of their conjunctions during the same period.

The difference between the revolutions of a planet and those of its apogee (*mandocca*) or *śīghrocca* gives the number of revolutions of the mean anomaly (in the first case) and of the *śīghra* anomaly in the second case (during the same period). (11). (BC)

युगान्तरकरणम्

—शकाब्दात् कल्यब्दः

7. 10. 1. 'धूसीकाल'-युतः शाकः कल्यब्द इति कीर्तितः ।
(*Vākyakaraṇa*, extra line before 1.2)

[1] Thus, according to Āryabhaṭa I,
Intercalary months in a *yuga* = 5,34,33,336—5,18,40,000 = 15,93,336
Omitted lunar days in a *yuga* = 1,60,30,00,080—1,57,79,17,500 = 2,50,82,580

Conversion of eras

—Śaka to Kali

3179 added to the Śaka year gives the Kali year. (KVS)

—शकाब्दात् प्रभवादि-बृहस्पत्यब्दानयनम्

7. 11. 1. शकाब्दा 'रुद्र'संयुक्ता यद्वा षष्ट्या समुद्धृताः ।
तत्रावशिष्टसंख्याः स्युर्विज्ञेयाः प्रभवादयः ।।
(Anon. Com. on Citrabhānu, *Karaṇāmṛta*, 1.1-2)

—Śaka years to Jovian years

Add 11 to the Śaka year and divide by 60. (Ignore the quotient.) The remainder will give the year in the Bṛhaspati era (Jovian years) as counted from Prabhava.[1] (KVS)

—कोलम्बवर्षात् कल्यब्दः

7. 12. 1. गतवर्षान्तकोलम्बवर्षाः 'तरळगा'न्विताः ।
कल्यब्दा, 'धीस्थकाला'द्याः शकाब्दा वा भवन्ति ते ।।
(Putumana Somayāji, *KP*, 1.8)

—Kollam year to Kali

The elapsed Kolamba (i.e. Kollam or Malabar) years added to 3926, or the elapsed Śaka years added to 3179 give the corresponding Kali year. (8). (KVS)

—विक्रमसंवत्-हिजिरी

7. 13. 1. विक्कमि जे वरिस मास चित्ताइ करिवि दिण
'छमुणिनंद'लद्धहियमास ते वच्छर जुय पुण ।
'नवनिहाणरस'-वरिस मास दुइ दुइ दिण ऊणय
ताजिय वच्छरु हवइ मास मुहरम माईणय
ताजिक्कु पुणेवं करिवि पर अहिय मास सोहेवि पुणि ।।१।।
'नवमुणिछ'-वरिस दुइ दिण अहिय पंडिय विक्कमसमउ भणि ।। २ ।।
Ṭhakkura Pherū, *Gaṇitasāra*, 4.17)

—Conversion of Vikrama-saṃvat to Hijri

Convert the years from the beginning of the Vikrama era and the months from Caitra into (solar) days. Divide it by 976. The quotient is the number of intercalary months. Convert them into years, and add (to the given date), and then subtract from it 699 years, 2 months and 2 days. The result is the Hijri year, the months starting from Muharram. (1)

Do the same for the Hijri date (i.e., convert the years and months into lunar days and from the latter obtain the number of intercalary months), and then subtract from it the intercalary months and add 679 years, 2

[1] The Śakābda commenced with the 12th Jovian year and hence 11 is directed to be added to the current Śakābda and the sum directed to be divided by 60.

months and 2 days, respectively. The result is the Vikrama date.[1] (2). (SRS)

7. 13. 2. अथ स्वशाकोपरि सन्नमासज्ञानोपाय:—

शक: 'खाभ्रपञ्चेन्दु'हीनोऽर्क'-निघ्नो
मधोर्मासियुक्तो ह्यध-'स्त्र्यश्वि'युक्त: ।
द्विनिघ्न: स्व-'खव्योमनन्दां'शहीन:
'शराङ्गै'-रवाप्ताधिमासैर्युगूर्ध्व: ॥ ३ ॥
'पतङ्गा'प्तशेषैर्महर्मादिमासो
'रसाष्टाङ्क'-युक्तेषु लब्धेषु सन्न: ॥ ४ ॥

अस्योदाहरणम्—

शाके 1565 'खाभ्रपञ्चेन्दु' (1500) हीने जातम् 65. 'अर्क'निघ्ने (12) जातम् 780. चैत्रशुक्लादेर्वर्तमानमास: 5 शुक्लादि श्रावण: । तेन युक्ते जातम् 785. अध: स्थाप्य 'त्र्यश्वि' (23) युक्ते जातम् 808. द्विनिघ्ने जातम् 1616. तत: स्व-'खव्योमनन्दांशेन' (900) हीने जातम् 1615. 'शराङ्गै' (65)-र्भक्ते लब्धाधिमास: 24. अधिमासैर्युते जातम् ऊर्ध्वम् 809. द्वादशभक्ते शेषे मुहर्मादिमास: । लब्धाङ्क (67) मध्ये 'रसाष्टाङ्कक' (986) युक्ते जातो वर्तमानसन्न: 1053.

Mālajit Vedāṅgarāya, *Pārasīprakāśa*, 1. 3-4

Conversion of a Śaka date to Hijri date

Now the method of calculating the Hijri year (*sanna*) and the month from a given date in the Śaka era:

Reduce the Śaka (year) by 1500, multiply it by 12 and add the months from Caitra. (The result is the number of solar months after Śaka 1500).

Take this number separately, add 23 and multiply by 2. This, when reduced by its 900th part and divided by 65, gives the intercalary months. (3)

Add these intercalary months to the above (number of solar months). Divide the sum by 12. The remainder is (the number of the months as counted from Muharram. The quotient increased by 986 gives the Hijri year. (4)

Its illustration: The Śaka year 1565, reduced by 1500, becomes 65. This, multiplied by 12, becomes 780. The current month Śrāvaṇa beginning with the bright fortnight is the fifth month from Caitra beginning with the bright fortnight. Adding this (i.e. 5), we get 785 (i.e. solar months after Śaka 1500).

Take this number separately and add 23 to it. It becomes 808. When multiplied by 2, it becomes 1616. When its 900th part is subtracted from it, it becomes 1615. Dividing it by 65, we get 24 intercalary months. The above number (i.e. 785), increased by the intercalary months, gives 809 (i.e. lunar months after Śaka 1500).

When divided by 12, the remainder is (the number of the Islamic) month as counted from Muharram (i.e. Jamadā'l awwal, the fifth month). The quotient 67, increased by 986, becomes the current Hijri year 1053. (SRS)

7. 13. 3. अथ सन्नोपरि हिन्दूक-शाकमास-ज्ञानोपाय:—

सन'स्तर्कवस्वङ्क'हीनोऽर्क'निघ्नो
महर्मादिमासान्वितोऽध'स्त्रिदस्रै:' ॥ ४ ॥
युतो 'नेत्र'निघ्न: स्व-'खाभ्राङ्क'-भागै-
र्युतोऽद्रचङ्ग'-लब्धाधिमासैर्विहीन: ।
'द्युनाथा'प्तशेषे मधोर्मास इष्ट:
'खखाक्षेन्दु'-युक्तो च लब्धे तु शाक: ॥ ५ ॥

अस्योदाहरणम्

वर्तमानसन्न: (1053) 'तर्कवस्वङ्क' (986) हीनो जात: 67. 'अर्क' (12) निघ्नो जात: 804. मुहर्मदादिवर्तमानमास: 5 जमादिलव्वल: । तेन युक्ते जातम् 809. अध'स्त्रिदस्रै:' (23) युते जातं 832. द्विनिघ्ने जातम् 1664. तत: स्व-'खाभ्राङ्क' (900) भागैर्युते जातम् 1665. 'अद्रचङ्गै' (67) र्भक्ते अधिमास: 24. अधिमासोपरि हीने जातम् 785. द्वादशभक्ते लब्धम् 65, शेषम् 5. तेन चैत्रादेर्वर्तमान: पञ्चम: शुक्लादि: श्रावणो मास: । लब्धमध्ये 1500 एतद्युक्ते जातो वर्तमान शाक: 1565.

(Mālajit Vedāṅgarāya, *Pārasīprakaśa*, 1. 4-5)

Conversion of a Hijri date to Śaka date

Now the method of calculating the Śaka year and month of the Hindus from the Hijri date:

Reduce the Hijri year by 986, multiply it by 12 and add the months from Muharram. (The result is the number of lunar months after A.H. 986.)

Take this number separately, add 23 and multiply by 2. This, when increased by its 900th part and divided by 67, gives the intercalary months.

Subtract the intercalary months (from the lunar months obtained above) and divide by 12. The remainder is the (number of the desired month as counted from Caitra. The quotient increased by 1500 gives the Śaka year. (4-5)

Its illustration: The current Hijri year 1053, reduced by 986, becomes 67. When multiplied by 12, it becomes 804. The current month Jamada'l awwal is the fifth from Muharram. Adding this (i.e. 5th month), we get 809 (i.e. lunar months after A.H. 986).

[1] The Hijri era commenced on V.S. 679 (expired) Śrāvaṇa śukla 2 (i.e., Friday, 16 July 622 A.D.), and the years are lunar, intercalation being prohibited by the Quran. Therefore, in order to convert a Vikrama date into Hijri one, Pheru first transforms the solar years in the Vikrama era into lunar years by adding the intercalary months from the beginning of the Vikrama era. From this sum he subtracts the difference between the commencement of the two eras (expressed in lunar years) to obtain the Hijri date. The reverse process yields the Vikrama date corresponding to a given Hijri date.

In the second operation 679 years 4 months and 2 days should be added to get the correct result.

Write this number separately and add 23 to it. It becomes 832. Multiply it by 2. It becomes 1664. By adding its 900th part, it becomes 1665. This, divided by 67, gives 24 intercalary months. Subtract the intercalary months from the above number (i.e. the lunar months). It becomes 785 (i.e. solar months after A.H. 986).

Divide it by 12. The quotient is 65 and the remainder 5. Therefore, the month is Śrāvaṇa beginning with the bright fortnight, which is the fifth month as counted from Caitra beginning with the bright fortnight. The quotient increased by 1500 gives the present Śaka 1565. (The date corresponds to A.D. 1643.)

(The above two rules are basically the same as given by Pherū. But instead of computing intercalary months for the whole period in the Śaka era or Hijri era, Vedāṅgarāya adopts Śaka 1500=A.H. 986 as the gauge year (since these two years commence at the same time, and makes calculations for the remaining period.) (SRS)

दैर्घ्यमानम्

7. 14. 1. जालान्तरगे भानौ यदणुतरं दर्शनं रजो याति ।
तद् विन्द्यात् परमाणुं प्रथमं तद्धि प्रमाणानाम् ।। १ ।।
परमाणुरजो बालाग्रलिक्षयूकं यवोऽङ्गुलं चेति ।
अष्टगुणानि यथोत्तरमङ्गुलमेकं भवति संख्या ।। २ ।।

(VM, *Bṛ. Saṃ.*, 58.1-2)

Linear measures

The smallest particle of dust that comes to sight, when the Sun passes through the interstice of a window, is to be understood as an *atom*. This is the smallest unit of all measurements. (1)

An atom, a dust particle, a tip of the hair, a nit, a louse, a barley corn and a digit are in order eight times bigger than the preceding measure. One digit becoms an integer.[1] (2). (M.R. Bhat)

7. 14. 2. रवेर्गृहान्तःस्थितरश्मितो ये
प्रकाशमायान्त्यणवोऽष्टभिस्तैः ।
कचाग्रमष्टौ खलु तानि लिक्षा
ताभिश्च यूकाऽष्टभिरेवमुक्ता ।। १ ।।
यवोऽष्टयूकोऽङ्गुलमष्टभिस्तै-
रथाङ्गुलद्वादशभिर्वितस्तिः ।
वितस्तियुग्मेन करः करैर्ना
चतुर्भिरेका नृसहस्रमुक्तः ।। २ ।।
क्रोशस्तु तैर्योजनमष्टसंख्यै-
स्तैर्व्योमवृत्तं कथयन्ति सन्तः ।
'खव्योमपूर्णर्तुनगेषुखाक्षि-
ग्रहाब्धिभूभृत्सुखपक्षचन्द्रैः' ।। ३ ।।

(Vaṭeśvara, *VSi.*, 1.7.1-3)

Eight *aṇus* or minute particles (of dust)seen in a beam of sunlight (entering through an aperture) in the interior of a house, make one *kacāgra* (or *bālāgra*); eight of them make one *likṣā*; and eight of them are said to make one *yūkā*. Eight *yūkās* make one *yava*; eight of them make one *aṅgula* (finger-breadth or digit); twelve *aṅgulas* make one *vitasti*; two *vitastis* make one *kara* (hand or cubit); four *karas* make one *nṛ* (*nara*, man's height). 1000 *nṛ* are said to make one *krośa*; 8 of them make one *yojana*; and 1,24,74,72,05,76,000 of them, say the learned, make (the circumference of the circle of the sky.[1] (1-3). (KSS)

दैर्घ्यमानम्

7. 14. 3. योजनाष्टसहस्रांशो दण्डस्तच्चरणः करः ।
त'ज्जिनां'शोऽङ्गुलं तस्य षष्ट्यंशो व्यङ्गुलं स्मृतम् ।।

(Śaṅkara Varman, *Sadratnamālā*, 2.11)

Linear measures

1 *yojana*	=	8000 *daṇḍas*
1 *daṇḍa*	=	4 *hastas*
1 *hasta*	=	24 *aṅgulas*
1 *aṅgula*	=	60 *vyaṅgulas*

घनमानम्

7. 15. 1. क्रमाच्चतुर्गुणं मानं कुडुबः प्रस्थ आढकः ।
द्रोणो वहः खारिकेति घनहस्तमितावधिः ।। १० ।।

(Śaṅkara Varman, *Sadratnamālā*, 2.10)

Volume measures

4 *kuḍubas*	=	1 *prastha*
4 *prasthas*	=	1 *āḍhaka*
4 *āḍhakas*	=	1 *droṇa*
4 *droṇas*	=	1 *vaha*
4 *vahas*	=	1 *khārikā*=one cubic *hasta*

---

[1] The result may be tabulated as follows :

8 atoms	=	1 dust particle.
8 dust particles	=	1 tip of hair.
8 tips of hair	=	1 nit.
8 nits	=	1 louse.
8 lice	=	1 barley grain.
8 barley grains	=	1 digit.

[1] Linear measures

8 *aṇus*	=	1 *kacāgra*
8 *kacāgras*	=	1 *likṣā*
8 *likṣās*	=	1 *yūkā*
8 *yūkās*	=	1 *yava*
8 *yavas*	=	1 *aṅgula* (digit)
12 *aṅgulas*	=	1 *vitasti*
2 *vitastis*	=	1 *kara* (cubit)
4 *karas*	=	1 *nṛ*
1000 *nṛ*	=	1 *krośa*
8 *krośas*	=	1 *yojana*
1,24,74,72,05,76,000 *yojanas*	=	1 circle of sky

उन्मानम्

7. 16. 1. तुलाशतांशः पलमेतदङ्घ्रिः
कर्षोऽस्य माषोऽ'वनिपाल'भागः ।
गुञ्जास्य पञ्चांशक एतदर्धं
यवो हि, गुञ्जात्रितयं तु वल्लः ।। २२ ।।
(Śaṅkara Varman, *Sadratnamālā*, 2.12)

Weight measures

1 *tulā*	=	400 *palas*
1 *pala*	=	4 *karṣas*
1 *karṣa*	=	16 *māṣas*
1 *māṣa*	=	5 *guñjas*
1 *guñja*	=	2 *yavas*
1 *malla*	=	3 *guñjas*

राश्यादिविभागः

7. 17. 1. विकलानां कला षष्टचा तत्षष्ट्या भाग उच्यते ।
तत्त्रिशता भवेद्राशिः भगणो द्वादशैव ते ।। २८ ।।
(*Sū. Si.*, 1.28)

Measures of angles

Sixty seconds (*vikalā*) make a minute (*kalā*); sixty of these a degree (*bhāga*); of thirty of the latter is composed a Sign (*rāśi*); twelve of these are revolutions (*bhagaṇa*). (28). (Burgess)

7. 17. 2. प्रतत्परा षष्टिगुणा हि तत्परा
विलिप्तिका सैवमसौ तथा कला ।
सैवं लवस्तत्त्रिदाहृतिर्भवेद्
राशिः स 'मार्तण्ड'गुणो भमण्डलम् ।। ३ ।।
(Śaṅkara Varman, *Sadratnamālā*, 2.4)

60 *pratatparās*	=	1 *tatparā*
60 *tatparās*	=	1 *viliptikā* (*viliptā* or *vikalā*)
60 *viliptikās*	=	1 *liptikā* (*liptā* or *kalā*)
60 *liptikās*	=	1 *lava* (or *bhāga*)
30 *lavas*	=	1 *rāśi*
12 *rāśis*	=	1 celestial circle

7. 17. 3. सप्तविंशति भं चक्रं, राशिः सांध्यृक्षयुग्मकः ।
राशौ नक्षत्रदिङ्नाडचः शतं त्रिंशच्च पञ्च च ।। ४ ।।
(Śaṅkara Varman, *Sadratnamālā*, 2.4)

27 asterismal segments	=	1 sidereal circle
1 Sign	=	$2\frac{1}{4}$ sidereal segments
1 Sign	=	($2\frac{1}{4} \times 60 =$) 135 sidereal *nāḍikā* segments

7. 17. 4. अल्पे हि मण्डलेऽल्पा महति महान्तश्च राशयो ज्ञेयाः ।
अंशाः कलास्तथैवं विभागतुल्याः स्वकक्ष्यासु ।।
(Āryabhaṭa I, *ABh.*, 3.14)

(The linear measures of) the Signs are to be known to be small in small orbits and large in large orbits; so also are (the linear measures of) the degrees, minutes, etc. The circular division is however, the same in the orbits of the various planets.[1] (14). (KSS)

ज्याः (१)—संख्या २४; व्यासार्धः ३४३८'; चापः ३° ४५'

7. 18. 1. मखि भखि फखि धखि णखि ञखि
ङखि हस्झ स्ककि किष्ग श्घकि किघ्व ।
घ्लकि किग्र हक्य धकि किच
स्ग श्झ ङ्व क्ल प्त फ छ कलार्धज्याः ।। १२ ।।
(Āryabhaṭa I, *ABh.*, 1.12)

R sine tables (1)—No. 24; R=3438'; arc=3°45'

225, 224, 222, 219, 215, 210, 205, 199, 191, 183, 174, 164, 154, 143, 131, 119, 106, 93, 79, 65, 51, 37, 22 and 7— these are the R sine-differences (at intervals of 225 minutes of arc) in terms of minutes of arc.[2] (12). (KSS)

7. 18. 2. क्रमार्धजीवाः 'शरनेत्रबाहवो'
'नवाब्धिवेदाः' 'कुशिलोच्चयर्तवः' ।
'खनन्दनागाः' 'शरशून्यशूलिनः'
'शरेन्दुविश्वे' 'नखबाणभूमयः' ।। १ ।।
'नवेन्दुसप्तक्षितयः' कलामया
'दिगङ्कचन्द्रा'-'स्त्रिनवाभ्रबाहवः' ।

[1] The non-equality of the linear measures of the circular divisions in the orbits of the various planets implies that although the planets have equal linear velocity, their angular velocities are different.

[2] **R sines and R sine-differences at the intervals of 225' or 3° 45'**

		Āryabhaṭa I's values		Modern values	
No.	Arc	R sines	R sine-differences	R sines	R sine-differencs
1.	225'	225'	225'	224'.856	224'.856
2.	450'	449'	224'	448'.749	223'.893
3.	675'	671'	222'	670'.720	221'.971
4.	900'	890'	219'	889'.820	219'.100
5.	1125'	1105'	215'	1105'.109	215'.289
6.	1350'	1315'	210'	1315'.666	210'.557
7.	1575'	1520'	205'	1520'.589	204'.923
8.	1800'	1719'	199'	1719'.000	198'.411
9.	2025'	1910'	191'	1910'.050	191'.050
10.	2250'	2093'	183'	2092'.922	182'.872
11.	2475'	2267'	174'	2266'.831	173'.909
12.	2700'	2431'	164'	2431'.033	164'.202
13.	2925'	2585'	154'	2584'.825	153'.792
14.	3150'	2728'	143'	2727'.549	142'.724
15.	3375'	2859'	131'	2858'.592	131'.043
16.	3600'	2978'	119'	2977'.395	118'.803
17.	3825'	3084'	106'	3083'.448	106'.053
18.	4050'	3177'	93'	3176'.298	92'.850
19.	4275'	3256'	79'	3255'.546	79'.248
20.	4500'	3321'	65'	3320'.853	65'.307
21.	4725'	3372'	51'	3371'.940	51'.087
22.	4950'	3409'	37'	3408'.588	36'.648
23.	5175'	3431'	22'	3430'.639	22'.051
24.	5400'	3438'	7'	3438'.000	7'.361

Similar values are given also in ABh. II. 3.4-8, *Mahā.*, 3.1-8; Bhāskara II, *Si.Śi.*, 1.2.2-6 etc.

'शिलोच्चयाङ्गाक्षियमा'स्ततः परं
'शशाङ्करामाम्बुधिबाहवः' पुनः ॥ २ ॥

'शराष्टतत्त्वानि' 'वसुद्व्यगाश्विन'-
स्तत'स्त्रिवर्गेषुभुजङ्गबाहवः' ।
'भुजङ्गशैलत्रिकवर्गबाहवः'
'समुद्रनागाभ्रगुणा' भटोदिताः ॥ ३ ॥

'शिलोच्चयाद्रीन्दुहुताशना'स्ततो
'रसेषुदन्ताः' 'कुयमाग्निवह्नयः' ।
'द्व्यगामरा' 'गोऽभ्रसमुद्रवह्नयः'
'कुत्र्यब्धिरामा'-ऽ'हिगुणाब्धिवह्नयः' ॥ ४ ॥

(Lalla, *ŚiDhVṛ.*, 2.1-4)

R sines

225, 449, 671, 890, 1105, 1315, 1520, 1719, 1910, 2093, 2267, 2431, 2585, 2728, 2859, 2978, 3084, 3177, 3256, 3321, 3372, 3409, 3431 and 3438 minutes are, respectively, the R sines (or *jyās* of arcs of 225′, 450′, 675′, etc.) as given by Āryabhaṭa. (1-4). (BC)

उत्क्रमज्याः

7. 18. 3. ततो विलोमा'स्तुरगा' 'नवाश्विनो'
'रसर्तवः' 'शैलशिवा' 'द्व्यहीन्दवः' ।
'कुषड्यमा' 'वेदशिलीमुखाग्नयः'
'खषड्युगान्यङ्क'-'शिलोच्चयेषवः' ॥ ५ ॥

'दिगद्रयो' 'रामशिलीमुखा हयः'
'स्वराम्बराशाः' 'शशिशैलशूलिनः' ।
'शराब्धिविश्वे''ऽष्टयमेषुभूमय'-
स्ततो 'नवेन्दुस्वरशीतरश्मयः' ॥ ६ ॥

'भुजङ्गशीतद्युतिनन्दभूमयो'
'हुताशनाक्षिक्षितिलोचना'न्यतः ।
ततो 'हुताशानलवह्निबाहवो'
'मतङ्गजाम्भोनिधिसायकाश्विनः' ॥ ७ ॥

'नगाङ्गभान्यङ्कगजाङ्कबाहव'-
'स्त्रिभूरदा' 'वस्वनलाब्धिवह्नयः' ।
इमां त्रिभज्यामथ चक्रलिप्तिका-
वृतेरपि व्यासदलं वदन्त्यतः ॥ ८ ॥

(Lalla, *ŚiDhVṛ.*, 2. 5-8)

R versed Sines

7, 29, 66, 117, 182, 261, 354, 460, 579, 710, 853, 1007, 1171, 1345, 1528, 1719, 1918, 2123, 2333, 2548, 2767, 2989, 3213 and 3438 minutes are, respectively, the R versed sines (or *utkramajyās* of 225′, 450′, etc.).

It is said that 3438, the R sine (*jyā*) of an arc of 90°, is also the radius of the circle whose circumference is 360° or 21600′. (5-8). (BC)

ज्यानयनम्

7. 18. 4. द्विघ्नान्त्यखण्डनिघ्नात्
तत्तज्यार्धात् त्रिभज्याप्तम् ॥ १३ ॥

अन्त्यादिखण्डयुक्तं
त्याज्यं स्यात् पूर्वपूर्वगुणसिद्ध्यै ।
न्यूनाधिकचापज्या-
श्रुतिदोःकोट्यादिभिस्त्वभीष्टज्या ॥ १४ ॥

(Nīla., *Golasāra*, 3. 13b-14)

The tabular R sines by computation

To get the R sines successively previous to the one already found, multiply this R sine by twice the very last difference, and divide by the radius. The result plus the difference (using which this R sine has been found) is the difference to be subtracted from this R sine to get the next previous R sine.[1]

14b. The R sine of any desired arc in excess or defect (of a given arc for which the tabular R sine is given) can be found by using the hypotenuse, the base, the perpendicular etc., (in the manner mentioned already).[2] (13b-14a). (KVS)

ज्यानयनपरिलेखः

7. 18. 5. समवृत्ते व्यासार्धं कृत्वा भूमिं, भुजे च तत्तुल्ये ।
लम्बं च भुजायोगाद्, बाह्वर्धसमे तदाबाधे ॥ ६ ॥

आबाधैवार्धज्या परिधेः पादेऽत्र शिष्टचापभवा ।
राशेरर्धज्या सा व्यासार्धदलेन सम्मिता तस्मात् ॥ ७ ॥

तस्याः कोटिर्लम्बः, कर्णो व्यासार्धमर्धमौर्विकयोः ।
कोट्यूनं व्यासार्धं बाहोर्बाणम्, ततस्तयोः कर्णात् ॥ ८ ॥

अर्धज्यादिकमेवं युक्त्या नेयं मुहुर्मुहुर्वृत्ते ।

(Nīla., *Golasāra*, 3.6-9a)

R Sines by the graphical method

In a circle, making the radius the base (of a triangle) and the other two sides equal to it, and dropping a

[1] The very last difference is supposed to be known, this being the difference between the radius and the penultimate R sine, which itself is the R cosine of the first R sine, which is again taken equal to the corresponding arc. For example, if there are 24 R sines in the quadrant, and if minutes of arc are taken as units, the radius is 3438 and is the 24th R sine. The 23rd is $\sqrt{3438^2 - 225^2} = 3430.6$ and the last difference is 7.4. Twice this is 14.8. The difference between the 23rd and the 22nd is $3431 \times 14.8 \div 3438 + 7.4 = 22$. Subtracting 22 from 3431, the 22nd is 3409. The difference between the 22nd and the 21st is $3409 \times 14.8 \div 3438 + 22 = 37$. The 21st is $3409 - 37 = 3372$. The difference between the 21st and 20th is $3372 \times 14.8 \div 3438 + 37 = 51$. The 29th is $3373 - 51 = 3321$. And so on.

[2] This is for ensuring accuracy. Otherwise, interpolation by the differences would suffice. The formula is: $R \sin(\theta \pm \alpha) = R \sin\theta . \cos\alpha \pm R \cos\theta . \sin\alpha = R \sin\theta \left(1 - \frac{\alpha^2}{2}\right) \pm \alpha R . \cos\theta$, where α is in radians, and sufficiently small. It is this formula that is the basis for the instruction in verses 13b-14a.

perpendicular from the intersection of the two sides, (viz., the vertex), the two segments of the base (thus formed) are each equal to half the side, (i.e., the radius). (6)

The segment itself is (thus seen to be) the R sine of the remaining arc of the quadrant. Therefore, R sine one *rāśi*, i.e., 30°, is half of the radius. (7)

The R cosine related to it is the perpendicular, (which is thus the R sine of 60°). The hypotenuse of (the triangle having for its sides) the two half-chords, is the radius. The radius *minus* R cosine is the arrow, (i.e., R versine), referring to the R sine. Then, from the hypotenuse of these two, (viz., the R sine and R versine), R sines (15°, 7½°) etc. should be found by the repeated working in the circle. (8-9a).[1] (KVS)

सूक्ष्मज्यानयनमार्गः

7. 18. 6. निहत्य चापवर्गेण चापं तत्तत्फलानि च ।
हरेत् समूलयुग्वर्गैस्त्रिज्यावर्गहतैः क्रमात् ।। ४४० ।।
चापं फलानि चाधोऽधो न्यस्योपर्युपरि त्यजेत् ।
जीवाप्त्यै, संग्रऽहोस्यैव 'विद्वान्' इत्यादिना कृतः ।।४४१।।
निहत्य चापवर्गेण रूपं तत्तत्फलानि च ।
हरेद् विमूलयुग्वर्गैस्त्रिज्यावर्गहतैः क्रमात् ।। ४४२ ।।
किन्तु व्यासदलेनैव द्विघ्नेनाद्यं विभज्यताम् ।
फलान्यधोऽधः क्रमशो न्यस्योपर्युपरि त्यजेत् ।। ४४३ ।।
शराप्त्यै, संग्रहोऽस्यैव 'स्तेन स्त्री'त्यादिना कृतः ।।

इष्टज्या-शरानयनोपयोगीनि 'विद्वान्' इत्यादिवाक्यानि

'विद्वांस्' 'तुन्नबलः' 'कवीशनिचयः' 'सर्वार्थशीलस्थिरो'
'निर्विद्धाङ्गनरेन्द्ररुङ' निगदितेष्वेषु क्रमात् पञ्चसु ।
आधस्त्याद् गुणितादभीष्टधनुषः कृत्या विहृत्यान्तिम-
स्याप्तं शोध्यमुपर्युपर्यथ घनेनैवं धनुष्यन्ततः ।।४३७।।
'स्तेनः' 'स्त्रीपिशुनः' 'सुगन्धिनगनुद्' 'भद्राङ्गभव्यासनो'
'मीनाङ्गो नरसिंह' 'ऊनधनकृद्भूरेव' षट्स्वेषु तु ।
आधस्त्याद् गुणितादभीष्टधनुषः कृत्या विहृत्यान्तिम-
स्याप्तं शोध्यमुपर्युपर्यथ फलं स्यादुत्क्रमज्यान्त्यजम् ।।

(Mādhava, Q by Śaṅkara in his com. in verse on Nīlakaṇṭha's *Tantrasaṅgraha*, 2.10-15)

माधवोदिताः तत्परादिमहाज्याः

'श्रेष्ठं नाम वरिष्ठानां' 'हिमाद्रिर्वेदभावनः' ।
'तपनो भानुः सूक्तज्ञो' 'मध्यमं विद्धि दोहनम्' ।। १ ।।

'धिगाज्योनाशनं कष्टं' 'छन्नभोगाशयाम्बिका' ।
'मृगाहारो नरेशोऽयं' 'वीरो रणजयोत्सुकः' ।। २ ।।

'मूलं विशुद्धं नाळस्य' 'गानेषु विरळा नराः' ।
'अशुद्धिगुप्ता चोरश्रीः' 'शङ्कुकर्णो नगेश्वरः' ।। ३ ।।

'तनूजो गर्भजो मित्रं' 'श्रीमानत्र सुखी सखे' ।
'शशी रात्रौ हिमाहारो' 'वेगज्ञः पथि सिन्धुरः' ।। ४ ।।

'छायालयो गजो नीलो' 'निर्मलो नास्ति सत्कुले' ।
'रात्रौ दर्पणमभ्राङ्गं' 'नागस्तुङ्गनखो बली' ।। ५ ।।

'धीरो युवा कथालोलः' 'पूज्यो नारीजनैर्भगः' ।
'कन्यागारो नागवल्ली' 'देवो विश्वस्थली भृगुः' ।। ६ ।।

तत्परादिकलान्तास्ता महाज्या माधवोदिताः ।। ७ ।।

(Mādhava, Q by Nīlakaṇṭha in his Com. on ĀBh, I, 2.12)

Method for accurate R sines correct to 9 places of decimals

Multiply the arc and the resulting products by the square of the arc and divide the results, in order, by the product of the square of even numbers increased by the numbers and the square of the radius. The arc and the results obtained from the above are placed one below the other and are subtracted systematically one from its above, to obtain the R sines represented by the expressions *vidvān* etc. (440-41)

Multiply the unit (i.e. the radius) and the resultant products by the square of the arc and divide the results by the product of the square of even numbers decreased by the numbers and the square of the radius. Place the results one below the other and subtract one from the above, for getting the Śara (i.e. R—R cosθ), represented by the expressions *stenastrī* etc.[1] (442-44)

Use of 'vidvān' etc. to derive any chosen sine or versine

Place the expressions 0′ 0″ 44‴, 0′ 33″ 6‴, 16′ 5″ 41‴, 273′ 57″ 47‴, and 2220′ 39″ 40‴ from below upwards. Multiply the lowest by the square of the chosen arc and divide by R^2 (i.e. 2,91,60,000). Subtract the quotient from the expression just above. Continue this operation through all the expressions above. The remainder got at the last operation is to be multiplied by the cube of the chosen arc and divided by R^3 (i.e. 1,57,46,40,00,000). Subtract the quotient from the chosen arc to get its R sine. (437)

Place the expressions 0′ 0″ 6‴, 0′ 5′ 12‴, 3′ 9′ 37‴, 71′ 43″ 24‴, 872′ 3″ 5‴, and 4241′ 9″ 0‴ from below

[1] For the rationale and graphical proof, see *Golasāra*: *KVS*, p. 16.

[1] For an elucidation in modern terms, see A.K. Bag, 'Mādhava's sine and cosine series', *IJHS*, 11 (1976) 54-57; T. A. Sarasvati Amma, *Geometry in Anc. and Med. India*, pp. 165-71.

upwards. Multiply the lowest by the square of the chosen arc and divide by R^2. Continue the operation through all the expressions above. The last quotient will be the versed sine of the chosen arc. (438). (For the results see footnote below).[1] (438). (KVS)

भोग्यखण्डस्पष्टीकरणम्

7. 18. 7. यातैष्ययोः खण्डकयोर्विशेषः
शेषांशनिघ्नो 'नख'हृत् तदूनम् ।
युतं गतैष्यैक्यदलं स्फुटं स्यात्
क्रमोत्क्रमज्याकरणेऽत्र भोग्यम् ॥
(Bhāskara II, *Si Śi.* 1.2.16)

Rectification of the next R sine difference

The difference of the previous and the following R sine differences being multiplied by the remaining degrees and divided by 20, the result is subtracted from the arithmetic mean of the two aforesaid differences, to obtain the rectified R sine difference.[2] (16). (AS)

ज्यातश्चापः

7. 18. 8. ज्यासङ्कलितात् क्रमशः शोधितजीवामखिर्मखेश्शेषम् ।
मख्या हतमन्त्याप्तं पूर्वयुतं तद्भवति चापः ॥ ६ ॥
(Bhāskara I, *MBh.*, 8.6)

Arc from a given R sine

From the given R sine subtract in serial order (as many tabulated R sine-differences as possible): multiply the number of the Rsine-differences subtracted by 225. Then multiply the residue (of the given R sine) by 225 and divide by the current R sine-difference. Add this result to the previous one. Thus is obtained the arc (corresponding to the given R sine in terms of minutes). (6). (KSS)

ज्यानां चतुर्विंशतिसंख्यानिर्देशे कारणम्

7. 18. 9. ननु चतुर्विंशति ज्यार्धानीति कथमेष नियमः, यतः बुहुधा विच्छेत्तुं शक्यते समवृत्तपरिधिपादः । उच्यते—अत्र हि भुजाकोटिकर्णकल्पनया जीवाखण्डान्युत्पाद्यन्ते । अतो यावद्धा खण्डिते परिधौ काष्ठं ज्यार्धं च समपरिमाणं भवति तावद्धा खण्डनमेव प्रयोजनवत्, ततो न्यूने चापे तदनुपातेन ज्यार्धसिद्धेः । अतो यावच्चापतुल्यमाद्यज्यार्धं भवति तावद्धा एव परिधिश्छेत्तव्यः । चतुर्विंशतिधा चापखण्डने कृते प्रथमज्यार्धं चापं च तुल्यसंख्यं जातम् । अतस्तत्रैव खण्डनं पर्यवसितम् ।
(Sūryadeva-yajvan, com. on *ABh.*, 2.11: ed. *KVS*, p. 50)

Reason to restrict the number of R sines to 24

Now, why should there a be rule that the number of R sines (in a quadrant) be restricted to 24, when the quadrant of a circle can be divided into any number of segments.

To state the reason: Here the R sine segments are designed as the basis of sine, cosine and hypotenuse. Hence, only that division would be useful according to which the segment and R sines are equal in length, for, if less, the R sine would be proportionally less. Hence the quadrant should be divided in such a manner that the first R sine and the corresponding arc are exactly equal. When the number of segments in a quadrant is 24, the first R sine is equal to the first arc. Hence division into segments has been restricted to the said number (of 24). (KVS)

ज्याः (२)—संख्या २४; व्यासार्धः १२०; चापः ३°४५′

7. 18. 10. षष्टिशतत्रयपरिधेर्वर्गदशांशात् पदं स विष्कम्भः ।
तदिहांशाशचतुष्कं सम्प्रकल्प्य राश्यष्टभागज्या ॥ १ ॥
व्यासार्धस्य कृतेर्ध्रुवसंज्ञाकृतांशस्ततः स मेषस्य ।
ध्रुवकरणी मेषोना द्वयोस्तु राश्योः पदं ज्याः स्युः ॥२॥
शेषेष्विष्टेषु धनुर्द्विगुणं पदात् प्रोज्झ्य शेषगुणहीनात् ।
व्यासस्यार्धाद्वर्गं द्विगुणकरण्यां समायोज्यम् ॥ ३ ॥
तत्पादोऽभिमता स्याद् ध्रुवा तदूनावशेषपिण्डस्य ।
ध्रुवकरणीदलमध्यर्धसंज्ञमन्योऽत्र विधिरुक्तः ॥ ४ ॥

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[1] **R sines of Mādhava derived by the above method**

No.	*Arc*	*R Sine*				*R sine in decimal*	*Modern value*
1.	225′	224′	50″	22‴		.06540	.06540
2.	450′	448′	42″	58‴		.13053	.13053
3.	675′	670′	40″	11‴		.19509	.19509
4.	900′	889′	45″	15‴		.25882	.25882
5.	1125′	1105′	1″	39‴		.32144	.32144
6.	1350′	1315′	34″	7‴		.38268	.38268
7.	1575′	1520′	28″	35‴		.44229	.44228
8.	1800′	1718′	52″	24‴		.50000	.50000
9.	2025′	1909′	54″	35‴		.55557	.55558
10.	2250′	2092′	46″	3‴		.60876	.60876
11.	2475′	2266′	39″	50‴		.65934	.65934
12.	2700′	2430′	51″	15‴		.70711	.70711
13.	2925′	2584′	38″	6‴		.75184	.75184
14.	3150′	2727′	20″	52‴		.79335	.79335
15.	3375′	2858′	22″	55‴		.83147	.83146
16.	3600′	2977′	10″	34‴		.86603	.86603
17.	3825′	3083′	13″	17‴		.89687	.89688
18.	4050′	3176′	3″	50‴		.92388	.92388
19.	4275′	3235′	18″	22‴		.94693	.94692
20.	4500′	3320′	36″	30‴		.96593	.96593
21.	4725′	3371′	41″	29‴		.98079	.98079
22.	4950′	3408′	20″	11‴		.99144	.99144
23.	5175′	3430′	23″	11‴		.99785	.99785
24.	5400′	3437′	44″	48‴		1.00000	1.00000

R sines in decimal and their modern values have been added for the sake of comparison and have been taken from A.K. Bag, *Mathematics in ancient and medieval India*, Varanasi-Delhi, 1979, pp. 247-50.

[2] This is a formula for interpolation which agrees with what is called the 'quadratic interpolation' given by Ball in his *Spherical Astronomy*, p. 18, in the form:

$$y = yo + \frac{x}{h}\,(y1 - yo) + \frac{x\,(x-h)}{2h^2}\cdot(y2 - 2y1 + yo).$$

For a detailed treatment, see *Si Śi*:*AS*, pp. 115-18.

इष्टांशद्विगुणोनत्रिभज्ययोना त्रयस्य चापज्या ।
षष्टिगुणा सा करणी तया ध्रुवोनाऽवशेषस्य ।। ५ ।।
शेषज्याः स्वरतिथयो गुणशिवधृतिभिश्च विंशतिः सहिताः ।
पञ्च नरकं शतार्धं त्रिसमेतं षष्टिरिति लिप्ताः ।। ६ ।।
सैकाजे पञ्चाशत् पञ्चाष्टकं पञ्चवर्गवेदाश्च ।
त्रिंशच्चतुर्भिरधिका षट्पञ्चाशच्छराः शून्यम् ।। ७ ।।
षट्त्रयोदशैकोना विंशतिस्त्र्यष्टकान्यतस्त्रिंशत् ।
युक्ताम्बरपञ्चनवातिजगतिभिर्लिप्तिका वृषभे ।। ८ ।।
चत्वारिंद् रामा मुनयोऽर्धशतं च सैकमतिजगती ।
द्वादश षष्टिर्हीना मनुभिर्विषयैर्वृषे विकलाः ।। ९ ।।
गुणरसनवका दशभिर्द्वित्रिभूतभूतरसयुक्ताः ।
ज्यापिण्डा पिण्डा ये द्वितीयराशावतो विकलाः ।। १० ।।
धृतिगुणधृतिपरिहीना षष्टिः शून्यं शतार्धमनलोनम् ।
वेदा व्येकार्धशतं पञ्चेति तदन्तरज्याः स्युः ।। ११ ।।
मुनयोऽजे व्येकान्ते रसत्रयं पञ्चकौ कृताच्च गवि ।
शिखिपक्षचन्द्रशून्याः द्विर्द्विर्मिथुने कला ज्यानाम् ।। १२ ।।
मेषे विकलार्धशतं सैकं व्येकेन्द्रियेश्वरं त्रिंशत् ।
द्वाविंशतिस्त्रिवर्गः . . . . . ।। १३ ।।
. . . . ।
. . . . खगुणकृतार्णवयमनवकसमुद्रशिखिवर्गैः ।।१४।।
मनुविषयतिथिरसाः स्युस्त्रिगुणाः पञ्चाष्टकं स्वरोपेतम् ।
सप्तदशनवपञ्चकं षोडश चेति क्रमान्मिथुनैः ।। १५ ।।

(Varāha, *PS*, 4.1-15)

R sines (2)—No. 24; R=120; arc=3° 45′

Take the circumference as measured in 360 units, square it, take the tenth part of the square, and find its square-root. The result is the diameter of the circle in the units taken. We assume the diameter to be 4°, i.e. 240′, and hereunder give the tabular sines of angles for 3° 45′ interval. (1)

The square of the radius, (*i.e.*, 14,400) is called *dhruva-karaṇī*, (literally, 'the fixed irrational'). The fourth part of it, (*i.e.*, 3600) is the *karaṇī* (irrational) related to the first sign, or 30°. *Dhruva-karaṇī minus* the *karaṇī* of Meṣa, (*i.e.*, 14,400—3600=10,800) is the *karaṇī* of two Signs, or 60°. The square root of a *karaṇī* is the tabular sine. (2)

The other tabular sines, (*i.e.*, sine 3° 45′, sine 7° 30′ etc. other than the 4 mentioned, total 24), are formed successively in the following manner: Let the angle or arc for which the sine is required be θ:

I. $\text{Sin}^2\theta = \frac{1}{4}\,[\sin^2 2\theta + \{120 - \sin(90° - 2\theta)\}^2]$

II. $\text{Sin}^2\theta = 60 \times \{120' - \sin(90° - 2\theta)\}$, where the sines are in minutes etc. Of the 24 sines, the *karaṇī* of the nth sine=14,400—the *karaṇī* of the (24—n)th sine. 7200 is the *karaṇī* of one and a half Signs, (*i.e.* 45°). (3-5)

The other sines are the following: In the first sign, the minutes parts are successively: 7, 15, 20+3, 20+11, 20+18, 5×9, 50+3, and 60. The seconds respectively are: 50+1, 5×8, 25, 4, 30+4, 56, 5 and 0.[1] (6-7)

Of the sines in the second Sign the minutes parts taking the increments in the current Sign above, are successively; 6, 13, 20—1, 3×8, 30+0, 30+5, 30+9, and 30+13. The respective seconds are: 40, 3, 7, 50+1, 13, 12, 60—14, and 60—5.[2] (8-9)

Of the sine increments given in the third Sign, above the second, the minutes are: 3, 6, 9, 10+2, 10+3, 10+5, 10+5, and 10+6. The respective seconds are: 60 — 18, 60 — 3, 60 — 18, 0, 50 — 3, 4, 50 — 1, and 5.[3] (10-11)

Of the intervals, the minute parts are, in the first Sign, 7, 7, 7, 7, 7, 7, 7, 6; in the second Sign, 6, 6, 6, 5, 5, 4, 4, 4; and in the third Sign 3, 3, 2, 2, 1, 1, 0, 0. The seconds in the first Sign are, 50+1, 50—1, 50—5, 50—11, 30, 22, 9, (and here is a break resulting in the loss of the 4th foot of the 13th verse, and the first two feet with 4 *mātrās* of the 3rd foot. From the values of the sines we can compute that the seconds in the 8th interval must be 55, which must have been given in the missing part. By examining the remaining part we can construct the meaning of this verse thus:

"The seconds of the intervals in the second Sign are, respectively: 10×(4, 2, 0, 4, 2, 5, 3, 0)+ (0, 3, 4, 4, 2, 9, 4, 9), added each to each".

The seconds of the intervals in the third Sign, are 3×14, 3×5, 3×15, 3×6, 5×8+7, 17, 9×5, and 16 (12-15).[4] (TSK)

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[1] Thus we have for the first sign—

Sine no:	1	2	3	4	5	6	7	8
Arc or angle :	3° 45′	7° 30′	11° 15′	15° 0′	18° 45′	22° 30′	26° 15′	30° 0′
Sine :	7′ 51″	15′ 40″	23′ 25″	31′ 4″	38′ 34″	45′ 56″	53′ 5″	60′ 0″

[2] Adding the minutes and seconds, and 60′ for the end of the first Sign, the sines are:

Sine No. :	9	10	11	12	13	14	15	16
Arc or angle:	33° 45′	37° 30′	41° 15′	45° 0′	48° 45′	52° 30′	56° 15′	60° 0
Sine :	66′ 40″	73′ 3″	79′ 7″	84′ 51″	90′ 13″	95′ 12″	99′ 46″	103′ 55″

[3] Adding the given minutes and seconds to 103′55″, the sine of 2 Signs, we have:

Sine No :	17	18	19	20	21	22	23	24
Arc or angle:	63° 45′	67° 30′	71° 15′	75° 0′	78° 45′	82° 30′	86° 15′	90° 0′
Sine :	107′ 37″	110′ 52″	113′ 37″	115′ 55″	117′ 42″	118′ 59″	119′ 44″	120′ 0″

[4] For explanation and rationale, see *PS*:*TSK*, 4.12-15.

ज्या: (३)—संख्या ९; व्यासार्धः ३००'; चापः १०°

7. 18. 11. दशभागज्या द्विशराः पञ्चाशद् वसुकृताः शिखिश्रुतयः ।
सप्तविंशत् त्रिंशद् द्वियमा लोकेन्दवः पञ्च ।। २२ ।।
रामो नु-रत्नाढ्य-नृमान्य-लुब्धको
नागाग्र-निस्तार-खजाग्र-माधुराः ।
ज्ञानाङ्ग-मित्यत्र नव प्रकीर्तिता
जीवा ह्यनन्ताप्तफलैः समन्विताः ।। २३ ।।
मुनि-र्दिव्यो नभो नाथः सनको नैशिकाननः ।
दुग्धको देवरोऽनङ्ग इति ज्या व्युत्क्रमोदिताः ।। २४ ।।
(Deva, *KR*, 1.22-24)

R Sines (3): No. 9; R=300'; arc=10°

R Sine differences:

The R sine-differences at the intervals of 10 degrees are: 52, 50, 48, 43, 37, 30, 22, 13 and 5 (minutes). (22)

R sines

52, 102, 150, 193, 230, 260, 282, 295, and 300 (minutes): these are stated to be the nine R sines.

To obtain the R sine of an arc, divide it into smaller arcs of 600 minutes each and add the R sine-differences corresponding to each of them. (23)

R versed-sines (for R 300')

5, 18, 40, 70, 107, 150, 198, 248 and 300 (minutes): these are the R versed-sines. (24). (KSS)

ज्या: (४)—संख्या ९; व्यासार्धः १५०; चापः १०°

7. 18. 12. प्रत्यब्दशुद्धिविधिना ग्रहमध्यतत्त्व-
मुक्तं सुखावहमतो लघुशिञ्जिनीभिः ।
वच्मि स्फुटत्वमपि ताश्च दशांशकेभ्यो
ज्ञेयश्च कार्मुकविधौ गुणको दशांशाः ।। १ ।।

खण्डज्याः

षड्विंशतिः शरयमा युगलोचनानि
रूपाश्विनो नवभुवस्तिथयस्त्रिनेत्राः ।
शैला यमौ च कथितानि नव क्रमेण
ज्याखण्डकानि कथयाम्यथ पिण्डितानि ।। २ ।।

पिण्डज्याः

अङ्गाश्विनः कुविषया विषयस्वराः स्यु-
स्तस्माच्चतुर्विरहितं च शतं शरेशाः ।
व्योमाग्निशीतकिरणाः शशिवेदचन्द्राः
स्तम्बेरमाब्धिशशिनो गगनेषुचन्द्रः ।। ३ ।।
(Lalla, *SiDhVr*, 13.1-3)

R Sines (4): No. 9; R=150; arc=10

I have given the easier way of calculating the mean places of planets by the method called *Pratyabdaśuddhi* or corrections given to the longitudes of planets at the end of the each year. Now, I shall explain how to calculate the true places of planets (more easily) by using the *Laghujyā* method. In this case, the R sine of arcs of 10° are used. It must be remembered that when the value of an arc corresponding to a given R sine is to be found, the multiplier must be 10°. (1).

The *Khaṇḍajyās and Piṇḍajyās*

The R sines of the nine consecutive arcs, (each of 10°, in a quadrant), are, respectively, 26, 25, 24, 21, 19, 15, 11, 7 and 2.

Now, I give the R sines of arcs of 10°, 20°, 30°, etc. upto 90°. They are, respectively, 26, 51, 75, 96, 115, 130, 141, 148 and 150.[1] (2-3). (BC).

ज्या: (५)—संख्या ६; व्यासार्धः ३४३८/८०; चापः १५°

7. 18. 13. पयो-नयो-धनं-सेना-वनं-यज्ञों-ऽशकाः क्रमात् ।। २ ।।
मध्यांशचापज्याखण्डा ज्यासभोज्यफलाहता ।
अर्धेन सहितौ खण्डौ द्वितीयान्त्यौ स्फुटौ मतौ ।। ३ ।।
ज्यापिण्डाच्छुद्धखण्डोनात् संख्या मध्यहता तथा ।
शेषान्मध्यहताल्लब्धं अशुद्धेन युतं धनुः ।। ४ ।।
भुजा, कोटी, भुजाकोटचः ग-ग-राशिक्रमात् स्मृताः ।
हीनं 'ग'भं, भुजाकोटचोः राश्याद्यैरन्यदीरितम् ।। ५ ।।
(*VK*, 3. 2b-5)

R Sines (5): No. 6; R=3438/80; arc=15°

For every increase of 15° of arc form 0, we have the sine-segments 11, 10½, 9, 7, 4 and 1½ in degrees. To find the sine of any arc, take as many sine segments as there are full 15° in the arc. Add to these the fraction of the sine-segment got from multiplying the sine-interval by the degrees left over, and dividing by 15. To find the arc of any sine, deduct from it as many sine segments as can be done and multiply the number by 15°. Add to this the degrees got by multiplying the sine left over by 15°, and dividing by the sine-segment next to the last full sine-segment taken. (2b-4a)

The part of the arc falling into the quadrant containing the *rāśis* 0 to 3, i.e. the first quadrant, is called *bhujā*. That falling into the quadrant containing *rāśis* 3+ to 6, i.e. the second quadrant, is called *koṭi*. That falling into the quadrant containing *rāśis* 6+ to 9, i.e. the third quadrant, is called *bhujā*. And that falling into the quadrant containing *rāśis* 9+ to 12, i.e., the fourth quadrant, is called *koṭi*. The *rāśis* etc. of *bhujā*, deducted from 3 *rāśis* is called *koṭi*; the *rāśis* etc. of *koṭi* deducted from 3ʳ is called *bhujā*. (4b-5). (TSK-KVS)

ज्या: (६)—संख्या ९६; व्यासार्धः ३४३७' ४४"; चापः ५६' १५"

क्रमज्या-कलाः

7. 18. 14. अर्धज्या रसबाणाः, करशशिशशिनो, गजाङ्गचन्द्रमसः ।
वेदाकृतयो, व्योमस्तम्बेरमबाहवो, रसाग्निगुणाः ।। २ ।।

[1] See the notes, *Śi Dh Vṛ*: *BC*, II. 243.

नेत्रनवहुतभुजो, गजजलधिकृताः, कृतनभोबाणाः ।
नन्दशिलीमुखबाणाः, शरशश्यृतवः, खपर्वताङ्गानि ।।३।।

तत्त्वागाः, खाष्टनगाः, शराग्निनागा, नवाष्टपवनभुजः ।
रामाब्ध्यङ्का, नगनवनन्दाः, कुशराभ्ररजनीशाः ।। ४ ।।

शरखशिवाः, स्तम्बेरम-
तिथिशशिनः, जलधरप्रकृतिशशांकाः ।
शिखिरससूर्याः, शरशशि-
वह्निधराः स्वराङ्गरामभुवः ।। ५ ।।

नागैकवेदशशिनो, नवरसशक्रा, नखेषुरजनीशाः ।
खागतिथयो, नखाष्टयो, नवरसभूपा, धृतिसप्तमृगाङ्काः ।।

सप्तर्तुसप्तशशिन,स्तिथिधृतयो, द्वयङ्गनागहरिणधृतः ।
नवखाङ्कभुवो, रसशरनवचन्द्राः करखशून्यकराः ।। ७ ।।

नगकृतखनया, द्विनवव्योमभुजाः, सप्तविश्वनेत्राणि ।
खधृतियमा, वेदभुजद्विभुजा, रसषड्भुजाक्षीणि ।। ८ ।।

वसुखाग्नियमाः, खशरत्रिभुजा, आकाशनन्दगुणयमलाः ।
खगुणजिनाः, खागजिना, नवाभ्रतत्त्वा,न्यगाब्धितत्त्वानि ।।

वेदाष्टेषुयमाः, शशिनेत्राङ्गभुजा, नगेषुरसयमलाः ।
द्विनवोत्कृतयः, सप्तद्विनगभुजा,श्चन्द्रषड्भानि ।। १० ।।

वेदाङ्कभानि, रसयमवसुनेत्रा,ण्यष्टपञ्चवसुयमलाः ।
नववस्वष्टभुजा, नवशशिनन्दयमा, गजाब्धिनवदस्राः ।।

नगसप्ताङ्कभुजाः, कृत-
खखरामाः, शशिगुणाभ्रहव्यभुजः ।
सप्तविशिखाभ्ररामा-
स्त्रिनागखगुणा नगाभ्रशशिरामाः ।। १२ ।।

भूगुणकुगुणा, जलधीष्वेकगुणा रसधराधरैकगुणाः ।
स्वरनवकुगुणाः सप्तप्रकृतिपुष्करा, रसगुणदन्ताः ।।१३।।

विशिखविशिखाक्षिरामा, बाहुधरित्रीधराक्षिहव्यभुजः ।
क्रमपरिपाटया जीवाश्छिद्रस्तम्बेरमद्विगुणाः ।। १४ ।।

शरखसुरा, नखदेवा, वेदत्रिसुरा, नगाब्धिगुणरामाः ।
खाङ्गत्रिगुणा, भूनगनाकगृहा, नेत्रनागगुणरामाः ।। १५ ।।

शशिनन्दाग्निगुणा नभ-
खाब्धिगुणा, गजव्योमाब्धिहुताशाः ।
तिथिजलरामाः, चन्द्र-
द्वयब्धिगुणा, रसकराब्धिहव्यभुजः ।। १६ ।।

खाग्निसमुद्रहुताशा,स्त्रित्र्यब्धिगुणाः, शराग्निकृतरामाः ।
सप्तगुणवेदरामा, नगगुणवेदाग्नयो लिप्ताः ।। १७ ।।

क्रमज्या-विकलाः

आसां विकलास्तिथयो,
नन्दभुजाः, क्वब्धयः, पयोदशराः ।
रसविशिखाः, सप्तशराः,
अग्निशरा,स्त्रिकृताः, शराक्षीणि ।। १८ ।।
नवविशिखाः, पञ्चयमाः,
खकृताः, पञ्चाब्धयो, द्विरदरामाः ।

धृति,रिषुवेदा, मङ्गल-
विशिखाः, पञ्चेषव,स्तुरङ्गगुणाः ।। १९ ।।

भू, नागा, रसबाणा,स्तत्त्वानि, जलाग्नयः, कुभुजाः ।
नगवेदा, नन्दकृता, वसुनेत्रा,ण्यग्निजलधयो, दन्ताः ।।

विशिखशरा, नेत्रशराः, कुभुजा, द्वियमा, हुताशशराः ।
वेदेषवो, ऽक्षनेत्रा,ण्यब्धियमा, द्वीषवो, रससमुद्राः ।।

अङ्गा,न्यग्निपृषत्का, वेदा, नववह्नयो, ऽङ्कगुणाः ।
रूपं, सायकवेदाः, कुशरा, गजभूमयः, शराः, सूर्याः ।।

गजरामा, नेत्रयमा,स्तत्त्वानि, कृताब्धयः, कुनेत्राणि ।
विश्वे, कुभुजाः, सायकनिगमा, गुणबाहव,स्तिथयः ।।

खभुजा, नन्दगुणा, दश,
त्रिशरा, नन्दाब्धयो, ऽक्षशराः ।
विश्वे, कुकृता, अतिधृति-,
रङ्गानि, गुणा, गो, ऽब्धिनेत्राणि ।।

सप्ताब्धयो, धृति,र्नगविशिखा गुणसागरा, ऋतुगुणाश्च ।
ऋतुरामा, रामकृता, रागेषवो, वासराः, कुकृताः ।।

सूर्या, नन्दसमुद्रा, रदा, नखा, वह्निचन्द्रमसः ।
ईशा, मनवो, ऽग्निभुजा, रसाग्नयो, वेदसायका, विधृतिः ।।

वेदकृताः . . . ।। २७-a ।।

उत्क्रमज्या-कलाः

. . . विपरीताः खं, भू,र्वेदा, नगा, रुद्राः ।
अष्टि,र्नेत्रभुजा, नवनेत्रा,ण्यगवह्नयो, विशिखवेदाः ।।

पञ्चशराः, षड्ऋतवो, नगमुनयो, नन्दकुञ्जरा,स्त्रिदशाः।
नगरुद्रा, रदचन्द्रा, वसुमनवो, वेदरसचन्द्राः ।। २८ ।।

द्वयष्टभुवः, शून्यनखाः, खाक्षिभुजाः, खाब्धिनेत्राणि ।
कूत्कृतय,स्त्र्यष्टभुजा, रसखगुणा, व्योमगीर्वाणाः ।।२९।।

वेदेषुगुणा, नवनग-
रामाः, शरखाब्धयो, रदसमुद्राः ।
खाङ्गाब्धयो, ऽङ्ककुञ्जर-
वेदा, धृतिसायका, गजाब्धिशराः ।। ३० ।।

नवनगविशिखा, जलधरशश्यृतवो, गुणकृताङ्गानि ।
रसनगरसाः, खशशधरनगाः, पृषत्काब्धिधरणिधराः ।।

खाष्टनगा, रसकुगजा,स्त्रिशरगजा, जलदनन्दवसवश्च ।
वसुभुजनन्दा, नगरसबिलानि, रसखाभ्रहरिणाङ्काः ।।३२।।

ऋत्वब्धिदिशो, नागा-
ऽष्टखभुवो, ऽङ्कनेत्रशशिचन्द्रमसः ।
कुनगशिवा, विश्वार्का,
रसतत्त्वभुवः, खखाग्निरूपाणि ।। ३३ ।।

वेदकृताग्निशशाङ्का,
नवाष्टविश्वे, शराग्निकृतचन्द्राः ।

क्वष्टमनवो, भतिथयो,
ऽब्ध्यगशरचन्द्रा, द्विबाहुरसचन्द्राः ।। ३४ ।।

खनगाष्टयो, ऽष्टभूनगशशिनो, ऽगरसागचन्द्रमसः ।
नगशशिधृतयो, ऽगरसद्विपशशिनो, ऽगैकनन्दरजनीशाः ।।

सप्ताङ्गाङ्कभुवो, ऽष्टकुखभुजा, व्योमागशून्यनेत्राणि ।
द्वीनभुजाः, कृतनगशशिनेत्रा,ण्यङ्गाक्षिबाहुरविपुत्राः ।।

अङ्कागाक्षिभुजा, रदरामभुजा, रसगजाग्निनयनानि ।
नवरामजिना, गुणनवसिद्धाः, सप्ताब्धितत्त्वानि ।। ३७ ।।

द्व्यभ्रोत्कृतयः, पर्वतशराङ्गनेत्राणि, रुद्रभानीह ।
सप्ताङ्गभानि, यमयमनागभुजा, नगनगाष्टकराः ।।३८।।

सुरनवभुजा, नवाष्टच्छिद्राक्षी,ण्यब्धिजलधिशून्यगुणाः ।
खखकुगुणा, रसपृषत्कभूरामा, द्विशशिभुजाग्नयः ।।३९।।

नवरसरदा, पञ्चाक्षिरामाग्नय,श्चन्द्रनागगुणरामाः ।
नगगुणवेदहुताशा . . .

विकलाः

. . . विकलाः भान्य,म्बुदपृषत्काः ।। ४० ।।

वसवः, कुभुजाः, खगुणाः सुराः, कुरामा, जिना, रवयः ।
पञ्चशरा, नेत्रगुणा, रामा, नवबाहवो, द्विपसमुद्राः ।।

भू,र्वसवो, ऽष्टौ, चन्द्रा, नगवेदाः, षट्भुजा, अचलबाणाः ।
विंशति,रिषुहव्यभुजः, कुकृता, वसुवह्नयो, ऽक्षभुजाः ।।

रामाः, कुगुणा, वर्गः सप्तानां, पञ्चशराः, शशिबाणाः ।
वेदगुणाश्च, पृषत्काः, सिद्धा, नवबाहवः, कुभुजाः ।।४३।।

नवविशिखा, रामभुजा
इलाग्नयो, वह्निनयनानि ।
खं, नवचन्द्रा, द्विभुजा,
रसा, रदा, नन्दवह्नयो, ऽङ्गभुजाः ।। ४४ ।।

त्रिशरा, नन्दपृषत्का, गुणाब्धयः सायका, विशिखाः ।
खकृताः, कुशरा, मङ्गलहव्यभुजो, वसुशरा, द्विशराः ।।

व्योमभुजा, नवचन्द्राः, खशराः, कुशरा, दृगक्षीणि ।
त्रिकरा, द्विशरा,श्छिद्रप्रणिम्नगेशा, इन,श्चन्द्रः ।। ४६ ।।

अष्टिः, पञ्चशरा, नग-
बाणा,ग्निभुजा, दिशो, ऽङ्कभुवः ।
अष्टकृता, रसरामा-
स्त्रिकृता, अचलो, ऽङ्काब्धयो, ऽङ्गकृताः ।। ४७ ।।

नवविशिखा, रसनेत्रा-
ण्यङ्गा,न्यङ्केषवो, ऽब्धयो, ऽङ्कभुवः ।
शरवेदा, नवचन्द्रा,
भू,रिन्दुशरा, नगाब्धयो, ऽष्टकृताः ।। ४८ ।।

वेदशरा, हव्यभुज,स्थितयो, ऽङ्कभुजाः, कृताब्धयः . . . ।

त्रिज्या, त्रिज्याकृतिः जिनांशज्या च

. . . . त्रिज्या ।
अगगुणवेदहुताशाः कलिका विकलाः समुद्रजलधयः ।।

सप्तजलखाष्टशशिधृति-
शशिनः कलिका शराग्नयो विकलाः ।
त्रिज्याकृतिरष्टनव-
त्रिभुवो विश्वे, जिनांशज्या ।। ५० ।।

गणितवशगास्तु जीवाः षण्णवतिः प्रोदिताः क्रमेणैव ।
करणीमूलग्रहणात् तुल्यत्वं प्रथमजीवया धनुषः ।। ५१ ।।

(Vaṭeśvara, *Vsi.*, 2.1.2-51)

R sines (6): No. 96; R=3437′ 44″; arc 56′ 15″

The following are the minutes of the R sines: 56, 112, 168, 224, 280, 336, 392, 448, 504, 559, 615, 670, 725, 780, 835, 889, 943, 997, 1051, 1105, 1158, 1210, 1263, 1315, 1367, 1418, 1469, 1520, 1570, 1620, 1669, 1718, 1767, 1815, 1862, 1909, 1956, 2002, 2047, 2092, 2137, 2180, 2224, 2266, 2308, 2350, 2390, 2430, 2470, 2509, 2547, 2584, 2621, 2657, 2692, 2727, 2761, 2794, 2826, 2858, 2889, 2919, 2948, 2977, 3004, 3031, 3057, 3083, 3107, 3131, 3154, 3176, 3197, 3217, 3236, 3255, 3272, 3289, 3305, 3320, 3334, 3347, 3360, 3371, 3382, 3391, 3400, 3408, 3415, 3421, 3426, 3430, 3433, 3435, 3437.

Of these R sines, the seconds are: 15, 29, 41, 50, 56, 57, 53, 43, 25, 59, 25, 40, 45, 38, 18, 45, 58, 55, 37, 1, 08, 56, 25, 34, 21, 47, 49, 28, 43, 32, 55, 52, 21, 22, 53, 54, 25, 24, 52, 46, 06, 53, 04, 39, 39, 01, 45, 51, 18, 05, 12, 38, 22, 25, 44, 21, 13, 21, 45, 23, 15, 20, 39, 10, 53, 49, 55, 13, 41, 19, 06, 03, 09, 24, 47, 18, 57, 43, 36, 36, 43, 56, 15, 41, 12, 49, 32, 20, 13, 11, 14, 23, 36, 54, 17, 44. (2—27a)

R versed-sines at intervals of 56′ 15″

The following are the minutes of the R versed-sines : 00, 01, 04, 07, 11, 16, 22, 29, 37, 45, 55, 66, 77, 89, 103, 117, 132, 148, 164, 182, 200, 220, 240, 261, 283, 306, 330, 354, 379, 405, 432, 460, 489, 518, 548, 579, 610, 643, 676, 710, 745, 780, 816, 853, 890, 928, 967, 1006, 1046, 1087, 1129, 1171, 1213, 1256, 1300, 1344, 1389, 1435, 1481, 1527, 1574, 1622, 1670, 1718, 1767, 1817, 1867, 1917, 1967, 2018, 2070, 2122, 2174, 2226, 2279, 2332, 2386, 2439, 2493, 2547, 2602, 2657, 2711, 2767, 2822, 2877, 2933, 2989, 3044, 3100, 3156, 3212, 3269, 3325, 3381, and 3437.

(Of these R versed-sines) the seconds are: 27, 50, 8, 21, 30, 33, 31, 24, 12, 55, 32, 3, 29, 48, 1, 8, 8, 1, 47, 26, 57, 20, 35, 41, 38, 25, 3, 31, 49, 55, 51, 34, 5, 24, 29, 21, 59, 23, 31, 23, 0, 19, 22, 6, 32, 39, 26, 53, 59, 43, 5, 5, 40, 51,

38, 58, 52, 20, 19, 50, 51, 22, 23, 52, 49, 12, 1, 16, 55, 57, 23, 10, 19, 48, 36, 43, 7, 49, 46, 59, 26, 6, 59, 4, 19, 45, 19, 1, 51, 47, 48, 54, 3, 15, 29, 44.[1] (27-49 a-b). (KSS)

[1] The above R sines and R versed-sines may be stated in the tabular form as follows :

Serial No.	Arc	R sine	R versed-sine
1.	56′ 15″	56′ 15″	0′ 27″
2.	112′ 30″	112′ 29″	1′ 50″
3.	168′ 45″	168′ 41″	4′ 08″
4.	225′ 00″	224′ 50″	7′ 21″
5.	281′ 15″	280′ 56″	11′ 30″
6.	337′ 30″	336′ 57″	16′ 33″
7.	393′ 45″	392′ 53″	22′ 31″
8.	450′ 00″	448′ 43″	29′ 24″
9.	506′ 15″	504′ 25″	37′ 12″
10.	562′ 30″	559′ 59″	45′ 55″
11.	618′ 45″	615′ 25″	55′ 32″
12.	675′ 00″	670′ 40″	66′ 03″
13.	731′ 15″	725′ 45″	77′ 29″
14.	787′ 30″	780′ 38″	89′ 48″
15.	843′ 45″	835′ 18″	103′ 01″
16.	900′ 00″	889′ 45″	117′ 08″
17.	956′ 15″	943′ 58″	132′ 08″
18.	1012′ 00″	997′ 55″	148′ 01″
19.	1068′ 45″	1051′ 37″	164′ 47″
20.	1125′ 00″	1105′ 01″	182′ 26″
21.	1181′ 15″	1158′ 08″	200′ 57″
22.	1237′ 30″	1210′ 56″	220′ 20″
23.	1293′ 45″	1263′ 25″	240′ 35″
24.	1350′ 00″	1315′ 34″	261′ 41″
25.	1406′ 15″	1367′ 21″	283′ 38″
26.	1462′ 30″	1418′ 47″	306′ 25″
27.	1518′ 45″	1469′ 49″	330′ 03″
28.	1575′ 00″	1520′ 28″	354′ 31″
29.	1631′ 15″	1570′ 43″	379′ 49″
30.	1687′ 30″	1620′ 32″	405′ 55″
31.	1743′ 45″	1669′ 55″	432′ 51″
32.	1800′ 00″	1718′ 52″	460′ 34″
33.	1856′ 15″	1767′ 21″	489′ 05″
34.	1912′ 30″	1815′ 22″	518′ 24″
35.	1968′ 45″	1862′ 53″	548′ 29″
36.	2025′ 00″	1909′ 54″	579′ 21″
37.	2081′ 15″	1956′ 25″	610′ 59″
38.	2137′ 30″	2002′ 24″	643′ 23″
39.	2193′ 45″	2047′ 52″	676′ 31″
40.	2250′ 00″	2092′ 46″	710′ 23″
41.	2306′ 15″	2137′ 06″	745′ 00″
42.	2362′ 30″	2180′ 53″	780′ 19″
43.	2418′ 45″	2224′ 04″	816′ 22″
44.	2475′ 00″	2266′ 39″	853′ 06″
45.	2531′ 15″	2308′ 39″	890′ 32″
46.	2587′ 30″	2350′ 01″	928′ 39″
47.	2643′ 45″	2390′ 45″	967′ 26″
48.	2700′ 00″	2430′ 51″	1006′ 53″
49.	2756′ 15″	2470′ 18″	1046′ 59″
50.	2812′ 30″	2509′ 05″	1087′ 43″
51.	2868′ 45″	2547′ 12″	1129′ 05″
52.	2925′ 00″	2584′ 38″	1171′ 05″
53.	2981′ 15″	2621′ 22″	1213′ 40″
54.	3037′ 30″	2657′ 25″	1256′ 51″
55.	3093′ 45″	2692′ 44″	1300′ 38″
56.	3150′ 00″	2727′ 21″	1344′ 58″
57.	3206′ 15″	2761′ 13″	1389′ 52″
58.	3262′ 30″	2794′ 21″	1435′ 20″
59.	3318′ 45″	2826′ 45″	1481′ 19″
60.	3375′ 00″	2858′ 23″	1527′ 50″

Radius, Square of Radius and R sin 24°

3437′ 44″ is the radius; and 1,18,18,047′ 35″ is the square of the radius; and 1398′ 13″ is the value of R sin 24°.[1] (49b-50)

Thus have been stated, in serial order, the ninety-six R sines (and R versed-sines) as obtained through mathematical computation, the equality of the first R sine and the elemental arc being taken as the basis of this computation.[2] (51). (KSS)

Serial No.	Arc	R sine	R versed-sine
61.	3431′ 15″	2889′ 15″	1574′ 51″
62.	3487′ 30″	2919′ 20″	1622′ 22″
63.	3543′ 45″	2948′ 39″	1670′ 23″
64.	3600′ 00″	2977′ 10″	1718′ 52″
65.	3656′ 15″	3004′ 53″	1767′ 49″
66.	3712′ 30″	3031′ 49″	1817′ 12″
67.	3768′ 45″	3057′ 55″	1867′ 01″
68.	3825′ 00″	3083′ 13″	1917′ 16″
69.	3881′ 15″	3107′ 41″	1967′ 55″
70.	3937′ 30″	3131′ 19″	2018′ 57″
71.	3993′ 45″	3154′ 06″	2070′ 23″
72.	4050′ 00″	3176′ 03″	2122′ 10″
73.	4106′ 15″	3197′ 09″	2174′ 19″
74.	4162′ 30″	3217′ 24″	2226′ 48″
75.	4218′ 45″	3236′ 47″	2279′ 36″
76.	4275′ 00″	3255′ 18″	2332′ 43″
77.	4331′ 15″	3272′ 57″	2386′ 07″
78.	4387′ 30″	3289′ 43″	2439′ 49″
79.	4443′ 45″	3305′ 36″	2493′ 46″
80.	4500′ 00″	3320′ 36″	2547′ 59″
81.	4556′ 15″	3334′ 43″	2602′ 26″
82.	4612′ 30″	3347′ 56″	2657′ 06″
83.	4668′ 45″	3360′ 15″	2711′ 59″
84.	4725′ 00″	3371′ 41″	2767′ 04″
85.	4781′ 15″	3382′ 12″	2822′ 19″
86.	4837′ 30″	3391′ 49″	2877′ 45″
87.	4893′ 45″	3400′ 32″	2933′ 19″
88.	4950′ 00″	3408′ 20″	2989′ 01″
89.	5006′ 15″	3415′ 13″	3044′ 51″
90.	5062′ 30″	3412′ 11″	3100′ 47″
91.	5118′ 45″	3426′ 14″	3156′ 48″
92.	5175′ 00″	3430′ 23″	3212′ 54″
93.	5231′ 15″	3433′ 36″	3269′ 03″
94.	5287′ 30″	3435′ 54″	3325′ 15″
95.	5343′ 45″	3437′ 17″	3381′ 29″
96.	5400′ 00″	3437′ 44″	3437′ 44″

[1] Assuming $\pi = 3.1416$, as stated by Āryabhaṭa I, we have

$$\text{Radius} = \frac{21600}{2\pi} = \frac{21600}{6.2832} = 3437' 44'', \text{ correct to seconds}$$

$$\text{and } (\text{Radius})^2 = \left(\frac{21600}{2\pi}\right)^2 = \left(\frac{21600}{6.2832}\right)^2 = 11818047' 35'', \text{ correct to seconds.}$$

The value of R sin 24° may be easily obtained from the above table of R sines by simple interpolation.

[2] For a detailed account of the development of the subject in India, see 'Hindu Trigonometry' by B.B. Datta and A.N. Singh, rev. by K.S. Shukla, *IJHS* 18 (1983) 39-108.

The following Table gives the essential details of the several Indian R sine Tables from early times to the 18th cent.

No.	Constructor of Table	Radius chosen	Interval taken	Sexagesimal places calculated
1.	Author of *SūSi*[1]	3438′	225′	1 (min. only)
2.	Āryabhaṭa I [2]	3438′	225′	1 (same as in *SūSi*)
3.	Varāhamihira[3]	120	225′	2 (min. and secs.)
4.	Brahmagupta (1)[4]	3270	225′	1
5.	(2)[5]	150	15°	1
6.	Deva[6]	300	10°	1
7.	Lalla (1)[7]	3438′	225′	1 (same as in Ā Bh.)
8.	(2)[8]	150	10°	1
9.	Sumati[9]	3438′	1°	1
10.	Govindasvāmi[10]	3437′ 44″ 19‴	225	3
11.	Vaṭeśvara[11]	3437′ 44″	56′ 15″	2
12.	Mañjula[12]	8° 8′	30°	2 (deg. and min.)
13.	Āryabhaṭa II[13]	3438′	225′	1
14.	Srīpati[14]	3415′	225′	1
15.	Udayadivākara[15]	12375859‴ or 3437′ 44″ 19‴	225′	1 (thirds only)
16.	Bhāskara II[16] (1)	3438′	225′	1
17.	(2)	120	10°	1
18.	Brahmadeva[17]	120	15°	1
19.	Vṛddha Vasiṣṭha[18]	1000	10°	1
20.	Malayendu Sūri[19]	3600	1°	2
21.	Madanapāla[20]	21600	1°	2
22.	Mādhava[21]	3437′ 44″ 48‴	225′	3
23.	Parameśvara[22]	3437′ 44″	225′	2
24.	Muniśvara[23]	191	1°	4
25.	Kṛṣṇa-daivajña[24]	500	3°	1
26.	Kamalākara[25]	60	1°	5
27.	Jagannātha Samrāṭ[26]	60	30′	5

[1] *SūSi*, ii. 17-22(a-b).
[2] *ĀBh.* i. 12.
[3] *PSi*, iv. 6-12.
[4] *BrSpSi*, ii. 2-5
[5] *KK*, Part 1, iii. 6; *DhGr*, 16.
[6] *KR*. i. 23.
[7] *Śi.DhVṛ*, I. ii. 1-8.
[8] *Ibid*, xiii. 3.
[9] *SMT* and *SK*.
[10] His com. on *MBh*, iv. 22.
[11] *VSi*, II. i. 2-26.
[12] *LMā*, ii. 2(c-d).
[13] *MSi*, iii. 4-6(a-b).
[14] *SiŚe*, iii. 3-6.
[15] His com. on *LBh*, ii. 2-3.
[16] *SiŚi*, Gaṇita, ii. 3-6; 13.
[17] *KPr*, ii. i.
[18] *VVSi*, ii. 10-11.
[19] *YR*, i. 5, commentary.
[20] His com. on *SūSi* xii. 83.
[21] See Nīlakaṇṭha's com. on *ĀBh*. ii. 12.
[22] His. com. on *LBh*, ii, 2(c-d)-3(a-b).
[23] *SiSā*, ii. 3-18.
[24] *KKau*, ii. 4-5.
[25] *SiTVi*, ii. pp. 244-5 (Lucknow Edition).
[26] *Siddhānta-samrāṭ*, ii, beginning.

मख्यादिना विना ज्यानयनम्

7. 19. 1. मख्यादिरहितं कर्म वक्ष्यते तत्समासतः ।
चक्राधांशकसमूहाद्विशोध्या ये भुजांशकाः ।। १७ ।।
तच्छेषगुणिता द्विष्ठाः शोध्याः 'खाभ्रेषुखाब्धि'तः ।
चतुर्थांशेन शेषस्य द्विष्ठमन्त्यफलं हतम् ।। १८ ।।
बाहुकोट्योः फलं कृत्स्नं क्रमोत्क्रमगुणस्य वा ।
लभ्यते चन्द्रतीक्ष्णांश्वोस्ताराणां वापि तत्त्वतः ।। १९ ।।
(Bhaskara I, *MBh.*, 7.17-19)

Direct computation of R sines

(Now) I briefly state the rule (for finding the *bhujaphala* and the *koṭiphala*, etc.) without making use of the R sine-differences, 225, etc. Subtract the degrees of the *bhuja* (or *koṭi*) from the degrees of half a circle (i.e. 180 degrees). Then multiply the remainder by the degrees of the *bhuja* (or *koṭi*) and put down the result at two places. At one place subtract the result from 40,500. By one-fourth of the remainder (thus obtained) divide the result at the other place as multiplied by the *antyaphala* (i.e. the epicyclic radius). Thus is obtained the entire *bāhuphala* (or *koṭiphala*) for the Sun, Moon or the star-planets. So are also obtained the direct and inverse R sines.[1] (17-19). (KSS)

7. 19. 2. भुजकोट्यंशोनगुणा भार्धांशास्तच्चतुर्थभागोनैः ।
'पञ्चद्वीन्दुखचन्द्रै'र्विभाजिता व्यासदलगुणिता ।।
तज्ज्ये परमफलज्यासंगुणितास्तत्फले विना ज्याभिः ।
इष्टोच्चनीचवृत्तव्यासार्धं परमफलजीवा ।। २४ ।।
(Brahmagupta, *BrSpSi.*, 14.23-24)

Multiply the degrees of the *bhuja* or *koṭi* by the degrees of half a circle diminished by the same. (The product so obtained) be divided by 10125 lessened by the fourth part of that same product. The whole multiplied by the semi-diameter gives the sine.[2] (24). (RCG)

7. 19. 3. चक्राधांशा भुजांशैर्विरहितनिहतास्तद्विहीनैर्विभक्ताः
'खव्योमेष्वभ्रवेदैः' 'सलिल'-निहताः पिण्डराशिः प्रदिष्टः ।

[1] In current mathematical symbols the rule implied can be put as

$$R \sin\phi = R\,\phi(180-\phi)/\tfrac{1}{4}\{40500-\phi(180-\phi)\}$$

i.e.

$$\sin\phi \; \frac{4\,\phi\,(180-\phi)}{40500-\phi(180-\phi)}$$

where ϕ is in degrees. For the rationale of the process see *MBh*:KSS, pp. 208-10. For the rationale of this method and its equivalents by later astronomers, given below, see R.C. Gupta, 'Bhaskara I's approximation to sine', *IJHS* 2 (1967) 121-36.

[2] The rule works out to:

$$R \sin\phi = \frac{R\,\phi\,(180-\phi)}{10125-\frac{1}{4}\phi\,(180-\phi)}$$

षड्भांशघ्ना भुजांशा निजकृति-रहितास्तत्तुरीयांशहीनै-
र्भक्ताः स्यात् पिण्डराशिर्विशिखनयनभूव्योमशीतांशुभिर्वा ।।
(Vaṭeśvara, *VSi*, *Spaṣṭa.*, 4.2)

Multiply the degrees of half a circle less the degrees of *bhuja* (by the degrees of *bhuja*). Divide (the product so obtained) by 40500 less that product. Multiplied by four is obtained the required sine. *Or* the *bhuja* in degrees be multiplied by 180 degrees and (the result) be lessened by its own square. A fourth part of the quantity (so obtained) be subtracted from 10125, and by this (new) result the first quantity be divided. The sine is obtained.[1] (2). (RCG)

7. 19. 4. दोःकोटिभागरहिताभिहताः 'खनाग-
चन्द्रा'स्तदीय-चरणोन-'शरार्कदिग्भिः' ।
ते व्यास-खण्ड-गुणिता विहृताः फले तु
ज्याभिर्विनैव भवतो भुजकोटि-जीवे ।। १७ ।।
(Śrīpati, *Si Śe.*, 3.17)

The degrees of *doḥ* or *koṭi* are multiplied by 180 less degrees of *doḥ* or *koṭi*. Semi-diameter times the product (so obtained) divided by 10125 less fourth part of that product becomes the *bhuja* or *koṭiphala*.[2] (17). (RCG)

7. 19. 5. चापोननिघ्नपरिधिः प्रथमाह्वयः स्यात्
पञ्चाहतः परिधिवर्गचतुर्थभागः ।
आद्योनितेन खलु तेन भजेच्चतुर्घ्न-
व्यासाहतं प्रथममाप्तमिह ज्यका स्यात् ।।
(Bhāskara II, *Līlāvatī*, *Kṣetra.*, 48)

Circumference less (a given) arc multiplied by that arc is *prathama*. Multiply the square of the circumference by five and take its fourth part. By the quantity so obtained, but lessened by *prathama*, divide the *prathama* multiplied by four times the diameter. The result will

[1] The two formulae obtained are:

$$\sin\phi = \frac{\phi\,(180-\phi)\times 4}{40500-\phi\,(180-\phi)}$$

and

$$\sin\phi = \frac{\phi\,.\,180-\phi^2}{10125-\frac{1}{4}(\phi\,.\,180-\phi^2)}$$

[2] The rule works out to:

$$R \sin\phi = \frac{R\,\phi\,(180-\phi)}{10125-\frac{1}{4}\phi\,(180-\phi)}$$

This form is exactly the same as that of Brahmagupta.

be the chord (*i.e. pūrṇajyā or double-sine*) of the (given) arc.[1] (RCG)

7. 19. 6. वृत्यर्धं धनुरूनितं स्वगुणितं तेनोनयुक्ते क्रमाद्
वृत्यर्धं च वृतिश्च ते स्वगुणिते तौ गुण्यहाराह्वयौ ।
व्यासे गुण्यहते हराङ्घ्रिविहृते ज्या स्यादथाद्यज्यया-
सन्ना ज्या रहिता ग्रहाख्यगणिते स्युर्व्यासखण्डानि च ।।

अथवा

वृत्ते धनू रहितनिघ्नवृतिर्द्विधा तां
व्यासाहतां च विभजेदितराङ्घ्रिहीनैः ।
वृत्यङ्घ्रिवर्गगुणितैर्विषयैश्च जीवा
स्यात् खेचराख्यगणितेऽप्युपयोग एषः ।। ७० ।।

(Nārāyaṇa Paṇḍita, *Gaṇitakaumudī*, *Kṣetravyavahāra*, 69-70)

'Multiply by itself half the circumference less (a given) arc. The quantity so obtained, when respectively subtracted from and added to the squares of half the circumference and circumference, respectively, gives the Numerator and Denominator. Multiply the diameter by the Numerator and divide by one-fourth of the Denominator to get the chord.'

Alternately,

Multiply the circumference less the given arc by the circumference and put the result in two places. In one place multiply it by the diameter and divide the result by five times the square of the quarter circumference less the quarter of the result in the other place. The final result is the chord.[1] (RCG)

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[1] The rule works out to:

$(360 - 2\phi).\, 2\phi = prathama$

$$2R \sin\phi = \frac{4 \times 2R \times (prathama)}{\frac{1}{4} \times 5 \times 360^2 - (prathama)}$$

$$= \frac{8R\,(360 - 2\phi)\,.\,2\phi}{\frac{1}{4} \times 5 \times 360^2 - (360 - 2\phi)\,.\,2\phi}$$

giving

$$\sin\phi = \frac{4\phi\,(180 - \phi)}{40500 - \phi\,(180 - \phi)}$$

[1] In symbols these can be expressed as follows. Let c stand for circumference.

First form:

$$\text{Numerator, } N = (\tfrac{1}{2}c)^2 - (\tfrac{1}{2}c - 2\phi)^2$$
$$\text{Denominator, } D = c^2 + (\tfrac{1}{2}c - 2\phi)^2$$

then

$$\text{Chord} = \frac{N \times 2R}{\frac{1}{4}\,.\,D}$$

i.e.

$$2R \sin\phi = \frac{2R\,\{180^2 - (180 - 2\phi)^2\}}{\frac{1}{4}\{360^2 + (180 - 2\phi)^2\}}$$

Second form:

$$\text{Chord} = \frac{(c - 2\phi)\, 2\phi\,.\,2R}{5(\tfrac{1}{4}c)^2 - \tfrac{1}{4}\,.\,2\phi\,(c - 2\phi)}$$

or

$$2R \sin\phi = \frac{2R\,(360 - 2\phi)\, 2\phi}{40500 - \phi\,(180 - \phi)}.$$

On simplification both the forms reduce to the rule of Bhāskara I.

8. गोलबन्धः – ARMILLARY SPHERE

गोलप्रशंसा

8. 1. 1. शशधरकिरणैर्विना प्रदोषः
कुचरहितं ललनाजनस्य वक्षः ।
मधुररसविवर्जितं च भोज्यं
न किमपि गोलविवर्जितं च तन्त्रम् ।। २ ।।

वादी यथाऽव्याकरणो न भाति
श्रुतिस्मृतिभ्यां रहितश्च यज्वा ।
भिषग् यथा सम्यगदृष्टकर्मा
सांवत्सरस्तद्वदगोलवेदी ।। ३ ।।

सङ्ख्याय शास्त्रविद एव वदन्ति गोलं
गोलागमे पटुधियो ग्रहचेष्टितज्ञाः ।
ये जानते न गणितं न च गोलतत्त्वं
जानन्ति ते ग्रहगतिं न कथंचिदज्ञाः ।। ४ ।।

यो वेत्ति सम्यगमुमर्कपुरःसराणां
नानाप्रकारवरचारुविचारचारुम् ।
प्रत्यक्षवत् सकलमेव स वेत्ति विश्वं
धर्मार्थमोक्षयशसामपि भाजनं स्यात् ।। ५ ।।

यन्मध्यादिकमुक्तं तत् प्रत्यक्षं च दृश्यते गोले ।
छेद्यकविधौ ग्रहाणां विशेषतस्तद्विदाचार्यः ।। ६ ।।

(Lalla, *ŚiDh Vr.*, 14.2-6)

Importance of the armillary sphere

No astronomical treatise (is complete) without a section on the sphere of the universe, just as the night is not (lovely) without the rays of the Moon, no woman without breasts (is pretty to look at), and no feast (is enjoyable) without sweets. (2)

Just as a disputant without the knowledge of grammar, a sacrificer without the knowledge of the *Śrutis* (Vedas) and *Smṛtis* (*Purāṇas*, etc.), and a physician without experience, so is an astronomer totally unsuccessful, without the knowledge of the armillary sphere. (3)

The astronomers stress that the armillary sphere is solely (an object) of mathematical calculations; and that those who wish to study the planets must be experts in the subject of the sphere. Those who know neither mathematics nor the essence of the sphere of universe are ignorant. How can they ever detect the motion of the planets? (4)

He who has a comprehensive knowledge of this sphere containing the Sun, etc. sees in front of his eyes, as it were, the whole universe, beautiful with a variety of exquisite situations. Moreover, he becomes the recipient of spiritual fruit, wealth, salvation and fame. (5)

The mean motions of the planets, etc., are clearly perceptible on the armillary sphere, especially to one who has mastered the science of their (geometrical) representation. (6). (BC)

गोलबन्धविधिः

8. 2. 1. अधऊर्ध्वयाम्यसौम्यगमिह वृत्तं दक्षिणोत्तराख्यं स्यात् ।
तन्मध्येऽप्यधऊर्ध्वं वृत्तं पूर्वापरं तु घटिकाख्यम् ।। ३ ।।

बहिरनयोस्तिर्यक् स्याच्चतुराशास्वस्तिकं परं वृत्तम् ।
विषुवत्संज्ञितमेतत् त्रितयं 'खरसा'ङ्कमत्र घटिकाख्यम् ।।४।।

'खरसाग्न्य'ङ्कमिहान्यद् द्वितयं तद्वत्पुनः परं वृत्तम् ।
पूर्वापरस्वस्तिकगमधऊर्ध्वाभ्यां च सौम्यदक्षिणयोः ।। ५ ।।

'जिन'भागे बध्नीयादपमाख्यं दक्षिणोत्तरे वृत्ते ।
घटिकाख्योभयपार्श्वेऽभीष्टक्रान्त्यन्तरे ततस्तद्वत् ।। ६ ।।

स्वाहोरात्राख्यानि च बध्नीयान्मिण्डलान्यतुल्यानि ।
अपमगपातद्वयगं सौम्ये याम्ये ततश्च भत्रितये ।। ७ ।।

परमक्षेपान्तरितं चन्द्रादेः क्षेपमण्डलं भवति ।
विषुवद्ध्रुवयाम्योदक्स्वस्तिकयोर्गोलदण्डकः प्रोतः ।। ८ ।।

नक्षत्रगोल एषः स्यादिह बाह्ये खगोलोऽस्ति ।
तस्मिन् विषुवत्रितयं प्राग्वत्, क्षितिजाख्यमेषु तिर्यक्स्थम् ।।

सममण्डलं तु पूर्वापरगं, याम्योत्तराख्यमपरं स्यात् ।
कल्प्या भूः समवृत्ता मृदादिना गोलदण्डमध्यगता ।। १० ।।

गोलस्थितिरेवं स्यात् निरक्षदेशे ह्यभीष्टदेशे तु ।
अधऊर्ध्वं च खगोले याम्योदक्स्वस्तिकात् पलज्यान्ते ।।

कृत्वा वेधद्वितयं तत्प्रोतं गोलदण्डकं कुर्यात् ।
उन्मण्डलाख्यवृत्तं गोलदण्डाग्रयुग्मकप्रोतम् ।। १२ ।।
कुर्यात् पूर्वापरगस्वस्तिकगं चेष्टदेशगोलोऽयम् ।
शरदण्डिके च योज्ये गोलान्तरे स्थिरत्वाय ।। १३ ।।

भ्रमति ह्यपराभिमुखं प्रवहाक्षेपात् सदा भगोलोऽयम् ।
स्थिर एव खगोलः स्याद् दिगादिसिद्ध्यै प्रकल्पितो ह्येषः ।।

लम्बाक्षज्ञानार्थं प्रकल्प्यते दण्डनाभिहरिजान्ते ।
अन्यद् द्युवृत्तमन्यैर्भूज्याक्षज्येह लम्बकः क्रान्तिः ।। १५ ।।

(Par., *Gola D*, 1. 3-15)

Armillary sphere

A circle kept (vertically, and therefore) extending from below upwards, (and, placed) in the north-south direction is (to represent) the Solstiital Colure (*Dakṣiṇottara*). (3a).

Another (similar) vertical circle (fixed) at the (upper and lower) middle (points) of the first, (and turned to) the east-west direction, is the Equinoctial Celestial Equator (*Ghaṭikā-maṇḍala*). (3b)

Still another circle, (*viz.*, the Equinoctial Colure), is (fixed) around these, cutting them at right angles and making crosses (*Svastika-s*) at the four cardinal points. (4a)

These three are called *viṣuvad* circles. (4b)

Of these the Celestial Equator is divided into 60 equal divisions, and the other two into 360 divisions. (4b-5a)

Another similar circle is fixed passing through the east and west crosses and inclined at 24° north and south of the zenith and nadir. This is the Ecliptic (*Apama-vṛtta*). (5a-6a).

Construct across the Solstitial Colure on either side of the Celestial Equator, (*Ghaṭikāvṛtta*), and parallel to it, at the required declinations, several diurnal circles (*ahorātra-vṛtta-s*), of different magnitudes. (6b-7a)

The orbits of the Moon and the other (planets) are (then to be constructed) crossing the ecliptic at the planets' two nodes (*pāta-s*) and diverging from it (north and south) by their maximum latitudes at three Signs (i.e., 90°) from these nodes. (7b-8a)

Through the north and south crosses formed by the intersection of the Equinoctial and the Solstitial Colures, the central axis (of the armillary sphere) is then inserted. (8b)

The sphere thus formed is called the Starry sphere (*Nakṣatragola* or *Bhagola*.) (9a)

The Celestial sphere (*Khagola*) is constructed outside this.[1] Herein too, (three circles corresponding to) the *Viṣuvad* circles (are to be constructed) as before. Of these, the horizontal circle is (termed) the prime vertical (*Samamaṇḍndala*); and the third is the north-south circle or Meridian (*Dakṣiṇottara*). (9a-10a)

A (model) of the Earth is (then) to be constructed with clay, etc. in the shape of a sphere, and placed at the centre of (the axis) of the armillary sphere. (10b).

[1] An inner Starry sphere and an outer Celestial sphere are conceived in the armillary sphere according to Indian astronomy to facilitate comparative measurements of the former with reference to the latter.

The situation of the armillary sphere is like this at places of zero latitude. (11a).

At any other required place, however, two holes are made in the Celestial sphere at a distance equal to the latitude (of the place),[1] below and above, respectively, the south or north crosses and the axis of the Starry sphere is made to pass through them. (11a-12a)

A circle termed *Unmaṇḍala* is now constructed (and fixed) so as to pass through the two ends of the axis and the east and west crosses. This will be the situation of the spheres at the desired place. (12b-13a).

Two pieces of reed (*śara-daṇḍikā-s*) are fitted (on the axis) between the two spheres to keep them in position. (13b).

The Starry sphere revolves constantly westwards, blown by the Pravaha wind. The Celestial sphere, however, remains stationary; it is constructed in order to reckon directions, etc. (14)

To measure the sine and cosine of the latitude, some (astronomers) construct a diurnal circle (with a radius equal to the sine latitude, *akṣa-jyā*,) with (a point on) the central axis as the centre and just touching the horizon. Here the *Bhū-jyā* (i.e., sin. decl. tan. lat.) is sine latitude and the sine declination is cosine latitude. (15). (KVS)

खगोलबन्धः

8. 3. 1. पूर्वापरमूर्ध्वमधः प्रथमं सममण्डलं जगुर्वृत्तम् ।
यांम्योत्तरं च तद्वत् तथैव वृत्तद्वयं च विदिशोः ।। १ ।।
आवेष्टयमानमेतान्यर्धच्छेदेन यद् भवेत् वृत्तम् ।
तत् क्षितिजमण्डलं स्यादुदयास्तमयाविह द्युसदाम् ।। २ ।।
वृत्ते प्रागपरे कुजाक्षभवयोः सक्तं द्वयोः संज्ञयोः
सौम्यक्ष्माभववृत्तयोगजनितादङ्कात् ध्रुवे चोपरि ।
स्वाक्षांशाङ्कगतेऽथ याम्यकुजयोर्योगादधोऽक्षांशकै-
र्वृद्धिहानी विधाय वासरनिशोरुन्मण्डलं मण्डलम् ।।
षडमूनि नभोगोले भगणांशैरङ्कितानि षष्टयङ्कम् ।
विषुवद्वृत्तं नाड्याह्वयं भगोले स्थितमस्याधः ।। ४ ।।

(Lalla, *ŚiDh.Vṛ.*, 15. 1-4)

Sphere of the sky

The great circle passing through the east and west points (of the horizon) is said to be the prime vertical or *samamaṇḍala*. The great circle passing through the north and south points (of the horizon) is the meridian or *yāmyottaramaṇḍala*. In the same manner, the great circles (passing through the north-east and south-west

[1] *Pala-jyā*, here, as also elsewhere below, means only the latitude, *akṣa*, and not the sine latitude, *akṣa-jyā*.

points and through the north-west and south-east points) are called verticals in the intermediate directions (or *koṇamaṇḍala*). (1)

The great circle that goes transversely through the middle of these is the horizon or *kṣitija*. The planets rise and set here. (2)

The six o'clock circle or *unmaṇḍala* passes through those two points on the prime vertical where the horizon and the celestial equator meet it; it also passes through the north pole at a distance equal to the latitude of the place, from that point on the horizon, where the meridian meets it; it also passes through the south pole at a distance equal to the latitude of the place, below that point on the horizon, where the meridian meets it.

The six o'clock circle indicates the increase and decrease in the lengths of day and night. (3)

These six circles of the celestial sphere are each divided into 360°.

The celestial equator or *viṣuvadvṛtta* (or *nāḍīvṛtta*) is divided into 60 (*nāḍis* or *ghaṭikās*). It is the sphere of the fixed stars which is placed below the celestial sphere. (4). (BC)

भगोलवन्धः

8. 4. 1. समवृत्तादक्षांशैर्दक्षिणतो भवति नाडिकावृत्तम् ।
खस्वस्तिकादधस्तादुत्तरतोऽक्षांशकैरेव ॥ ५ ॥
याम्योत्तरमत्रान्यद् विवेष्टमानं समन्ततः कुजवत् ।
ध्रुवयाम्योदक्स्वस्तिकसक्ता ध्रुवयष्टिरन्तर्भूः ॥ ६ ॥
मेषतुलादौ लग्नं नाडीवृत्तेऽपमण्डलं तदुदक् ।
'जिन'भागैः कर्क्यादौ याम्यैस्तैरेव मकरादौ ॥ ७ ॥
भ्रमति रविरत्र वलये ग्रहाश्च चन्द्रादयः स्वपातयुताः ।
भूभा भार्धे भानोः स्वशीघ्रवृत्ते ज्ञसितपातौ ॥ ८ ॥
विक्षेपमण्डलदलं पूर्वं क्षेपांशकैरुदक् पातात् ।
षड्भयुताद् दक्षिणतो दिङ्मण्डलार्धं द्वितीयं स्यात् ॥ ९ ॥
वृत्तत्रयमपमांशैर्नाडीवृत्ताद् भवत्यजादीनाम् ।
व्यस्तं कर्क्यादीनामेवं षण्णां तुलादीनाम् ॥ १० ॥
इष्टक्रान्तेरग्रे तद्द्युज्यामण्डलं च बध्नीयात् ।
मध्येऽस्य ग्रहगोला भवन्ति वृत्तैर्भगोलस्य ॥ ११ ॥
(Lalla, *ŚiDhVṛ.*, 15. 5-11)

Sphere of asterisms

The celestial equator in the *Bhagola* is to the south of the prime vertical (at a distance equal to the) latitude of the place. (It intersects the meridian at a distance equal to the) latitude of the place, to the south of the zenith and (again at a distance equal to the) latitude of the place to the north of the nadir. (5)

There is to be another circle known as meridian which completely encircles the (other circles) like the horizon. The *dhruvayaṣṭi* or the polar axis is the line joining the north and south poles and passing through the earth's centre. (The earth is in position loosely in the middle of it). (6)

The ecliptic or *apamaṇḍala* intersects the celestial equator at the first points of Aries and Libra. It is to the north of the equator by 24° at which is the first point of Cancer. It is also to the south of the equator by 24° at which is the first point of Capricorn. (7)

The Sun moves along the ecliptic and so do the Moon, the planets and the Nodes of the Moon and the superior planets. The shadow of the Earth also moves on the ecliptic at a distance of 6 signs from the Sun. The Nodes of Mercury and Venus move along their respective *śīghravṛttas*. (8)

The first half of the orbit or *vikṣepamaṇḍala* (of a planet) measured from the Node is to the north of the ecliptic and (the greatest distance between them) is the greatest latitude of the planet to the north.

The second half of the orbit measured from the same Node and increased by 6 signs is to the south of the ecliptic and (the greatest distance between them) is the greatest latitude of the planet to the south. (9)

The three day-circles of the first three signs beginning with Aries are drawn parallel to the celestial equator, at distances equal to their respective declinations. (The same circles considered in a) contrary direction are the day-circles of the next three Signs. In the same way the day-circles of the other six Signs beginning with Libra can be drawn. (10)

The day-circle for any given declination should be drawn (parallel to the celestial equator) at a distance equal to that declination. The spheres of the planets are drawn by means of great circles in the *Bhagola*. (11). (BC)

ग्रहगोलबन्धः

8. 5. 1. विषुवद्वृत्तस्थित्या खचराणां कक्षया समं वृत्तम् ।
याम्योत्तरं च कार्यं परिकरसंस्थं क्षितिजवच्च ॥ १२ ॥
नाडीवृत्तादपमण्डलं भवेद् यत् तदेव दातव्यम् ।
कक्षावृत्तात् खचरापमण्डलं ग्रहभवृत्तं तत् ॥ १३ ॥
सममृदुकेन्द्रवृत्तान्यन्यानि च शीघ्रकेन्द्रवृत्तानि ।
प्रतिमण्डलानि तद्वद् ग्रहगोलाधो महीगोलः ॥ १४ ॥
उन्मण्डले विलग्नं क्रान्तिज्याग्रे परे च पूर्वे च ।
याम्योत्तरे नतांशैश्चलग्रहभ्रमभवं वलयम् ॥ १५ ॥
(Lalla, *SiDhVṛ.*, 15. 12-15)

Sphere of the planets

In the plane of the equator, make a circle equal to the planet's orbit. (This is the equator in the sphere of the planet). Also draw the meridian circle going round (this circle) like the horizon. (12)

Now, fix a great circle at the same distance from the circle already drawn equal to the planet's orbit as is the ecliptic from the celestial equator in the celestial sphere. This will be the ecliptic in the sphere of the planet. The planet moves along this (circle). (13)

(Then, fix) epicycles for mean anomaly and for *śīghra* anomaly and also eccentrics for mean anomaly and for *śīghra* anomaly.

The Sphere of the Earth is beneath that of the planets. (14)

The circle attached to the six o'clock circle at a distance equal to the planet's declination, in the eastern and the western hemisphere, and to the planet's meridian at a distance equal to the planet's meridian zenith distance, is the diurnal circle of the moving planet. (15). (BC)

गोलबन्धः तदुपयोगश्च

8. 6. 1. याम्योदग् वा मध्यज्यावृत्तं मध्यलग्नमत्र वृतौ ।
दृङ्मण्डलमुपरिष्टाद् दृष्टः स्यात् तद्वृतौ खचरः ॥ १६ ॥

दृग्गतिजीवावलयं मध्यविलग्नस्य संस्थितं पुरतः ।
छायाग्रभ्रमवृत्तं ग्रहभ्रमव्यत्ययं च स्यात् ॥ १७ ॥

क्षितिजेऽग्रा परपूर्वद्युज्यावृत्तान्तरालभागज्या ।
अग्रायाः परिवृत्तं प्रथमक्षितिजे च पूर्वे च ॥ १८ ॥

क्षितिजोन्मण्डलमध्ये द्युज्यावृत्तस्थिता महीजीवा ।
मध्ये चरार्धजीवा त्रिज्यावृत्ते तयोरेव ॥ १९ ॥

पूर्वक्षितिजापममण्डलयोर्योगाद् विलग्नमुद्दिष्टम् ।
पाश्चात्यकुजवलयापवृत्तयोगोऽस्तलग्नं स्यात् ॥ २० ॥

पूर्वापरकुजवृत्तादुन्नतलवशिञ्जिनी शङ्कुः ।
तस्याग्रे दिनकरो नरोऽर्कबिम्बावलम्बो वा ॥ २१ ॥

समशङ्कुः समपूर्वाद् विदिक्कुजात् कोणशङ्कुरेव स्यात् ।
समदक्षिणोत्तरकुजादुन्नतजीवा द्युदलशङ्कुः ॥ २२ ॥

अम्बरमध्यांशुमतो मध्यांशज्या भवेन्नतज्या खे ।
शङ्कोर्मूलाद् दिङ्मध्यगामिनि भूतले दृग्ज्या ॥ २३ ॥

विषुवद्दिवसे द्युदले पलो नतांशाः समुन्नता लम्बः ।
उन्नतनतभागज्ये तदावलम्बाक्षजीवे स्तः ॥ २४ ॥

लम्बाग्राद् दिङ्मध्यप्रापि तलाद् यत् खमध्यगं सूत्रम् ।
सा त्रिज्याऽन्यत्रापि च कर्णः क्षेत्रे चतुर्बाहौ ॥ २५ ॥

सौम्ये ध्रुवोन्नतिसमा कार्या नतिरुन्नतिश्च दक्षिणतः ।
सोन्नतिनतिबद्धायां रज्जोरर्धं पलज्या स्यात् ॥ २६ ॥

उन्नत्योर्नत्योर्वा संसक्ता तस्य सूत्रस्य ।
अर्धमवलम्बजीवा नत्युन्नत्योर्दलं त्रिज्या ॥ २७ ॥

लङ्कावृत्ते द्युसदां कक्षाव्यासार्धमुच्यते द्युज्या ।
दृग्ज्या तु सा सुराणामपक्रमज्या भवेच्छङ्कुः ॥ २८ ॥

याम्योदग् या गगने कक्षान्तरमथ नतिर्नभोमध्यात् ।
पूर्वापरं यदन्तरमुभयोस्तल्लम्बनं ग्रहयोः ॥ २९ ॥

कक्षावृत्तं पूर्वापरं तथा दक्षिणोत्तरं क्षितिजम् ।
विषुवद्वलयोद्वृत्ते स्थिराणि भानां ग्रहाणां च ॥ ३० ॥

मन्दोच्चनीचवृत्तानि सप्त, पञ्च स्वशीघ्रवृत्तानि ।
प्रतिमण्डलान्यपि तथा दिग्दृक्षेपापवलयानि ॥ ३१ ॥

प्रत्येकमिनादीनां चन्द्रादीनां विमण्डलानि च षट् ।
पञ्चाशदेकयुक्ता च वृत्तानां भवत्येवम् ॥ ३२ ॥

(Lalla, *ŚiDhVṛ.*, 15. 16-32)

The Armillary sphere and its use

The meridian is also called *madhyajyāvṛtta*. The *madhyalagna* or meridian ecliptic point is on this circle. The *dṛṅmaṇḍala* is above it. The planet is seen on this circle. (16)

The *dṛggatijīvāvalaya* is just ahead of the meridian ecliptic point.

The circle on which the extremity of the shadow (of the gnomon caused by a planet) moves, is opposite to that on which the planet moves. (17)

The amplitude or *agrā* is (measured) on the celestial horizon. Its R sine is the R sine of the arc of the horizon intercepted between the east point and the day-circle of the rising planet or the west point and the day-circle of the setting planet. That is, its extreme points are the east (or west) point and the point where the planet rises (or sets). (18)

The earth-sine or *kujyā* is the R sine of the arc of the day-circle intercepted between the horizon and the six o'clock circle. The R sine of the same arc, when converted to the great circle or *trijyāvṛtta* is the R sine of the ascensional difference. (19)

The orient ecliptic point or *lagna* is the point on the ecliptic where the eastern horizon meets it. The *astalagna* or the setting ecliptic point is the point on the ecliptic where the western horizon meets it. (20)

The R sine of the degrees of the arc (or altitude) by which the Sun is raised either above the eastern or the western horizon, is called *śaṅku*. The Sun is at its

extremity. Or, (it can be said that the) *śaṅku* is the perpendicular support of the Sun's disc. (21)

The *samaśaṅku* is the R sine altitude of the Sun when it is on the prime vertical; and the *koṇaśaṅku* is the R sine altitude when it is on the *koṇamaṇḍala* or the vertical circle in an intermediate direction. When the Sun is equidistant from the north and south points of the horizon, the R sine of the degrees by which it is raised above the horizon is called '*śaṅku* at midday' or *dyudalaśaṅku*. (22)

When the Sun is on the meridian, the R sine of the degrees measured from the zenith is called *natajyā* or R sine zenith distance . The *dṛgjyā* is the distance on the earth between the foot of the R sine altitude and the centre of the earth, that is, the point where the directions meet. (23)

When the Sun is on the equinox, its zenith distance at midday is equal to the latitude of the place, and its altitude is the colatitude of the place. Thus, the R sines of the zenith distance and of the altitude are, respectively, the R sines of the latitude and the colatitude of the place. (24)

The line that joins the zenith to the point of intersection of the lines of direction is the radius or *trijya* and so is also the line that joins the top of the R cosine latitude to the same point. The same *trijyā* is the hypotenuse in a rectangular figure. (25)

In the north, measure out an arc below the horizon equal to the elevation of the pole. (Let it be the lower point.) In the south, measure out an arc of the same length above the horizon. (Let it be the upper point.) One half of each of the vertical lines joining one of the poles to one of the points measures the R sine latitude of the place; and one half of the horizontal lines measures R sine colatitude. One half of the oblique line joining one upper point to one lower point is the radius or *trijyā*. (26-27)

When seen from the equator, the radius of the orbit of a planet is the radius of its diurnal circle. The same is the R sine of the zenith distance of a planet when seen from the pole of the Earth. The R sine of its declination is its R sine altitude. (28)

The latitudinal parallax or *nati* is the north-south difference of the orbits of two planets from the middle of the sky. The parallax in longitude or *lambana* is their east-west difference. (29)

The prime vertical, meridian, horizon, celestial equator and the six o'clock circle (all of the same size as the planet's orbit) are fixed for the stars and planets (at any given latitude). (30)

There are 7 epicycles, 7 eccentrics, 7 *dṛṅmaṇḍalas*, 7 *dṛkkṣepamaṇḍalas* and 7 *apamaṇḍalas* or *kakṣāmaṇḍalas*, one each for each planet beginning with the Sun. Then, there are 5 *śīghra* epicycles and 5 *śīghra* eccentrics, one for each of the five planets. Then, there are 6 *vimaṇḍalas*, one for the Moon and five for the five planets. Thus, there are 51 great circles (which are not fixed). (31-32). (BC)

8. 6. 2. अथ लग्नकालसिद्धयै पूर्वापरपरिकरोत्तरैर्नवभिः ।
निर्मापयेद् भगोलं प्राग्विधिना क्रान्तिवृत्तमिह ।। ३ ।।

तस्य बहिश्च खगोलं समवृत्तक्षितिजदक्षिणोत्तरगैः ।
उन्मण्डलेन च तथा ध्रुवयष्ट्या पूर्ववत् सभुवा ।। ४ ।।

षष्टयाङ्कयेद् भगोलं प्रागपराणीतराणि चक्रांशैः ।
कुर्याद् दृढं खगोलं श्लथं खगोलं च नलिकाभ्याम् ।। ५ ।।

यस्मिन्नंशे सविता तत्र शलाकां क्षिपेदपमवृत्ते ।
नाडीवृत्तस्थां तामुदये क्षितिजाद्रविवशेन ।। ६ ।।

भ्रमयेच्छश्वत् तद्वत् यथा न केन्द्रं त्यजेच्छलाकाभा ।
रविचिह्नक्षितिजान्तरमुदितांशस्तृणकुजान्तरा घटिकाः ।।७।।

गोलः समैश्चतुर्भिः क्षितिजस्वस्तिकगतैः समो विधृतः ।
गर्तार्धनिमग्नेन करणैर्बीजेन वा भ्राम्यः ।। ८ ।।

ध्रुवयष्टयां च शलाका नाडीवृत्तानुसारिणी शिथिला ।
अग्रद्वयावलम्बा स्पृशति घटीर्या गतास्ता वा ।। ९ ।।

(Lalla, *ŚiDhVṛ.*, 21. 3-9)

Now, in order to determine the *lagna* (i.e., the longitude of the rising point of the ecliptic) and the *kāla* (i.e., the time elapsed since sunrise in the forenoon, or to elapse before sunset in the afternoon), one should get a *Bhagola* ('Sphere of the asterisms') constructed with the help of nine circles, the prime vertical etc. The ecliptic should also be exhibited in it, in the manner stated before. (3)

Outside the *Bhagola* should be constructed the *Khagola* ('Sphere of the sky' or the Celestial sphere) by means of the prime vertical, the horizon, the meridian, and the 6 o'clock circle, and having the polar axis and in its middle the globe of the Earth, as before. (4)

(The equator lying on) the *Bhagola* should be graduated with the 60 marks (of *ghaṭīs*), whereas the prime vertical and the other circles should be graduated with the 360 marks (of degrees). The *Khagola* should be fastened firmly (to the polar axis,) whereas the *Bhagola* should be kept loose by tying it to two loose pipes inserted into the polar axis. (5)

Now, one should fix one end of a pin to that point of the equator which rises with the Sun and should point

the other end of the pin towards the degree-mark of the ecliptic occupied by the Sun. (6)

Now, rotate the *Bhagola* continuously until the shadow of the pin passes through (*lit.* does not leave) the centre (of the *Bhagola*). This being done, the degrees of the ecliptic lying between the point denoting the Sun and the horizon denote the degrees of the ecliptic which have risen since sunrise, and the *ghaṭīs* (on the equator) lying between the pin and the horizon denote the *ghaṭīs* (elapsed since sunrise). (7)

Alternative method

Or, the *Khagola* should be mounted on four vertical pillars of equal height erected at the four cardinal points, whereas the *Bhagola* should be rotated by the instruments of action (such as rope etc.) or *bījas* (such as mercury, oil and water) by (the astronomer) placing himself in the hemispherical cavity underneath. (8)

In the polar axis, one should fix a pin in the plane of equator, which should be free to move with the *Bhagola*. The *ghaṭī* marks of the equator with which its two ends are in contact (at the moment) indicate the *ghaṭīs* elapsed (since sunrise). (9). **(KSS)**

8. 6. 3. भूभगोलस्य रचनां कुर्यदाश्चर्यकारिणीम् ।
अभीष्टं पृथिवीगोलं कारयित्वा तु दारवम् ।। ३ ।।

दण्डं तन्मध्यगं मेरोरुभयत्र विनिर्गतम् ।
आधारकक्ष्याद्वितयं कक्ष्यां वैषुवतीं तथा ।। ४ ।।

भगणांशाङ्गुलैः कार्या दलितास्तिस्र एव ताः ।
स्वाहोरात्रार्धकर्णैश्च तत्प्रमाणानुपाततः ।। ५ ।।

क्रान्तिविक्षेपभागैश्च दलिता दक्षिणोत्तरा ।
स्वैःस्वैरपक्रमैः कार्या मेषादीनामपक्रमात् ।। ६ ।।

कक्ष्याः प्रकल्पयेत्ताश्च कर्क्यादीनां विपर्ययात् ।
तद्वत्तिस्रस्तुलादीनां मृगादीनां विलोमतः ।। ७ ।।

याम्यगोलाश्रिताः कुर्यात् कक्ष्याधारद्वयोपरि ।
याम्योदग्भागसंस्थानां भानामभिजितस्तया ।। ८ ।।

सप्तर्षीणामगस्त्यस्य ब्रह्मादीनां प्रकल्पयेत् ।
मध्ये वैषुवती कक्ष्या सर्वासामेव संस्थिता ।। ९ ।।

तदाधारयुतेः भार्धमयने विषुवद्वये ।
अयनादयने चैव कक्ष्या तिर्यक्तथाऽपरा ।। १० ।।

क्रान्तिसंज्ञा तया सूर्यः सदा पर्येति भासयन् ।
चन्द्राद्याश्च स्वकैः पातैरपमण्डलमाश्रितैः ।। ११ ।।

ततोऽपकृष्टा दृश्यन्ते विक्षेपाग्रेष्वपक्रमात् ।
विषुवस्थानतो भागैः स्फुटैर्भगणसञ्चरात् ।। १२ ।।

क्षेत्राण्येवमजादीनां तिर्यग्जानि प्रकल्पयेत् ।
उदयं क्षितिजे लग्नमस्तं गच्छति तद्वशात् ।। १३ ।।

लङ्कोदयैस्तथा सिद्धं खमध्योपरि मध्यगम् ।
मध्यक्षितिजयोर्मध्ये या ज्या साऽन्त्याऽभिधीयते ।। १४ ।।

ज्ञेया चरदलज्या च विषुवत्क्षितिजान्तरम् ।
कृत्वोपरि स्वकं स्थानं मध्ये क्षितिजमण्डलम् ।। १५ ।।

वस्त्रच्छन्नं बहिश्चापि लोकालोकेन वेष्टितम् ।
अमृतस्रवयोगेन कालभ्रमणसाधनम् ।। १६ ।।

गुणबीजसमाकृष्टं गोलयन्त्रं प्रकल्पयेत् ।
गोप्यमेतत्प्रकाश्योक्तं सर्वगम्यं भवेद्यतः ।। १७ ।।

तस्माद् गुरूपदेशेन रचयेद् गोलमुत्तमम् ।। १८a ।।

(*SūSi.*, 13. 3-18a)

Prepare the wonder-working fabric of the terrestrial and stellar sphere (*Bhūbhagola*). Having fashioned an earth-globe of the desired size, fix a staff passing through the midst of it and protruding at either side, for Meru; and likewise a couple of sustaining hoops (*kakṣa*), and the equinoctial hoop. (3-4)

These are to be made with graduated divisions (*aṅgula*) of degrees of the circle (*bhagaṇa*). Farther—by means of the several day-radii, as adapted to the scale established for those other circles, and by means of the degrees of declination and latitude (*vikṣepa*) marked off upon the latter—at their own respective distances in declination, according to the declination of Aries etc., three hoops are to be prepared and fastened: these answer also inversely for Cancer, etc. In the same manner, three for Libra etc., answering also inversely for Capricorn, etc.; and situated in the southern hemisphere, are to be made and fastened to the two hoop-supporters. (5-8a)

Those likewise of the asterisms (*bha*) situated in the southern and northern hemisphere, of Abhijit, of the Seven Sages (*saptarṣayas*), of Agastya, of Brahmā etc., are to be fixed. Just in the midst of all, the equinoctial (*vaiṣuvatī*) hoop is fixed. (8b-9)

Above the points of intersection of that and the supporting hoops are the two solstices (*ayana*) and the two equinoxes (*viṣuvat*). From the place of the equinox, with the exact number of degrees, as proportioned to the whole circle, fix, by oblique chords, the spaces (*kṣetra*) of Aries and the rest; and likewise another hoop, running obliquely from solstice (*ayana*) to solstice, and called the circle of declination (*krānti*): upon that the Sun constantly revolves, giving light; the Moon and the other planets also, by their own nodes, which are situated in the ecliptic (*apamaṇḍala*), being drawn away from it, are beheld at the limit of their removal in latitude (*vikṣepa*) from the corresponding point of declination. The orient ecliptic point (*lagna*) is that at the

orient horizon; the occident point (*astamgacchat*) is similarly determined. (10-13)

The meridian ecliptic-point (*madhyama*) is as calculated by the equivalents in right ascension (*laṅkodayas*) for mid-heaven (*khamadhya*) above. The sine which is between the meridian (*madhya*) and the horizon (*kṣitija*) is styled the day-measure (*antyā*). (14)

And the sine of the Sun's ascensional difference (*caradala*) is to be recognized as the interval between the equator (*viṣuvat*) and the horizon. Having turned upward one's own place, the circle of the horizon is midway of the sphere. (15)

As covered with casing (*vastra*) and as left uncovered, it is the sphere surrounded by Lokāloka. By the flow of water is ascertained the revolution of time. (16)

One may construct a sphere-instrument combined with quicksilver; this is a mystery; if plainly described, it would be generally intelligible in the world. (17)

Therefore, let the supreme sphere be constructed only according to the instruction of the preceptor (*guru*). (18). (Burgess)[1]

[1] For notes, see *Sū. Si*: *Burgess*, pp. 298-305.

9. वेधशालाः – OBSERVATORIES

केरलेषु महोदयपुरे वेधशाला

9. 1. 1. गोलान्महोदयपुरे रविवर्मदेव
सम्बन्धयन्त्रवलयाङ्कितराशिचक्रात् ।
भानोः कुलीरदशभागगते तुलान्त्यं
लग्नं मया विदितमाशु वदेह कालम् ।।

अपि च—
सुरपतिदिशि दृष्टं गोलयन्त्राद्विलग्नं
घनपटलनिरुद्धे भास्करे सिंहराशौ ।
गतवति दशभागांश्चापराश्यर्धयातं
वद झटिति रवे ! त्वं नित्यकर्मोक्तकालम् ।।

(Śaṅkaranārāyaṇa, Com. on *LBh.*, 3. 20)

Observatory at Mahodayapuram in Kerala, c. A. D. 860

(To the King): Oh Ravivarmadeva, now deign to tell us quickly, reading off from the armillary sphere installed (at the observatory) in Mahodayapura, duly fitted with all the relevant circles and with the Sign (-degree-minute) markings, the time of the rising point of the ecliptic (*lagna*) when the Sun is at 10° in the Sign of Capricorn, and also when the Sun is at the end of the Sign Libra, which I have noted.

Then again—

Oh Ravi, deign to tell us immediately, reading off from the armillary sphere, by means of the reverse *vilagna* method, the time for offering the daily oblations, when the Sun, shrouded under thick clouds, is 10° in the Sign Leo and also when it is the middle (i.e. 15°) in the Sign Sagittarius. (KVS)

काशी-मानमन्दिर-वेधालयः

9. 2. 1. अस्ति श्री-वाराणस्यां भागीरथीतीरे मणिकर्णिकाघट्टान्नातिदूरे दक्षिण-पश्चिम-दिग्भागे राजपुतानाख्यप्रसिद्ध-देशान्तर्वर्त्यामेर-संज्ञ-नगर्याः स्वामिना मानसिंहाभिधेयेन राज्ञा विनिर्मितं सदनमेकं मानमन्दिरसंज्ञकं नाम । तत्र तस्य नृपतेर्वंश्येन प्रतापवता जयसिंहेन भूपेनेतः-पूर्वं सार्धशतवर्षासन्नकाले ग्रहनक्षत्रवेधार्थं रचितानि ज्यौतिषयन्त्राणि तदुपयोगजिज्ञासूनां सहृदयपुरुषाणां रञ्जनायेदानीमहं संक्षेपतो वर्णयामि ।

(Bapudeva, *Mānamandira*, p. 15)

Kāśī Mānamandira Observatory

In Vārāṇasi, on the banks of the Ganges, not far removed from Maṇikarṇikā ghāṭ there is a building, named Mānamandira in the north-west direction, which was constructed by the king named Mānasiṃha of Āmer city in the province of Rājputānā. Approximately 150 years ago,[1] his (Mānasiṃha's) famous descendant Jaya Siṃha installed certain astronomical instruments in that building for observing the planets and stars. Here I am describing them in brief for the pleasure of those scholars who want to know their use. (SDS)

याम्योत्तरयन्त्रम्

9. 3. 1. तत्र 'याम्योत्तर'भित्तिसंज्ञं यन्त्रं दक्षिणोत्तररेखायां चूर्णेष्टका-प्रस्तरैर्निर्मितं कुड्यरूपं सत्र्यंशसप्तहस्तोच्छ्रायं सत्रिलवद्व्यैकाङ्गुलाधिकषड्हस्तपरिणाहं सार्धषोडशांगुलपिण्डं समीचीन-सुधालेपे चिक्कणीकृते तत्पूर्वपार्श्वे भूमेरङ्गुलद्वयोन-'नग'करोच्छ्रितयोरन्योन्यं हस्तपञ्चकेन चतुर्विंशांशाधिकेन विप्रकृष्टयोः तत्पार्श्वोर्ध्वकोणद्वयासन्नदेशयोः रोपितौ लोहकीलौ केन्द्रे कृत्वा कीलान्तरमितव्यासार्धेन लिखिताभ्यां मिथः सम्पातं कुर्वद्भ्यां वृत्तचरणाभ्यां स्वाधोभागे तत्केन्द्रकैरेव क्रमेण पञ्चदशभिर्नवत्यां नवशतेन च समं विभक्तौ त्रिभिस्त्रिभिर्वृत्त-चतुर्थांशैः सहिताभ्यामङ्कितं मन्दिरान्तःप्रविशद्भिः यन्त्रदिदृक्षुभिः प्रथममेवोपलभ्यते ।

अस्मिश्च यन्त्रेऽत्र दक्षिणोत्तरमण्डलं याते सहस्रकिरणे वृत्तचरणे पतितां तत्केन्द्रस्थकीलच्छायां यावद् अपरकीलमूलात् तिग्मरश्मेरुन्नतांशास्तलतश्च नतांशाः प्रत्यहम् अहर्दलेऽवगम्यन्ते । परमिह काश्यामक्षांशानां परमक्रान्तितोऽधिकत्वात् उष्णगभस्तेः खस्वस्तिकादूर्ध्वभागे सञ्चारासम्भवादस्तीह तस्योन्नतलवानामवगमाय याम्यकेन्द्रकस्यैव वृत्तचरणस्योपयोगः । यत् पुनः उदग्-दक्षिण-केन्द्रकयोः वृत्तचरणयोरत्रोल्लेखनं तत् खमध्याद् उत्तरे दक्षिणे च याम्योत्तरमण्डलं सर्वेषामुडूनां वेधाय इत्यतिरोहितं मतिमताम् ।

यन्त्राच्चास्माद् भवति तरणि-परमक्रान्तेरक्षांशानां चावगमः सुगमः । तथाहि—प्रतिदिनं दिनदलेऽर्कस्यावनतलवेष्ववलोकितेषु यावन्तः परमाधिकाः नतांशाः, यावन्तश्च परमाल्पाः अवगम्येरन्, तद्विश्लेषदले परमापमांशाः भवेयुः । तांश्च श्रीजयसिंहमहीपतिरष्टाविंशतिकलाधिकत्रयोविंशतिमितान् निरणयत् । एभिश्चोनीकृतेषु परमाधिकनतलवेषु अधिकीकृतेषु परमाल्पनतांशेषु वा सम्पद्यन्तेऽक्षांशाः ।

एवमवगताक्षांशेऽस्मिन् विषयेऽभीष्टदिने मध्याह्नेऽर्कस्य-नतांशानवलोक्य तांश्चाक्षांशैर्विश्लेष्यावशेषमितं तस्मिन् दिने दिनेशस्य पलांशेभ्यो नतांशानाम् अल्पत्वे सौम्यम् अधिकत्वे च याम्यम् अवगच्छेत् । ततश्चरस्याहर्निशोः प्रमाणस्यारुणभोगस्यावगमः सिद्धान्तविदोऽतिस्पष्टतरः ।।

(Bapudeva, *Mānamandira*, pp. 15-16)

[1] That is about A.D. 1720, being approximately 150 years prior to A.D. 1866 when the present account was written. For an account of the astronomical endeavours of Sawai Jai Singh, see K.V. Sarma, Foreword, and S.D. Sharma, Introduction to *Mānamandira*: *SDS*, pp. ix-xxiv, 1-13.

Yāmyottara-yantra: Mural quadrant

There is a *Yamyottara-yantra* in the North-South direction in the form of a wall made of lime-brick-stone. It is 7 1/3 hands[1] in height, 6 hands and 1 2/3 *aṅgulas* in breadth and 16½ *aṅgulas* in thickness. One of its sides, facing east, is whitewashed and smoothened. On this side, at the points near the top corners, there are fixed two gnomons (*śaṅkus*) made of iron, which are separated by 5 5/24 hands from each other and are fixed at a height of 7 hands less 2 *aṅgulas* from the ground. With the gnomons as centres and the radii equal to the distance between them, there are drawn two intersecting quadrants. Below both these quadrants are the concentric quadrants which are uniformly divided consecutively into 15 parts (of 6° each), 90 parts (of 1° each), and 900 parts (of 6′ each). This instrument can be seen by the visitors at first sight while entering the Mandira.

In this instrument, every day at midday, when the Sun is on the meridian, the shadow of the gnomon falls on the quadrant whose centre is the former. From the base of the other gnomon up to the shadow are the altitude degrees and from the bottom (up to the shadow) are the co-latitude degrees. But in Vārāṇasi the latitude is more than the maximum declination (the obliquity) of the Sun. So it cannot go higher (towards north) than the zenith there. Thus for the Sun, only the quadrant with the centre towards south is useful. Here the graduations of both the north-south (gnomon)-centred quadrants are for the observations of all the stars north or south of the zenith, when they are on the meridian. This (fact) would be evident to the scholars.

With the help of this instrument, the maximum declination of the Sun and the latitude (of the place of observation) can be known easily. Observing the co-altitudes (zenith distances) every day at midday, whatever maximum and minimum zenith distances are observed, half of their difference is the maximum declination. King Jaya Siṃha has determined its value to be 23° 28′. The maximum zenith distance derived by subtracting the maximum declination or the minimum zenith distance by adding to the maximum declination yields the latitude.

Thus, knowing the latitude of the place, note the zenith distance of the Sun on any desired day at midday time. Subtract the latitude from this. The remainder is the declination of the Sun on that day. If the latitude is less than the zenith distance, then the declination is to the north and if the latitude is greater than the zenith distance, then the declination is to the south. From this, the ascensional difference (and hence) the lengths of day and night and also the celestial longitude of the Sun are easily understood by astronomers. (SDS)

एकमज्ञातयन्त्रम्

9. 4. 1. अस्ति चास्याः दक्षिणोत्तरभित्तेः प्राग्भागे तत्सक्तैव तत्तुल्यविस्तारा दैर्घ्ये सप्तकरा पूर्वं जलवत् समीकृता अपि इदानीं किंचिदुच्चावचत्वं गता भुमिः यस्यां तत्पूर्वविदिक्कोणासन्नदेशयोर्भित्तिस्थकीलद्वयसंसाधित-प्राग्दिशि निखातौ सरन्ध्राग्रौ लोहकीलौ द्वौ आस्ताम् । तयोरेक उत्तरदिक्क एवाधुनाऽवशिष्यते । अस्याः भूमेश्च समीपदेशे एव चूर्णेष्टकानिर्मितं साङ्घ्रिनवाङ्गुलाधिकैकहस्तव्यासं समीकृतभूमिकमेकं मण्डलमन्यच्च प्रस्तरमयं समभूमिकं सप्तांगुलाधिकहस्तद्वयव्यासमस्ति । तन्निकट एव चैकं पाषाणघटितं सममहीकमेकमेकादशाङ्गुलोत्तरैकहस्तपरिमितबाहुकं वर्गाकारं चतुरस्रं वर्तते । एतन्मण्डलद्वयं वर्गक्षेत्रं चेदानीं परिमृष्टविभागचिह्नं पूर्वं शङ्कुच्छायां दिगंशांश्चावगन्तुं निरमायीति प्रतिभाति ।

(Bapudeva, *Mānamandira*, p. 16)

An unknown instrument

Towards the east of this north-south wall (where the *Yāmyottarayantra* is placed) there is an adjacent plane with breadth equal to that of the wall and length equal to seven hands. Although it was initially levelled like (the surface of) water, now it is uneven. On this plane there were fixed two pegs of iron with holes at the top. These are fixed along the eastern direction determined by the two gnomons in the wall. At present only one of them, in the east, exists there. Quite near this plane, there is a levelled circular plane made of lime-bricks and with diameter equal to one hand and 9¼ *aṅgulas*. There is also another, (a second), levelled circular plane made of stones and having diameter equal to 2 hands and 7 *aṅgulas*. Nearby this plane there is a levelled square with side equal to 1 hand and 11 *aṅgulas*. The graduations on these two circular platforms and the square are erased now. But it seems that during earlier times these were made use of to ascertain the gnomonic shadow and the azimuth. (SDS)

सम्राट्–यन्त्रम्

9. 5. 1. अस्ति चास्य भित्तियन्त्रस्योत्तरदिशः किंचित् पूर्वदिग्भागे खमध्यस्थया संसाधितदक्षिणोत्तरकाष्ठायां चूर्णेष्टिकापाषाणैर्निर्मितया हस्तत्रयविस्तारया चतुर्विंशतिकरया प्रस्तरमयध्रुवाभिमुख-तिरश्चीनोपरितनभागया दक्षिणदिशि मूलभागे साङ्घ्रिवेदकरैरुत्तरतोऽग्रभागे चाङ्गुलत्रितयोनपञ्चदशहस्तैः उच्छ्रितयान्तर्गतसोपानया शंकुसंज्ञिकया भित्त्या विशिष्टं पूर्वपश्चिमभागयोः प्रत्येकमेकैकेन वृत्तपरिधिचतुर्थांशाभ्यधिकदैर्घ्येण किंचिदूनकरचतुष्टयविस्तारेण सार्धनवांगुलपिण्डेन शंकुपालिस्थितकेन्द्रेण षड्हस्तपरिमितव्यासार्धेन पाषाणमयचापेन सहितं चान्यदेकं यन्त्रसम्राडाख्यं महद्यन्त्रम् । यत्र प्रतिचापमुभयतः पालौ वृत्तचतुर्थांशाः समैः पञ्चदशभिर्घटीविभागैरङ्किताः । सोऽप्येकैको विभागस्तुल्यैः किंचिदूनांगुलत्रयायामैः षड्विभागैः चिह्नितोऽस्ति । शंकुपालौ च प्रतिचापपालिकेन्द्रस्थानमेकैकं लघुलोहमयवलयं वर्तते ।

[1]Hand (*hasta*) refers to the linear measure 'cubit'.

अस्मिन् च यन्त्रे दिवा चापपालौ शंकुमूलात् तत्पालिच्छायां यावत् यावन्तो घटीविभागास्तावत्यो नतघटिकाः शंकोः पाश्चात्यभागे छायापाते मध्याह्नात् पूर्वं पौरस्ये च ततोऽनन्तरमवगम्याः । एतच्छायापातस्थानस्य समीपतः सम्यगवलोकनाय प्रतिचापमुभयतः प्रास्तरा आरोहणमार्गा निर्मिताः सन्ति । किन्त्वत्रेदानीमुक्तचापयोः स्वाग्रभागे भारातिशयेन वास्तवस्थानादीषन्नमितत्वादत्रत्यच्छायातः कालज्ञानं किंचित् स्थूलं भवति ।

एवमत्रार्कात् पतिता शंकुपालिच्छाया यद्वद् व्यक्तमवलोक्यते न तद्वत् हिमांशोः स्पष्टा भवति । भौमादिग्रहनक्षत्रेभ्यस्तु छायैव नोत्पद्यते इत्यस्माद् यन्त्रात् कथं तेषां नतकालावगमः स्यादित्येतदर्थमुच्यते—

अत्यन्तमृज्वीं धातुमयीं सूक्ष्मां नलिकां निर्माय तस्याः एकमग्रं चापपालावपरं शंकुपालौ तथा न्यसेत् यथा यस्य ग्रहस्य नक्षत्रस्य वा नतकालो ज्ञातव्यः स चापाधःपालिगतया दृष्ट्या नलिकासुषिरे दृश्यते । तथा च नलिकासक्तचापपालिस्थानात् नतकालावगमः स्फुटः स्यात् । शंकुपालौ च यत्र नलिकासंगस्तस्मात् स्थानात् तच्चापपालिकेन्द्रं यावत् तस्य ग्रहस्य नक्षत्रस्य वा क्रान्तेः स्पर्शरेखा स्यात् । एवमस्माद् यन्त्रात् सूर्यादिग्रहाणां च नतकाला अपमाश्च सम्यगवगन्तुं शक्यन्ते ।

नक्षत्रस्य विषुवकालोऽप्यस्मादवगम्यते । स चैवम्—

अस्तं जिगमिषोर्भास्करस्य नतकालं ततो यावता कालेनावगन्तव्यविषुवकालं नक्षत्रमाकाशे सम्यगवलोक्येत तावान् घटीयन्त्रेणावगम्यः । तस्यार्कनतकालस्य च योगस्तात्कालिकोऽहर्पतेर्नतकालो भवति । स च भास्वतस्तात्कलिकेन विषुवत्कालेन युक्ते दशमलग्नस्य विषुवकालो जायते । सोऽपि तस्य नक्षत्रस्य तत्कालेऽवगतेन नतकालेन प्रागपरकपालक्रमेण युतोनितस्तन्नक्षत्रविषुवकालो भवति ।

(Bapudeva, *Mānamandira*, pp. 16-17)

Samrāṭ-yantra

Towards the north of the Bhitti-yantra somewhat in the eastern part, there is another big instrument called *Samrāṭ-yantra*. This has a gnomon wall which is set in the precisely determined north-south direction and is made of lime-brick stones. This is 3 hands in thickness and 24 hands in length. Its upper part, which is, made of stones, is inclined and points towards the polaris. In the south, the base is $4\frac{1}{4}$ hands in height and on the front side in the north, it is 15 hands less 3 *aṅgulas* in height. It has ladders in between. On the eastern and the western sides there are two stony arcs with radii 6 hands and somewhat more than a quadrant each in size. These have breadth somewhat less than 4 hands and their thickness is $9\frac{1}{2}$ *aṅgulas*. Their centres lie on the *śaṅkupālis* (the edges of the gnomon wall). On each arc, there are quadrants which, at the edges, have divisions in 15 equal *ghaṭīs*. Each division is further graduated in uniform six subdivisions which are somewhat less than 3 *aṅgulas* in measure. On the *śaṅkupālis* (on both edges of the gnomon wall) there are two small iron rings, whose centres coincide with the centres of the *cāpapālis* (the edges of the arcs).

In this instrument, during daytime, from the base of the gnomon upto the shadow of the *cāpa-pāli* whatever are the *ghaṭī* divisions, these will be the hour angle (in units of *ghaṭī*-timings or the *nata-ghaṭīs*). If the shadow of the gnomon falls on the western side, then take these (*nataghaṭīs*) to be the time left for midday and if the shadow falls on the eastern side then take these to be the time past midday. In order to observe keenly the shadow from nearby positions stony riding passages are made on both the arcs. But because of excessive weight, the arcs are a little inclined from the actual positions near the terminal edges. So, in these parts, the shadow gives time which is somewhat erroneous.

The shadow of the *śaṅkupāli* from the Moon is not so clear as it is from the Sun. Moreover, the planets and the stars do not produce any shadow at all. That being so, how can the hour angles of the planets Mars etc. and those of stars be determined with the help of this instrument? For this, we give the method as follows:

Construct a fine narrow and exactly straight tube out of metal. Place one of its ends on the edge of the arc and the other end on the *śaṅku-pāli* in such a way that the planet or the star whose hour angle is to be determined is visible through the hole of the tube by placing the eye below the *cāpa-pāli*. In this way the hour angle is evidently determined from the point of contact of the tube on the *cāpa-pāli*. From the point of contact of the tube with the *śaṅku-pāli* upto the centre of the *cāpa-pāli* is the tangent line of the declination of the planet or the star. In this manner, the hour angles and declinations of Sun and other planets can be well determined with the help of this instrument.

Also, the *viṣuva-kāla* (right ascension expressed in units of time) of a *nakṣatra* can be determined with the help of this instrument. The method is as follows:

Determine the hour angle of the Sun at the time of its setting. Using a chronometer, note the time interval by which the star whose *viṣuva-kāla* is to be determined, is visible in the sky. The sum of the two is the hour angle of the Sun at that time (i.e., the time when the star is just visible); on adding to this the *viṣuva-kāla* (right ascension in time units) of the Sun, one gets the *viṣuva-kāla* of the *daśama-lagna* (the point of contact of the ecliptic with the meridian). Determine the hour angle of the star at that time and add to or subtract it from the same (i.e., from the *viṣuva-kāla* of *daśama-lagna*), depending upon whether the star is in the eastern or western hemisphere. The result is the *viṣuva-kāla* of the star.[1] (SDS)

[1] For the rationale, see *Mānamandira*: *SDS*, pp. 29-32.

भित्तियन्त्रम्

9. 6. 1. अस्मिन्नपि यन्त्रे शंकोः पूर्वपार्श्वेऽन्यदेकं भित्तियन्त्रं पूर्वोक्तवदेवं विरचितमास्ते । किन्त्वत्र कीलयोर्विप्रकर्षोऽङ्गुलद्वयोनसप्तकरमितोऽस्ति ।

(Bapudeva, *Māmandira*, p. 17)

Bhitti-yantra

In this very instrument, on the eastern side, there is made another *Bhitti-yantra* of the type described earlier, but here the distance between the two gnomons is seven hands less 2 *aṅgulas*. (SDS)

नाडीमण्डलम्

9. 7. 1. अस्माच्च यन्त्रात् पूर्वस्यां दिशि प्रस्तरमेकं नाडीमण्डलाभिधं यन्त्रं विषुवद्वृत्तक्षेत्रे वर्तते । यस्योत्तरपार्श्वेऽङ्गुलद्वयाधिककरत्रयव्यासमूर्ध्वाधरतिर्यग्रेखाभ्यां विहिततुल्यचतुर्विभागं प्रतिपदं नवत्या भागैरङ्कितं केन्द्रमध्यस्थ-ध्रुवाभिमुखलोहकीलं वृत्तं कृतमस्ति । तस्य च कीलस्य छायया सौम्यगोले स्थितस्य रवेर्नक्षत्रस्य वा नतकालः ज्ञायते । एतद्यन्त्रदक्षिणपार्श्वमपि याम्यगोलस्थस्य भस्य भास्वतो वा नतकालावगमाय त्रयोदशाङ्गुलाधिकैकहस्तव्यासेन पूर्ववदूर्ध्वाधरतिर्यग्रेखाभ्यां भांशैश्चाङ्कितेन मण्डलेन मण्डितं वर्तते ।

(Bapudeva, *Mānamandira*, pp. 17-18)

Nāḍī-maṇḍala : Equinoctial circle

Towards the east of the *yantra* described last, there is, in the plane of the celestial equator, another stone instrument called *Nāḍī-maṇḍala*. On its northern side, there is a circle whose diameter is 3 hands and 2 *aṅgulas*. It is divided into four parts by vertical and horizontal lines. Each quadrant is graduated into 90 degrees. At the centre of the circle there is an iron peg pointing towards the polaris. From the shadow of this rod, when the Sun or the *nakṣatra* is in the northern hemisphere, its hour angle is known. Also the south-facing side of this instrument has a circle with diameter equal to 1 hand and 13 *aṅgulas*. This too, like the north-facing side, is divided into quadrants by vertical and horizontal lines and graduated in degrees. This circle is intended to determine the hour angle if the *nakṣatra* or the Sun is in the southern hemisphere. (SDS)

चक्र-यन्त्रम्

9. 8. 1. अस्ति चास्य यन्त्रस्य निकट एव भित्तिद्वयमध्यवर्ति लोहमयं चलमण्डलं बाह्यपालौ पित्तलपत्रेणावृतं ध्रुवाभिमुखव्यासकं अङ्गुलत्रयविस्तारं अर्धाङ्गुलपिण्डं पार्श्वयोर्भांशैः प्रत्यंशं तुल्यैश्चतुर्भिर्विभागैश्चाङ्कितं केन्द्रस्थकीले प्रोतया अङ्गुलत्रयविस्तारया पित्तलपट्टया विशिष्टं चक्रयन्त्रम् नाम ।

अस्माच्च यन्त्रात् खेटस्य भस्य वा क्रान्तेरवगमाय चक्रं तन्मध्यगतपट्टीं च तथा चालयेद् यथा यस्य क्रान्तिर्ज्ञातुमिष्यते तत्पट्टयधोगतया दृष्टया विलोकितं सत् पट्टीमध्यगतसूत्रे दृश्येत । तथा चास्य ध्रुवाभिमुखव्यासे योऽन्यो लम्बरूपो व्यासः स्यात् तस्य पट्टयाश्च मध्ये यावन्तः परिधावंशास्तावन्तस्तस्य खेटस्य नक्षत्रस्य वा क्रान्त्यंशा वेदितव्याः । अत्रैव यन्त्रे ग्रहनक्षत्राणां नतकालाद्यवगमाय आधारवृत्तादीन्यन्यान्यपि वलयान्यासन् । परमिदानीं तानि नष्टानीति प्रतिभाति । या चात्र पट्टी वर्तते तयेदानीं वक्रत्वंगतया प्रोक्तवत् क्रान्तिज्ञानं न भवति ।

(Bapudeva, *Mānamandira*, p. 18)

Cakra-yantra

Near the very instrument (described last) there is *cakra-yantra*. It lies in between two walls. It is made of iron and is capable of rotating. On the outer periphery, there is a covering with bronze foil. Its diameter points towards the polaris. The periphery is 3 *aṅgulas* broad and ½ *aṅgula* in thickness. On the edges it is graduated in degrees and each degree is further divided into four equal parts. There is a *paṭṭi* (tube device) made of bronze. It is 3 *aṅgulas* broad and passes through a peg at the centre. The same has a thread with a mark (index) in the middle.

To know the declination of a planet or star with the help of this instrument, move the instrument and the *paṭṭi* in such a way that the celestial body, whose declination is to be determined, is visible along the thread to the eye placed below the *paṭṭi*. From the diameter perpendicular to the one facing the pole, the degrees on the periphery up to the *paṭṭi* is the declination of the planet or the star.

In this very instrument there were base circles etc. for determining hour angles of planets and stars, but these have been erased now. The *paṭṭi* is now bent and hence with the help of this instrument described above, the declination cannot be determined accurately. (SDS)

लघुसम्राट्–यन्त्रम्

9. 9. 1. अस्य च प्राग्भागे यन्त्रसम्राडाकारमेवान्यल्लघु यन्त्रं दैर्घ्ये सप्तदशाङ्गुलोत्तरषट्हस्तमितेन विंशत्यंगुलविस्तारेण याम्यभागे सत्र्यंशकराभ्यां सौम्ये च सार्धपञ्चहस्तैरुच्छ्रितेन शङ्कुना विशिष्टमंगुलचतुष्टयाधिकैकहस्तविस्ताराभ्यां पञ्चांगुलपिण्डाभ्यां सत्र्यंशहस्तद्वयविष्कम्भार्धबाह्यपालिभ्यां चापाभ्यां सहितं च वर्तते ।

(Bapudeva, *Mānamandira.*, p. 18)

Small Samrāṭ-yantra

Towards the eastern side of the (*yantra* described above) there is a small *yantra* having its construction similar to the *Samrāṭ-yantra*. This has a *śaṅku*-(wall) 6 hands and 17 *aṅgulas* in length and 20 *aṅgulas* in thickness. On the southern side, the height is 2½ hands and on the northern side, it is 5½ hands high. The arcs are 1 1/6 in breadth and 5 *aṅgulas* in thickness. Their edges (*cāpa-pālis*) have a radii equal to 2 1/3 hands. (SDS)

दिगंश-यन्त्रम्

9. 10. 1. अस्ति चास्मात् यन्त्रात् प्राच्यां दिशि विपुलं दिगंशयन्त्रम् । यन्मध्यगतो वृत्ताकारो दशांगुलोत्तरहस्तद्वयव्यासः, पादोनहस्तत्रयोत्सेधः, स्वकेन्द्रस्थाननिखातसरन्ध्राघःकीलः स्तम्भः स्वस्मात् चतुरं-

गुलोनकरपञ्चकान्तरे वर्तमानया स्तम्भतुल्योच्छ्राययया रूपहस्तमितविस्तारया भित्त्यावृत्तस्तद्भित्तेरपि किंचिदधिकहस्तद्वयान्तरे बहिर्भागे स्थितयोच्छ्राये पूर्वभित्तेर्द्विगुणया, सत्र्यंशैकहस्तविस्तारयापरभित्त्या चावेष्टितोऽस्ति । अनयोश्च भित्त्योः प्रत्येकमुपरितनभागो दिग्भिर्भांशैश्चांकितो वर्तते । तस्य बाह्यभित्तेश्चोपरि चतुर्दिक्षु चत्वारो लोहकीला निखाताः सन्ति ।

एवंविधमिदं महायन्त्रं दिगंशज्ञानार्थमुपयुज्यते । तथाहि—बाह्यभित्तेरुपरि रोपितयोः पूर्वपश्चिमकीलयोः बद्धस्य सूत्रस्य तथाविधयोर्दक्षिणोदक्कीलयोः बद्धेन सूत्रेण सह सम्पातो मध्यस्तम्भकेन्द्रस्योपरितनदेशे एव वर्तते । ततस्तृतीयस्य प्रगुणस्य सूत्रस्यैकमग्रं स्तम्भकेन्द्रे दृढं बध्वान्यदग्रं बाह्यभित्तिपालौ नीत्वा तथा चालयेद्यथा यस्य ग्रहस्य दिगंशा वेद्यास्तस्मिन् मध्यभित्तिबाह्यपालिगतदृष्टया विलोक्यमाने स ग्रहः प्रोक्तसूत्रद्वयसम्पातश्च तृतीयसूत्रगतौ स्याताम् । तथा च बाह्यभित्तिपालौ प्राक्चिह्नात् पश्चिमचिह्नाद् वा तृतीयसूत्रपर्यन्तं यावन्तोंऽशास्तावन्तस्तस्य ग्रहस्य दिगंशा भवन्ति ।

(Bapudeva, *Mānamandira*, pp. 18-19)

Digaṁśa-yantra

On the eastern side of this instrument there is a big *Digaṁśa-yantra* (an instrument for knowing *digaṁśa* or azimuthal angle). At its middle there is a pillar having a diameter equal to two hands and 10 *aṅgulas* and height equal to $2\frac{3}{4}$ hands. At its centre there is fixed a gnomon with a hole at its bottom. From this pillar at a distance of five hands minus four *aṅgulas* there is an enclosing (circular) wall. Its height is equal to that of the pillar, and its thickness is one hand. From this wall too somewhat more than 2 hands away towards the outer side, there is another enclosing wall. Its height is double that of the first one and the breadth is $1\frac{1}{3}$ hands. The upper sides of both the walls are graduated in directions and degrees. On the outer wall there are pivoted iron pegs in the four directions.

The instrument of this type is used to determine the azimuth. (For this purpose) a thread is fastened to the eastern and western pegs and another to similar pegs fixed at the north and the south points. The two threads intersect at the centre of the pillar in the middle. Then a third thread is fastened tight to the centre at the pillar; the other end of this straightened thread is moved in such a way that (i) the planet whose azimuth is to be determined and (ii) the intersection of the two threads are in line with the third thread as seen by placing the eye on the outer edge of the middle wall. The azimuth of the planet is given by the degrees on the edge of the outer wall from the east or west point up to the third thread.

On all these instruments in general the graduations are found erased. Some of the instruments are now tilted or broken at places. (SDS)

अथ वृत्तषष्ठांशसंज्ञं यन्त्रमुच्यते

9. 11. 1. अथ जलसमीकृतायां भूमौ दिक्साधनं कृत्वा याम्योत्तरा भित्तिः पूर्ववत् समकोणसमचतुर्भुजा कार्या । तस्यां दक्षिणतः किञ्चित् प्रदेशं त्यक्त्वा ऊर्ध्वा रेखा कार्या । एवमूर्ध्वपालौ याम्योत्तरा तत्समा रेखा कार्या । तद्योगे केन्द्रं कृत्वा किञ्चिदधिकचतुर्थांशमितं वृत्तं कार्यम् । तद्वृत्तसंलग्नं चतुरङ्गुलपुष्टं चूर्णेष्टकादिना कार्यम् । तत् श्लक्ष्णं कृत्वा तत्रांशाः यथासम्भवं कलाश्चाङ्क्याः । पुनस्तद्भित्तिसमा समानान्तरहस्तद्वयान्तरेण अन्या भित्तिः पूर्वतः कार्या । तत्रापि वृत्तं तथैव विधेयम् । उत्तरतः द्वारं स्थापयित्वा सर्वं भित्त्यन्तरं मुद्रितं कार्यम् । एवं दक्षिणतोऽपि भित्त्यन्तरं मुद्रितं कार्यम् । एवमुपरितनप्रदेशेऽपि पट्टिकादिना मुद्रितः कार्यः । भित्तिद्वयस्य दक्षिणकोणद्वयं ऊर्ध्वाधररेखातः कर्णाकारं छित्वा पट्टिकया पिहितं कार्यम् । छिन्नोभयकोणगतयाम्योत्तररेखायां ऊर्ध्वाधररेखासमीपे सूक्ष्मं सुषिरं कार्यम् । यथा तद्गता रविप्रभा मध्याह्ने वृत्ते पतति तथा कार्यम् ।

अनेन प्रकारेण यन्त्रं कृत्वा तदन्तर्गत्वा कपाटं दत्त्वा मध्याह्ने सुषिरगतां रविप्रभां वृत्ते प्रत्यहं विलोक्य क्रान्त्यक्षयोर्निश्चयः कार्यः । शेषं भित्तियन्त्रवद् ज्ञेयम् । इदं यन्त्रं यथा यथा महद्भवति तथा तथा वेधः भवति । इदमेव यवनैः 'सुद्शफकरी'त्युच्यते ।

(Samrāṭ Jagannātha, *SiSam.*,pp.7-8)

Aperture gnomon

On ground, made level by means of water, erect a square wall as before, in the direction determined as North-South. On the southern side of it, at some point, a vertical line is drawn, and at its upper end, a North-South line equal to it is drawn. Around their intersection, as centre, a circle is constructed with a width somewhat greater than four inches (*aṃśa*). Make then a circle of plaster adhering to it, exactly four inches (*aṅgula*) in width. This is to be polished, and the degrees and minutes marked on it and numbered. Make another wall parallel to the first and separated from it by two cubits (*hasta*). Make a circle on this in just the same way. At the north end another wall is built to cover these two completely, and the south end is also so covered. The upper part is then covered by boards, etc. At the two southern corners of the two walls cut openings at the vertical line, and cover these with plates. Along the North-South line, where each of these corners is cut, make a fine aperture (in the plate) at the vertical line. When this is done the light of the Sun falls at Noon, as required.

In order to use the instrument, a door giving entry is made. When at noon, the light of the Sun enters the aperture, it is visible each day on the circle, and fixes exactly the declination and the latitude. In other respects this is to be understood like the Wall Instument (*bhittiyantra*). The larger the instrument, the more precise it is. By foreigners (Muslims) this is called Suds al-Fakhri.

(Raymond Mercier, *IJHS* 19 (1984) p. 171).

10. यन्त्राणि – INSTRUMENTS

यन्त्राणि तत्साधनानि च

10. 1. 1. गोलो भगणश्चक्रं धनुर्घटी शङ्कुशकटकर्तर्यः ।
पीठकपालशलाका द्वादशयन्त्राणि सह यष्ट्या ।।
(Lalla, *ŚiDhVr.*, 21.53)

Instruments and their accessories

The *Gola*, the *Bhagaṇa*, the *Cakra*, the *Dhanu*, *the Ghaṭī*, the *Śaṅku*, the *Śakaṭa*, the *Kartarī*, the *Pīṭha*, the *Kapāla*, the *Śalākā*, together with the *Yaṣṭi*, are the twelve (astronomical) instruments. (53). (KSS)

10. 1. 2. कर्णश्छाया द्युदलं रविरक्षो लम्बको भ्रमः सलिलम् ।
स्युर्यन्त्रसाधनानि प्रज्ञा च समुद्यमश्चैवम् ।। ५४ ।।
(Lalla, *ŚiDhVr.*, 21-54)

The hypotenuse (*karṇa*), the shadow, the semi-duration of the day, (the longitude of) the Sun, the latitude (of the place), the plumb, the compass (or the revolving machine, *bhrama*), water, as well as intelligence and effort are the accessories required in the use of (astronomical) instruments. (54). (KSS)

10. 1. 3. वृत्तं भ्रमत् त्रिचतुरस्रमुपैति कर्णा-
ल्लम्बाच्च सिद्धिमध ऊर्ध्वमिला समाद्भिः ।। २ ।।
(Lalla, *ŚiDhVr.*, 21. 2b)

A circle is constructed by means of a compass (or a circular plate is made by means of a revolving machine), and a triangle or quadrilateral by means of hypotenuses (*karṇa*); verticality is achieved by means of a plumb, and the ground is levelled by means of water. (2b). (KSS)

स्वयंभ्रमद् गोलयन्त्रम्

10. 2. 1. काष्ठमयं समवृत्तं समन्ततः समगुरुं लघुं गोलम् ।
पारदतैलजलैस्तं भ्रमयेत् स्वधिया च कालसमम् ।।
(Āryabhaṭa I, *ABh.*, 4.22

Automatically rotating sphere

The Sphere (*Gola-yantra*) which is made of wood, perfectly spherical, uniformly dense all round but light (in weight) should be made to rotate, keeping pace with time, with the help of mercury, oil and water, by the application of one's own intellect. (KSS)

10. 2. 2. भूमौ स्तम्भद्वयं दक्षिणोत्तरं निधाय तयोरुपरि अयःशलाकाग्रे स्थापयेत् । गोलदक्षिणोत्तरच्छिद्रे च तैलेन सिञ्चेत्, यथा निस्सङ्गो गोलो भ्रमति । ततो गोलस्यापरतोऽवटं खात्वा तस्मिन् गोलपरिधिसम्मितदैर्घ्यं साधश्छिद्रं जलपूर्णं नलकं निदध्यात् । गोलस्यापरस्वस्तिके कीलं सन्निधाय तस्मिन् सूत्रस्यैकमग्रं बध्वा अधो विषुवन्मण्डलपृष्ठेन प्राङ्मुखं नीत्वा तत उपर्याकृष्य प्रत्यङ्मुखं तेनैव नीत्वा तदग्रबद्धं पारदपूरितमलाबु जलपूर्णे नलके निदध्यात् ।

ततो नलकस्याधश्छिद्रं विवृतं कुर्यात् । तेन जले निस्रवति नलकस्थजलमधो गच्छति । तद्वशात् तत्रस्थमलाबु पारदपूर्त्या गुरुत्वाद् जलं मुञ्चत् गोलं प्रत्यङ्मुखमाकर्षति । एवं त्रिंशद्घटिकाभिरर्धसम्मितं यथा जलं स्रवति गोलस्य चार्धं भ्रमति, तथा स्वबुद्ध्या जलनिस्रावो योज्यः । एवमपराभिस्त्रिंशद्घटिकाभिः नलकस्थं जलं यथा निश्शेषं स्रवति, अलाबु च नलकस्थले भवति, गोलश्च सकलो भ्रमति तथा च स्वधिया कालसमं गोलं भ्रमयेत् ।

(Sūryadeva-yajvan, Com. on *ABh.*, 4.22:
Edn. KVS, pp. 143-44)

Fix two posts on the ground in the North-South direction and fix on them the two ends of an iron rod (which passes through the centre of an armillary sphere). Apply oil to the holes at the north and south poles of the sphere (through which the iron rod passes) so that the sphere might rotate smoothly. Then, on the ground below the west point of the sphere dig a pit and fix in it a (narrow) cylinder with a (closed) hole at its lower end and as high as the circumference of the sphere. Fill the cylinder with water. Then fix a nail at the west point of the sphere and fasten to it one end of a string. Pass the string downwards along the equator towards the east point. Then stretch it upwards and take it to the west point (again). Then attach to the string a dry hollow gourd filled with mercury and place it on the surface of the water in the cylinder kept filled with water.

Now, open the hole at the bottom of the cylinder. As the water flows out, its level falls. Consequently, the gourd, weighed down as it is by mercury, (also goes down) not leaving (the surface of) the water, and (the string attached to it) pulls the sphere westwards. The outflow of water should be regulated in such a manner that in 30 *ghaṭikās* (12 hours), half the water in the cylinder flows out and the sphere rotates one half. Similarly, in the next 30 *ghaṭikās* the entire water flows out, the gourd reaches the bottom of the cylinder and the sphere completes one full rotation. In this manner, the sphere could be rotated intelligently keeping pace with time. (KVS)

10. 2. 3. इष्टं सुवृत्तवलयं लघुशुष्कदारु
निर्मापितं विविधशिल्पविदाप्ततक्ष्णा ।
गोलं समं सलिलतैलवृषाङ्कबीजैः
कालानुसारिणममुं भ्रमयेत् स्वबुद्ध्या ।। १ ।।

त्रिंशत् क्षणं तरति यद्रसतैलकेषु
तत् सार्यते त्रिभिरिदं स्ववहस्य बीजम् ।
(Lalla, *ŚiDhVṛ.*, 21. 1-2a)

The armillary sphere, with radius of one's liking, having perfect circles, made of light and dry timber, by a skilled carpenter who is proficient in the various crafts, should be set to rotate uniformly with time, with the help of water, oil and mercury, by applying one's intellect. (1)

A weight which floats for 30 *kṣaṇas* (=24 hours) in mercury, oil or water, rotates it (i.e., the armillary sphere) with the help of the three (fluids, mercury, oil and water). This is the basic principle of self-rotation.[1] (2a). (KSS)

चक्रयन्त्रम्

10. 3. 1. श्रीपर्णीदारुमयं चक्रं सदरं सुनेमि सन्नाभि ।
अर्धं पारदपूर्णं युक्त्याराणां स्वयं भ्रमति ।। १८ ।।
अक्षोऽस्य ध्रुवयष्टिर्नाभिः श्रोणी भगोलनलिकेव ।
एवंकृतो भगोलः स्वयमेव भ्रमति चक्रवशात् ।। १९ ।।
(Lalla, *ŚiDhVṛ.*, 21. 18-19)

Self-rotating wheel

A 'Wheel', made of the timber of Śrīparṇī (the silk-cotton tree), having a hole in the centre, bearing excellent rim and beautiful nave, and skilfully half-filled with mercury in its (curly) spokes, rotates by itself. (18)

Let the polar axis, which is the same as the axis of rotation of the *Bhagola*, be made the Wheel's axle passing through its nave. This being done, the *Bhagola* rotates automatically due to the rotation of the Wheel. (19). (KSS)

छायायन्त्राणि

10. 4. 1. आर्यभटसिद्धान्तोक्तयन्त्रानुसारेण तत्कृतयन्त्राध्यायश्लोका विलिख्यन्ते । छायायन्त्राणि—

दिङ्मध्यात् सप्तपञ्चाशदङ्गुलैस्त्रिज्यकांशकैः ।
लिखेद् वृत्तं च चक्रांशचिह्नितं सममण्डलम् ।। १ ।।
चराग्रज्याद्युनाडीभिः छायायन्त्राणि साधयेत् ।
समवृत्तविदिक्छायाकर्णाभ्यां क्रान्तिदोर्गुणाः ।। २ ।।
समवृत्ते स्वदिश्यग्रां दद्यात् प्राच्यपराशयोः ।
चरज्यानामथाग्राङ्कान् दिङ्मध्यात् स्वदिशि न्यसेत् ।।३।।
तदग्रबिन्दुतो वृत्तं वृत्तान्ताग्रं लिखेत्तु तत् ।
स्वाहोरात्रदलं तत्र स्पष्टा नाड्यः स्वशंकुभिः ।। ४ ।।
स्वाहोरात्रदलेंऽशाः स्युः षड्गुणा दिननाडिकाः ।
अग्रान्तेऽस्तोदयार्कौ च याम्यार्धे पूर्वपश्चिमे ।। ५ ।।
तत्पूर्वापररेखातो दक्षिणार्धं च तत् स्मृतम् ।
(Āryabhaṭa, *ABh. Siddhānta* Q. By Rāmakṛṣṇa Ārādhya in his com. on *SūSi*)

Shadow instruments

According to the astronomical instruments mentioned in the *Siddhānta* of Āryabhaṭa are indicated (below) the verses (34 in number) from the chapter on the astronomical instruments, written by him: (First) the shadow instruments.

Construct a perfect circle (*samamaṇḍalaṁ vṛttaṁ*) with radius equal to 57 *aṅgulas*, being the number of degrees in a radian, and on (the circumference of) it mark the 360 divisions of degrees. (1)

Then construct shadow-instruments (for every day of the year) with the help of the R sine of the Sun's ascensional difference, the R sine of the Sun's *agrā*, and the *nāḍīs* of the duration of the day (in the following manner). (2)

Determine the R sines of the Sun's declination and of the Sun's longitude from the *samavṛttacchāyākarṇa* (i.e., the hypotenuse of the shadow of the gnomon when the Sun is on the prime vertical) or from the *vidik-chāyākarṇa* (i.e., the hypotenuse of the shadow when the Sun is in a mid-direction).

On the perfect circle (drawn above), lay off the (Sun's) *agrā* in its own direction (north or south) in the east as well as in the west (and at each place put down a point). Again lay off the (Sun's) *agrā* corresponding to the Sun's ascensional difference in its own direction from the centre of the circle. (3)

With that end of the (Sun's) *agrā* (as centre), draw a circle passing through the (two) points marked on the circle: this circle denotes the Sun's diurnal circle. On (the southern half of) that circle, put down marks indicating true *ghaṭīs* with the help of the corresponding positions of the gnomon. (4)

These *ghaṭīs* of the day, multiplied by six, are the degrees on the diurnal circle. At the two points marked at the ends of the Sun's *agrā* in the east and the west, are the positions of the Sun at rising and setting. (5)

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[1] The method may be stated in full as follows:

One should fasten one end of a string to the west cardinal point of the armillary sphere and carry the string round the armillary sphere along the equator (from below). Coming to the west point again, he should tie to it a dry gourd containing mercury and throw it into the cylindrical basin, full of water, kept below the west point, and then open the hole in the bottom of the basin. With the outflow of the water, the gourd goes down and the armillary sphere rotates. In 24 hours the gourd reaches the bottom of the basin and the armillary sphere makes one complete rotation. When necessary, one should apply oil to the ends of the axis of the armillary sphere, so that the axis may rotate smoothly.

The weight of mercury put into the hollow of the dry gourd should be so adjusted that the gourd may float on water and, with its motion downwards, the armillary sphere may rotate.

[1] In each position the gnomon is to be held in such a way that the end of the shadow may lie at the centre of the circle.

Half of the diurnal circle lying towards the south of the rising-setting line (of the Sun) is called the southern half of the diurnal circle. (6a).[1] (KSS)

तोययन्त्राणि

10. 5. 1. स्तम्भं सद्बिलसम्पूर्णं तोयं रन्ध्रे तु योजयेत् ।
तन्मुक्तकालसम्भाज्यः स्तम्भायामोऽङ्गुलात्मकम् ॥१८॥
अङ्गुलानां मितिः स्तम्भे प्रतिनाडीं तु यन्त्रके ।
नाड्याख्याद् भूतलच्छिद्रात् पूर्यादम्बुघटीतलम् ॥ १९ ॥
बीजमेतत् घटीमानं यन्त्रेषु स्तम्भसूत्रयोः ।
यन्त्रे बद्धनरे शिल्पे युद्धे मेषादिकेऽपि च ॥ २० ॥
अन्तःसुषिरमेवं तन्मयूरं वानरं तथा ।
स्थाप्य स्तम्भे तु सम्पूर्णे यन्त्रे षष्ट्यङ्गुलोच्छ्रिते ॥२१॥
सुश्लक्ष्णकीलकं सूक्ष्मं यन्त्राङ्कपरिकल्पितम् ।
षष्ट्यङ्गुलेन सूत्रेण वेष्टयेत् षष्टिवेष्टनैः ॥ २२ ॥
तं प्रक्षिपेन्नरे मूर्ध्नि निर्गच्छन् कर्णरन्ध्रयोः ।
पार्श्वयोर्निक्षिपेत् सूत्रं मयूरे वानरेऽपि वा ॥ २३ ॥
मध्यवेष्टितसूत्राग्रे बध्वाऽलाबुं सपारदाम् ।
नरोपरि जले क्षिप्त्वा गुदच्छिद्रेऽम्बु मोचयेत् ॥ २४ ॥
मयूरे वानरे वेत्थं बध्वाऽलाबु सपारदाम् ।
नाभिरन्ध्रादधः स्तम्भजले क्षिप्त्वाऽम्बु मोचयेत् ॥ २५ ॥
प्रतिनाडीं जलं छिद्रान्निर्गच्छत्येकमङ्गुलम् ।
स्तम्भेऽलाबुर्बिलान्तस्याऽधो याति तथांऽगुलम् ॥ २६ ॥
तद्यन्त्रमध्यकीलस्थवेष्टनं चैकमङ्गुलम् ।
अलाबुकर्षणे सूत्रं अधो याति बिलोन्मुखम् ॥ २७ ॥
तत्कीलाग्रेऽपरं सूत्रं नाडीज्ञानाय लम्बयेत् ।
तद्वेष्टनानि यावन्ति तावन्त्यो घटिका गताः ॥ २८ ॥

(Āryabhaṭa I, *ABh.Siddhānta*, Q by Rāmakṛṣṇa Ārādhya in his com. on *SūSi.*)

Water instruments

Construct a pillar with an excellent (cylindrical) cavity inside. Fill up the cavity with water (and then open the hole at the bottom of the pillar so that water may flow out). By the time (in *ghaṭis*) taken by the water to flow out completely, divide the whole length of the pillar. From this can (be calculated) the measure of an *aṅgula* (which corresponds to a *ghaṭi*). (18)

On the pillar, mark the *aṅgulas* corresponding to each *ghaṭi*. The water corresponding to one *ghaṭī* flowing out from the hole (at the bottom of the pillar) in the level of the ground, completely fills a *ghaṭikā* vessel (in one *ghaṭi*). (19)

This measure of *ghaṭi* is the basis (*bīja*) (for the determination) of (the height of) the pillar and of (the length of) the cord to be used in connection with the (time-)instruments. Having tied around the pillar a man or a pair of fighting rams of craftsmanship (keeping the head of a figure just above the top of the pillar), or having surmounted the pillar by the figures of a peacock or a monkey, bearing a cavity inside it, thus making the whole instrument sixty *aṅgulas* in height, take a smooth fine (cylindrical) needle with its periphery equal to a unit of graduation (i.e. one *aṅgula*) on the instrument, and on it wrap a cord of sixty *aṅgulas* in sixty coils. (20-22)

Place this needle within the head of 'man' passing it through the holes of the ears, or, in the case of peacock or monkey, support the needle (over the holes) on the two sides (of the body). (23)

Having tied a gourd containing (an appropriate quantity of) mercury to the end of the cord wrapped around the needle, place it on the water (inside the pillar, through the hole) at the top of the man, and then let the water flow out through the hole at the anus. (24)

Similarly, in the case of the peacock or the monkey, tying a gourd containing mercury (as before), throw it on the water of the pillar through the navel hole and release the flow of water. (25)

An *aṅgula* of water now flows out in a *nāḍī*, as also the gourd within the pillar goes down by an *aṅgula*. The cord wrapped around the needle, within the instrument, also goes down towards the hole underneath due to the pull of the gourd. (26-27)

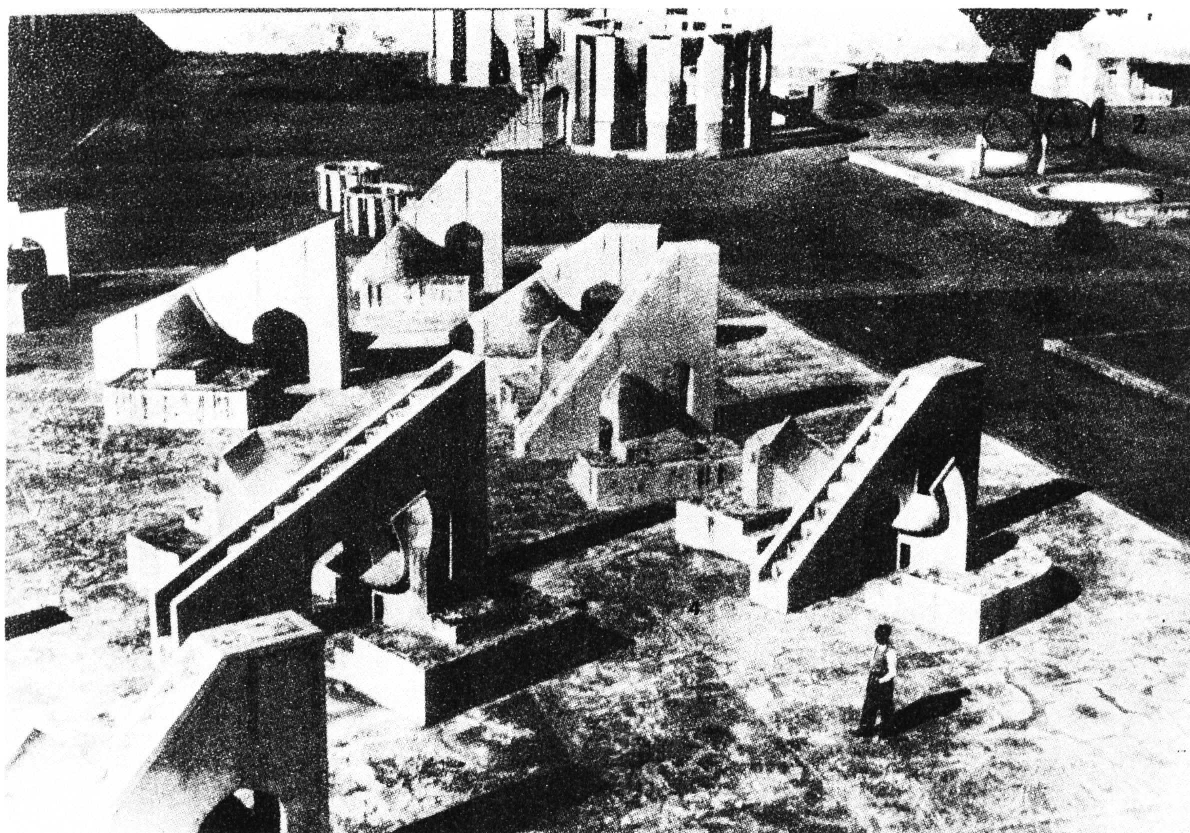
At one extremity of the needle, protruding outside the instrument, suspend another cord to know the *nāḍīs* elapsed. The number of coils made by this cord on the needle will indicate the *nāḍīs* elapsed. (28). (KSS)

10. 5. 2. जलकुण्डेऽधश्छिद्रे घटिकाकालाङ्किते जलस्रुत्या ।
गोले वेष्टनसूत्राग्रबद्धतुम्बं क्षिपेत् सरसम् ॥ १० ॥
स्रवति च यथा यथाम्भस्तथा तथालाबु गच्छमानमधः ।
भ्रमयति गोलकमम्भो मुक्ताङ्का नाडिका याताः ॥११॥

(Lalla, *ŚiDhVṛ.*, 21. 10-11)

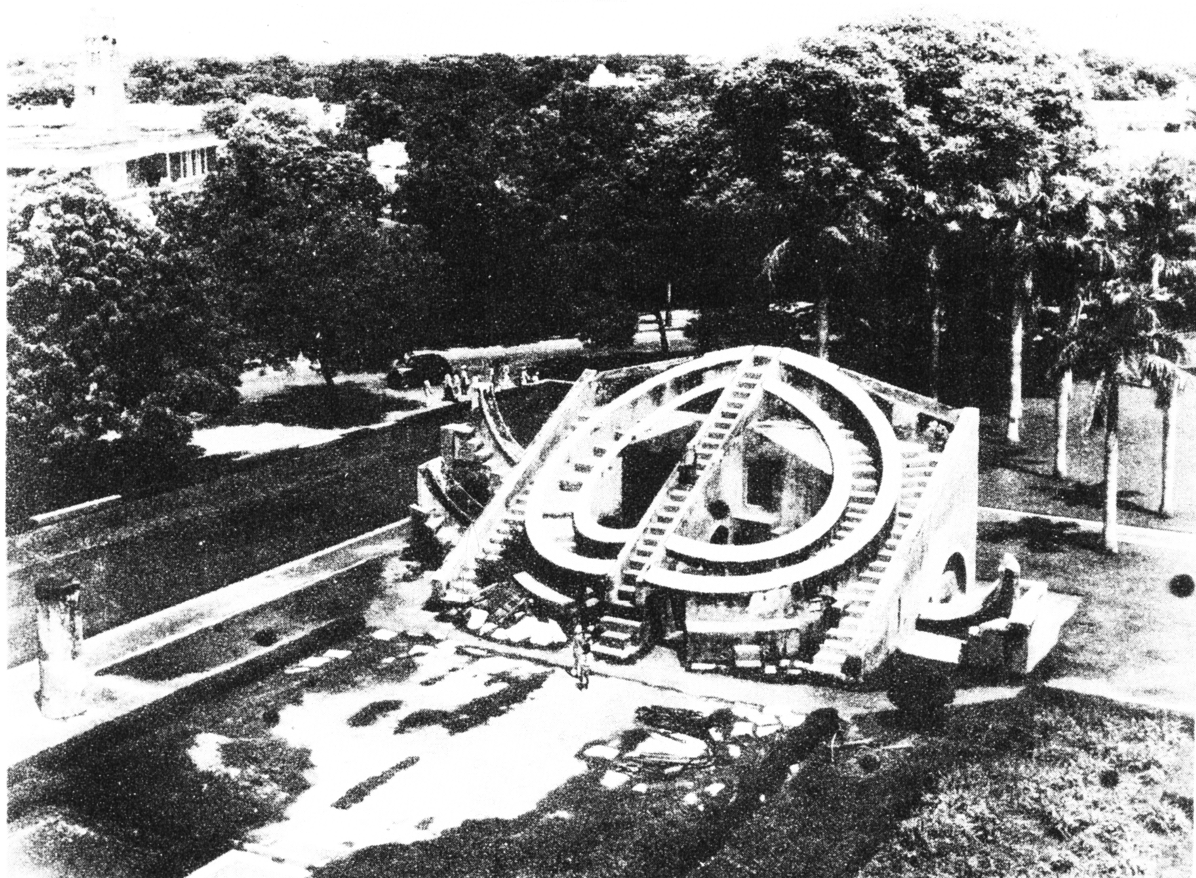
(One should fasten one end of a string to the west cardinal point and carry it round the *Bhagola* along the equator from below). Where the string completes one round of the *Bhagola*, tie to it a (dry hollow) gourd containing mercury and throw it into the (cylindrical) basin of water (kept below the west point), having a hole in

[1] The *Chāyā-yantra* thus constructed, was used for determining the *ghaṭi-s* and degrees elapsed (since sunrise) at any time of the day. For this purpose, one had to find the point where the instantaneous shadow of the gnomon crossed the dirunal circle and to read the graduations between that point and the initial point of the Sun's *agrā*. One *Chāyā-yantra* served the purpose of giving time for one day only. Hence, 365 such instruments were constructed, one for each day of the year. Shadow-instruments of the type described above are unusual, as they are not found to occur in any other work on Indian astronomy.



Maharaja Sawai Jai Singh's Observatory, Jaipur: A view

1. Rāma-yantra
2. Declination instruments
3. Jayaprakāśa-yantra
4. Rāśi-yantra complex



Maharaja Sawai Jai Singh's Observatory, Delhi: Miśra-yantra

प्रणिपत्यैकमनेकं सत्यां देवतां परं ब्रह्म ॥ आर्यभटस्त्रीणि गदति
गणितं कालक्रियां तथा गोलं ॥१॥ वर्गाक्षराणि वर्गे ऽवर्गे
ऽवर्गाक्षराणि कात् ङ्मौ यः ॥ खद्विनवके स्वरा नव वर्गेऽवर्गे
नवान्त्यवर्गे वा ॥२॥ युगरविभगणाः ख्युघृ ४३२००००
शशि चयगियिङुशुछ्लृ ५७७५३३३६ कु ङिशिबुण्लृ
१५८२२३७५०० प्राक् ॥ शनि ढुङ्विघ्व १४६५६४ गुरु
खिच्युभ ३६४२२४ कुज भद्लिझ्नुखृ २२९६८२४ भृगु
बुध सौराः ॥३॥ चन्द्रोच्च र्जु ध ४८८२१९ बुध सुगु
शिन १७९३७०२० भृगु जषबिखुछृ ७०२२३८८ शे
षार्काः ॥ बुफिनच २३२२२६ पात विलोमा बुधाह्न्यजार्को
दयाच्च लङ्कायां ॥४॥ काहो मनवो ढ १४ मनुयुगस्ख ७२
गतास्ते च ६ मनुयुगछ्ना २७ च ॥ कल्पादेर्युगपादा ग ३
च गुरुदिवसाच्च भारतात्पूर्वं ॥५॥ शशिराशयष्ठ १२
चक्रं तेंशकला योजनानि य ३० व ६० ञ १० गुणाः ॥ प्राणे
नैति कलां भं खयुगांशे ग्रहजवो भवांशेऽर्कः ॥६॥ नृषि
८० योजनं ञिला १०५० भूव्यासोऽर्कन्दोर्घ्रिञ ४४१० गि
ण ३१५ कं १ मेगोः ॥ भृगु गुरु बुध शनि भौमाः शशि ङ ५
ञ १० ण १५ न २० मां २५ शाकाः समार्कसमा ४३२०००
॥७॥ भा २४ परमा ग्रहांशाः शशिविक्षेपोपमण्डलांशार्द्धं ४॥
शनि गुरु कुज ख २ क १ गार्द्धं भृगु बुध ख २ स्वा
ंशा य ४ हस्ताना ॥८॥ बुध भृगु कुज गुरु शनि न २० व
र ४० पा ८० ह १०० गत्वांशकान्प्रथमपाताः ॥ सवितुरमी
षां च तथा द्वा ७८ ञखि २१० सा ९० ह्रा ८०० ल्ध
खिच्य २७० मन्दोच्चं ॥९॥ झार्द्धानि मन्दवृत्तं शशि न छ
७ ग ३ घ ४ ट १४ छ ७ झ ९ यथोक्ताभ्यः ॥ झ ९ ग ल
५३ झ ५१ दु ३१ तथा शनिगुरुकुजभृगुबुधोच्चशीघ्रे
भ्यः ॥१०॥ मन्दात् ङ ५ ख २ द १८ ज ८ दा २८ वक्रिणा

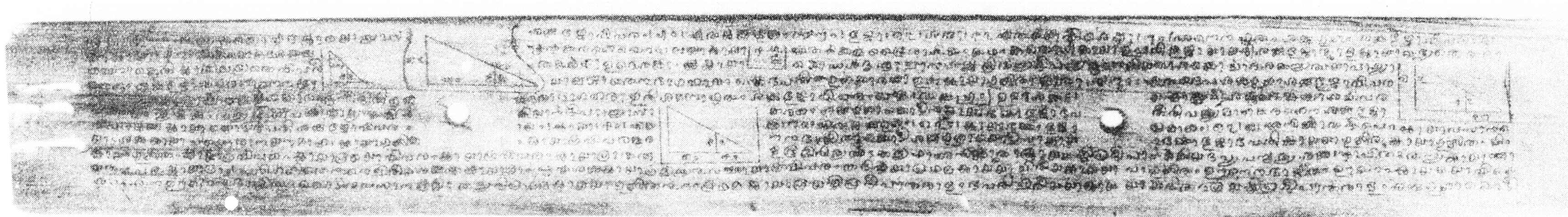
Āryabhaṭīya of Āryabhaṭa I
Facsimile of page 1 from Bombay University ms.

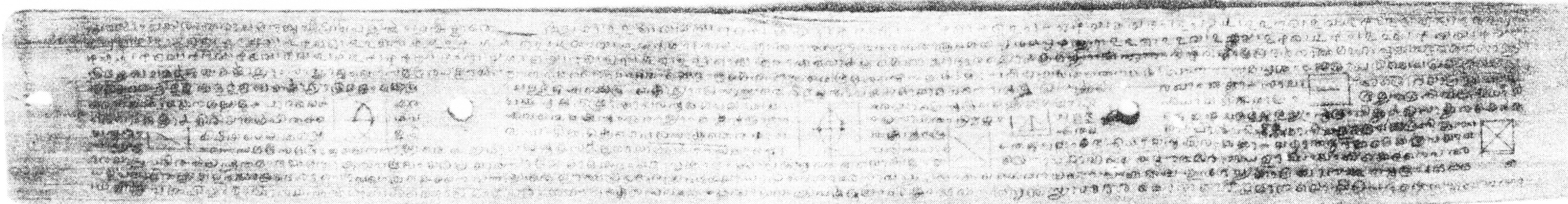
Sūryasiddhānta

Facsimile of page 1 from Bombay University ms.

Siddhāntaśiromani of Bhāskara II

Facsimile of a page from Bombay University ms.





Aryabhatīya-bhāṣya of Govindasvāmin
Facsimile of two pages from palmleaf ms. with K.V. Sarma



Laghumānasa of Muñjāla with the commentary of Sūryadevayajvan
Facsimile of palmleaf ms. with K.V. Sarma

the bottom and graduated with the marks of *ghaṭīs* on the basis of the outflow of water. (10)

As the water flows out (through the hole in the bottom of the basin), the gourd, going downwards, rotates the *Bhagola*; and the *nāḍī* marks left behind by the water denote the *ghaṭīs* elapsed. (11). (KSS)

नरयन्त्रम्

10. 5. 3. घटिकाङ्गुलसङ्ख्यां बुध्वा चीर्यां निवेशयेद् घटिकाः ।
तदनेन ता निरुद्धयादुदरेण नतवदनमनुजस्य ॥ १२ ॥
चीर्येत बद्धसूत्रे तिर्यक्स्थितवदनकीलकनलेन ।
नीत्वा जठरच्छिद्रेण केनचिद् तद् बहिः कुर्यात् ॥ १३ ॥
तत्र निबद्धमलाबु प्राग्वत् सलिलेन नीयमानमधः ।
चीरीमाकृष्यान्यां जपत्यमुं नाडिकां गुटिकाम् ॥ १४ ॥

Nara-yantra

Knowing the number of *aṅgulas* corresponding to one *ghaṭī* (from the fall in water level as a result of the outflow of water in one *ghaṭī*) insert *ghaṭikā* beads at that distance in a strip of cloth and keep them in position by applying wax (or any other adhesive) and throw it inside the body of a human figure with face bent down. (12)

To one end of the cloth-strip tie a string and carry it round a rod fitted horizontally within the mouth of the man, and take it out through a certain hole in the belly of the man. (13)

Tie to it a dry gourd (containing mercury) (and throw it into the basin of water kept underneath, as before). As the gourd goes down, the man, pulling the cloth-strip, releases a new *nāḍī* bead after the lapse of every *ghaṭī*. (14). (KSS)

कूर्ममेषमयूरादियन्त्राणि

10. 5 4. कूर्मादयश्चैवं घटिकां जह्नुर्यथेष्टकालेन ।
मेषादीनां युद्धं सूत्रे सक्ते भवेदुभयोः ॥ १५ ॥
परिकल्पितकालाध्वनि युक्त्या योगो भवेद्वधूवरयोः ।
घटिकाङ्गुलाङ्कितं वा ग्रसति मयूरः क्रमादुरगम् ॥१६॥
हन्ति मनुष्यः पटहं छादयति छादकस्तथा छाद्यम् ।
एवंविधानि यन्त्राण्येवमनेकानि सिद्धयन्ति ॥ १७ ॥
(Lalla, *ŚiDhVṛ*., 21. 12-17)

Kūrma-Meṣa-Mayūra-yantra

The tortoise etc. also, in the same manner, can be made to release the *ghaṭikā*-(beads) at the desired intervals of time. Similarly, there occurs a fight between two rams (after every *ghaṭī*), when they are connected by the string (carrying beads). (15)

By the proper coordination of time and distance, there occurs a union of husband and wife (after every *ghaṭī*), a peacock gradually devours a 'serpent' marked with the divisions of *aṅgulas* denoting *ghaṭīs*, a 'man' beats a drum (after every *ghaṭī*), and an 'eclipser' eclipses the body to be eclipsed (after the lapse of every *ghaṭī*). Several other (time-measuring) instruments like these may be (designed and) constructed. (16-17). (KSS)

कपालयन्त्रम्

10. 5. 5. स्वेष्टं वाऽन्यदहोरात्रे षष्ट्याऽम्भसि निमज्जति ।
ताम्रपात्रमधश्छिद्रमम्बुयन्त्रं कपालकम् ॥ ३१ ॥
(Āryabhaṭa I, *ABh.Siddhānta*, Q by Rāmakṛṣṇa Ārādhya in his com. on *SūSi*.)

Kapāla-yantra

Any copper vessel made according to one's liking with a hole in the bottom, which sinks into water 60 times in a day and night, is the water instrument called *Kapāla*. (31). (KSS)

चक्रयन्त्रम्

10. 6. 1. वृत्तं कृत्वा फलकं षड्वर्गाङ्कं तथा च षष्टयङ्कम् ।
मध्यस्थितावलम्बं मध्यस्थित्या प्रविष्टोष्णम् ॥ २० ॥
तदधोलम्बविमुक्तं गृहादि यत्तदुदितं दिनकरांशात् ।
नाड्यः पूर्वकपाले द्युगतास्ताः पश्चिमे द्युदलात् ॥ २१ ॥
(Lalla, *ŚiDhVṛ*., 21. 20-21)

Cakra-yantra (Circle instrument)

Take a circular board, graduate it with the 360 divisions (of degrees) as well as the 60 divisions (of *ghaṭīs*), fix a vertical needle at its centre and set it up in such a way in that the Sun may lie centrally in its plane. (20)

Then the Signs etc. left behind (since sunrise) by the shadow of the vertical needle denote the Signs etc. by which the Sun has ascended (above the horizon). The *nāḍīs* left behind denote the *nāḍīs* elapsed since sunrise, provided the Sun is in the eastern hemisphere. When the Sun is in the western hemisphere, the *nāḍīs* thus obtained should be subtracted from half the duration of the day (to get the *nāḍīs* to elapse before sunset).[1] (21). (KSS)

[1] This rule is true for places on the equator only. At other places, it will give the Sun's altitude and the *ghaṭīs* corresponding to it. There are reasons to believe that observation was made at a place away from the equator, since sunrise was deduced from the Sun's altitude by applying the following formula:

$$\textit{nāḍīs}\text{ elapsed} = \frac{\text{semi-duration of day} \times \text{Sun's altitude}}{\text{Sun's meridian altitude}}$$

Brahmagupta has criticised the astronomers who used this formula. See *Br SpSi*, 22.11.

धनुर्यन्त्रम्

10. 7. 1. चक्राख्यं यन्त्रमिदं दलं धनुर्यन्त्रमाहुरस्यैव ।
ज्याकार्मुकभृच्छिद्रप्रविष्टदिनकरकरं धार्यम् ।। २२ ।।
मध्यस्थलम्बमुक्ताः कोटेरारभ्य नाडिका द्युगताः ।
उदिताश्च दिनकरांशादारभ्य भवन्ति गृहभागाः ।।२३।।
(Lalla, *SiDhVṛ.*, 21. 22-23)

Dhanur-yantra (Semi-circle instrument)

What has been described above is the *Cakra-yantra*, half of which is called *Dhanuryantra* ('Semi-circle'). This latter instrument should be held (with its chord horizontal) in such a way that a ray of the Sun passing through the hole in the middle of the chord may fall on the arc. (22)

Then the *nāḍīs* left behind by the shadow of the central needle, as measured from the arc-end, denote the *nāḍīs* elapsed during the day (since sunrise), and the signs and degrees left behind denote the signs and degrees by which the Sun has (ascended above the horizon). (23). (KSS)

10. 7. 2. वृत्तव्यासो धनुर्ज्या स्याद् व्यासार्धं धनुषः शरः ।। ६ ।।
शङ्कुच्छाया धनुर्ज्यायां दिङ्मध्यात्त्विष्टभा सदा ।
प्रागग्रं धनुषो वृत्ते भ्रामयेदर्कदिङ्मुखम् ।। ७ ।।
चापाग्रोदयमध्यांशाः षड्भिर्भाज्या दिने गताः ।
(Āryabhaṭa I, *ABh.Siddhānta*, Q by Rāmakṛṣṇa in his com. on *SūSi.*)

The chord of the *Dhanuryantra* is equal to the diameter of the circle (i.e., the perfect circle), and its arrow is equal to the radius. It is mounted on the circle vertically with the two ends of its arc coinciding with the east and west points. (6 b)

The eastern end of the *Dhanuryantra* should be moved along (the circumference of) the circle until the *dhanuryantra* is towards the Sun.[1] The shadow of the gnomon will then fall along the chord of the *Dhanuryantra*, and (the shadow-end being at the centre of the circle) the distance of the gnomon, as measured from the centre of the circle, will always be equal to the shadow at the desired time.[2] (7)

The degrees intervening between the (eastern) end of the *Dhanuryantra* and the rising point of the Sun divided by six, give the *ghaṭīs* elapsed in the day. (8a). (KSS)

[1] The *Dhanuryantra* resembles a semi-circular arc. It is held vertically with the two ends of its arc coinciding with the east and west points of the circle drawn on the ground.

[2] The gnomon is represented in this case by means of a cord suspended from the Sun's position on the arc of the *Dhanuryantra*.

कर्तरीयन्त्रम्

10. 8. 1. समपूर्वापरमेतत् स्थिरं भवति कर्तरीयन्त्रम् ।
ज्यामध्यस्थिततिर्यक्कीलच्छायोज्झिता घटिकाः ।।२४।।
(Lalla, *ŚiDhVṛ.*, 21.24)

Kartarī-yantra (Scissors instrument)

When the (*Dhanur-yantra*) is permanently fixed in the plane of the equator (forming the lower half of the equator) (and a needle pointing towards the north pole is fixed at the middle of the chord), it is known as *Kartarī-yantra*. In this case, the *ghaṭīs* left behind by the shadow of the needle fixed at the middle of the chord denote the *ghaṭīs* elapsed since sunrise. (24). (KSS)

कपालयन्त्रं पीठयन्त्रं च

10. 9. 1. इदमेवोर्ध्वशलाकं भुवि स्थितं स्यात् कपालकम् यन्त्रम् ।
चक्रे चोर्ध्वशलाकं वदन्ति पीठं सुसिद्धाशम् ।। २५ ।।
अनयोः कीलच्छाया मुक्ता घटिका भवन्ति वारुण्याः ।
मीनार्कोदयवेधादग्रश्चापांशकाश्चापि ।। २६ ।।
(Lalla, *SiDhVṛ.*, 21.25-26)

Kapāla-yantra and Pīṭha-yantra

This very *Kartarī-yantra*, with its dial set horizontally on the ground, and its needle vertical, is called *Kapāla-yantra*; and the *Cakra-yantra*, with its dial having the directions marked in its rim, and set on the ground with its axis vertical, is called *Pīṭha-yantra*. (25)

In the case of these two instruments, the *ghaṭīs* left behind up to the west point by the shadow of the needle denote the *ghaṭīs* elapsed. Moreover, in the case of the *Pīṭha-yantra*, the degrees on the rim from the point where the Sun is observed at sunrise (up to the east point) give (the degrees of) the Sun's *agrā*. (26). (KSS)

भगणयन्त्रम्

10. 10. 1. अथवा चक्रे स्वविषयराश्युदयविनाडिकादशांशभवैः ।
कालांशैर्दीर्घलघून् राशींश्च लिखेत् स्वभगणाङ्कान् ।।
क्षितिजवलयस्य मध्ये लम्बं सुसमीकृतस्य स्वस्थस्य ।
ध्रुवयष्टिस्थं कृत्वा समपूर्वं कारयेत् तदतः ।। २८ ।।
यस्मिन् राशावर्कस्तस्य यथा सप्तमो भवत्युदये ।
ध्रुवयष्टिभा विमुक्ता विद्धचुदितांशान् भगणयन्त्रे ।।
(Lalla, *SiDhVṛ.*, 21. 27-29)

Bhagaṇa-yantra

On the rim of the circular plate of the *Cakra-yantra*, mark the Sign segments, large or small, as the case may be, depending on the time-degrees obtained by dividing the *vināḍīs* of the oblique ascensions of the Signs by 10, and also graduate each Sign segment with the divisions of degrees. (27)

Keep it horizontally on well levelled ground in open space and fix a vertical needle at its centre. Then hold the *yantra* in the equatorial plane with its needle pointing to the north pole in such a way that at sunrise the Sun is in the seventh Sign (just 6 Signs ahead of its actual position) (so that the shadow of the needle is directed towards the Sun's position at sunrise). Then the degrees left behind by the shadow of the needle on the *Bhagaṇa-yantra* will denote the degrees traversed by the Sun since sunrise. (28-29). (KSS)

घटिकायन्त्रम्

10. 11. 1. वृत्तं ताम्रमयं पात्रं कारयेद्दशभिः पलैः ।
षडङ्गुलं तदुत्सेधो विस्तारो द्वादशानने ।। २९ ।।
तस्याधः कारयेच्छिद्रं पलेनाष्टाङ्गुलेन तु ।
इत्येतद् घटिकासंज्ञं पलषष्ट्यम्बुपूरणात् ।। ३० ।।

(Āryabhaṭa I, *ABh.Siddhāntu*, Q by Rāmakṛṣṇa Ārādhya in his com. on *SūSi*)

Clepsydra

One should get a hemispherical bowl made of copper, 10 *palas* in weight, six *aṅgulas* in height, and twelve *aṅgulas* in diameter at the top. At the bottom thereof, let a hole be made by a needle eight *aṅgulas* in length and one *pala* in weight.

This is the *ghaṭikā-yantra*, (so named) because it is filled by water in a period of 60 *palas* (i.e., one *ghaṭī*). (29-30). (KSS)

10. 11. 2. दशभिः शुल्बस्य पलैः पात्रं कलशार्धसन्निभं घटितम् ।
हस्तार्धमुखव्यासं समघटवृत्तं दलोच्छ्रायम् ।। ३४ ।।
सत्र्यंशमाषकत्रयकृतनलया सुसमवृत्तया हेम्नः ।
चतुरङ्गुलया विद्धं मज्जति विमले जले नाड्या ।।३५।।
अथवा स्वेच्छाघटितं घटीप्रमाभिः प्रसाधितं भूयः ।
त्रैराशिकसिद्धं वाङ्गुलवद् गुरु विपुलरन्ध्रं यत् ।।३६।।
इष्टदिनार्धघटीभिः सममथवापं निमज्जति घटी सा ।
षष्टैः शतैस्त्रिभिर्वा विंशतिलघ्वक्षरासूनाम् ।। ३७ ।।

(Lalla, *SiDhVṛ.*, 21. 34-37)

The *Ghaṭikā* vessel, looking like one-half of a (spherical waterpot called) *kalaśa*, made of ten *palas* of copper, half a hand in diameter at the top and half as high, and having a hole bored (at the bottom) by a uniformly circular needle of 4 *aṅgulas* in length made of three and one third *māṣas* of gold, sinks into limpid water exactly in one *nāḍī*. (34-35)

Or, it is a vessel made according to one's liking (with a hole in the bottom) later adjusted by the measure of a *ghaṭī*.

Or, it is a vessel having a finger-wide hole (in the bottom) and of size determined by proportion in such a way that it may sink (in water) as many times in a day as there are *ghaṭīs* in a day.

A *ghaṭī* is also equal to 360 *asus*, each *asu* (being equal to the time of pronunciation) of 20 short syllables (or 10 long syllables). (36-37). (KSS)

घटिकायन्त्राङ्कनम्

10. 11. 3. यन्त्रे दिनगतनाड्यो दिनमानगुणाः प्रमाणघटिकाभिः ।
यन्त्रभवाभिर्भक्ताः स्फुटा भवन्त्यन्यथा स्थूलाः ।।३०।।

(Lalla, *SiDhVṛ.*, 21.30)

Graduation of a clepsydra

The *ghaṭīs* elapsed during the day (since sunrise) as indicated by the instrument (used), when multiplied by the actual measure of the day and divided by the number of *ghaṭīs* in a day as adopted in the graduation of the instrument, give the true *ghaṭīs* elapsed since sunrise. Otherwise, they are gross.[1] (30). (KSS)

शङ्कुः दिग्ज्ञानं च

10. 12. 1. भ्रमसिद्धः सममूलाग्रपरिधिरतिसुगुरुसारदारुमयः ।
ऋजुरव्रणराजिलाञ्छनस्तथा च समतलः शङ्कुः ।।
वृत्तः षडङ्गुलानि द्वादशदीर्घश्चतुर्भिरवलम्बैः ।
स्थाप्यः सुसमः प्रथमे जलेन च सुसमीकृते फलके ।।
छायास्या बहिः परिधेर्ग्राह्या गोपुच्छसंस्थिते केन्द्रात् ।
छायाग्राच्छङ्क्वग्रप्रापी कर्णो भवेत् तिर्यक् ।। ३३ ।।

(Lalla, *SiDhVṛ.*, 21. 31-33)

Gnomon and orientation

The gnomon, constructed with the help of the revolving machine, having equal periphery at the top and bottom, made of very heavy and strong timber, perfectly straight and free from scars, streaks and spots, uniformly circular, six *aṅgulas* in circumference and twelve *aṅgulas* in height, should be set up on a (horizontal) board, (already) levelled by means of water, in the vertical direction with the help of four plumbs. (31-32)

The shadow of this (cylindrical gnomon), resembling the tail of a cow, should be observed outside the circular base and should be measured from (the circular base at) the centre. From the tip of the shadow up to the (upper) end of the gnomon lies obliquely the hypotenuse (of the shadow). (33). (KSS)

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[1] For graduating the *ghaṭī* divisions in the instrument, it is generally assumed that there are 60 *ghaṭīs* in a day. Actually, it is not exactly the case. Hence the above rule. Śrīpati, too, gives this rule. See *Si Śe.*, 19. 17.

10. 12. 2. शिलातलेऽम्बुसंसिद्धे वज्रलेपेऽपि वा समे ।
तत्र शङ्क्वङ्गुलैरिष्टः समं मण्डलमालिखेत् ॥ १ ॥
तन्मध्ये स्थापयेच्छङ्कुं कल्पितद्वादशाङ्गुलम् ।
तच्छायाग्रं स्पृशेद्यत्र वृत्तं पूर्वापराह्णयोः ॥ २ ॥
तत्र बिन्दू विधातव्यौ वृत्ते पूर्वापराभिधौ ।
तन्मध्ये तिमिना रेखा कर्तव्या दक्षिणोत्तरा ॥ ३ ॥
याम्योत्तरदिशोर्मध्ये तिमिना पूर्वपश्चिमे ।
दिङ्मध्यमत्स्यैः संसाध्या विदिशस्तद्वदेव हि ॥ ४ ॥
चतुरश्रं बहिः कुर्यात्सूत्रैर्मध्याद्विनिर्गतैः ।
भुजसूत्राङ्गुलैस्तत्र दत्तैरिष्टप्रभाग्रतः ॥ ५ ॥
प्राक्पश्चिमाश्रिता रेखा प्रोच्यते सममण्डलम् ।
उन्मण्डलं च विषुवन्मण्डलं परिकीर्त्यते ॥ ६ ॥
(*SūSi.*, 3. 1-6)

On a stony surface, made water-level, or upon hard plaster, made level, there draw an even circle, of a radius equal to any required number of the digits (*aṅgula*) of the gnomon (*śaṅku*). (1)

At its centre, set up the gnomon of twelve digits of the measure fixed upon; and where the extremity of its shadow touches the circle in the former and later parts of the day, (2)

There fixing two points upon the circle, and calling them the forenoon and afternoon points, draw midway between them, by means of a fish-figure (*timi*), a north and south line. (3)

Midway between the north and south directions draw, by a fish figure, an east and west line: and in like manner also, by fish-figures (*matsya*) between the four cardinal directions, draw the intermediate directions. (4)

Draw a circumscribing square, by means of the lines going out from the centre; by the digits of its base-line (*bhujasūtra*) projected upon that is any given shadow reckoned. (5)

The east and west line is called the prime vertical (*samamaṇḍala*); it is likewise denominated as the east and west hour circle (*unmaṇḍala*) and equinoctial circle (*viṣuvanmaṇḍala*).[1] (6) (Burgess)

दिग्ज्ञानम्

10. 12. 3. यन्त्रेणावनतादिना च निपुणो यद्वाम्बुसम्पूरणे-
नोर्वीं चारु समीकरोत्वथ दृढं शङ्कुं कराध्र्यायतम् ।
मूले द्व्यङ्गुलविस्तृतं क्रमवशादग्रे तदर्धोन्मित-
व्यासं वृत्ततरं सरोजमुकुलाग्राकारमाकल्पयेत् ॥१॥
शङ्कुदीर्घयुगसम्मितसूत्रेणाकलय्य परिवृत्य सुवृत्तम् ।
वृत्तमध्यमवधार्य सुसूक्ष्मं शङ्कुमत्र सुदृढं निवेशयेत् ॥२॥
शङ्कुच्छायाग्रभागे त्ववहितहृदयो वृत्तलग्नेऽङ्कयित्वा
प्राह्णान्ते पश्चिमस्यां दिशि तदितरदिश्येवमेवापराह्णे ।
पाश्चात्येऽन्येद्युरप्यङ्कनमपि च विधायाङ्कयोरेतयोर-
प्यन्तर्भागत्रिभागं नयतु गतदिनाङ्कं तदेवेह सूक्ष्मम् ॥
पूर्वापरेद्युप्रभवाङ्कयुग्ममेवं सुसूक्ष्मं परिकल्पितं यत् ।
तदङ्कयुग्माहितसूत्रमेव पूर्वापराशाप्रभवं सुसूक्ष्मम् ॥४॥
(Nīlakaṇṭha, *Manuṣyālayacandrikā*, 2. 1-4)

The Cardinal directions

The expert (astronomer) should first level the ground perfectly by means of levelling instruments or water. He should then pick up a gnomon, cylindrical, made of hard (wood), 12 digits in length, two digits in diameter at the bottom, tapering to a diameter of one inch towards the top, the tip, however, being trimmed to a point like a rose bud. (1)

He should, then, draw (on the levelled ground) a perfect circle with a string of twice the length of the gnomon. The centre of the circle should be correctly identified and the gnomon planted there, firmly. (2)

With a concentrated mind, he should, in the forenoon, mark the shadow of the tip of the shadow of the gnomon as it reaches the circumference of the circle (in the west). He should mark the circle, in the same manner, (in the east), in the afternoon. Another mark should be made (in the west) (in the same manner) next day forenoon also. The first day's mark should be brought towards the second day's mark by a third of the distance between the two marks; then that will be the correct point (in the west). (3)

When markings have been done correctly as above for the previous and current days, the line joining the two points would be the exact East-West line. (The North-South lines should be drawn by constructing a fish-figure from the east-west points on the circumference). (4). (KVS)

दिक्ज्ञानविधयः

10. 12. 4. समभुवि वृत्ते शङ्कोर्मध्यस्थस्थं प्रभा क्रमाद्यत्र ।
प्रविशत्यपैति ककुभौ क्रान्तिवशात् स्तोऽपरैन्द्रयाख्ये ॥२॥
तुल्यप्रभाग्रयोर्वा पूर्वापरयोः कपालयोर्बिन्दू ।
कार्याविपक्रमवशादपरैन्द्रयाख्ये दिशौ भवतः ॥ ३ ॥
वृत्तं रवौ प्रविष्टे सममण्डलसंज्ञितं प्रभा या स्यात् ।
समपूर्वापरगा सा सौम्या यत्र ध्रुवः सा स्यात् ॥ ४ ॥
इष्टाभाभुजकोटीरचितत्रिभुजस्य वा श्रवणतुल्या ।
यत्रेष्टाभा यावत् तावत्पूर्वापरा कोटिः ॥ ५ ॥

[1] For elucidation see, *Sū.Si*: *Burgess*, pp. 108-11.

यत्रास्तमेति कश्चिद् द्युचरः क्रान्त्या विनोदयं याति ।
वरुणामरपत्योर्दिशौ पतेते क्रमादथवा ।। ६ ।।
उदयति पौष्णं यत्र श्रवणो वा सा दिगिन्द्रस्य ।
स्थूलाऽथवा प्रदिष्टा चित्रास्वात्यन्तरं विबुधैः ।। ७ ।।
छायात्रयाग्रजमीनद्वयमध्यगसूत्रयोर्युतिर्यत्र ।
याम्या सोत्तरगोले सौम्या याम्ये ककुब् नृतलात् ।। ८ ।।
छायात्रितयाग्रस्पृक्सूत्रयुतेर्वृत्तमालिखेत्तस्य ।
लेखां न जहात्याभा वनितेव कुलस्थितिं कुलोत्पन्ना ।। ९ ।।
याम्योत्तरलेखायां द्युदलाभा वृत्तशंकुविवरं यत् ।
याम्यमुदक्चाक्षाभाऽजतुलादिगते च दिनद्युपतौ ।। १० ।।
शङ्कुप्रमाणवर्गाच्छायावर्गान्वितात्पदं कर्णः ।
कर्णकृतेर्वाऽर्ककृतिं विशोध्य मूलं प्रभा भवति ।। ११ ।।

(Vaṭeśvara, *VSi.*, 3. 1. 2-11)

Methods for cardinal directions

Method 1

(The points) where the shadow of the (vertical) gnomon, set up at the centre of a circle drawn on level ground, enters into (the circle in the forenoon) and passes out (of the circle in the afternoon), give (respectively) the west and east directions (with respect to each other), when due allowance is made for the variation of the Sun's declination. (2)

Method 2

Or, put down points at the extremities of two equal shadows, one (in the forenoon) when the Sun is in the eastern half of the celestial sphere and the other (in the afternoon) when the Sun is in the western half of the celestial sphere. These, too, give (respectively) the west and east directions, provided due allowance is made for the change in the (Sun's) declination. (3)

Method 3

When the Sun enters the circle called the prime vertical, the shadow (of a vertical gnomon) falls exactly east to west. Towards the north pole lies the north direction. (4)

Method 4

As long as the shadow (of a vertical gnomon), for the desired time, is equal (in magnitude and direction) to the hypotenuse of the right-angled (shadow) triangle formed by that shadow and the *bhuja* (base) and *koṭi* (upright) for that shadow, so long is the *koṭi* (upright) directed east to west. (5)

Method 5

(The points of the horizon), where any heavenly body, with zero declination, rises and sets, are (respectively) the east and west directions (relative to the observer). (6)

Method 6: *Ancient method*

(The point of the horizon) where the star Revatī (Zeta Piscium) or Śravaṇa (Altair or Alpha Aquilae) rises is the east direction. Or, as stated by the learned, it is roughly that point (of the horizon) which lies midway between the points of rising of Citra (Spica) and Svāti (Arcturus). (7)

Method 7

The junction of the two threads which passes through the two fish-figures which are constructed with the extremities of three shadows (taken two at a time) as centre, is the south or north relative to the foot of the gnomon, according as the Sun is in the northern or southern hemisphere. (8)

Wth the junction of the (two) threads as centre, draw a circle passing through the extremities of the three shadows. (The tip of) the shadow (of the gnomon) does not leave this circle in the same way as a lady born in a noble family does not discard the customs and traditions of the family. (9)

Method 8

The midday shadow of the gnomon lies on the north-south line between the circle (denoting the locus of the shadow-tip) and (the foot of) the gnomon (situated at the centre). When the Sun is at the first point of Aries or Libra, the equinoctial midday shadow, too, lies south to north. (10)

Hypotenuse of Shadow

The square-root of the sum of the squares of the length of the gnomon and the length of the shadow is the hypotenuse of shadow. The square-root of the difference between the squares of the hypotenuse of shadow and the gnomon is (the length of) the shadow. (11) (KSS)

अग्रान्तरसंस्कारेणानीता स्फुटा पूर्वापररेखा

10. 12. 5. छायानिर्गमनप्रवेशसमयार्कक्रान्तिजीवान्तरं
क्षुण्णं स्वश्रवणेन लम्बकहृतं स्यादङ्गुलाद्यं फलम् ।
पश्चाद् बिन्दुमनेन रव्ययनतः सञ्चालयेत् व्यत्ययात्
स्पष्टा प्राच्यपराऽथवायनवशात् प्राग्बिन्दुमुत्सारयेत् ।।

(Śrīpati, *SiŚe.*, 4. 3)

Accurate determination of the cardinal directions

The result obtained by the difference between the sines of the declinations of the Sun between the moments when the gnomonic shadow equals the radius of the circle drawn with the foot of the gnomon as centre and any arbitrary radius, multiplied by the *chāyā-karṇa* (shadow-hypotenuse) of those moments and divided by $\cos\phi$ (which difference is in *aṅgulas*, the gnomon's height being 12 *aṅgulas*) is the amount by which the western

point where the shadow crosses the circle is to be shifted in the opposite direction of the Sun's motion in declination. Then we have the true East-West line. (The North-South line is drawn therefrom by means of a fish-figure.) (3). (AS)

शलाकायन्त्रम्

10. 13. 1. त्रिज्याङ्गुलां शलाकां कृत्वा पूर्वापरां विनिदध्यात् ।
इष्टाङ्गुलां तदग्रे दद्यादन्यां तिरश्चीनाम् ।। ३८ ।।

यद्वत्तदग्रसंस्थस्त्रिज्याशामूलसंस्थया दृष्ट्या ।
संदृश्यते खगामी क्षितिजस्थः सा तु तस्याग्रा ।। ३९ ।।

त्रिज्याविद्धैकं खगमन्यं तिर्यक्स्थया ग्रहं विद्धयेत् ।
तद्वर्गयोगमूलं त्रिज्या कर्णो भवत्यनयोः ।। ४० ।।

तिर्यक्स्थिता शलाका याम्योदग् या परेतरा या स्यात् ।
तत्काष्ठमन्तरांशा ग्रहयोर्ज्ञेया यथास्थितयोः ।। ४१ ।।

(Lalla, *ŚiDhVṛ*, 21. 38-41)

Śalākā-yantra (Needle instrument)

Agrā

Having made a needle of as many *aṅgulas* as there are (minutes) in the radius, stretch it from east to west; and at its (western) end, at right angles to it, lay off another needle of arbitrary length such that an observer, with his eye at the tip of it and his line of sight passing through the other end of (the former needle equal to) the radius, sees a planet exactly on the horizon. Then that (latter needle) is the *agrā* (of that planet). (38-39)

Angular distance between two planets

Observe one planet in the direction of (the needle equal) to the radius, and another planet from the tip of the vertical needle. Then the square root of the squares of the two (needles), which is really equal to the radius, is the hypotenuse (of the needle-triangle). (40)

Of the two needles, the (latter) vertical needle stands directed north-to-south; the other one, east to west. (The degrees) corresponding to that (vertical needle) should be known as the degrees between the two planets, as situated at the time of observation. (41) (KSS)

शकटयन्त्रम्

10. 14. 1. यच्छङ्कुभ्रमवृत्ते शङ्कुस्थानं तदूर्ध्वगा शलाका ।
सूर्यभ्रमसंलग्ना कथयति तत्रोद्गतान् राशीन् ।। ५२ ।।

(Lalla, *ŚiDhVṛ.*, 21. 52)

Śakaṭa-yantra (Cart instrument)

The *Śakaṭa-yantra*, of which one tube is directed towards the position of the gnomon in the circle denoting its path and the other towards the Sun in its diurnal circle, enables one to know the signs etc. of the Sun's altitude. (52). (KSS)

10. 14. 2. शकटाकृतियष्टिभ्यां विद्धा रविशीतगू तदवलम्बे ।
भगणांशाङ्के वृत्ते मुक्त्वा संलक्षयेत् स्थाने ।। ४२ ।।

अन्तरमनयोर्भागा हि सूर्यराशिनोर्दिवाकरविभक्ताः ।
तिथयः शुक्ले याताः कृष्णे शेषाः फलं भवति ।। ४३ ।।

नतजीवाग्राजीवे षष्टिहते त्रिज्यया हृते न्यासे ।
षष्ट्यङ्गुलकृतवृत्ते केन्द्रात् प्रागपरतश्चापि ।। ४४ ।।

स्वदिशि प्राग्वद् बिन्दुत्रयेण शङ्कुभ्रमं लिखेद् वृत्तम् ।
छायावृत्तं दिग्व्यत्ययेन दिक्साधनं कृत्वा ।। ४५ ।।

आग्राग्राच्छङ्कुभ्रमवृत्ते कालांशकैर्लिखेद् राशिम् ।
दिङ्मध्यच्छायाग्रं कृत्वात्र स्थापयेच्छङ्कुम् ।। ४६ ।।

अग्राग्राच्छङ्कुतलान्तरस्थिता वास्तमुद्गता भागाः ।
कालांशाः षट्कहृता भवन्ति घटिका दिनस्य गताः ।।

(Lalla, *ŚiDhVṛ.*, 21. 42-47)

(Tithi From Observation)

(Construct a circle on level ground and graduate its circumference with the 360 divisions of degrees). Then, having made the observation of the Sun and the Moon through the V-shaped (lit. cart-shaped) tubes, (the Sun through one tube and the Moon through the other), place the tubes (on the ground) with their tips on the circumference of the circle graduated with the 360 divisions of degrees (and the common end of the tubes at the centre of the circle), and show by means of points the places (where the tips fall). (42)

The degrees between these (two points) are the degrees between the Sun and the Moon. These, when divided by 12, give the *tithis* elapsed in the light half of the month, or the *tithis* to elapse in the dark half of the month. (43)

Time from Observation

(Severally) multiply the R sines of the (Sun's meridian) zenith distance and *agrā* by 60 and divide (each product) by the radius. Lay them off, in their own directions, in a circle drawn (on level ground) with 60 *aṅgulas* as radius (and having the directions marked in it), the former from the centre and the latter from the east as well as the west points. And, through the three points (thus obtained) draw, as before, the circle denoting the path of the gnomon. By laying off the same in the contrary directions, one may draw the circle (denoting the path of the tip) of the shadow (of the gnomon). (44-45)

Beginning with the end of the *agrā* (laid off in the east), graduate the circle denoting the path of the gnomon with the (appropriate) Signs by means of the degrees of time

(of their oblique ascension). And then set up a gnomon (on that circle) in such a way that the tip of the shadow may fall at the centre. (46)

Then the degrees of time lying from the end of the *agrā* (in the east) up to the foot of the gnomon are the degrees of time of the (Sun's) ascension. These degrees of time divided by six, are the *ghaṭīs* elapsed in the day (since sunrise). (47). (KSS)

यष्टियन्त्रम्

10.15.1. वृत्तव्यासदलं यष्टिस्त्रिज्यांशाङ्गुलसम्मिता ।। ८ ।।
दिङ्मध्येऽर्कोन्मुखी धार्या यष्टिः कर्णस्तदुन्नतिः ।
शङ्कुस्तस्यैव मूलात्तु छाया दिङ्मध्यगा सदा ।। ९ ।।
यष्ट्यग्रोदयमध्यांशाः षड्भिर्भाज्या दिने गताः ।
(Āryabhaṭa I, *ABh.Siddhānta*, Q by Rāmakṛṣṇa Ārādhya in his com on *SūSi.*)

Yaṣṭi-yantra (The graduated tube)

The *Yaṣṭi-yantra*[1] which is equal in length to the semi-diameter of the (perfect) circle, with as many graduations of *aṅgulas* as there are degrees in a radian (i.e. 57), should be held at the centre of the circle towards the Sun. The *yaṣṭi* then denotes the hypotenuse, its elevation denotes the gnomon, and the distance from the foot of the gnomon up to the centre of the circle always denotes the shadow (of the gnomon.) The degrees intervening between the end of the *yaṣṭi* and the rising point of the Sun, divided by six, give the *ghaṭīs* elapsed in the day. (8b-10a). (KSS)

10.15.2. दिङ्मध्यस्थितमूला यष्टिर्नष्टप्रभा त्रिगुणतुल्या ।
धार्या तदीयलम्बकाष्ठांशा वोदिता भागाः ।। ४८ ।।
यष्टिस्त्रिज्या कर्णो लम्बो ना कृतिविशेषपदमनयोः ।
दृग्ज्या छाया प्राक्परलम्बनिपातान्तरं बाहुः ।। ४९ ।।
प्रागपराग्रासक्तं सूत्रं शङ्क्वन्तरं हतं 'सूर्यैः' ।
षष्ट्यवलम्बविभक्तं यष्ट्यवलम्बेन विषुवद्भा ।। ५० ।।
सूर्येन्द्वोरिव विवरं ग्रहयोर्विज्ञाय शङ्कुयष्टिभ्याम् ।
तद्युक्तः पाश्चात्यः प्राच्यः स्यात् सोऽपि हीनेन ।। ५१ ।।
(Lalla, *ŚiDhVṛ.*, 21. 48-51)

Altitude, Zenith Distance and Bāhu. Hold the *Yaṣṭi*, equal to the radius (of the circle drawn on level ground), with its lower end at the centre (of the circle), in such a way that it may not cast any shadow (on the ground). Then the degrees in the arc corresponding to the perpendicular dropped on the ground from the other end) of it are the degrees of the (Sun's) altitude. (48)

The *Yaṣṭi*, equal to the radius, is the hypotenuse; the perpendicular (dropped on the ground from the upper end of the *Yaṣṭi*) is the (great) gnomon (i.e., the R sine of the Sun's altitude); the square root of the difference of their squares is the (great) shadow, i.e., the R sine of the (Sun's) zenith distance; and the distance between the east-west line and the foot of the perpendicular is the *bāhu*. (49)

Equinoctial midday shadow. Multiply the distance between the rising-setting line (lit. the line joining the ends of the *agrās* laid off in the east and west) and the gnomon by 12 and divide by the perpendicular dropped from the tip of the *Yaṣṭi*. This is how the equinoctial midday shadow is obtained from the perpendicular dropped from the tip of the *Yaṣṭi*. (50)

Distance between two planets and their longitudes. Determine the distance between the two planets with the help of the gnomon or the *Yaṣṭi*, like that between the Sun and the Moon. This added to (the longitude of) the westward planet gives (the longitude of) the eastward planet and the same subtracted from the latter gives the former. (51). (KSS)

नलकयन्त्रम्

10.16.1. कोटचग्रात् स्वदिशि भुजं निधाय सम्यक्
तस्याग्रे नरमथ केन्द्रशङ्कुप्रभाग्रे ।
कृत्वा ना नयनमुदग्रकर्णगत्या
शङ्क्वग्रे ग्रहमवलोकयेन्निविष्टम् ।। ४७ ।।
विपुलनलकं कर्णस्थित्या दृगुच्छ्रितदीर्घया
स्फुटतरककुप्सिद्धिर्बध्वा यथाविधि वंशयोः ।
ग्रहणमुदितं शीतांशुं वा ग्रहं च विलोकये-
न्नभसि नलकच्छिद्रेणैवं जले च विलोमतः ।। ४८ ।।
(Lalla, *ŚiDhVṛ.*, 4. 47-48)

Nalaka instrument

(In the *chāyāvṛtta* or a circle with the shadow as radius, mark the north-south and east-west lines. From their point of intersection along the east-west line measure off the *koṭi* of the shadow cast by the planet at the time of observation. If the planet is in the eastern hemisphere, the *koṭi* must be marked to the west; but to the east, if the planet is in the western hemisphere). At the extremity of the *koṭi* draw the *bhuja* in its own direction. Place a bamboo joining the extremity of the *bhuja* to the centre, (to represent the shadow). Connect the point of intersection of the *bhuja* and the shadow to the top of the gnomon by a bamboo. Then, the planet will be visible along this line on the top of the gnomon (after fixing the *nalaka* instrument).

Then, at the end, fix the *nalakayantra* on two bamboo sticks, so that it (i.e., its hollow) is in the same direction as the joining line, and its height is that of the observer's eye. Now, the observer shall look through the instrument and he will see in the sky the eclipse, the rising

[1] The *Yaṣṭi-yantra* resembles a cylindrical stick.

Moon or the planet. The reverse process must be followed for visibility in water. (47-48).[1] (KSS)

10. 16. 2. विधाय बिन्दुं समभूमिभागे
ज्ञात्वा दिशः कोटिरतः प्रदेया ।
प्रत्यङ्मुखी पूर्वकपालसंस्थे
पूर्वामुखी पश्चिमगे ग्रहे सा ।। १०५ ।।
कोटचग्रतो दोरपि याम्यसौम्यो
बिन्दोश्च भा भाग्रभुजाग्रयोगात् ।
सूत्रं च बिन्दुस्थनराग्रसक्तं
प्रसार्यं कर्णाकृतिसूत्रगत्या ।। १०६ ।।
दृगुच्चमूलं नलकं निवेश्य
वंशद्वयाधारमथास्य रन्ध्रे ।
विलोकयेत् खे खचरं किलैवं
जले विलोमं तदपि प्रवक्ष्ये ।। १०७ ।।
निवेश्य शङ्कुं भुजभाग्रयोगे
बिन्दोर्नराग्रानुगते च सूत्रे ।
तथैव धार्यो नलको विलोक्यो
बिन्दुस्थतोये सुषिरेण खेटः ।। १०८ ।।
(Bhāskara II, *SiŚi.*, 1. 3. 105-8)

On a horizontal plane mark a point and through it draw the east-west line and also the north-south; if the planet is in the east, mark off the computed *koṭi* of the shadow towards the east-west line; if the planet is in the western hemisphere, make this *koṭi* towards the east. (105)

From the extremity of the *koṭi* mark the computed *bhuja* perpendicular to the east-west line and draw the computed shadow from the point so as to form a right angled triangle with the *bhuja* and *koṭi*. Extend a thread from the point of intersection of the *bhuja* and shadow to meet the gnomon's tip so as to form the *chāyākarṇa* or the hypotenuse of the right angled triangle of which the other sides are the gnomon and the shadow. (106)

Along this thread place the Nalaka so that the lower extremity of the Nalaka coincides with the eye. Seeing through the Nalaka, the planet is to be seen. I shall state how the planet could be seen in water as well.[1] (107)

Place the Śaṅku at the point of intersection of the *bhuja* and shadow and holding the Nalaka along the joint of the tip of the Śaṅku and the point, the planet can be seen in a basin of water placed at the point. (108). (AS)

[1] The Nalaka is a simple tube formed generally of bamboo. The purpose of this is to verify the correctness of the computation of the shadow and its *bhuja*. If the computation is wrong the planet will not be seen in that direction. It might be asked how the shadow and *bhuja* are pertinent with respect to a planet whose shadow cannot be observed as that of the Sun. It must be noted the computation of the shadow and *bhuja* are made similar to that of the Sun, knowing the declination etc. as in the case of the Sun. The computation does not depend on the observation of the actual shadow. By computing the magnitudes of the *bhuja* and *koṭi*, the direction of the *chāyākarṇa* points to the planet in the sky.

छत्रयन्त्रम्

10. 17. 1. छत्रं वेणुशलाकाभिः कृत्वा चक्रांशसंख्यया ।
दिङ्मध्ये समवृत्तं च कल्पयेच्छत्रयन्त्रकम् ।। १३ ।।

छत्रदण्डं च तन्मध्ये व्यासार्धं शङ्कुरेव सः ।
स्वाहोरात्रदलं सौम्यं व्यस्ताग्रं भाभ्रमाह्वयम् ।। १४ ।।

षड्गुणा दिननाड्योंऽशाः सौम्यार्धे छत्रयन्त्रतः ।
अग्रान्तेऽर्कोदयास्ते च प्रत्यक्प्राक् तु प्रभा स्थिता ।। १५ ।।

तत्प्रत्यगन्तमस्ताख्यं प्रागन्तमुदयाह्वयम् ।
अस्ताख्यादुदयस्यान्तं छायाकालांशकाः स्थिताः ।। १६ ।।

छत्रमध्यस्थशङ्कोस्तु छायैवेष्टप्रभा सदा ।
छायाग्रास्ताख्यमध्यांशा षड्भिर्नाड्यो दिवा गताः ।।
(Āryabhaṭa I, *ABh.Siddhānta*, Q by Rāmakṛṣṇa Ārādhya in his com. on *SūSi*)

Chatra-yantra (Umbrella instrument)

Construct a *Chatra-yantra* (an instrument resembling an umbrella) by bamboo-needles, mark (the circumference of) it with the 360 divisions of degrees, and set it at the centre of the (perfect) circle. Or, treat the perfect circle itself as the *Chatra-yantra*. (13)

The rod of the *Chatra-yantra*, in the middle of it, equal to the radius, is the gnomon; the northern half of the diurnal circle drawn through the end-points of the (Sun's) *agrā*, laid off in the contrary direction (in the west and the east), is the so called 'path of shadow'.[1] (14)

The *nāḍīs* of the day, multiplied by six, are the degrees in (the diurnal circle lying in) the north half of the *Chatra-yantra*. Towards the end-points of the (Sun's) *agrā*, in the west and the east the shadow falls at sunrise and sunset, respectively. (15)

The end (of the Sun's *agrā*) in the west is (therefore) called the 'setting point (*asta*)', and the end (of the Sun's) *agra* in the east the 'rising point (*udaya*)'. From the 'setting point' to the 'rising point' (on the northern half of the diurnal circle) lie (the graduation of) the degrees of time in a *Chatra-yantra*. (16)

The shadow cast by the gnomon, situated in the middle of the *chatra*, is always the shadow at the desired time. The degrees (on the diurnal circle) intervening between the end of the shadow and the 'setting point', divided by six, give the *nāḍīs* elapsed in the day. (17) (KSS)

[1] In fact, this is not the path of shadow.

फलकयन्त्रम्

10. 18. 1. कर्तव्यं चतुरस्रकं सुफलकं 'खाङ्का'ङ्गुलैर्विस्तृतं
विस्ताराद् द्विगुणायतं सुगणकेनायाममध्ये तथा ।
आधारः श्लथशृङ्खलादिघटितः कार्या च रेखा तत-
स्त्वाधारादवलम्बसूत्रसदृशी सा लम्बरेखोच्यते ॥१८॥

लम्बं नवत्यङ्गुलकैर्विभज्य
प्रत्यङ्गुलं तिर्यगतः प्रसार्य ।
सूत्राणि तत्रायतसूक्ष्मरेखा
जीवाभिधानाः सुधिया विधेयाः ॥ १९ ॥

आधारतोऽधः 'खगुणा'ङ्गुलेषु
ज्यालम्बयोगे सुषिरं च सूक्ष्मम् ।
इष्टप्रमाणा सुषिरे शलाका
क्षेप्याक्षसंज्ञा खलु सा प्रकल्प्या ॥ २० ॥

षष्टयङ्गुलव्यासमतश्च रन्ध्रात्
कृत्वा सुवृत्तं परिधौ तदङ्क्यम् ।
षष्ट्या घटीनां भगणांशकैश्च
प्रत्यंशकं चाम्बुपलैश्च दिग्भिः ॥ २१ ॥

अग्रे सरन्ध्रा तनुपट्टिकैका
षष्टयङ्गुला दीर्घतया तथाङ्क्या ।
यत्खण्डकैः स्थूलचरं पलाद्यं
तद् 'गीकु'हृत् स्याच्चरशिञ्जिनीह ॥ २२ ॥

यष्टिसाधनम्

'वेदा' 'भवा' 'शैलभुवो' 'धृति'श्च
'विश्वे' च 'बाणा' फलकर्णनिघ्नाः ।
'अर्को'द्धृताः स्युः क्रमशः स्वदेशे
राश्यर्धलभ्यानि हि खण्डकानि ॥ २३ ॥

तैः क्रान्तिपाताढ्यरवेर्भुजज्या
षष्टयुद्धृताक्षश्रवणेन युक्ता ।
'दिग्'घ्नी 'कृता'प्ता भवतीह यष्टिः
सा पट्टिकायां सुषिरात् प्रदेया ॥ २४ ॥

धार्यं तथा फलकयन्त्रमिदं यथैव
तत्पार्श्वयोर्लगति तुल्यमिनस्य तेजः ।
छायाक्षजा स्पृशति तत्परिधौ यदंशं
तत्रांशके मतिमता तरणिः प्रकल्प्यः ॥ २५ ॥

अक्षप्रोतां रविलवगतां पट्टिकां न्यस्य तस्माद्
यष्टेरग्रादुपरि फलकेऽधश्च गोलक्रमेण ।
यत्नाद् देयश्चरदलगुणस्तत्र या ज्या तयात्र
छिन्ने वृत्ते तलगघटिका स्युर्नता लम्बकान्ताः ॥ २६ ॥

(Bhāskara II, *SiŚi*, 2. 10. 18-26)

Phalaka-yantra[1]

Prepare a rectangular plate, either of metal or of wood, having a breadth of 90″ and length double thereof, (i.e. 180″). Mark the midpoint (M) of the plate at a longer side wherefrom it can be hung with a chain in the plane of a vertical. Draw a perpendicular through the midpoint (M) downwards, to be called *lamba-rekhā*. (18)

Divide this perpendicular into 90 equal parts, one *aṅgula* each, and draw thin parallels through the divisions, to be called *jyā-rekhās* (or *jīvā-rekhās*). Below the midpoint (M), at distance of 30″ make a small hole and insert into it a peg, to be called *akṣa*, of an arbitrary length (whose shadow will be cast on the plate. Let the hole be called O.) (19-20)

With O as centre and with diameter 60″ (i.e. radius 30″), draw a circle. Let the circle be graduated into 60 *ghaṭīs* and 360 degrees, each degree being divided into 10 equal parts to be called *ambupalas* (or *vighaṭīs*). (21)

Hang a thin slice (of copper or bamboo) of length 60″, divided into 60 equal parts from O (along the vertical line ON. (The breadth of this at O shall be $\frac{1}{2}$″ and at N be 1″, so that) it will be in the form of an axe. Make a small hole in this slice (at N). (22)

Since the rough *cara* derived from the said divisions is in *palas*, that divided by 19 would give Sine *cara*.[1] (22b)

The segments 4, 11, 17, 18, 13 and 5 being multiplied by the hypotenuse of the shadow, K, ($K^2 = S^2 + 12^2$ being the shadow), and divided by 12, will give the respective segments at the local place for arcs of 15°, (so that there would be six segments for 90°). (23)

From these obtain the *bhuja* of the longitude of the *sāyana* Sun and divide by 60 and add the hypotenuse of the local shadow (K of verse 23 above). Multiply the sum by 10 and divide by 4, the result being called the *yaṣṭi* in *aṅgulas*. This *yaṣṭi* is to be hung from the midpoint (O, along ON). (24)

When a *Phalaka-yantra* is held in the plane of the Sun's vertical, the point where the shadow cast by the *akṣa* (at O) on the circle (O) is to be taken as the point representing the Sun. (25)

Now, place the *paṭṭikā*, i.e. the slice hanging from O, on that point. Note the distance between the mark, as said before, on the *paṭṭikā*, by laying off the *yaṣṭi*. Mark

[1] The *Phalaka-yantra* is a versatile instrument which enables one to read off the astronomical time by noting the length of the shadow of the Sun and also other astronomical measures like *Hṛti*, *Antyā*, *Cara* etc. It can be used in respect of planets and stars as well.

[1] This is so since for 1″ of equinoctial shadow, the so-called *cara*-segments are 10, 8, 3/10, taking the latitude to be that of Ujjain where the equinoctial shadow is 4-30. The *cara*-segments for that latitude will be 45, 36 and 15, the *cara*-segments for any other place being derived from these. This *cara* is generally expressed in *vighaṭīs*, and each *vighaṭī* is equal to 6 *asus* or *prāṇas*. Since an *asu* is very small its R sine will be the same. Hence the following rule of three might be used: If for a radius equal to 3438, the *cara* is given by the above segments, what will be *cara* for 30×6 *asus*. Now 3438 divided by 30 × 16 is 19, approximately. Hence the rule given here.

off the *carajyā* above that mark when the Sun has northern declination, or below otherwise. Then, the distance from the point where the parallel line (lengthwise) meets the circle to the parallel line through O gives the hour angle in *ghaṭīs*. (26). (AS)

यन्त्रैः ग्रहनिरीक्षणम्

10. 19. 1. क्षितिरवियोगाद् दिनकृद्
रवीन्दुयोगात् प्रसाधितश्चेन्दुः ।
शशिताराग्रहयोगात्
तथैव ताराग्रहाः सर्वे ॥ ४८ ॥
(Āryabhaṭa I, *ABh.*, 4. 48)

Observation of planets with instruments

The Sun is determined from the conjunction of the Earth and the Sun, the Moon from the conjunction of the Sun and the Moon, and all the other planets from the conjunctions of the planets and the Moon. (48). (KSS)

अर्काग्रा

10. 19. 2. ग्रीवासमां भगणभागविभक्तवृत्तां
कुर्यात् स्थलीं समतलां कृतदिग्विभागाम् ।
तस्या जलेशदिशि मण्डलमध्यदृष्टि-
र्विध्याद्रविं परिधिलग्नमनाकुलात्मा ॥ ५६ ॥

पूर्वरेखाग्रवेधस्य रविवेधस्य चान्तरम् ।
अर्काग्राचापनिर्माणं परिधौ भागलक्षिते ॥ ५७ ॥

अर्काग्रा ज्या भवेत्तस्य तन्नतिज्याविशेषजा ।
लिप्ता शङ्क्वग्रजीवाया दक्षिणे चोत्तरेऽन्यथा ॥ ५८ ॥

दक्षिणाभिमुखी छाया यदा भवति भास्वतः ।
नतिज्यारहिताकाग्रा शङ्क्वग्रं कथ्यते तदा ॥ ५९ ॥

विद्धि तेन विषुवत्प्रभां सतीं
पूर्ववच्च पललम्बकौ पुनः ।

अज्ञातग्रहः

वेदितव्यविदितग्रहान्तरं
नाडिकाभिरवगम्य तत्त्वतः ॥ ६० ॥

षड्गुणास्तु घटिका लवास्तु तैः
पूर्वपश्चिमदिशि स्थिते क्षयः ।
उच्यते धनमपि क्रमेण त-
ज्ज्ञातचारनिचयैः सदा बुधैः ॥ ६१ ॥

एवं नक्षत्रतारणां ग्रहैस्ताराभिरेव च ।
साधितं क्षेत्रनिर्माणं युक्त्या सर्वत्र सर्वदा ॥ ६२ ॥
(Bhāskara I, *MBh.*, 3. 56-62)

Sun's agrā etc. by observation

One should erect a (circular) platform, as high as one's neck, with its floor in the same level, and its circumference graduated with the divisions of Signs, degrees, etc., and bearing the marks of the directions. (Then standing) on the western side thereof, one, having undisturbed of mind, should, with the line of sight passing through the centre of the circular base, make the observation of the Sun when (at sunrise) it appears as if clinging to the circumference, (and mark there a point).

The (actual) distance, measured along the circumference graduated with the marks of degrees, between the end of the line drawn eastwards (i.e., the east point) and the point where the Sun is observed is the arc of the Sun's *agrā*. The R sine of that (arc) is (the R sine of) the Sun's *agrā*. The minutes of the difference between that (R sine of the Sun's *agrā*) and the R sine of the Sun's meridian zenith distance are the minutes of the *śaṅkvagra*, provided that the Sun is in the southern hemisphere; when the Sun is in the northern hemisphere (and the shadow of the gnomon falls towards the north), the process is otherwise (i.e., the addition of the two).

When, however, (the Sun being in the northern hemisphere), the shadow (of the gnomon) due to the Sun falls towards the south, the Sun's *agrā* minus the R sine of the Sun's meridian zenith distance is stated to be (the value of) the *śaṅkvagra*. From that (*śaṅkvagra*) determine the true value of the equinoctial midday shadow (of the gnomon), and then calculate as before the latitude and colatitude (for the place). (56-60a)

Longitude of an unknown planet

Having correctly ascertained in terms of *nāḍikās* (i.e., *ghaṭīs*) the difference between (the times of rising or setting of) the planets known and to be known, multiply those *ghaṭīs* by six. Thus are obtained the degrees (of the difference between the longitudes of the two planets). By those degrees diminish or increase the longitude of the known planet according as it is to the east or west of the planet to be known. This is stated by the learned people well versed in planetary motions (to be the method for getting the longitude of the planet to be known). (60b-61)

Longitudes of the prominent stars of the nakṣatras

In this way from (the known longitudes of) the planets or stars, have been determined, at all places and at all times, the celestial longitudes of the (prominent) stars of the *nakṣatras*. (62). (KSS)

वेधप्रकारः

10. 19. 3. सलिलेन समं कृत्वा तुङ्गं फलकं यथादिशं दृष्ट्वा ।
दक्षिणकोटयां शङ्कुं फलकप्रमितं व्यवस्थाप्य ॥ ३० ॥

ऋजुशङ्कुबुध्नविन्यस्तलोचनो नमयेत् तथा शङ्कुम् ।
भवति यथा शङ्क्वग्रं ध्रुवता दृष्टिमध्यस्थम् ॥ ३१ ॥

पतितेन भवति वेधो लङ्कायामूर्ध्वगेन तु सुमेरौ ।
विनतेन च तथान्तराले फलके चाक्षोर्ध्वसूत्रसमम् ।।३२।।

तत्रावलम्बको यः सोऽक्षज्या तस्य शङ्कुविवरं यत् ।
विषुवदवलम्बकोऽसौ याम्योत्तरदिक्प्रसिद्धिकरः ।। ३३ ।।

स्वप्रयत्नेन सन्तः विज्ञायैवं वदन्ति भूमध्यम् ।
सकलमहिमानं वा रसमिव लवणाम्भसाल्पेन ।। ३४ ।।

(Varāha, *PS*, 13. 30-34)

Method of observation

Take a plank, with its surface plane, as verified by dropping water on it. Set it so as to have its surface horizontal and level with the eye, and its parallel sides north-south and east-west. At the southern edge, in the middle, hinge a sighting tube equal in length to the north-south length of the plank. With the eye at the hole of the rigid sighting instrument, at the hinge, raise the instrument to such an extent that the north-pole-star is sighted through the hole of the instrument[1]. (30-31)

When so sighting the pole-star, the perpendicular dropped on to the plank from the end of the sight is the R sine of the latitude of the place. The base so formed is the R cos of the latitude of the place. The R cos line, i.e. the base, coincides with the north-south-direction line. (32-33)

Learned men, observing things for themselves thus, determine the North pole, the dimensions of the whole Earth, etc. as one would determine the salty taste of the whole quantity of a solution by tasting a drop thereof.[2] (34). (TSK)

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[1] From the next verse we can understand that Varāhamihira implies here that the north-south length of the plank is 120 units, so that the length of the sighting instrument also is 120 units=R. So taken, the perpendicular dropped on to the plank from the end of the instrument will be equal to R sine angle raised, and the base from the foot of the perpendicular to the hinge will be R cos raised angle).

[2] What is meant here is that an observation made at a small area on the Earth can give us, by suitable reasoning, knowledge of the whole Earth.

11. पञ्चाङ्गम् – CALENDAR

पञ्चाङ्गम्

11. 1. 1. नक्षत्र-वार-तिथयः करणानि योगाः ।
पञ्चाङ्गम् . . .

(Vidyāmādhava, *Vidyāmādhavīya* or *Muhūrta-darśana*)

The five constituents of the Calendar

The *Nakṣatra* (constellation or asterism), *Vāra* (weekday), *Tithi* (Lunar day), *Karaṇa* and *Yoga* constitute the five limbs (of the daily calendar). (KVS)

11. 1. 2. वासराः सूर्यवाराद्यास्तिथयः प्रतिपन्मुखाः ।
करणं कृमिसिंहाद्यं योगा विष्कम्भपूर्वकाः ।। ७ ।।

(Śaṅkaravarman: *Sadratnamālā*, 2. 7)

(Of the five-faced almanac), the weekdays are Sunday and others (being, Monday, Tuesday, Wednesday, Thursday, Friday and Saturday).

(The fifteen lunar days of the fortnight are) Pratipad and others (viz., Dvitīyā, Tṛtīyā, Caturthī, Pañcamī, Ṣaṣṭhī, Saptamī, Aṣṭamī, Navamī, Daśamī, Ekādaśī, Dvādaśī, Trayodaśī, Caturdaśī and Pañcadaśī or Parva, being Amāvāsyā at the end of the dark fortnight and Pūrṇimā or Paurṇamāsī at the end of the bright fortnight).

(The eleven) *karaṇas*, (four immovable and seven movable) are Krimi, Siṃha etc.[1]

The twentyseven *yogas* begin with Viṣkambha.[2] (KVS)

नक्षत्रानयनम्

11. 2. 1. ऋक्षं लिप्ताष्टशति व्यर्कच्चिन्द्रात्तिथिर्द्विषट्कांशैः ।
भुक्त्यनुपाताद् वेला रवीन्दुभुक्त्यन्तराच्च तिथेः ।। १६ ।।

(Varāha, *PS*, 3. 16)

Nakṣatra computation

For every 800 minutes of arc in the Moon's longitude there is one *nakṣatra* (asterismal segment). Deduct the Sun's longitude from the Moon's. For every twelve degrees of the remainder there is one *tithi*. The time of the ending moment of the *nakṣatra* should be found by proportion using the Moon's daily motion. The time of the ending moment of the *tithi* should be found by proportion using the difference in the daily motions of the Sun and the Moon. (16) (TSK)

[1] For the list, see below, 11.16.1

[2] For the list, see below, 11.13.1 fn.

11. 2. 2. शश्यर्धदलं त्रिकृतिघ्नमृक्षमंशस्थिता मुहूर्ताः स्युः ।
व्यर्केन्दुदलं 'विषया'हतं तिथिस्तद्वदेवोक्तः ।। ७ ।।

(Varāha, *PS*, 2. 7)

Divide the true Moon by 4 and multiply by 9. What we get in the *rāśi* column is the *nakṣatra*. What is got in the degree column are the *muhūrtas*. Deduct the true Sun from the true Moon, divide the result by 2 and muliply by 5. *Tithis* are obtained in the *rāśi* column and thirtieths in the degree column. (7). (TSK)

सूक्ष्मनक्षत्रानयनम्

11. 3. 1. स्थूलं कृतं भानयनं यदेतज्-
ज्योतिर्विदां संव्यवहारहेतोः ।। ७१ ।।

सूक्ष्मं प्रवक्ष्येऽथ मुनिप्रणीतं
विवाहयात्रादिफलप्रसिद्ध्यै ।
अध्यर्धभोगानि षडत्र तज्ज्ञाः
प्रोचुर्विशाखादितिभध्रुवाणि ।। ७२ ।।

षडर्धभोगानि च भोगिरुद्र-
वातान्तकेन्द्राधिपवारुणानि ।
शेषाण्यतः पञ्चदशैकभोगा-
न्युक्तो भभोगः शशिमध्यभुक्तिः ।। ७३ ।।

सर्वर्क्षभोगोनितचक्रलिप्ता
वैश्वाग्रतः स्यादभिजिद्भभोगः ।
कलीकृतादिष्टखगाद्विशोध्य
दास्रादिभोगान् गतभानि विद्यात् ।। ७४ ।।

विशुद्धसंख्यानि गतं तु शेष-
मशुद्धभोगात् पतितं तदेष्यम् ।
गतागते षष्टिगुणे विभक्ते
ग्रहस्य भुक्त्या घटिका गतैष्या ।। ७५ ।।

(Bhāskara II, *SiŚi*., 1. 2. 71b-75)

Accurate computation of asterisms

The computation of the *nakṣatras* done as prescribed before, is only approximate. Now I shall give the method of obtaining what are called *Sūkṣma-nakṣatras* as prescribed by the ṛṣis, which are required for noting the auspicious occasions regarding marriages, journeys etc. (71b-72a)

People who knew about it, have said that the six stars Viśākhā, Punarvasu, Rohiṇī, and the three Uttaras, viz. Uttaraphalgunī, Uttarāṣāḍha, and Uttarabhadrā have the duration of one and half stars, i.e. 3/2 × 790′ 35″ = 1185′ 52″. The six stars Āśleṣā, Ārdrā, Svātī, Bharaṇī,

Jyeṣṭhā and Śatabhiṣak have half the duration of a star i.e. 395′ 17″. The remaining 15 alone have one nakṣatra duration, i.e. 790′ 35″. A star's duration is the mean daily motion of the Moon, i.e. 790′ 35″. (72b-73)

The sum total of all the above 27 stars being subtracted from 360°, give the duration of the star that is called Abhijit which occurs after Uttarāṣāḍhā and before Śravaṇa.

To obtain the position of the star in which a planet is situated, convert its longitude in minutes of arc and subtract the durations of the stars from Aśvinī as many as could be subtracted. The number of stars whose durations are thus subtracted are deemed to have elapsed. (74)

The remainder is called the *gata* or elapsed portion of the current star and the difference of this *gata* and the duration of the current *nakṣatra* is called the *eṣya*, i.e. unelapsed portion. To obtain the elapsed time or the unelapsed time of the current star, the *gata* or the *eṣya* is to be multiplied by 60, and divided by the daily motion of the planet concerned, the result being in *nāḍīs*. (75). (AS)

रविनक्षत्रम्

11. 4. 1. कलितं रविमष्टशतैर्विभजेद्भान्यंशविरहितच्छेदात् ।
अंशाच्च षष्टिगुणितात् पुनरपि तैरेव घटिकाः स्युः ॥
(Deva, *KR*, 1. 43)

Nakṣatra of the Sun

Reduce the longitude of the Sun to minutes, and then divide by 800: the quotient gives the (number of) *nakṣatras* passed over (by the Sun). Multiply the remainder and the divisor minus remainder (severally) by 60 and divide both of them by the Sun's daily motion: the results obtained give the *ghaṭīs* elapsed and the *ghaṭīs* to elapse (in the curent *nakṣatra*).[1] (43) (KSS)

[1] The stars lying near the ecliptic are divided into 27 groups called *nakṣatras*. The names of these *nakṣatras*, in the order in which they occur, are as follows:

1. Aśvinī
2. Bharaṇī
3. Kṛttikā
4. Rohiṇī
5. Mṛgaśirā
6. Ārdrā
7. Punarvasu
8. Puṣya
9. Āśleṣā
10. Maghā
11. Pūrvā Phalgunī
12. Uttarā Phalgunī
13. Hasta
14. Citrā
15. Svātī
16. Viśākhā
17. Anurādhā
18. Jyeṣṭhā
19. Mūla
20. Pūrvāṣāḍhā
21. Uttarāṣāḍhā
22. Śravaṇa
23. Dhaniṣṭhā
24. Śatabhiṣak
25. Pūva-Bhādrapadā
26. Uttara-Bhādrapadā
27. Revatī

The first *nakṣatra* is supposed to begin at the first point of the star Zeta Piscium. (p. 30a: read 'δ' as 'z' Piscium).

Starting from the first point of the *nakṣatra* Aśvinī, the ecliptic is also divided into 27 equal parts, each of 800′. These divisions of the ecliptic are also called *nakṣatras* and given the same names as the twenty-seven *nakṣatras* mentioned above. The *nakṣatras* referred to in the above stanza are these 27 divisions of the ecliptic.

चन्द्रनक्षत्रम्

11. 5. 1. शशिनं कलितं विभजेच्छताष्टकैरागतानि भानि स्युः ।
शेषे गतभाविन्यो नाड्यस्तिथिवत् स्वभुक्तिहृते ॥ ४४ ॥
(Deva, *KR*, 1. 43-44)

Nakṣatra of the Moon

Reduce the Moon's longitude to minutes and then divide by 800: the quotient obtained gives the (number of) *nakṣatras* passed over (by the Moon). The remainder and the divisor minus remainder (multiplied by 60 and) divided by the (Moon's) own daily motion give the *nāḍīs* elapsed and *nāḍīs* to elapse (in the current *nakṣatra*), as in the case of the *tithi*. (44). (KSS)

वारः वाराधिपाश्च

11. 6. 1. तिथेरेकगुणः प्रोक्तो नक्षत्रं च चतुर्गुणम् ।
वारस्याष्टगुणः प्रोक्तः करणं षोडशान्वितम् ॥ ७.२१ ॥
आदित्यः सोमो भौमश्च तथा बुधबृहस्पती ।
भार्गवः शनैश्चरश्चैव एते सप्त दिनाधिपाः ॥ ८.२ ॥
(*Atharvaṇa-Jyotiṣa*, 7. 21; 8. 1.)

Weekday

(In the matter of their potency for bestowing benefits for rituals performed during their currency), the lunar day ranks onefold, the asterism twofold, the weekday eightfold and the *karaṇa* sixteenfold. (7.21)

The lords of the weekdays are (in order) the seven (planets), the Sun, the Moon, Mars, Mercury, Jupiter, Venus and Saturn. (8.1). (KVS)

दिनपाः होरेशाश्च

11. 6. 2. सप्तैते होरेशाः शनैश्चराद्या यथाक्रमं शीघ्राः ।
शीघ्रक्रमाच्चतुर्था भवन्ति सूर्योदयाद् दिनपाः ॥
(Āryabhaṭa I, *ABh.*, 3.16)

Lords of Weekdays and Hours

The (above mentioned) seven planets beginning with Saturn, which are arranged in the order of increasing velocity, are the lords of the successive hours. The planets occurring fourth in the order of increasing velocity are the lords of the successive days, which are reckoned from sunrise (at Laṅkā).[1] (16). (KSS)

[1] That is to say, the lords of the twenty-four hours (the hours being reckoned from sunrise at Laṅkā) are:

Saturn, Jupiter, Mars, Sun, Venus, Mercury, Moon, Saturn, Jupiter, Mars, Sun, Venus, Mercury, Moon, Saturn, Jupiter, Mars, Sun, Venus, Mercury, Moon, Saturn, Jupiter, and Mars, respectively.

and the lords of the seven days are:

Saturn, Sun, Moon, Mars, Mercury, Jupiter, and Venus, respectively.

The Lord of a day is the Lord of the first hour of that day, the day being measured from sunrise at Laṅkā.

दिनाधिपानयनम्

11. 6. 3. अहर्गणोऽब्दाधिपशुद्धिनाडिका-
युतो यथा स्वं स्वरभक्तशेषकम् ।
भवेत् सितादिर्दिनपस्तदह्निजाः
सहस्ररश्मेरुदये नभश्चराः ।। ३२ ।।
(Lalla, *ŚiDhVr.*, 1. 32)

Lords of the days

To the *ahargaṇa* should be added the days and hours (*ghaṭikās*) of the *śuddhi*. The sum, which is the *ahargaṇa* from the beginning of *Caitra*, should be divided by 7. The remainder counted from this day gives the lord of the day. The longitudes of the planets, etc. calculated from the *ahargaṇa* will be for the sunrise on the current day. (32). (BC)

वर्षाधिपानयनम्

11. 6. 4. गताः समा 'नन्दकृतेन्दु'ताडिता
'रसाद्रिबाणा'पहृता दिनादिकम् ।
फलं युतास्तेन समाः समाधिपो
'नगा'प्तशेषा भृगुनन्दनाद् भवेत् ।। २७ ।।
(Lalla, *ŚiDhVr.*, 1. 27)

Lord of the year

The number of solar years elapsed (since the beginning of the *Kaliyuga*) multiplied by 149 and divided by 576, gives the result in days etc. Add this to the number of solar years and divide the sum by 7. The remainder counted from Friday gives the Lord of the year.[1] (27). (BC)

अहर्मानम्

11. 7. 1. गतमयनादुत्तरतो द्यूनां गन्तव्यमपि च याम्यस्य ।
द्विघ्नं 'शशिरस'भक्तं द्वादशसहितं दिवसमानम् ।। ५ ।।
(Varāha, *PS*, 12.5)

Duration of day

Take the days elapsed in the Uttarāyaṇa, (*i.e.* the northward course of the Sun), and the days to go in the Dakṣiṇāyana, (*i.e.*, the southward course). Multiply by two, divide by 61, and add 12. The duration of the day time, (in *muhūrtas*), is obtained.[2] (5). (TSK)

स्वदेशोदयः

11. 8. 1. रविचरदलमुदक् क्षेप्यमथो दक्षिणे व्यस्तम् ।। ४२ ।।
(Deva, *KR*, 1. 42b)

Local time

To find the time at the local place, add the (Sun's) ascensional difference to the time at the local equatorial place, provided the Sun is in the northern hemisphere; and subtract the same, when the Sun is in the southern hemisphere. (It is assumed that the local place is in the northern hemisphere).[1] (42b). (KSS)

तिथिस्वरूपम्

11. 9. 1. चन्द्रार्कान्तरचक्रे तिथयस्त्रिंशद् यतस्ततो राशिः ।
सार्धतिथिद्वययुक्तस्तत्र हि नाड्यः शतं सपञ्चाशत् ।।५।।
(Śaṅkaravarman : *Sadratnamālā*, 2.5)

Lunar day

Since in the difference circle of the Moon and the Sun (containing 12 Signs) there are 30 lunar days, a Sign would contain $2\frac{1}{2}$ lunar days, the *nāḍīs* therein being ($2\frac{1}{2} \times 60 =$) 150. (5). (KVS)

तिथ्यानयनम्

11. 10. 1. स्फुटार्कोनः शशी छेद्यो लिप्ताभिः 'खद्विभूधरैः' ।
तिथयस्तत्र लभ्यन्ते शेषं षष्ट्या समाहृतम् ।। ३१ ।।
छिन्द्याद् भुक्तिविशेषेण घटीविघटिकासवः ।
तिथेश्शेषो मतो वापि निर्दिष्टो भास्करोदयात् ।। ३२ ।।
(Bhāskara I, *MBh.*, 4. 31-32)

Calculation of the lunar day

Divide the true longitude of the Moon as diminished by that of the Sun by 720 minutes (of arc): the quotient (obtained) denotes the number of *tithis* (elapsed). Multiply the remainder by 60 and divide (the product) by the difference between the (true) daily motions of the Sun and the Moon: then are obtained the *ghaṭīs*, *vighaṭīs*, and *asus* (elapsed of the current *tithi*). (The time in *ghaṭīs*, *vighaṭīs*, etc. of) the current *tithi* to elapse or elapsed is measured from sunrise. (31-32). (KSS)

रविचन्द्रस्फुटं विना तिथ्यानयनम्

11. 11. 1. शशिवत्सरताडिते गणेऽह्नां युगभूवासरभाजिते समादि ।
अधिकाब्दगुणे तथैव विद्यादुभयोरन्तरमर्कवर्षपूर्वम् ।।१।।
रविवर्षगणेन नास्ति कृत्यं
परिशेषीकृतराशितोऽथ मासौ ।
'धृति'सम्मितवासरान् दिनेभ्यः
परिशोध्यैव ततो भुजादि कार्यम् ।। २ ।।
रविसंहृतमर्ककाललब्धं
विपरीतं तु धनर्णमिन्द्वहेषु ।
शशिकेन्द्रजमप्यथाशु चैवं
शशिवत्तच्छशिवासरेषु कार्यम् ।। ३ ।।
परिनिष्ठितनाडिका व्यतीताः
शशिनो या दिवसस्य षष्टिनिघ्नाः ।
स्फुटभोगविशेषसूर्यभागै-
र्भजितास्ताः स्फुटनाडिकास्तदाप्ताः ।। ४ ।।
(Bhāskara I, *MBh.*, 8. 1-4)

[1] For notes, see *ŚiDhVr.*: *BC*, II. 18-19.

[2] For elucidation, see *PS* : *TSK*, 12.5.

[1] The time at the local equatorial place is obtained by applying the longitude correction to the time at Laṅkā. This correction is additive, if the local place is to the east of the prime meridian, and subtractive if the local place is to the west of the prime meridian.

True lunar day without Sun and Moon

Multiply the *ahargaṇa* by the number of lunar years[1] (in a *yuga*) and divide by the number of civil days[2] (in a *yuga*). Then are obtained the mean lunar years, etc. (corresponding to the *ahargaṇa*). Also multiply the *ahargaṇa* by the number of intercalary lunar years[3] (in a *yuga*) and divide by the number of civil days (in a *yuga*). (Thus are obtained the mean intercalary years, etc., corresponding to the *ahargaṇa*). The difference between the two denotes the mean solar years, etc. (i.e., years, months, days, and *ghaṭīs*) (corresponding to the *ahargaṇa*). (1)

The solar years are not required; (so they are to be omitted). From the remaining quantity (in months, days, and *ghaṭīs*) subtract two months and eighteen days. Then (treating the months, days, etc., of the remainder thus obtained as the Signs, degrees, etc. of the Sun's mean anomaly) calculate the (Sun's) equation of the centre. (2)

Divide the (corresponding) solar time by 12 and apply that to the (mean) lunar days (and *ghaṭīs*) (obtained from the *ahargaṇa*) contrararily (i.e., add when the Sun's equation of the centre is subtractive, and subtract when the Sun's equation of the centre is additive). Also apply one-twelfth of the time corresponding to the Moon's equation of the centre[4] to the resulting lunar days (and *ghaṭīs*) in the same way as it is applied to the Moon's longitude. (3)

(The lunar days thus obtained are the true lunar days which have elapsed at sunrise since the beginning of the current month). The *ghaṭīs* obtained above denote the elapsed portion of the current lunar day in terms of *ghaṭīs*. Multiply those *ghaṭīs* by 60 and divide by one-twelfth of the difference between the true daily motions of the Sun and Moon, in degrees: the quotient denotes the true time in *ghaṭīs* (which has elapsed at sunrise since the beginning of the current lunar day).[5] (4). (KSS)

दर्शकाल:

11. 12. 1. गततिथिघटिकाघ्ने सूर्यशीतांशुभुक्ती
'गगनरस'विभक्ते शोधयेत् तत्कलाभ्य: ।
धनमपि च विदध्याद् भोग्यनाडीभिरेवं
समगृहलवलिप्तालिप्तिकौ तौ भवेताम् ।। २६ ।।
मासान्ते रविशशिनौ समौ भवेतां
पक्षान्ते लवकलिकाविलिप्तिकाभि: ।
अन्यस्यामपि च तिथौ तदावसाने
तुल्यौ स्त: खलु कलिकाविलिप्तिकाभि: ।। २७ ।।
(Lalla, *ŚiDhVṛ.*, 2. 26-27)

New and Full Moon

The true motions of the Sun and Moon should be multiplied by the *ghaṭikās* passed since the end of the fifteenth *tithi* and divided by 60. One should subtract the results (in minutes) from the respective minutes (in their true longitudes). The longitudes then have the same Signs, degrees, minutes and seconds, if the fifteenth *tithi* is *Amāvāsyā* (new moon) and the same degrees. minutes and seconds, if it is Pūrṇimā (full moon).

(The results obtained in the same manner from the) *ghaṭikās* to elapse till the fifteenth *tithi*, should be added.

(The same procedure should be followed for any other *tithi*). At the end of every other *tithi*, the Sun and Moon are equal in respect of minutes and seconds. (26-27). (BC)

पर्वराशि:

11. 12. 2. निरेकं द्वादशाभ्यस्तं द्विगुणं गतसंयुतम् ।
षष्टचा षष्टचा युतं द्वाभ्यां पर्वणां राशिरुच्यते ।। १३ ।।
स्यु: पादोऽर्धं त्रिपाद्या या त्रिद्व्येकेऽह्न: कृता स्थितिम् ।
साम्येनेन्दो: स्त्वृणोऽन्ये तु पर्वका: पञ्च सम्मता: ।।१४।।
भांशा: स्युरष्टका: कार्या: पक्षद्वादशकोद्भवा: ।
एकादशगुणश्चोन: शुक्लेऽर्धं चैन्दवा यदि ।। १५ ।।
नवकैरुद्गतोंऽश: स्यादून: सप्तगुणो भवेत् ।
आवापस्त्वयुजेऽर्धं स्यात् पौलस्त्येऽस्तं गते परम् ।। १६ ।।
जावाद्यंशै: समं विद्यात् पूर्वार्धे पर्वसूत्तरे ।
भाऽऽदानं स्याच्चतुर्दश्यां द्युभागेभ्योऽधिके सति ।। १७ ।।
जौ द्रा ग: खे श्वे ही रो षा चिन् मू ष ण्य: सू मा धा ण: ।
रे मृ धा स्वा पो ऽज: कृष्यो हृज्ये ष्ठा इत्यृक्षा लिङ्गै: ।।
कार्या भांशाष्टकस्थाने कला एकान्नविंशति: ।
ऊनस्थाने त्रिसप्ततिमुद्वपेदूनसंभवे ।। १९ ।।
तिथिमेकादशाभ्यस्तां पर्वभांशसमन्विताम् ।
विभज्य भसमूहेन तिथिनक्षत्रमादिशेत् ।। २० ।।
या: पर्वभाऽऽदानकलास्तासु सप्तगुणां तिथिम् ।
युक्त्या तासां विजानीयात् तिथिभाऽऽदानिका: कला: ।।
अतीतपर्वभागेभ्य: शोधयेद् द्विगुणां तिथिम् ।
तेषु मण्डलभागेषु तिथिनिष्ठांगतो रवि: ।। २२ ।।

. . . .

एकादशभिरभ्यस्य पर्वाणि नवभिस्तिथिम् ।
युगलब्धं सपर्व स्याद् वर्तमानार्कभं क्रमात् ।। २५ ।।

[1] i.e.,44,52,778.

[2] i.e., 1,57,79,17,500.

[3] i.e., 1,32,778.

[4] To obtain the Moon's equation of the centre, the mean longitude of the Moon may be obtained as follows: Multiply the mean lunar days, etc., (corresponding to the *ahargaṇa*) by 12, convert the resulting days etc. into months, etc., and add to them the mean solar months, etc., (corresponding to the *ahargaṇa*); treat the months, etc. thus obtained as the Signs, etc. of the mean longitude of the Moon.

[5] For exposition and rationale, see *MBh:KSS*, pp. 214-17.

सूर्यर्क्षभागान् नवभिर्विभज्य
शेषं द्विरभ्यस्य दिनोपभुक्तिः ।
तिथेर्युता भुक्तिदिनेषु कालो
योगो दिनैकादशकेन तद्भम् ॥ २६ ॥
त्र्यंशो भशेषो दिवसांशभाग-
श्चतुर्दशश्चाप्यपनीय भिन्नम् ।
भार्धेऽधिके चापि गते परोंशो
द्वावुत्तमे तन्नवकैरवेत्य ॥ २७ ॥

. . . .

द्यूनं द्विषष्टिभागेन हेयं सौर्यात् सपार्वणम् ।
यत्कृतावुपजायेते मध्येऽन्ते चाधिमासकौ ॥ ३७ ॥

. . . .

यदर्धं दिनभागानां सदा पर्वणि पर्वणि ।
ऋतुशेषं तु तद्विद्यात् संख्याय सह पर्वणाम् ॥ ४१ ॥
यदुत्तरस्यायनतो गतं स्यात्
शेषं तथा दक्षिणतोऽयनस्य ।
तदेकषष्ट्या द्विगुणं विभक्तं
सद्वादशं स्याद्दिवसप्रमाणम् ॥ ४० ॥

(*YV-VJ*, 13-40)

Parva-rāśi

Take the ordinal number of the year in the *yuga*. Lessen this by 1, multiply by 12, again multiply by 2, add the *parvas* gone in the year, for every 60 of the total *parvas* add 2, and the number obtained is the *parva-rāśi* or group of *parvas* (*i.e.*, the total number of *parvas* gone at the time, for which calculations are to be made). (13)

On account of the civil days of the *yuga* being divided into quarters, halves and three quarters corresponding to the divisibility of the Moon's asterisms in the *yuga*, the Moon's asterismal parts also are a quarter (i.e., 31 parts), two quarters (i.e., 62 parts), three quarters (i.e., 93 parts), without residue. But the other parts of the asterisms have to be measured in units of fifth divisions of the parts. (14)

Eight *nakṣatra* parts (*bhāṁśas*) are to be put down for every unit in the quotient of the number of *parvas* divided by 12. The remainder is to be multiplied by 11 and added. If the *parva* in question is full moon, 62 parts more are to be added if the parts refer to the Moon's *nakṣatras* (and not the Sun's). (15)

Dividing the number of *parvas* by 9, take one part for each quotient. Each of the remainder should be multiplied by 7 and added. If the quotient is odd, add half, i.e., 62 parts. If the Moon is setting when the Sun rises, i.e., if full moon *parva*, add another half, i.e., 62 *parvas*. (16)

Take the (twentyseven) *nakṣatras* represented by the abbreviations (of their names, viz., (1) *jau* (for Āśvayujau), (2) *drā* (for Ārdrā), (3) *gaḥ* (for bhagaḥ or Pūrva-Phalgunī), (4) *khe* (for Viśākhe), (5) *śve* (for Viśvedevāḥ or Uttarāṣāḍhā), (6) *hiḥ* (for Ahiḥ or Uttaraproṣṭhapadā), (7) *ro* (for Rohiṇī), (8) *ṣā* (for Āśleṣā), (9) *cit* (for Citrā), (10) *mū* (for Mūlā), (11) *ṣa* (for Śatabhiṣak), (12) *ṇyaḥ* (for Bharaṇyaḥ), (13) *sū* (for Punarvasū), (14) *mā* (for Aryamā or Uttara-Phalgunī), (15) *dhāḥ* (for Anūrādhāḥ), (16) *ṇaḥ* (for Śravaṇaḥ), (17) *re* (for Revatī), (18) *mṛ* (for Mṛgaśirāḥ), (19) *ghāḥ* (for Maghāḥ), (20) *svā* (for Svātī), (21) *paḥ* (for Āpaḥ or Pūrvāṣāḍhā), (22) *jaḥ* (for Aja Ekapāt or Proṣṭhapadā), (23) *kṛ* (for Kṛttikā), (24) *ṣyaḥ* (for Puṣyaḥ), (25) *ha* (for Hastaḥ), (26) *jye* (for Jyeṣṭhā) and (27) *ṣṭhāḥ* (for Śraviṣṭhā), in the order of one, two, three etc. of the parts of the *nakṣatras* (found in verse 15) each to each. Then so many parts of that *nakṣatra* has gone at the end of that *parva* for which that *bhāṁśa* has been found. If the *parva* falls within the first half of the *parva-nakṣatra*, (i.e., if the *nakṣatra* part is 62 or less at *parva*), the beginning of the *nakṣatra* to be found (*bhādānam*) will fall in the *parvatithi* (i.e., the 15th *tithi*) itself. If the *nakṣatra* parts is greater than the parts of the day at which the *parva* falls, the beginning of the *nakṣatra* falls in *caturdaśī tithi* day. (17-18)

For every eight *nakṣatra* parts, 19 *kalā*-s are to be set down for work. For the rest (i.e., the remainder), when the remainder occurs, take away 73 *kalā*-s for each of the number remaining. (We then get the time of the day, in *kalā*-s, at the beginning of the *parva nakṣatra*, i.e., the Moon's *nakṣatra* at new or full moon). (19)

Multiplying the *tithis* elapsed after a *parva* by 11, and adding it to the parts of the *nakṣatra* current at the end of the *parva*, and dividing out by 27 and taking the remainder, and using this in the *jāvādi* series (set out in verses 17-18), the *nakṣatra* current at the *tithi* can be found. (20)

Adding *kalā-s* equal to seven times the *tithis* elapsed after the *parva* to the *bhādāna-kalā* of the *parva*, we get the *bhādāna-kalā* pertaining to the *tithis* (i.e., the times of the beginning of the *nakṣatras* current at the end of the *tithis*). (21)

Subtract twice the number of *tithis* after a *parva* from the parts of the day ending the *parva*. We get the parts of the day when the *tithi* ends, which is the same as the position of the Sun in the diurnal circle (technically called *nāḍīmaṇḍala*) divided into 124 parts. (22)

Nakṣatra of the Sun

Multiplying the number of *parva-s* elapsed by 11, and the *tithis* elapsed after that by 9, adding the two and dividing the total by the number of *parva-s* in the *yuga*, (viz., 124) taking the quotients and parts, and adding the number of the *parva-s* to be quotient, we get the total number and parts of the Sun's *nakṣatra* which have

elapsed, counted from the *nakṣatra* Śraviṣṭhā in the regular order. (25)

Yoga and its Nakṣatra

Divide the parts of the Sun's *nakṣatra* by 9. Multiply the remainder by 2. This is termed the partial complement for the day's parts. Add to this the complement for the *tithi-s*, got as quotient, to obtain the total complement, to be added to the day's parts. (26)

Add the total complement to the *nakṣatra* parts and divide by 11. The result is the day before the completed day's parts, when the *nakṣatra* began.

The excess in the rising of the zodiac over the diurnal circle in terms of its 124th parts is a third of the number of days elapsed in the year, rounding off the fraction up to fourteen days in any *parva*. When (nearly) half a zodiac has been found to have risen, add one more part and as the second half of the zodiac is (nearly) completed, add another one. This result can be obtained by extending the *navaka* rule (already given for the ending moments of the parvas, verses 16-17). (27)

Adhimāsa

The lunar day is less than the civil day by its 62nd part. The civil day is subtractable from the solar day (i.e., less than the latter). This defect, combined with the defect in the lunar day causes one extra month at the end of half-*yuga* and another month at the end of the full *yuga*. (37)

Ṛtu-śeṣa

Adding all the half *tithi-s* occurring after each *parva* of all the *parva-s* (that pass normally at the rate of 4 per *parva*) we get what is called *ṛtu-śeṣa* (that is, the *tithi-s* which remain in the last *ṛtu* and have to be passed to complete it). (41)

Length of day-time

The number of days which have elapsed in the northward course of the Sun (*uttarāyaṇa*), or the remaining days in the southward course (*dakṣiṇāyana*), doubled and divided by 61 plus 12, is the day-time (in *muhūrta-s*) of the day taken. (40). (TSK)

योगः

11. 13. 1. शशिमित्रैक्यं कृत्वा हत्वाऽष्टशतैस्तु योगभानि स्युः ।
शशिमित्रभुक्तियुत्या हृतशेषे नाडिका ज्ञेयाः ।। ४५ ।।

(Deva, *KR*, 1.45)

Yoga

Add the longitudes of the Sun and the Moon (and reduce the sum to minutes) and then divide it by 800: the quotient gives the (number of) *yoga-nakṣatras* passed over. Divide the remainder and the divisor *minus* remainder by the sum of daily motions of the Sun and the Moon (in degrees): the results obtained give the *nāḍīs* elapsed and the *nāḍīs* to elapse (of the current *yoga*).[1] (45). (KSS)

गोलायनसन्धिः

11. 14. 1. चक्रे चक्रार्धे च व्ययनांशेऽर्कस्य गोलसन्धिः स्यात् ।
एवं त्रिभे च नवमेऽयनसन्धिर्व्ययनभागेऽस्य ।। २ ।।
अयनांशोनितपाताद्दोःकोटिज्ये लघुज्यकोत्थे ये ।
ते 'गुणसूर्यै''रश्वैः' गुणिते भक्ते 'कृतैः' 'सूर्यैः' ।। ३ ।।
अयनांशोनितपाते मृगकर्क्यादिस्थिते 'द्विषड्रामैः' ।
कोटिफलयुतविहीनैर्बाहुफलं भक्तमाप्तांशैः ।। ४ ।।
मेषादिस्थे गोलायनसन्धी भास्करस्योनौ ।
तौ चन्द्रस्य स्यातां तुलादिषड्भस्थिते तु संयुक्तौ ।। ५ ।।
गोलायनसन्ध्यन्तं पदं विधोरत्र धीमता ज्ञेयम् ।
रविगोलवदस्पष्टा स्पष्टा क्रान्तिः स्वगोलदिक् शशिनः ।।

क्रान्तिसाम्यज्ञानम्

स्वायनसन्धाविन्दोः क्रान्तिस्तत्कालभास्करक्रान्तेः ।
ऊना यावत् तावत् क्रान्त्योः साम्यं तयोर्नास्ति ।। ७ ।।

(Bhāskara II, *SiŚi.*, 1.12.2-7)

Gola-sandhi and Ayana-sandhi

When the tropical longitude of the Sun, i.e. its longitude measured from the first point of Aries along the ecliptic, is equal to 0° or 180°, then the Sun is said to be at its *gola-sandhi*. In other words, when the Sun which moves along the ecliptic comes to the celestial equator, it will be at the *gola-sandhi*. Similarly, when its longitude is 90° or 270°, it is said to be at its *ayana-sandhi*. (2)

Speciality in Moon

Let the R sine and R cosine of the tropical longitude of the *Pāta* (Rāhu, the ascending node of the lunar orbit)

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[1] The *yogas*, like the *nakṣatras*, are 27 in number, and their names in the order of their occurrence, are as follows:

1. Viṣkambha	15. Vajra
2. Prīti	16. Siddhi
3. Āyuṣmān	17. Vyatīpāta
4. Saubhāgya	18. Variyān
5. Śobhana	19. Parigha
6. Atigaṇḍa	20. Śiva
7. Sukarmā	21. Siddha
8. Dhṛti	22. Sādhya
9. Śūla	23. Śubha
10. Gaṇḍa	24. Śukla
11. Vṛddhi	25. Brahmā
12. Dhruva	26. Indra
13. Vyāghāta	27. Vaidhṛta or Vaidhṛti.
14. Harṣaṇa	

It should be noted that in calculating the *tithi*, *nakṣatra*, *yoga* and *karaṇa*, the precession of the equinoxes is not applied to the longitude of the Sun. But in calculating the *Vyatīpāta* it is applied.

as per the smaller table of R sines when the radius is taken to be 120′, be respectively multiplied by 123 and 7 and divided respectively by 4 and 12. The results are known as the *bāhuphala* and *koṭiphala*, respectively. According as the tropical longitude of Rāhu, (say, λ), be such that as $270° \angle \lambda \angle 90°$ or $90° \angle \lambda \angle 270°$, the *bāhuphala* is to be divided by $362 + koṭiphala$. Subtract the result from the *gola-sandhis* and *ayana-sandhis* of the Sun, to get those of the Moon. (3-6)

Occurrence of a pāta

If the *sphuṭakrānti* of the Moon, when it is maximum, be less than that of the Sun, then there could be no occasion for their declinations to become equal in the near future.[1] (7). (AS)

व्यतीपातवैधृतौ

11. 15. 1. सूर्येन्दुयोगे चक्रार्धे व्यतीपातोऽथ वैधृतः ।
चक्रे च मैत्रपर्यन्ते विज्ञेयः सार्पमस्तकः ।। २६ ।।

(Bhāskara I, *LBh.*, 2. 29)

Vyatīpāta and Vaidhṛta

When the sum of the (true) longitudes of the Sun and the Moon amounts to half a circle (i.e., 180°), the phenomenon is called *(Lāṭa)-vyatīpāta*; when that (sum) amounts to a circle, (i.e., 360°), the phenomenon is called *Vaidhṛta- (vyatīpāta)*; and when that (sum) extends to the end of the *nakṣatra* Anurādhā (i.e., when the sum amounts to 7 signs, 16 degrees, and 40 minutes), the phenomenon is called *Sārpamastaka-(vyatīpāta)*. (29). (KSS)

व्यतीपात-वैधृतगणनम्

पातमध्यकालः

11. 15. 2. चक्रार्धचक्राधिकहीनलिप्ताः
'खषड्'गुणा भुक्तिसमासभक्ताः ।
भवन्ति नाड्यो गतगम्यसंज्ञा-
स्तात्कालिकौ तौ च तमश्च कुर्यात् ।। २ ।।
तयोरपि क्रान्तिकलाः समाश्चेत्
तदा भवेन्मध्यमपातकालः ।
कल्प्योऽधिकोऽप्यूनक एव चन्द्रः
स्फुटोऽपमश्चान्द्रमसोऽन्यदिक्स्थः ।। ३ ।।
सूर्यापमादोजपदोद्भवाच्चेद्
युग्मादिगश्चान्द्रमसो लघीयान् ।
अपक्रमः स्यान्न तदस्ति पात-
स्तदन्यथात्वेऽपमयोः समत्वम् ।। ४ ।।
अयुग्मजश्चान्द्रमसोऽपमश्चे-
दपक्रमाद् भानुमतोऽधिकः स्यात् ।
समोद्भवो वापि लघुस्तदेतो
विपातकालो भवितान्यथातः ।। ५ ।।
क्रान्त्योर्युतिरेकदिक्कयोर्विवरं भिन्नदिशोस्तु वैधृते ।
विवरं समदिक्कयोस्तयोर्व्यतिपातेऽन्यदिशोः समागमः ।।
प्रथमः स तथापरो युतै रहितैरिष्टघटीफलेन तैः ।
गतयोरथवापि गम्ययोर्विवरं संयुतिरन्यथा तयोः ।। ७ ।।
प्रथमेष्टघटीवधेऽमुना विहृते लब्धघटीमितेऽन्तरे ।
पातः प्रथमे गतागते गतगम्यः प्रथमाख्यकालतः ।। ८ ।।
तात्कालिकैरसकृदिष्टघटीफलोन-
युक्तैस्तथा प्रथमराशिरनेन भक्तम् ।
मानैक्यमिष्टघटिकाहतमाप्तनाड्यः
पातस्थितिर्ग्रहणवत् प्रथमान्त्यकालौ ।। ६ ।।

(Lalla, *ŚiDhVṛ.*, 12. 2-9)

Computation of Vyatīpāta and Vaidhṛta
Time of mid-pāta

Find the difference in minutes (between the sum of the true longitudes of the Sun and the Moon at any given time) and 6 Signs or 12 Signs. Multiply it by 60 and divide by the sum of the true daily motions of the two. The result gives in *ghaṭikās* etc. the time elapsed since the *vyatīpāta* or *vaidhṛta* took place or to elapse, (according as the sum is greater than 6 or 12 Signs or less). Then find the true longitudes of the Sun, the Moon and its Node for that time. (2)

At that time, if the true declination of the Moon's centre is equal to the declination of the Sun's centre, both expressed in minutes, the time is called middle of *pāta*. (3a-b)

Equality of declination

When the Moon's true declination is obtained by subtracting the mean declination from its latitude, even if the true declination is numerically greater than that of the Sun, it must be considered as less. (3c-d)

If the Moon's true declination at the beginning of an even quadrant, (that is, at the end of an odd quadrant), is less than that of the Sun in an odd quadrant, there is no possibility of *pāta*. If, in this case, the Moon's true declination is greater, there is a possibility of the declination being equal and hence of *pāta*. (4)

When the true declination of the Moon in an odd quadrant is greater or in an even quadrant less than that of the Sun, the *pāta* has taken place. In the contrary cases, it will take place. (5)

Time of Vyatīpāta and Vaidhṛta

(Find the Sun's declination and the Moon's true declination at the time when the sum of their true longitudes is 6 or 12 Signs). In the case of *vaidhṛta*, find the sum of the declinations, if in the same direction: but difference, if in opposite directions. In the case of *vyatīpāta*, find the difference, if in the same direction, and the sum, if in opposite directions. (6)

[1] For exposition and the rationale, see *SiŚi:AS*, pp. 521-33.

In each case, the result is called *prathama* (or 'the first'). In the same manner find *apara* (or 'the other'), by assuming some time (before or after that when the sum of the longitudes is 6 or 12 signs, by calculating the longitudes of the Sun, the Moon and its node for that time and hence the declination) and by taking their sum or difference (as above).

If both the groups of declinations indicate that the *pāta* has either taken place or will take place, find the difference of the first (*prathama*) and the other (*apara*). But find their sum, if one group indicates that *pāta* has taken place, and the other indicates that the *pata* will take place. (7)

By this sum or difference divide the product of the first and the assumed time. The result in *ghaṭikās* gives the time of the middle of *pāta*, before or after the time when the sum of the longitudes of the Sun and the Moon is 6 or 12 Signs, *i.e.*, before, if the first group of declinations shows that the *pāta* has taken place, and after, if it shows that the *pāta* will take place. (8)

Iterate the above process by repeatedly finding the true longitudes (of the Sun, the Moon and its node for the corrected time and hence their declination, till the time is fixed). Multiply the sum of the diameters of the Sun and the Moon by the fixed time and divide the product by the *prathama* or 'first'. The result in *ghaṭikās*, is the duration of the *pāta*. Thus find the times for the beginning and end of the *pāta*, as in the case of an eclipse. (9).[1] (BC)

11. 15. 3. सूर्येन्दुयोगे चक्रार्धे व्यतीपातोऽथ वैधृतः ।
चक्रे च मैत्रपर्यन्ते विज्ञेयः सार्पमस्तकः ।। ४६ ।।

. . . .

सूर्याचन्द्रमसोर्गोलं ज्ञात्वा लिप्तीकृतावमू ।
'अनन्ते'न हरेल्लब्धं तयोर्वाक्यप्रमा भवेत् ।। ४८ ।।

तद्वाक्ये पृथगेवात्र स्थापयेच्छेषमिश्रिते ।
'प्रभारत्नं' 'धीसवनं' 'गानस्थानं' 'जने धनम्' ।। ४९ ।।

'देहि नित्यं' 'सुगुप्रायः' 'सर्वलोकं' 'तटिद्वपुः' ।
'नव वाक्ये'ति वाक्यनि प्रोक्तानि रविसोमयोः ।। ५० ।।

भूयो हीनतमश्चन्द्रगोलवित्तमसः कलाम् ।
कृत्वा पूर्वविधानेन वाक्यान्येतानि चिन्तयेत् ।। ५१ ।।

'सवनानि' 'प्रधानानि' 'शालायां' 'वासुकी ननु' ।
'सेना राज्ञो' 'भृगुप्रायो' 'वामश्री''श्चतुरो''ऽसुरः' ।।५२।।

एतास्तात्कालिका गोले समभिन्ने युतायुताः ।
चन्द्रस्य स्थानिके मध्ये तदा चान्द्रं फलं स्फुटम् ।। ५३ ।।

रविक्रान्तेर्भुजाच्चन्द्रो महांश्चेत् स गतो ध्रुवम् ।
अल्पः कोटिशशी तद्वद्विपरीते विपर्ययः ।। ५४ ।।

यदा समानता क्रान्त्योः सूर्याचन्द्रमसोस्तदा ।
चक्रार्धं तद्विनिर्दिष्टं सर्वकार्यविगर्हितम् ।। ५५ ।।

[1] For detailed elucidation, see *ŚiDhVṛ:BC.*, II. 201-05.

यदा विषमता क्रान्त्योः विवरेण तयोस्तदा ।
'अनन्तं' गुणयेत्पूर्ववाक्यहीनेतरेण तु ।। ५६ ।।

हृत्वा लब्धकलाभिस्तु गतैष्ये हीनयुक् शशी ।
पुनर्गोलं परिज्ञाय स्फुटेद्यावत्समं भवेत् ।। ५७ ।।

(Deva, *KR*, 1. 46-57)

The Three Vyatīpātas

When the sum of the (true) longitudes of the Sun and the Moon amounts to half a circle (i.e., 180°), the phenomenon is called (Cakrārdha-Vyatīpāta or Lāṭa-)Vyatīpāta; when that sum amounts to a circle (i.e., 360°) the phenomenon is called Vaidhṛta-(Vyatīpāta); and when that (sum) extends to the end of the *nakṣatra* Anurādhā (i.e., when that sum amounts to 7 signs 16° 40′), the phenomenon is called Sārpamastaka-(Vyatīpāta). (46)

Calculation of Vyatīpāta: Declination

In the case of the Sun and the Moon, having determined its distance from the first point of Aries or Libra, (whichever is nearer), reduce it to minutes and divide by 600: the quotient gives the serial number of the tabular declination (crossed). Set down that declination in a separate place, and add to it the declination difference corresponding to the remainder of the division. (This sum is the desired declination). (48-49a)

The Declination table

242′, 479′, 703′, 908′, 1088′, 1237′, 1347′, 1416′, and 1440′—these are stated to be the declinations (at the intervals of 10 degrees of the ecliptic, from the first point of Aries) for the Sun and the Moon. (49-50).

Moon's latitude

Now, the astronomer should subtract the longitude of the Moon's ascending node from the longitude of the Moon and reduce the resulting distance (of the Moon) from the Moon's ascending node to minutes. Then (dividing it by 600) find, as before, the serial number of the Moon's tabular latitude crossed. Then make use of the following table of latitudes. (51)

Table of Moon's latitudes

47′, 92′, 135′, 174′, 207′, 234′, 254′, 266′, and 270′—(these are the Moon's latitudes at the successive intervals of 10° from the Moon's ascending node.)[1] (52)

[1] Mallikārjuna Sūri (A.D. 1178), in his commentary on the *Sūryasiddhānta* (9. 14-15), quotes another list of Moon's latitudes at the successive intervals of 10°, from his own work *Gaṇaka-hitārtha*. This list has 173′ in place of 174′; otherwise it is the same as the above one. Viṣṇu Śarmā (*c.* A.D. 1363), in his commentary on the *Vidyā-Mādhavīya* (2.30), quotes a similar list of the Moon's latitudes at the successive intervals of 10°. This list has 93′ in place of 92′; otherwise it is exactly the same as the above one, given by Deva. Makaranda (*c.* A.D. 1478) gives the Moon's latitude to two decimal places for every degree of the ecliptic as measured from the Moon's ascending node.

Moon's true declination

The instantaneous latitude of the Moon added to or subtracted from the declination of the Moon's projection on the ecliptic, according as the two are of like or unlike directions, gives the true declination of the Moon. (53).

Time of Vyatīpāta

When the *bhujā* of the Moon's longitude is greater than the *bhujā* of the longitude corresponding to the Sun's declination, (it should be understood that) the phenomenon of *Vyatīpāta* has already occurred. Similar is the case when the *koṭi* of the Moon's longitude is less then the *koṭi* of the longitude corresponding to the Sun's declination. In the contrary case, it is just the reverse (i.e. it should be understood that the phenomenon of *Vyatīpāta* is to occur.) (54)

When there is equality of the Sun's and Moon's declinations, (both in magnitude and direction), then the *Vyatīpāta* is called *Cakrārdha* (or *Lāṭa-Vyatīpāta*). It is prohibited for all (auspious) acts. (55)

When the declinations of the Sun and the Moon are unequal, multiply their difference by 600 and divide (the product) by the succeeding tabular declination *minus* the preceding tabular declination (given in verses 49-50) and subtract the (resulting) quotient from or add that to the Moon's longitude, according as the *Vyatīpāta* has occurred or is to occur. Again calculate the Moon's declination and iterate the process until the declinations (of the Sun and Moon) become equal.[1] (56-57). (KSS)

करणानि

11..16. 1. बवबालवकौलवतैतिलाख्यगरवणिजविष्टिसंज्ञानाम् ।
पतयः स्युरिन्द्रकमलजमित्रार्यमभूश्रियः सयमाः ।। १ ।।
कृष्णचतुर्दश्यर्धाद् ध्रुवाणि शकुनिश्चतुष्पदं नागम् ।
किंस्तुघ्नमिति च तेषां कलिवृषफणिमारुताः पतयः ।।२।।

(Varāha, *Bṛ.Saṃ.*, 100. 1-2)

Karaṇas

The lords of the seven movable karaṇas, viz. Bava, Bālava, Kaulava, Taitila, Gara, Vaṇija and Viṣṭi, are Indra, Brahman, Mitra, Aryaman, Bhū (Earth), Śrī (Goddess of Wealth) and Yama (Death), respectively. (1)

The fixed or *Dhruva-karaṇas*, viz. Śakuni, Catuṣpada, Nāga and Kiṁstughna, begin from the latter half of the 14th day of the dark fortnight and are presided over by Kali, Vṛsa, Pha;;in (Serpent) and Māruta (Wind), respectively. (2).[2] (M.R. Bhat)

करणगणनम्

11. 16. 2. सितबहुलयोः क्षयधनं षड्भागाः शीतगोर्विरविभोगात् ।
लिप्ता 'खर्तुहुताशै'र्लब्धं करणं तिथिवदन्यत् ।। १८ ।।
बहुलचतुर्दश्यर्धाद् ध्रुवाणि शकुनिश्चतुष्पदं नागः ।
किंस्तुघ्नमिति च करणान्यर्धे चराणि प्रवर्तन्ते ।। १९ ।।

(Varāha, *PS*, 3. 18-19)

Karaṇas—Computation

In the light fortnight, take the Moon *minus* the Sun and subtract from it 6°. For the dark fornight take the Moon *minus* the Sun from the beginning of the dark fortnight, i.e., take the Moon *minus* the Sun with 6 *rāśis* subtracted from it, and add 6°. Convert it into minutes and divide by 360′. What are obtained are the *cara-karaṇas* Bava etc. coming one after another repeatedly. Take the remainder and treat it as the remainder in calculating the *tithi*, (i.e., multiply 60, and divide by the difference of the daily motions of the Sun and the Moon in minutes etc.) and thus get the ending moment of the last *karaṇa*. In each *tithi* the first half is one *karaṇa* and the second half another. (18)

In the four half *tithis*, viz., the second half of Bahula-caturdaśī, the two halves of Amāvāsyā, and the first half of Śukla-pratipad, are the *sthira-karaṇas*, Śakuni, Catuṣpāda, Nāga and Kiṁstughna, respectively. Then from the remaining half, the *cara-karaṇas* come in the order Bāva, Bālava, Kaulava, Taitila, Gara, Vaṇijya (Vaṇija) and Viṣṭi (Bhadra), (repeating eight times). (19). (TSK)

11. 16. 3. तिथ्यर्धंह्रारलब्धानि करणानि बवादितः ।
विरूपाणि सिते पक्षे सरूपाण्यसिते विदुः ।। ३३ ।।

(Bhāskara I, *MBh.*, 4. 33)

The *karaṇas* (elapsed) are obtained by taking "half the measure of a *tithi* (i.e., 360 minutes)" for the diviser, and are counted with Bava. But the number of *karaṇas* elapsed in the light half of the month should be diminished by *one*, whereas those elapsed in the dark half of the month should be increased by *one*. This is what has been stated.[3] (33). (KSS)

11. 16. 4. व्यर्केन्दुकला भक्ताः 'खरसुगुणै'र्लब्धमूनमेकेन ।
चलकरणानि बवादीन्यगहृतशेषे तिथिवदन्यत् ।। ५८ ।।

[1] For further details regarding *Vyatīpāta* (also called *Pāta Mahāpāta*) see Appendix 1, *Mahāpātādhyāya, KR:KSS*, pp. 102-110.

[2] The fixed *karaṇas* appear only once in a lunar month. A *karaṇa* is equal to half a *tithi*. So there must be 60 *karaṇas* in a month. But the fixed ones are assigned to the latter half of the dark 14th day, the two halves of the new moon and the first half of the first lunar day of the bright fortnight. Hence 56 *karaṇas* remain to be filled with the 7 *karaṇas* repeated 8 times. Bava prevails in the latter half of the bright *pratipad*; Bālava and Kaulava in the two halves of the second; Taitila and Gara in the two parts of the third *tithi*; *Vaṇij* and Viṣṭi in those of the fourth. In this manner these seven movable *karaṇas* have to be repeated in both the fortnights, without touching the provinces of the four fixed *karaṇas*.

[3] For the rationale, see *MBh:KSS*, p.131.

कृष्णचतुर्दश्यन्ते शकुनिः पर्वणि चतुष्पदं प्रथमे ।
तिथ्यर्धेऽन्त्ये नागं किंस्तुघ्नं प्रतिपदाद्यर्धे ॥ ५९ ॥

(Deva, *KR*, 1. 58-59)

The longitude of the Moon minus the longitude of the Sun should be reduced to minutes and then divided by 360. The (resulting) quotient should be diminished by 1 and then divided by 7. The remainder, counted from *Bava*, gives the (number of) movable *karaṇas* (passed). To find the rest (i.e., the *ghaṭīs* elapsed of the current *karaṇa* or *ghaṭīs* to elapse before the beginning of the next *karaṇa*), one should proceed as in the case of the *tithi*. (58). (KSS)

(Of the four immovable *karaṇas*) Śakuni occurs in the second half of the 14th *tithi* in the dark half of the month; Catuṣpada occurs in the first half and *Nāga* in the second half of the new moon *tithi*; Kiṁstughna in the first half of the first *tithi* (in the light half of the month.)[1] (59). (KS)

[1] Śakuni, Catuṣpada, Nāga, and Kiṁstughna are called immovable karaṇas because their positions are fixed with respect to the *tithis*. Śakuni is a hawk; Catuṣpada is a quadruped; *Nāga* is a serpent; and Kiṁstughna is an animal. Their lords are Kaliyuga, Rudra, serpents and wind, respectively.

The seven movable *karaṇas*, in the order in which they occur, along with their lords, are as follows;

1. Bava or Baba (=lion; Babara in Hindi): Indra; 2. Bālava (=tiger): Dhātā; 3. Kaulava (=Boar, Derived from *Kola*, meaning boar): Mitra; 4. Taitila (=donkey): Aryamā; 5. Gara or Gaja (=elephant): Earth; 6. Vaṇij or Vaṇik (=businessman): Lakṣmī; 7. Viṣṭi or Bhadrā (=cow): Yama.

This cycle of the seven movable *karaṇas* starts from the second half of the first *tithi* in the light half of the month and repeats itself eight times, whereafter the four immovable *karaṇas* commence.

12. नक्षत्राणि – STARS AND ASTERISMS

उद्यत्ताराणां वेदेऽभिधानम्

12. 1. 1. उदगातां भगवती विचृत्तौ नाम तारके ।
विक्षेत्रियस्य मुञ्चातां अधमं पाशमुत्तमम् ॥ १ ॥
(*AV*, 2. 8. 1)

Rising stars mentioned in the Vedas

Arisen are the two blessed stars called *Vicṛtta-s* (i.e. Mūla). Let them unfasten the *kṣetriya* (disease), the lowliest, the highest fetter.

12. 1. 2. सुपर्णा एत आसते मध्ये चाराधने दिवः ।
ते सेधन्ति पथो वृकं तरन्तं यह्वतीरपः ॥ ११ ॥
(*RV*, 1. 105. 11)

High in the mid ascent of the heaven, those birds of beautious pinion (the stars) sit. Back from his path, they drive the Wolf (Sirius) who would cross the restless floods.

वेदे नक्षत्राणां स्थाननिश्चयः

12. 2. 1. यत् पुण्यं नक्षत्रम् । तत् बद् कुर्वीत उपव्युषम् । यदा वै सूर्य उदेति अथ नक्षत्रं नैति । यावति तत्र सूर्यो गच्छेत् यत्र जघन्यं पश्येत् तावति कुर्वीत । यत्कारी स्यात् । पुण्याह एव कुरुते । एवं ह वै यज्ञेषुं च शतद्युम्नं च मात्स्यो निरवसाययांचकार ।
(*Taitt. Brāhmaṇa*, 1. 5. 2. 1)

Fixing positions of stars in the vedas

The auspicious star, its position has to be determined at sunrise. But when the Sun rises, that star would not be visible (on account of the brightness of the Sun). So, before the Sun rises, watch for the adjacent star. By performing the rite with due time adjustment, one would have performed the rite at the correct time. Thus it was that sage Mātsya perform the sacrifices of Yajñeṣu and Śatadyumna to perfection. (KVS)

वेदे नक्षत्राणां परिसंख्यानम्

12. 3. 1. चित्राणि साकं दिवि रोचनानि
सरीसृपाणि भुवने जवानि ।
तुर्मिशं सुमतिमिच्छमानो अहानि
गीर्भिः सपर्यामि नाकम् ॥ १ ॥

सुहवमग्ने कृत्तिका रोहिणी
चास्तु भद्रं मृगशिरः शमार्द्रा ।
पुनर्वसू सूनृता चारु पुष्यो
भानुराश्लेषा अयनं मघा मे ॥ २ ॥

पुण्यं पूर्वा फल्गुन्यौ चात्र हस्त-
श्चित्रा शिवा स्वाति सुखो मे अस्तु ।
राधे विशाखे सुहवानुराधा
ज्येष्ठा सुनक्षत्रमरिष्ट मूलम् ॥ ३ ॥
अन्नं पूर्वा रासतां मे अषाढा
ऊर्जं देव्युत्तरा आ वहन्तु ।
अभिजिन्मे रासतां पुण्यमेव
श्रवणः श्रविष्ठाः कुर्वतां सुपुष्टिम् ॥ ४ ॥

आ मे महच्छतभिषग् वरीय
आ मे द्वया प्रोष्ठपदा सुशर्म ।
आ रेवती चाश्वयुजौ भगं म
आ मे रयिं भरण्य आ वहन्तु ॥ ५ ॥
(*AV*, 19. 7.1-5)

Vedic enumeration of asterisms

The brilliant lights shining in heaven together, which through the world glide on with rapid motion.

And days, and firmament with songs I worship seeking (the twentyeight-fold) for its favour. (1)

Kṛttikās, Rohiṇī be swift to hear me. Let Mṛgaśiras bless me, help me Ārdrā.

Punarvasu and Sūnṛtā, fair Puṣya, the Sun, Āślcṣas, Maghā, lead me onward. (2)

May bliss be Svāti and benignant Citrā, may right Pūrvaphalgunīs, and present Hasta.

Rādhās, Viśākhās, gracious Anurādhā, Jyeṣṭha and happy starred uninjured Mūla. (3)

Food shall be the Pūrva-āṣāḍhā-s grant me; let those that follow bring me strength and vigour.

With virtuous merit Abhijit endow me; Śravaṇa and Śraviṣṭhās make me prosper. (4)

Śatabhiṣak afford me ample freedom, and both the Proṣṭhapadas guard me safety.

Revatī and Aśvayujas bring me luck and Bharaṇīs abundant riches.[1] (5)

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[1] The seer of this hymn is Gārgya, another of whose hymns, *AV* 19.8, is also on the asterisms. The number of the asterisms as 28 occurs in that hymn:

aṣṭāviṃśāni śivāni śugmāni saha yogam bhajantu me |

'Propitious, let the eight and twenty together deal me out my share of profit.'

The *Taittirīya Saṃhitā* of the *Yajurveda* (4.4.10) also enumerates the asterisms, while the *Ṛgveda* mentions some of them in dfferent contexts, as required for prayer and worship, e.g., Rohiṇī (*RV* 1.62.9; 8.93.13; 8.101.13); Punarvasu (*RV* 10.19.1) and Citrā several times.

For a documented study see, Satya Prakash, *Founders of Sciences in Ancient India.*, ch. IV. 'Gārgya, th first enumerator of constellations', pp. 125-51.

12. 3. 2. अथ नक्षत्रकल्पं व्याख्यास्यामः ।

कृत्तिका रोहिणी मृगशिरा आर्द्रा पुनर्वसू पुष्य आश्लेषा मघाः फाल्गुनीफाल्गुन्यौ हस्तः चित्रा स्वाती-विशाखे अनुराधा ज्येष्ठा मूलम् पूर्वाषाढा उत्तराषाढा अभिजित् श्रवणः श्रविष्ठा शतभिषः पूर्वप्रोष्ठपदोत्तरप्रोष्ठपदौ रेवती आश्वयुजौ भरण्यः ।

(*AV Pariśiṣṭa* 1: *Nakṣatrakalpa*, 1)

Now we shall expound the science of constellations.

(The constellations are:) Kṛttikā, Rohiṇī, Mṛgaśiras, Ārdrā, Punarvasū, Puṣya, Āśleṣā, Maghā, (Pūrva-) phalgunī, (Uttara-)phalgunī, Hasta, Citrā, Svāti, Viśākhā, Anurādhā, Jyeṣṭhā, Mūla, Pūrvāṣāḍhā, Uttarāṣāḍhā, Abhijit, Śravaṇa, Śraviṣṭhā, Śatabhiṣak, Pūrva-proṣṭhapada, Uttara-proṣṭhapada, Revatī, Aśvayuk (Aśvinī) and Bharaṇī. (KVS)

प्रतिनक्षत्रं ताराणां संख्या

12. 4. 1. षट् कृत्तिका, एका रोहिणी, तिस्रो मृगशिराः, एकाऽऽर्द्रा, द्वे पुनर्वसू, एकः पुष्यः, षड् आश्लेषाः, षण् मघाः, चतस्रः फाल्गुन्यः, पञ्च हस्तः, एका चित्रा, एका स्वातिः, द्वे विशाखे, चतस्रोऽनुराधा, एका ज्येष्ठा, सप्त मूलम्, अष्टावाषाढा, एकोऽभिजित्, तिस्रः श्रवणः, पञ्च श्रविष्ठा, एका शतभिषा, चतस्रः प्रोष्ठपदौ, एका रेवती, द्वे आश्वयुजौ, तिस्रो भरण्यः ।। इति संख्यापरिमितं ब्रह्म ।।

(*AV Pariśiṣṭa* 1: *Nakṣatrakalpa*, 2)

Number of stars in the asterisms

(The number of stars in the several constellations are:) 6 for Kṛttikā; 1 for Rohiṇī; 3 for Mṛgaśiras; 1 for Ārdrā; 2 for Punarvasū; 1 for Puṣya; 6 for Āśleṣā; 6 for Maghā; 4 for the Phalgunis (2 for Pūrva-phalgunī and 2 for Uttara-phalgunī); 5 for Hasta; 1 for Citrā; 1 for Svāti; 2 for Viśākhā; 4 for Anurādhā; 1 for Jyeṣṭhā; 7 for Mūla; 8 for the Aṣāḍhas (7 for Pūrva-aṣāḍhā; and 1 for Uttara-aṣāḍhā); 1 for Abhijit; 3 for Śravaṇa; 5 for Śraviṣṭhā; 1 for Śatabhiṣak; 4 for the Proṣṭhapadas; (2 for Pūrva-proṣṭhapada and 2 Uttara-proṣṭhapada); 1 for Revatī; 2 for Āśvayuk (Aśvinī); and 3 for Bharaṇī. Thus are the constellations limited in the number of stars.[1]

कृत्तिकायास्ताराः

12. 4. 2. अम्बा दुला नितत्निरभ्रयन्ती मेघयन्ती वर्षयन्ती चुपुणीका नामासि ।

(*Taitt. Saṃhitā*, 4. 4. 5)

Names of the stars in the asterism Pleidas

(The names of the stars in the asterism Pleidas are): Ambā, Dulā, Nitatni, Abhrayantī, Meghayantī, Varṣayantī and Cupuṇikā. (KVS)

[1] For a documented study, see B.R. Modak, '*Nakṣatrakalpa*', *Proc. of the 26th International Congress of Orientalists, New Delhi*, 1964, vol. III, pt. i, pp. 119-22.

प्रतिनक्षत्रं नक्षत्रसंख्या

12. 4. 3. मुलाजाहिर्बुध्न्याश्वयुगदितीन्द्राग्नीफल्गुनीद्वितयम् ।
त्वाष्ट्रगुरुवारुणार्द्रानिलपौष्णान्येकताराणि ।। १ ।।
ब्रह्मेन्द्रयमहरीन्दुत्रितयं षड्वह्निभुजगपित्र्याणि ।
मैत्राषाढचतुष्कं वसुरविरोहिण्य इति पञ्च ।। २ ।।

(Brahmagupta, *KK*, 1. 9. 1-2)

Each of the *nakṣatras*, Mūla, Pūrva-Bhadrapadā, Uttara-Bhadrapadā, Aśvinī, Punarvasu, Viśākhā, Pūrva-Phalgunī, and Uttara-Phalgunī has two stars. The *nakṣatras*, Citrā, Tiṣya, Śatabhiṣaj, Ārdrā, Svātī, and Revatī, have one star each. Abhijit, Jyeṣṭhā, Bharaṇī, Śravaṇa and Mṛgaśiras have three stars each. Kṛttikā, Āśleṣā and Maghā have six stars each. Anurādhā, Pūrvāṣāḍhā and Uttarāṣāḍhā have four each. Dhaniṣṭhā, Hasta and Rohiṇī have five stars each. (1-2). (BC)

योगताराणां ध्रुवाः

12. 5. 1. स्वे तारागणमध्ये या तारा दृश्यतेऽतिदीप्ततरा ।
ध्रुवविक्षेपौ तस्याः कथितौ सा योगताराख्या ।। ३ ।।
अष्टनखैर्मेषे गवि रदलिप्तोनैर्गुणस्वरैर्मिथुने ।
कर्कटके गुणषोडशधृतिभिः सिंहे नवत्रिघनैः ।। ४ ।।
कन्यायां पञ्चनखैस्तुलिनि त्र्यतिधृतिभिरलिनि सेषुकलैः ।
द्विचतुर्दशातिधृतिभिर्धनुषि शशाङ्कमनुनखतत्त्वैः ।। ५ ।।
मकरेऽष्टनखैः कुम्भे नखषड्विंशैर्झषे मुनित्रिंशैः ।
पृथगश्विन्यादीनां ध्रुवकांशैर्योगताराः स्वैः ।। ६ ।।

Brahmagupta, *KK*, 1.9.3-6)

Longitudes of Junction stars

That star, which is the brightest of all the stars in each *nakṣatra*, is called its *yogatārā*. The *dhruvakas* of the *yogatārās* are given below. (3)

The following are respectively, the *dhruvakas* of the *yogatārās* in the *nakṣatras* beginning with Aśvinī, by means of which their conjunction with planets is considered.

8°; 20°; 1 sign 7° 28′; 1 sign 19° 28′; 2 signs 3°; 2 signs 7°; 3 signs 3°; 3 signs 16°; 3 signs 18°; 4 signs 9°; 4 signs 27°; 5 signs 5°; 5 signs 20°; 6 signs 3°; 6 signs 19°; 7 signs 2° 5′; 7 signs 14° 5′; 7 signs 19° 5′; 8 signs 1°; 8 signs 14°; 8 signs 20°; 8 signs 25°; 9 signs 8°; 9 signs 20°; 10 signs 20°; 10 signs 26°; 11 signs 7°, and 12 signs. (4-6). (BC)

12. 5. 2. 'गजाब्धयः' 'शून्यकृता' 'रसेषवः'
'कृतेषवो' 'युग्मशरा' 'नखा' 'द्व्यगाः' ।
'खवाजिनो'ऽ'ब्धिश्रुतयो' 'गजाब्धयो'
'रसाग्नयो'ऽ'ब्धिश्रुतयो' 'गजाद्रयः' ।। १ ।।
'रसर्तवो' 'युग्मरसा' 'यमाचला'
'यमेषवो'ऽ'ष्टावृतवो' 'ऽब्धयो' 'ऽब्धयः' ।

'नवेन्दवो' 'नागभुव:' 'खसागरा'
'यमाब्धय''स्तिग्मकरा:' 'कृताद्रय:' ।। २ ।।

क्रमेण सर्वा: 'खशशाङ्क'ताडिता
भभोगलिप्ता मुनिभि: प्रकीर्तिता: ।
शताष्टकं व्येकभसङ्ख्यया हतं
भवेगलिप्ताढ्यमुशन्ति भध्रुवम् ।। ३ ।।
(Lalla, *ŚiDhVṛ.*, 11. 1-3)

It has been said by the ancient sages that the positions of *nakṣatras*, *Āśvinī* etc., in their respective portions are, respectively, 48′, 40′, 56′, 54′, 52, 20′, 72′, 70′, 44′, 48′, 36′, 44′, 78′, 66′, 62′, 72′, 52′, 8′, 6′, 4′, 4′, 19′, 18′, 40′, 42′, 12′, and 74′, each multiplied by 10.

They also say that the polar longitude of each *nakṣatra* is equal in minutes to the sum of its portion and 800 times the number of the *nakṣatras* preceding it (that is, its own number less 1).[1] (1-3). (BC).

12. 5. 3. अष्टादश दिशो मनवोऽर्का द्वयोर्धन: ।
द्वाविंशतिश्च विश्वे च नव शक्रास्त्रयोदश ।। १ ।।

विश्वे विंशतिकोना द्वादशार्का दिनानि च ।
दिशो रसाश्च विश्वे च विश्वे सूर्या धृतिस्तथा ।। २ ।।

रुद्रा: सूर्यास्त्रिसप्ताथ शैलेन्दुतिथयस्तथा ।
पूर्वपूर्वयुता ज्ञेया योगभागा यथोदिता: ।। ३ ।।

आप्यवैष्णवमूलानां पित्र्यवासवयोरपि ।
त्रिंशल्लिप्ता सयाम्यानां क्षेप्या वैश्वस्य शेषत: ।। ४ ।।
(Bhāskara I, *LBh.*, 8. 1-4)

Eight, eighteen, ten, fourteen, twelve, eight, twenty-two, thirteen, nine, fourteen, thirteen, thirteen, nineteen, twelve, twelve, fifteen, ten, six, thirteen, thirteen, twelve, eighteen, eleven, twelve, twenty-one, seventeen, and fifteen—each of these numbers being increased by (the sum of) the preceding numbers, in the order in which they have been stated above, are to be taken as the degrees of the longitudes of the junction-stars of the (twenty-seven) *nakṣatras*. To the longitudes of (the junction-stars of) Pūrvāṣāḍhā, Śravaṇa, Mūla, Maghā, Dhaniṣṭhā, Bharaṇī, and Uttarāṣāḍhā (thus obtained), one should further add thirty minutes (of arc).[2] (1-4). (KSS)

12. 5. 4. अष्टौ नखा गजगुणा: खशरास्त्रिषट्का:
सप्तर्तवस्त्रिनव चाङ्गदिशोऽष्टकाष्ठा: ।
गोऽर्कास्तथाद्रिमनव: शरबाणचन्द्रा:
खात्यष्टयस्त्रिधृतयो नवनन्दचन्द्रा: ।। १ ।।

अर्काश्विनो जिनयमा नवबाहुदस्रा:
क्वब्ध्यश्विनो जलधितत्त्वमिताश्च भागा: ।
षष्ट्यश्विनश्च पवनोत्कृतयोऽष्ट भानि
खाङ्काश्विनो नखगुणा रसदन्तसंख्या: ।। २ ।।

सप्तामरा: खमिति भध्रुवका निरुक्ता
दृक्कर्मणायनभवेन सहाश्विधिष्ण्यात् ।
ब्रह्माग्निभध्रुवलवा रदलिप्तिकोना
मैत्रेन्द्रयोर्द्वचधिपभस्य च सेषुलिप्ता: ।। ३ ।।
(Bhāskara II, *SiŚi.*, 1. 11. 1-3)

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[1] See mathematical notes, *ŚiDhVṛ*: *BC*, II. 189.

[2] The junction-stars (*yogatārā*) of the *nakṣatras* are the prominent stars of the *nakṣatras* which were used for the study of the conjunction of the planets, especially the Moon, with them.

The longitudes of the aforesaid junction-stars are, in seven cases, slightly different from those given in the author's earlier work, the *Mahā-Bhāskarīya*, (obviously, as a result of further observation and study). The differences are exhibited by the following table:

Differences between the longitudes of the junction-stars in the two works of Bhaskara I

Junction-star of	longitude given in *Mahā Bhāskarīya*	longitude given in *Laghu-Bhāskarīya*	Celestial-latitude
1. Aśvinī	8°	8°	10° N
2. Bharaṇī	27°	26° 30′	12° N
3. Kṛttikā	1s 6°	1s 6°	5° N
4. Rohiṇī	1s 19°	1s 20°	5° S
5. Mṛgaśirā	2s 2°	2s 2°	10° S
6. Ārdrā	2s 10°	2s 10°	11° S
7. Punarvasu	3s 2°	3s 2°	6° N
8. Puṣya	3s 15°	3s 15°	0
9. Āśleṣā	3s 24°	3s 24°	7° S
10. Maghā	4s 8° 30′	4s 8° 30′	0
11. Pārva Phālgunī	4s 21°	4s 21°	12° N
12. Uttara Phālgunī	5s 4°	5s 4°	13° N
13. Hasta	5s 23°	5s 23°	7° S
14. Citrā	6s 5°	6s 5°	2° S
15. Svātī	6s 17°	6s 17°	37° N
16. Viśākhā	7s 2°	7s 2°	1° 30′ S
17. Anurādhā	7s 12°	7s 12°	3° S
18. Jyeṣṭhā	7s 18°	7s 18°	4° S
19. Mūla	8s 1°	8s 1° 30′	8° 30′ S
20. Pūrvāṣāḍhā	8s 14°	8s 14° 30′	7° S
21. Uttarāṣāḍhā	8s 27°	8s 26° 30′	7° S
22. Śravaṇa	9s 15°	9s 14° 30′	30° N
23. Dhaniṣṭhā	9s 26°	9s 25° 30'	36° N
24. Śatabhiṣak	10s 7°	10s 7°	18′ S
25. Pūrva-Bhādrapada	10s 28°	10s 28°	24° N
26. Uttara-Bhādrapada	11s 15°	11s 15°	26° N
27. Revatī	12s	12s	0

The polar longitudes of the stars from Aśvinī including Abhijit are as follows:

		r .d m
1.	Aśvinī	0- 8- 0
2.	Bharaṇī	0-20- 0
3.	Kṛttikā	1- 7-18
4.	Rohiṇī	1-19-18
5.	Mṛgaśīrṣa	2- 3- 0
6.	Ārdrā	2- 7- 0
7.	Punarvasū	3- 3- 0
8.	Puṣya	3-16- 0
9.	Āśleṣā	3-18- 0
10.	Maghā	4- 9- 0
11.	Purvāphalgunī	4-27- 0
12.	Uttarāphalgunī	5- 5- 0
13.	Hasta	5-20- 0
14.	Citrā	6- 3- 0
15.	Svāti	6-19- 0
16.	Viśākhā	7- 2- 5
17.	Anurādhā	7-14- 5
18.	Jyeṣṭhā	7-19- 5
19.	Mūlā	8- 1- 0
20.	Purvāṣāḍhā	8-14- 0
21.	Uttarāṣāḍhā	8-20- 0
22.	Abhijit	8-25- 0
23.	Śravaṇa	9- 8- 0
24.	Dhaniṣṭhā	9-20- 0
25.	Śatabhiṣak	10-20- 0
26.	Purvābhādrā	10-26- 0
27.	Uttarābhādrā	11- 7- 0
28.	Revatī	0- 0- 0

योगताराविक्षेपाः

12. 6. 1. सौम्या दशार्कविषया याम्याः शरदशभवा रसाः सौम्याः ।
खं सप्त दक्षिणाः खं सौम्याः सूर्यत्रयोदशकाः ॥ ८ ॥
दक्षिणतो भवयमलाः सप्तत्रिंशदुदगंशका याम्याः ।
अध्यर्धत्रिचतुष्कार्धनवमसत्र्यंशविषयशराः ॥ ९ ॥
सौम्या द्व्यधिका षष्टिस्त्रिंशत् षट्त्रिंशदितरतो लिप्ताः ।
अष्टादशोत्तरा जिनषड्विंशत्यम्बराण्यंशाः ॥ १० ॥
प्राजेशयोगतारा विक्षेपांशैः कलात्रिघनहीनैः ।
आग्नेयस्य कलानामेकोनत्रिंशता हीनैः ॥ ११ ॥
पञ्चदशकलाहीनैश्चित्रायाः सप्तभिर्विशाखायाः ।
षट्सप्तत्या मैत्र्यस्यैन्द्रस्य त्रिंशता हीनैः ॥ १२ ॥
त्रीणि ब्राह्मात् सार्पं द्वितयं हस्ताद् द्विदैवतात् षट् ।
एतानि दक्षिणदिशि विक्षिप्तान्यन्यानि चोत्तरतः ॥ १३ ॥
(Brahmagupta, *KK*, 1. 9. 8-13)

Latitudes of the Junction stars

The following are respectively, the *vikṣepas* of the *yogatārās* in the *nakṣatras* beginning with Aśvinī:

10° N, 12° N, 5° N, 5° S, 10° S, 11° S, 6° N, 0°, 7° S, 0°, 12° N, 13° N, 11° S, 2° S, 37° N, 1½° S, 3° S, 4° S, 8½° S, 5 1/3° S, 5° S, 62° N, 30° N, 36° N, 18′ S, 24° N, 26° N and 0°. (8-10)

The *vikṣepas* of the *yogatārās* of Rohiṇī, Kṛttikā, Citrā, Viśākhā, Anūrādhā and Jyeṣṭhā should be, respectively, decreased by 27′, 29′, 15′, 7′, 76′, and 30′. (11-12). (BC)

Three stars from Rohiṇī (that is, Rohiṇī, Mṛgaśiras and Ārdrā), Āśleṣā, two stars from Hasta, (that is, Hasta, and Citrā) and six stars from Viśākhā, (that is, Viśākhā, Anurādhā, Jyeṣṭha, Mūla, Pūrvāṣāḍha, and Uttarāṣāḍha), have *vikṣepas* to the south. (Śatabhiṣaj also has *Vikṣepa* to the south). The remaining stars, (that is, Aśvinī, Bharaṇī, Kṛttikā, Punarvasu, Pūrva Phalguṇī, Uttara Phalguṇī, Svāti, Abhijit, Śravaṇa, Dhaniṣṭhā, Pūrva Bhādrapadā and Uttara Bhādrapadā), have *vikṣepas* to the north. (Tiṣya, Maghā and Revatī have no *vikṣepa*). (BC). (13). (BC)

12. 6. 2. भशरा भध्रुवकालापमादिमे ॥ ४d ॥

'दश' 'लोचनभूमयः' 'शरा'
'विषया' 'व्योमभुवः' 'पिनाकिनः' ।
'षड'था'म्बरम्' 'अद्रयो''ऽम्बरं'
'रवयो' 'रामभुवो' 'गजा' 'यमौ' ॥ ५ ॥

'नगाग्नयो'ऽर्धेन विवर्जितं 'द्वयं'
'त्रयो 'ऽ'ब्धयो'ऽर्धेन युता 'भुजङ्गमाः' ।
त्रिभागयुक्ता 'विषयाः' 'शिलीमुखाः'
'खवह्नयः' 'षट्कगुणा' लवाः स्मृताः ॥ ६ ॥

ततस्त्रिभागोऽथ जिनाः षडश्विनो
नभश्च भागस्य मितिर्जिनाङ्गुलाः ।
(Lalla, *ŚiDhVṛ.*, 11. 4-8)

The following are, respectively, the polar latitudes of the *nakṣatras*, Aśvinī etc., measured from the ends of their respective declinations on the ecliptic corresponding to their polar longitudes. (4d)

(They are) 10°, 12°, 5°, 5°, 10°, 11°, 6°, 0°, 7°, 0°, 12°, 13°, 8°, 2°, 37°, 1°, 30°, 3°, 4°, 8° 30′, 5° 20′, 5°, 30°, 36° 20′, 24°, 26° and 0°.

1° is equivalent to 24 *aṅgulas*. (5-7). (KVS)

12. 6. 3. उदग् दिशोऽर्कभूतानि याम्ये पञ्च दिशो भवाः ।
उदग् रसास्तथा व्योम दक्षिणे मुनयोऽम्बरम् ॥ ६ ॥

[1] The *Vikṣepa* of a *yogatārā* is the distance of the *yogatārā* from its *dhruvaka* measured on its *dhruvaprota*.

उदगर्कास्तथा विश्वे दक्षिणे मुनयोऽश्विनौ ।
सौम्ये रसकृतिः सैका याम्ये सार्धास्तथाग्नयः ॥ ७ ॥
अब्धयो वसवः सार्धाः सप्तशैलास्ततः परम् ।
उदक् त्रिंशत् कृतिः षण्णां याम्ये लिप्तास्त्रिषट्ककाः ॥ ८ ॥
उदगर्कश्च विश्वे च द्विरभ्यस्ता नभस्तथा ।
विक्षेपांशाः क्रमाद् दृष्टाः पण्डितैर्वाजिभादितः ॥ ९ ॥

(Bhāskara I, *LBh.*, 8. 6-9)

North, ten, twelve, five; south, five, ten, eleven; north, six, zero; south, seven, zero; north, twelve, thirteen; south, seven, two; north, thirty-seven; south, one and a half, three, four, eight and a half, seven, seven; north thirty, thirty-six; south eighteen minutes of arc; north, twenty-four, twenty-six, and zero—these have been stated by the learned to be the degrees (unless otherwise stated) of the latitudes of the junction stars of the *nakṣatras* beginning with Aśvinī in their serial order.[1] (6-9). (KSS)

12. 6. 4. दिशोऽर्काश्च सार्धाब्धयः सार्धवेदा
दशेशा रसाः खं स्वराः खं च सूर्याः ।
त्रिचन्द्राः कुचन्द्रा विपादौ च दस्रौ
तुरङ्गाग्नयः सत्रिभागं च रूपम् ॥ ४ ॥
विपादं द्वयं सार्धरामाश्च सार्धा
गजाः सत्रिभागेषवो मार्गणाश्च ।
द्विषष्टिः खरामाश्च षड्वर्गसंख्या-
स्त्रिभागो जिना उत्कृतिः खं च भानाम् ॥ ५ ॥
निरुक्ताः स्फुटा योगताराशरांशा-
स्त्रयं ब्रह्मधिष्ण्याद्विशाखादिषट्कम् ।
करो वारुणं त्वाष्ट्रभं सार्पमेषां
शरा दक्षिणा उत्तराः शेषभानाम् ॥ ६ ॥

(Bhāskara II, *SiŚi.*, 1. 11. 4-6)

Rectified latitudes of the stars

1.	Aśvinī	10°- 0 north
2.	Bharaṇī	12°- 0 north
3.	Kṛttikā	4-30 north
4.	Rohiṇī	4-30 south
5.	Mṛgaśirā	10- 0 south
6.	Ārdrā	11- 0 south
7.	Punarvasu	6- 0 north
8.	Puṣya	0- 0 north
9.	Āśleṣā	7- 0 south
10.	Maghā	0- 0 north
11.	Pūrvāphalgunī	12- 0 north
12.	Uttarāphalgunī	13- 0 north
13.	Hasta	11- 0 south
14.	Citrā	1°-45′ south
15.	Svātī	37°- 0 north
16.	Viśākhā	1-20′ south
17.	Anurādhā	1-45 south
18.	Jyeṣṭhā	3-30 south
19.	Mulā	8-30 south
20.	Purvāṣāḍhā	5-20 south
21.	Uttarāṣāḍhā	5- 0 south
22.	Śravaṇa	30- 0 north
23.	Abhijit	62- 0 north
24.	Dhaniṣṭhā	36- 0 north
25.	Śatabhiṣak	0-20 south
26.	Purvābhādrā	24- 0 north
27.	Uttarābhādrā	26- 0 north
28.	Revatī	0- 0 north

[1] For elucidatory table, see *LBh.:KSS*, p. 97-98.

अगस्त्यादिताराणां ध्रुवविक्षेपाः

12. 7. 1. अगस्त्यनामा मिथुनोंऽशकैस्त्रिभि-
विवर्जिते योगमुपैति खेचरैः ॥ ७ ॥
चतुर्भिरंशैर्मृगयुर्विवर्जितै-
स्त्रिभिश्च भागैरभिजिच्च कार्मुकैः ।
क्रमाच्छरांशाः खगजाः खसागरा
यमस्य दिश्युत्तरतः कृतर्तवः ॥ ८ ॥

(Lalla, *ŚiDhVṛ.*, 11.5-8)

Longitudes and Latitudes of Canopus etc.

The polar longitude of Agastya is 2^r 27°. (When the true longitude of a planet is the same as this), there is conjunction of the planet and Agastya. (7c-d)

The polar longitude of *Mṛgavyādha* (Lubdhaka) is 2^r 26° and that of Abhijit is 8^r 27°. The polar latitudes of Agastya, Mṛgavyādha and Abhijit are respectively 80° S, 40° S and 64° N. (8). BC)

12. 7. 2. अशीतिभागैर्याम्यायामगस्त्यो मिथुनान्तगः ।
विंशे च मिथुनस्यांशे मृगव्याधो व्यवस्थितः ॥ १० ॥
विक्षिप्तो दक्षिणे भागैः खार्णवैः स्वादपक्रमात् ।
हुतभुग्ब्रह्महृदयौ वृषद्वाविंशभागगौ ॥ ११ ॥
अष्टाभिः त्रिंशता चैव विक्षिप्तावुत्तरेण तौ ।
गोलं बध्वोपरिक्षेत्तं विक्षेपध्रुवकान् स्फुटान् ॥ १२ ॥

(*SūSi.*, 8. 10-12)

Agastya is at the end of Gemini, and eighty degrees south; and Mṛgavyādha is situated at the twentieth degree of Gemini; (10)

Its latitude (*vikṣepa*), reckoned from its point of declination (*apakrama*), is forty degrees south; Agni (*hutabhuj*) and Brahmahṛdaya are Taurus, the twenty-second degree: (11)

And they are separated in latitude (*vikṣipta*), northward, respectively, eight and thirty degrees. Having constructed a sphere, one may examine the corrected (*sphuṭa*) latitude and polar longitude (*dhruvaka*) (12). (Burgess)[1]

12. 7. 3. अगस्त्यध्रुवः सप्तनागास्तु भागा-
'स्तुरङ्गाद्रय'स्तस्य याम्याः शरांशाः ।
षडष्टौ लवा लुब्धकस्य ध्रुवोऽयं
'नभोऽम्भोधि' भागाः शरस्तस्य याम्यः ।। ७ ।।
अगस्त्यस्य नाडीद्वयं प्रोक्तमिष्टं
सषड्भागनाडीद्वयं लुब्धकस्य ।
त्रिभागाधिकं स्थूलभानामणूनां
ततश्चाधिकं तारतम्येन कल्प्यम् ।। ८ ।।
(Bhāskara II, *SiŚi.*, 1.11.7-8)

The polar longitudes of Agastya (Canopus) is 87° and its polar latitude is 77° south. The polar longitude of Lubdhaka (Sirius) is 86° and its polar latitude is 40° south. (7)

The *Iṣṭanāḍis* for Agastya are said to be two, for Lubdhaka $2\frac{1}{6}$, for other stars which are next in size, $2\frac{1}{3}$ and for still smaller ones the *Iṣṭanāḍis* are to be taken still more.[2] (8). (AS)

सप्तर्षिचारः

12. 8. 1. आसन् मघासु मुनयः शासति पृथ्वीं युधिष्ठिरे नृपतौ ।
षट्द्विकपञ्चद्वियुतः शककालस्तस्य राज्ञश्च ।। ३ ।।
एकैकस्मिन्नृक्षे शतं शतं ते चरन्ति वर्षाणाम् ।
प्रागुदयतोऽप्यविवराद्दृजून्नयति तत्र संयुक्ताः ।। ४ ।।
पूर्वे भागे भगवान् मरीचिरपरे स्थितो वसिष्ठोऽस्मात् ।
तस्याङ्गिरास्ततोऽत्रिस्तस्यासन्नः पुलस्त्यश्च ।। ५ ।।
पुलहः क्रतुरिति भगवानासन्ना अनुक्रमेण पूर्वाद्यात् ।
तत्र वसिष्ठं मुनिवरमुपश्रितारुन्धती साध्वी ।। ६ ।।
(Varāha, *Bṛ Saṃ.*, 13. 3-6)

The Great Bear

The Seven Sages were stationed in the asterism *Maghā* when king *Yudhiṣṭhira* was ruling over the earth. The commencement of the *Śaka* era took place 2526 years after the period of that monarch. (3)

The Sages traverse through each lunar mansion for a period of 100 years. Whichever constellation makes them conspicuous when they rise to the east of it, in that they are said to be situated.[3] (4)

Among the Sages, the revered *Marīci* is situated in the east; to his west is *Vasiṣṭha*; to his west is *Aṅgiras*; to his west is situated *Atri*; and close to him is *Pulastya*. Next to him are in order *Pulaha* and *Kratu*. Arundhatī, the paragon of virtue, is close to the great Sage *Vasiṣṭha* among them. (5-6) (M.R. Bhat)

नक्षत्रानयनम् (पैतामहम्)

12. 9. 1. अथ नक्षत्रानयनं पैतामहमुच्यते सम्यक् ।। ६ ।।
अध्यर्धानि च भवन्ति षड्नक्षत्राण्युडूनि षडर्धानि ।
पञ्चदश समक्षेत्राण्यभिजिद्भोगो भवत्येकः ।। ७ ।।
केशादित्यविशाखाप्रोष्ठपदार्यम्णवैश्वदेवानि ।
षट् षड्ज्येष्ठाभरणीस्वात्यार्द्रावारुणाश्लेषाः ।। ८ ।।
पञ्चदशात्रानुक्तान्येकोऽभिजिदुक्त ऋक्षभोगोऽन्यः ।
तन्मानं नाक्षत्रं दुरधिगमं मन्दबुद्धीनाम् ।। ९ ।।
अध्यर्धार्धसमक्षेत्राणां मध्यगतिलिप्तिका शशिनः ।
अध्यर्धार्धैकगुणा भभोगलिप्तास्तदैक्योनाः ।। १० ।।
मण्डललिप्ताः, शेषो भोगोऽभिजितो भभोगलिप्तोनाः ।
भानि ग्रहभुक्तकला गतगम्या गतिहृता दिवसाः ।।११।।
चापानयने नवशतविकलवधाद् भोग्यलब्धलिप्ताभिः ।
कृत्वा खण्डकमसकृत् तल्लब्धकला विकलचापम् ।। १२ ।।
(Brahmagupta, *KK*, 2.1.6-12

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[1] For notes, see *SūSi*: *Burgess*, pp. 245-48.

[2] For the assumptions involved, see *SiŚi*:*AS*, pp. 506-12.

[3] It was believed by the authors of the ancient *Jyotiṣa Saṃhitās* that the Seven Sages had a motion of their own among the other stars, just like the planets, the rate of motion being 100 years per asterism (13° 20′). It has, however, to be stated that there is no such motion as claimed, and that the theory of their motion is wrong, howsoever it might have originated. That is why many standard astronomers like Āryabhaṭa, Brahmagupta, Śrīpati, Bhāskaras I and II, the author of the *Sūryasiddhānta* etc. do not deal with the subject at all, as being outside the pale of real astronomy. That is also why Kamalākara is constrained to say in his *Siddhānta Tattvaviveka*, (edn. Banaras, 1880-85), *Bhagrahayutyadhikāra*, vv. 25ff.:

Śākalyasaṃjñamuninā kathitās sabāṇāḥ
saptarṣitārakabhavā dhruvakāś calāś ca || 25a ||

. . . .

yair golatattvam vivṛtam hi taiś ca
Sūryādibhir naiva viśeṣa eṣaḥ |
proktaḥ svaśāstre 'sti gatir munīnām
ato na yuktā divi golarītyā || 30 ||

. . . .

adyāpi kairapi narair gatir āryavaryaiḥ
dṛṣṭā na yātra kathitā kila Saṃhitāsu | 32 |

. . . .

prayo tha te ca munayaḥ kila devatāṃśā
dṛggocarā nahi nṛṇām iha satphalāptyai | 36a |

'Sage Śākalya has given the motion of the Sages with their positions in his time. . . . Sūrya and others who explain the nature of the celestial sphere in their works do not give it, and therefore the theory cannot be sustained astronomically. . . . Even today this motion mentioned in the Saṃhitās is not observed by astronomers. . . . Therefore the seven real Sages who are the *presiding deities* (*of these stars*) *are only to be supposed to be moving unobserved by men, for the prediction of the fruits thereof.*'

But the motion has been accepted as a fact by certain common people and the authors of the Purāṇas, and an era called *Laukika Era* by the people belonging to the Kashmir region and the *Saptarṣi Era* by the Purāṇas has been founded upon this theory. For a full discussion on the subject, see T.S.K. Sastri and K.V. Sarma, 'The untenability of the postulated Śaka era of 550 B.C.', *Jl of Indian History*, 37 (1959) 201ff., see esp. pp. 208-18.

Asterism computation according to Paitāmaha Siddhānta

Now I shall correctly describe the computation of *nakṣatras* according to the *Paitāmahasiddhānta*. (6b)

Six *nakṣatras* are *adhyardhabhogi* (that is, each of them occupies $1\frac{1}{2}$ of 790′ 35″, the mean daily motion of the moon, along the zodiac); 6 are *ardhabhogi* (that is, each occupies $\frac{1}{2}$ of 790′ 35″); and Abhijit has a special extension of its own. (7)

Rohiṇī, Punarvasu, Viśākhā, Uttara Bhādrapadā, Uttara Phalgunī and Uttarāṣāḍhā are the six *adhyardhabhogi nakṣatras*. Jyeṣṭhā, Bharaṇī, Svāti, Ārdrā, Śatabhiṣaj and Āśleṣā are the six *ardhabhogi*. The remaining 15, excluding Abhijit, are *samabhogi*. Abhijit is the twenty-eighth *nakṣatra* and its extension is different from that of the above 27 *nakṣatras*. The extension of each *nakṣatra* along the zodiac is not the same, and not generally known to the common people. (8-9)

Multiply the number of minutes in the mean motion of the moon by 3/2, $\frac{1}{2}$ and 1. The products give, respectively, the extension in minutes of an *adhyardhabhogi*, *ardhabhogi* and *samabhogi nakṣatra*.

Add together the extensions of all the 27 *nakṣatras*. Subtract the sum from 21,600. The remainder is the extension of Abhijit in minutes.

Subtract as many extensions of the *nakṣatras* as possible, beginning with Āśvinī, from the longitude of a planet converted into minutes. (The planet is in the next *nakṣatra* after having passed the *nakṣatras* whose extensions have been subtracted.) The remainder is the portion of the current *nakṣatra* passed by the planet. Divide the portions passed and to be passed by the planet's daily motion. The results are, respectively, the number of days since the planet entered the current *nakṣatra* and the number of days the planet will remain in that *nakṣatra*. (10-11)

To find the arc corresponding to a given *jyā*, first subtract from the *jyā* as many tabular differences as possible. Multiply the remainder, called *vikala*, by 900 and divide the next tabular difference. The result gives roughly the arc in minutes corresponding to the *vikala*, as the *jyā*. Find the *bhogyakhaṇḍa* from this arc (see KK. II. 1. 4.). Repeat the process tilll the *bhogyakhaṇḍa* is fixed. This will give the correct value of the arc in minutes corresponding to the *vikala* as the *jyā* (and hence the required arc).[1] (12). (BC)

नक्षत्रदेवताः

12. 10. 1. अग्निः प्रजापतिः सोमो रुद्रोऽदितिर्बृहस्पतिः ।
सर्पाश्च पितरश्चैव भगश्चैवार्यमापि च ॥ ३२ ॥
सविता त्वष्टाथ वायुश्चेन्द्राग्नी मित्र एव च ।
इन्द्रो निर्ऋतिरापो वै विश्वेदेवास्तथैव च ॥ ३३ ॥
विष्णुर्वसवो वरुणोऽज एकपात् तथैव च ।
अहिर्बुध्नयस्तथा पूषा अश्विनौ यम एव च ॥ ३४ ॥
नक्षत्रदेवता ह्येता एताभिर्यज्ञकर्मणि ।
यजमानस्य शास्त्रज्ञैर्नाम नक्षत्रजं स्मृतम् ॥ ३५ ॥

(*YV-VJ* 32-35; *RV-VJ* 25-28)

Asterisms and presiding deities

Agni (is the presiding deity of Kṛttikā), Prajāpati (of Rohiṇī), Soma (of Mṛgaśīrṣa), Rudra (of Ārdrā), Aditi (of Punarvasu), Bṛhaspati (of Puṣya), Serpents (of Āśleṣā), Pitṛ-s (of Maghā), Bhaga (of Pūrva-phalgunī), Aryamā (of Uttara-phalgunī), Savitṛ (of Hasta), Tvaṣṭā (of Citrā), Vāyu (of Svātī), Indrāgnī (of Viśākhā), Mitra (of Anurādhā), Indra (of Jyeṣṭhā), Nirṛti (of Mūla), Āpaḥ or Waters (of Pūrvāṣāḍhā), Viśvedevāḥ (of Uttarāṣāḍhā) (X) Varuṇa (of Śatabhiṣaj), Aja-Ekapād (of Pūrva-Bhādrapadā), Ahirbudhnya (of Uttara-Bhādrapadā), Pūṣā (of Revatī), Aśvins (of Aśvinī), and Yama (of Bharaṇī). People learned in the religious lores say that these deity-names are to be used as the sacrificer's (sacrificial) name in sacrifices. (32-35). (KVS)

[1] For the rationale, see *KK:BC* I, 138-39.

(X) Vishnu (of

13. ग्रहाः – PLANETS

ग्रहाः

13. 1. 1. प्रकाशकौ द्वौ प्रथमं ग्रहाणां
तारग्रहाः पञ्च परे, ततो द्वौ ।
तमोग्रहौ, तेषु शुभास्तु मध्ये
त्रयो बलीन्दुश्च, परे तु पापाः ।। १९ ।।
(Vidyāmādhava, *Vidyāmādhavīya*, 1. 19)

The planets

From among the (nine) planets, the first two (viz., Sun and Moon) are luminous, the five (viz., Mars, Mercury, Jupiter, Venus and Saturn) are star-planets, and the last two (*viz.*, Rāhu and Ketu) are dark (and, so, invisible) planets. Of the star-planets, the middle three (*viz.*, Mercury, Jupiter and Venus) and also the (astrologically) strong planets and the Moon are benificent; the rest are baneful. (19). (KVS)

वेदेऽभिहिताः ग्रहाः

13. 2. 1. अमी पञ्चोक्षणो मध्ये तस्थुर्महो दिवः ।
देवत्रा नु प्रवाच्यं सध्रीचीना नि वावृतुः ।। १० ।।
(*RV*, 1.105.10)

Planets referred to in the Vedas

These five mighty (gods) were seen on the vast expanse of the sky. Even though they were seen coming together when I composed the hymns in honour of the gods, they have all gone away today. (10)

बृहस्पतिः

13. 2. 2. बृहस्पतिः प्रथमं जायमानो
महो ज्योतिषः परमे व्योमन् ।
सप्तास्यस्तु वि जातो रवेण
सप्तरश्मिरधमत् तमांसि ।। ४ ।।
(*RV*, 4.50.4)

Jupiter

Bṛhaspati, when being born in the highest heaven of supreme light, seven-mouthed, multiform (combined) with sound, and seven-rayed, has subdued the darkness. (4)

13. 2. 3. बृहस्पतिः प्रथमं जायमान-
स्तिष्यं नक्षत्रमभिसंबभूव ।। ५ ।।
(*Tait. Br.*, 3.1.5)

Bṛhaspati, when first appearing, rose in front of the Tiṣya (Puṣya) constellation. (5)

वेनः

13. 2. 4. अयं वेनश्चोदयत् पृश्निगर्भा
ज्योतिर्जरायू रजसो विमाने ।
इममपां संगमे सूर्यस्य
शिशुं न विप्रा मतिभी रिहन्ति ।। १ ।।
(*RV*, 10.123.1)

Venus

This Vena, enfoldened in the membrane of light, urges on (the waters) the germs of the Sun in the firmament of water; the sages cherish him at the confluence of waters, and the Sun with endearments, like a child. (1)

अहर्गणः

—पैतामहः

13. 3. 1. रविशशिनोः पञ्च युगं वर्षाणि पितामहोपदिष्टानि ।
अधिमासस्त्रिंशद्भिर्मासैरवमो द्विषष्ट्याह्नाम् ।। १ ।।
द्व्यूनं शकेन्द्रकालं पञ्चभिरुद्धृत्य शेषवर्षाणाम् ।
द्युगणं माघसिताद्यं कुर्याद्युगभानि वह्न्युदयात् ।। २ ।।
(Vahāra, *PS*, 12.1-2)

Days from Epoch—Paitāmaha

The Siddhānta of Pitāmaha teaches that the luni-solar *yuga* is made up of five years. After every thirty synodic months there is an intercalary month, and there is an omitted day for every 62 lunar days or *tithis*. (1)

Subtract two from the years of the elapsed Śaka era, and divide out the remaining years by five. The 'days from epoch' are to be calculated for the remaining years etc., the first day being the *śuklapratipad* of the month of Māgha. The *nakṣatras* of the Sun and the Moon, calculated by using the days, are for sunrise.[1] (2). (TSK)

—रोमक-पौलिशौ

13. 3. 2. 'सप्ताश्विवेद'संख्यं शककालमपास्य चैत्रशुक्लादौ ।
अर्धास्तमिते भानौ यवनपुरे सोमदिवसाद्यः ।। ८ ।।
मासीकृते समासे द्विष्ठे सप्ताहते'ऽष्टयमपक्षैः' ।
लब्धैर्युतोऽधिमासैस्त्रिंशद्घ्नस्तिथियुतो द्विष्ठः ।। ९ ।।
'रुद्र'घ्नः स'मनुशरो' लब्धोनो 'गुणखसप्त'भिर्द्युगणः ।
रोमकसिद्धान्तेऽयं नातिचिरे पौलिशेऽप्येवम् ।। १० ।।
'दिग्'घ्नाः 'साष्टनवरस'दिवसा 'एकर्तुसप्तनव'भक्ताः ।
पौलिशमतेऽधिमासाः 'विकृत'दिनान्यवमसंक्षेपः ।।११।।

[1] For detailed elucidation, rationales etc., see *PS*:*TSK*, 12.1-2.

तिथिदशमांशं दद्यादधिमासार्थं 'स्वराम्बरैका'ब्दैः ।
अवमार्थं 'पञ्चकृतद्वि'सम्मितैस्तिथि'शिवां'शं च ॥१२॥

अधिमासकेषु भूयोऽप्येकीकर्तुं 'खपञ्चकेन्द्रियां'शेषु ।
देयोऽवमेषु हेयो नवसप्तद्वित्रिखयमेषु ॥ १३ ॥

(Varāha, *PS*, 1.8-13)

—Romaka and Pauliśa

Deduct 427 from the Śaka year (elapsed) of the time taken. Multiply the remainder by 12. Add the months gone, counting from Caitra. Put this result in two places. In one place, multiply it by 7, divide by 228 and take the quotient which constitutes the intercalary months. Add this to the result kept in the second place. (The total are synodic months gone). Multiply this by 30 and add the *tithis* counted from *śukla-pratipad* to the current *tithi*. Put the sum in two places. In one place multiply by 11, add 514, divide by 703 and take the quotient, (which constitute the elided days or *avamas*). Deduct this from the sum put in the other place. The remainder are the 'Days from Epoch' (*dyugaṇa*), the moment of epoch being mid-sun-set at Yavanapura, beginning Monday, when the first *tithi* of Caitra was about to begin.

This rule is according to the Romaka. It can be taken as the Pauliśa rule also provided the time taken for computation is not very far from the epoch. (*or*, the part of the rule for *avama* may be used for the Pauliśa also, provided the taken days is not very far from the epoch; *or* in the Pauliśa too the movement of epoch is mid-sun-set at Yavanapura, beginning Monday). (8-10)

(The formula for 'Days from Epoch' according to the Pauliśa is as follows:) Deduct 427 from the Śaka year (elapsed). Multiply by 12 and add the months gone from Caitra. Multiply by 30. The 'Solar days' to the end of the current solar month are got). Multiply the 'Solar days' by 698, and divide by 9761. The quotient are the intercalary months. (Multiply the months got by 30 and add to the 'Solar days', and also the *tithis* from *śuklapratipad*, inclusive of the current *tithi*. The sum is the *tithis* gone from epoch. Multiply this by 11), add 444, (and divide by 703. The quotient are the elided days. Deduct this from the *tithis* gone. The remainder are the 'Days from epoch'.) (11)

For greater accuracy in finding the intercalary months by the above formula, add 1/10 'Solar days' to the 'Solar days' for every 107 years, and for still greater accuracy, add an extra 1/10 'Solar day' for every 550 such 1/10 'Solar days' added, (i.e., for 55 'Solar days' added). In the same way, in getting the elided days, for every 245 years add 1/11 *tithi* to the *tithis* gone and for every 2,03,279 such additions omit one addition. (12-13). (TSK)

—आर्यभटीयार्धरात्रिकपक्षः

13. 3. 3. शाकोऽ'गवसुशरो'नो'ऽर्क'गुणश्चैत्रादिमाससंयुक्तः ।
त्रिंशद्गुणस्तिथियुतः पृथ'गिषु'सहितो द्विधा भक्तः ॥३॥

'पञ्चाम्बुधिनवमनुभि''र्लब्धोनो भाजितः 'षडगनन्दैः' ।
लब्धाधिमासकदिनैरधिकोऽधो 'रुद्र'संगुणितः ॥ ४ ॥

'स्वरनववेद'युतोऽध'स्त्र्यगतिथिरुद्रै'र्हृतः फलविहीनः ।
'त्रिखनग'हृतः फलावमरात्रोनोऽहर्गणोऽर्कादिः ॥ ५ ॥

अधिमासावमशेषे घटिका'सप्तदशमनुभि'रभ्यधिके ।
इन्दूच्चचन्द्रपातावूनौ 'शरदि'ग्विलिप्ताभिः ॥ ६ ॥

(Brahmagupta, *KK*, 1.1.3-6)

—Midnight system of Āryabhaṭa

Deduct 587 from the Śaka year. Multiply the remainder by 12. Add to the product the number of months elapsed since the light half of Caitra. Multiply the sum by 30. Add to the product the number of *tithis* elapsed since the last Amāvāsyā. (The result is taken as the total number of *sauradinas* elapsed). Add 5 to this result. Put down the sum in two places. (3)

At one place, divide it by 14,945. Subtract the result thus obtained from the sum in the other place. Divide the remainder by 976. The quotient gives the number of *adhimāsas*. Convert it into days. Add the result to the number of *sauradinas* obtained above. (The sum is the total number of *cāndradinas* elapsed.) Multiply this number by 11. (4)

Add 497 to the product. Put down the sum in two places. At one place divide it by 1,11,573. Subtract the result thus obtained from the sum in the other place. Divide the remainder by 703. The quotient gives the number of (*avamarātras*). Subtract the quotient from the number of *cāndradinas* obtained above. The result is the *ahargaṇa* in *sāvana* units beginning from Sunday. (5)

Adhimāśaśeṣa and *Avamaśeṣa*, calculated in the above manner, should be increased by 17 and 14 *ghaṭikās*, respectively.

The Moon's *Ucca* and *Pāta* (calculated, according to I. 13, 14) when decreased by 5″ and 10″, respectively, (become equal to (those given by Āryabhaṭa in his Ārdharātrika or Midnight System).[1] (6). (BC)

—भास्करः १

13. 3. 4. 'नवाद्र्येकाग्नि'संयुक्ताः शकाब्दा द्वादशाहताः ।
चैत्रादिमाससंयुक्ताः पृथग्गुण्या युगाधिकैः ॥ ४ ॥

ते च 'षट्त्रिकरामाग्निनवभूतेन्दवो' युगे ।
भागहारोऽ'ब्धिवस्वेकशराः' स्युरयुताहताः ॥ ५ ॥

[1] For the rationale and formulae involved, see *KK:BC*, I.90-95.

अधिमासान् पृथक्स्थेषु प्रक्षिप्य त्रिशताहते ।
युक्त्वा दिनानि यातानि प्रतिराश्य युगावमैः ।। ६ ।।

सङ्गुण्या'म्बराष्टेषुद्वयष्टशून्यशराश्वि'भिः ।
छेदः 'खाष्टवियद्व्योमखखाग्निखरसेन्दवः' ।। ७ ।।

लब्धान्यवमरात्राणि तेषु शुद्धेष्वहर्गणः ।
वारः सप्तहृते शेषे शुक्रादिर्भास्करोदयात् ।। ८ ।।
(Bhāskara I, *LBh.*, 1.4-8)

—Bhāskara I

Add 3179 to the (number of elapsed) years of the Śaka era, (then) multiply (the resulting sum) by 12, and (then) add the (number of lunar) months) expired (since the commencement of Caitra. Set down (the result thus obtained) at (two) separate places; multiply (one) by (the number of) intercalary months in a *yuga*, which are 15,93,336 in a *yuga*; and divide (the product) by 5184 × 10,000 (i.e., by 5,18,40,000). (4-5)

Add the (resulting complete) intercalary months to the result placed at the other place. Then multiply (that sum) by 30 and (to the product) add the (lunar) days (i.e., *tithis*) expired (of the current month). Set down (the result thus obtained) in two places; multiply (one) by the (number of) omitted lunar days in a *yuga*, i.e., by 2,50,82,580, and divide by 1,60,30,00,080. (6-7)

The result forms the (complete) omitted lunar days, which when subtracted from the result put at the other place gives the (required) *ahargaṇa*. The remainder obtained on dividing (the *ahargaṇa*) by 7 gives the day beginning with Friday at sunrise (at Laṅkā). (6-8). (KSS)

—लल्लः

13. 3. 5. 'नवाद्रिचन्द्रानल'संयुतो भवे-
च्छकक्षितीशाब्दगणो गतः कलेः ।
'दिवाकर'घ्नो गतमाससंयुतः
'खवह्नि'निघ्नस्तिथिभिः समन्वितः ।। १२ ।।

पृथक् कृतः सङ्गुणितोऽधिमासकै-
स्ततो विभक्तो दिवसैः सहस्रगोः ।
फलाधिमासैः सहितो दिनीकृतैः
पुनः पृथक्स्थो गुणितस्तिथिक्षयैः ।। १३ ।।
निशाकराहैः स हृतः फलावमै-
र्विवर्जितोऽर्काभ्युदयादहर्गणः ।
भवत्ययं भार्गववारपूर्वकः
सदैव लङ्काविषये कलेर्गतः ।। १४ ।।
अतीतमासैः कलिजैः समाहता
'नवाष्टरामाङ्गयमद्विबाहवः' ।
'खखाभ्रपूर्णाङ्गमहीयमो'द्धृता
भवन्ति मासाः शशिनः कलेर्गताः ।। १५ ।।
दिनीकृतास्ते सदिनाः 'शराचल-
द्विपेषुनन्दाष्टवसुस्वरा'हताः ।
'कृताभ्रखाभ्रेषुकुखाष्ट'भाजिताः
फलं सरूपं द्युगणोऽथवा भवेत् ।। १६ ।।
(Lalla, *ŚiDhVṛ.*, 1.12-16)

—Lalla

The Śaka year, (when the civil days are required), added to 3179 gives the solar years elapsed since the beginning of the Kaliyuga. Multiply the number by 12 and add the months (beginning from Caitra) passed in the current year. Multiply the sum by 30 and add the number of *tithis* elapsed (since the last *amāvāsyā*). (12)

Put down the result at two places. (At one place) multiply it by the number of intercalary months (in a *yuga*) and divide by the number of solar days. The result is the corresponding number of intercalary months. Change it into days and add to the result at the second place. Put down the sum at two places. (At one place), multiply it by the number of omitted *tithis* (in a *yuga*) and divide the number of *tithis* (in the same period). The result is the corresponding number of omitted *tithis*. Subtract it from the sum at the second place. The remainder is always the number of civil days or *ahargaṇa* elapsed since the beginning of the *Kaliyuga* (till the given day) and is counted from Friday, at mean sunrise at Laṅkā. (13-14)

Alternate method

Or, the solar months elapsed since the beginning of the *Kaliyuga* multiplied by 22,26,389 and divided by 21,60,000 give the corresponding lunar months. Convert them into days and add the *tithis* elapsed (since the last *amāvāsyā*). Multiply the sum by 7,88,95,875 and divide by 8,01,50,004. The result increased by 1 is the *ahargaṇa* required.[1] (15-16). (BC)

—करणरत्नम्

13. 3. 6. शकवर्षं 'रुद्ररसै' रहितं रविसङ्गुणं सगतमासम् ।
त्रिशद्गुणं तिथियुतं तृतीय'मिषुवेदनवमनुभिः' ।। ५ ।।
लब्धं मध्ये त्यक्त्वा युक्त्वाऽत्र 'स्वरदहनवसून्' हृत्वा ।
'रसमुनिनव'भिर्लब्धान् मासान् त्रिशद्गुणानधिकान् ।।
उपरि क्षिप्त्वा त्रिरधः 'शरयमगुणखाग्नि'भिर्हृतं मध्ये ।
'रुद्रगुणे' संयोज्यं 'वेदाम्भोनिधिरसा'श्चात्त ।। ७ ।।
लब्धानि 'त्रिखशैलै'र्नष्टदिनानि व्यपोह्य तान्युपरि ।
शुद्धदिनं तं 'मुनि'हृतशेषे शुक्रादिदिनपः स्यात् ।। ८ ।।
(Deva, *KR*, 1.5-8)

—Karaṇaratna

Diminish the (current) *Śaka* year by 611, then multiply by 12, then add the number of months elapsed (since the

[1] For the rationale, see *SiDhVṛ.*: *BC*, II. 8-13.

beginning of the month of Caitra), then multiply by 30, and then add the number of *tithis* (lunar days) elapsed (of the current month). (Set down this result in three places one below the other). Divide the result in the third (lowest) place by 14,945, then subtract the quotient (obtained) from the result occupying the middle place, then add 837, then divide by 976, then multiply the resulting intercalary months by 30, and then add what is obtained to the result in the uppermost place. Set down this result (again) in three places. Divide the result in the lowest place by 30,325, add it to 11 times the result in the middle place, then add 644, then divide the sum by 703, and then subtract the resulting omitted lunar days from the result in the uppermost place. This is known as *Śuddha-dina* (or *Ahargaṇa*). This being divided by 7, the remainder counted from Friday gives the lord of the current day.[1] (5-8).

—वाक्यकरणम्

13. 3. 7. कल्यब्दो 'मातुल'गुणः वर्ष'वां'शेन संयुतः ।
पुनरब्दा'न्मान'गुणात् 'सालप्रिय'विवर्जितात् ॥ २ ॥
'तत्समा'प्तैर्दिनैर्युक्तं शुक्रवारादिकं दिनम् ।
स्फुटार्कचक्रावधिकम्

(*VK*, 1. 2-3)

—Vākyakaraṇa

(The Śaka year *plus* 3179 gives the Kali year.) Multiply the Kali year elapsed by 365. Add a fourth of the years. Add to this the days, (*nāḍikās* etc.) got by multiplying the years by 5, deducting 1237 and dividing by 576. This gives the Kali days to the end of the True Solar year,[2] the first (Kali) day being a Friday. (2-3). (TSK-KVS)

—ग्रहलाघवम्

13. 3. 8. 'द्व्यब्धीन्द्रो'नितशक 'ईश'हृतफलं स्या-
च्चक्राख्यं 'रवि'हतशेषकं तु युक्तम् ।
चैत्राद्यैः पृथगमुतः स'दृग्'घ्नचक्राद्
'दिग्'युक्ता'दमर'फलाधिमासयुक्तम् ॥ ४ ॥
'खत्रि'घ्नं गततिथियुङ्निरग्रचक्रा-
ङ्गांश'शा'द्यं पृथगमुतो'ऽब्धिषट्क'लब्धैः ।
ऊनाहैर्वियुतमहर्गणो भवेद्वै
वारः स्या'च्छर'हतचक्रयुग्गणोऽब्जात् ॥ ५ ॥

(Gaṇeśa, *GL*, 1.4-5)

—Grahalāghava

Subtract 1442 from the given Śaka year. Divide by 11. The quotient is termed *cakra*. Multiply the remainder by 12. Add to this the number of months that have elapsed from *cakra*. Keep the result, separately (x). Find $x+2$ *cakra*$+10$. Divide by 33. The quotient gives the *adhimāsas*, intercalary months. (y). (4)

Find $(x+y).30$. Add the number of *tithis* that have elapsed. To the result add one-sixth of *cakra*. Let it be z. Divide z by 64. The quotient gives the elapsed days. Subtract the *kṣayadina* from z to get the *ahargaṇa* (a).

(To determine the day of the week:) Find the remainder in $(a+5\times \text{cakra})\div 7$. The remainder gives the days that have elapsed from Monday.[1] (5). (VSN)

ग्रहभगणादिः

—आर्यभटीयानुसारि

13. 4. 1. युगरविभगणाः ख्युघृ, शशि
चयगियिङुशुछ्लृ, कु ङशिबुण्लृख्‌ प्राक् ।
शनिढुङ्विघ्व, गुरु खि-
च्युभ, कुज भद्लिझ्नुखृ, भृगुबुधसौराः ॥ ३ ॥
चन्द्रोच्च र्जुष्खिध, बुध
सुगुशिथृन, भृगु जषबिखुछृ, शेषार्काः ।
बुफिनच पातविलोमा,
बुधाह्न्यजार्धोदयाच्च लङ्कायाम् ॥ ४ ॥

(Āryabhaṭa 1, *ABh.*, 1.3-4)

Planetary revolutions

—Āryabhaṭan school

In a *yuga*, (aeon), the eastward revolutions of the Sun are 43,20,000; of the Moon, 5,77,53,336; of the Earth, 1,58,22,37,500; of Saturn, 1,46,564; of Jupiter, 3,64,224; of Mars, 22,96,824; of Mercury and Venus, the same as those of the Sun; of the Moon's apogee, 4,88,219; of (the *śighrocca* of) Mercury, 1,79,37,020; of the *śighrocca* of) Venus, 70,22,388; of (the *śighroccas* of) the other planets, the same as those of the Sun: of the Moon's ascending node in the opposite direction (*i.e.*, westward), 2,32,226. These revolutions commenced at the beginning of the sign Aries on Wednesday at sunrise at Laṅkā (when it was the commencement of the current *yuga*). (3-4). (KSS)

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[1] Thus, to find the *Ahargaṇa* for (T+1)th *tithi* in the (M+1)th month of *Śaka* year S elapsed), one has to proceed along the following steps:

Step 1: S—611=A, say.
Step 2: 12A+M=B, say.
Step 3: 30B+T=C, say.

Step 4: $C+\left[\left\{(C-C/14945)+837\right\}\frac{1}{976}\right]\times 30=D$, say.

Step 5: $D-\left[(11\,D+D/30325)+644\right]\frac{1}{703}=$ Ahargaṇa.

For the rationale see *KR*: *KSS*, *pp*. 3-5.

[2] For worked out example see *VK*: *TSK-KVS*, pp. 250-251.

The Kali days are usually computed for sunrise, so as to be full days. For this purpose the fraction is left out or considered as full according as the New Year begins in the daytime or night-time. To get the Kali days for any date, add to this the days gone in the year.

[1] Thus zero as remainder implies that Sunday is gone and that Monday is current, and say 4 as remainder implies that Thursday is gone and Friday is current.

—लल्लः

13. 4. 2. 'दन्ताब्धयो'ऽयुतहता युगवत्सराः स्युः
सूर्यज्ञशुक्रभगणा अपि तत्समानाः ।
तावन्त एव कुजजीवशनैश्चराणां
शीघ्राख्यतुङ्गभगणा गणकैर्निरुक्ताः ॥ ३ ॥

चन्द्रस्य 'षड्गुणसुरेष्वचलाद्रिबाणा'
भौमस्य 'सङ्कृतिगजाङ्गनवाश्विदस्राः' ।
प्रालेयरश्मितनयस्य चलोच्चचक्रा-
'ण्यभ्राक्षिखाद्रिगुणनन्दतुरङ्गचन्द्राः' ॥ ४ ॥

जीवस्य 'सागरयमाश्विकृताङ्गरामाः'
शीघ्रोच्चपर्ययमितिर्भृगुनन्दनस्य ।
'वस्वष्टरामयमनेत्रवियत्तुरङ्गा'
मन्दस्य 'सागररसेषुरसाब्धिचन्द्राः' ॥ ५ ॥

पातस्य 'षट्द्वियमदन्तयमा' विलोमा
'गोकुद्विवस्वहियुगानि' मृदूच्चमिन्दोः ।
'अङ्गामरानलनवेषुभुवो'ऽधिमासाः
'खाष्टेषुपक्षवसुखेषुयमाः' क्षयाहाः ॥ ६ ॥

(Lalla, *ŚiDhVr.*, 1. 3-6)

—Lalla

432 multiplied by 10,000, (i.e., 43,20,000), gives the number of solar years in a *yuga*. This is also the number of revolutions of the Sun, Mercury and Venus and that of the *śighroccas* of Mars, Jupiter and Saturn during the same period. This has been specified by astronomers. (3)

(The number of revolutions) of the Moon (in a *yuga*) is 5,77,53,336, that of Mars is 22,96,824; that of the *śighrocca* of Mercury is 1,79,37,020; that of Jupiter is 3,64,224; that of the *śighrocca* of Venus is 70,22,388; that of Saturn is 1,46,564; that of the Moon's node is 2,32,226 in the opposite direction; and that of the Moon's apogee is 4,88,219. (The number of) intercalary months (in a *yuga*) is 15,93,336 and (the number of) omitted *tithis* is 2,50,82,580.[1] (4-6). (BC)

—सूर्यसिद्धान्तः

13. 4. 3. युगे सूर्यज्ञशुक्राणां 'खचतुष्करदार्णवाः' ।
कुजार्किगुरुशीघ्राणां भगणाः पूर्वयायिनाम् ॥ २९ ॥

इन्दो 'रसाग्नित्रित्रीषुसप्तभूधरमार्गणाः' ।
'दस्रत्र्यष्टरसाङ्काक्षिलोचनानि' कुजस्य तु ॥ ३० ॥

बुधशीघ्रस्य 'शून्यर्तुखाद्रित्र्यङ्कनगेन्दवः' ।
बृहस्पतेः 'खदस्राश्विवेदषड्वह्नय'स्तथा ॥ ३१ ॥

सितशीघ्रस्य 'षट्सप्तत्रियमाश्विखभूधराः ।
शने'र्भुजङ्गषट्पञ्चरसवेदनिशाकराः' ॥ ३२ ॥

शशाङ्कोच्चस्य 'रुद्राश्विवसुसर्पार्णवा' युगे ।
वामं पातस्या'र्णवाग्नियमाश्विशिखिदस्रकाः' ॥ ३३ ॥

भोदया भगणैः स्वैस्स्वैरूनास्तस्योदया युगे ॥ ३४a ॥

कल्पभगणाः

अधिमासोनरात्र्यर्क्षचान्द्रसावनवासराः ।
एते सहस्रगुणिताः कल्पे स्युर्भगणादयः ॥ ३९ ॥

मन्दोच्चपातानां कल्पभगणाः

प्राग्गतेस्सूर्यमन्दस्य कल्पे 'सप्ताष्टवह्नयः' ।
कौजस्य 'वेदखयमा' बुधस्याष्टर्तुवह्नयः ॥ ४० ॥

'खखरन्ध्राणि' जैवस्य शौक्रस्या'र्थगुणासवः' ।
'गोऽग्नय'श्शनिमन्दस्य पातानामथ वामतः ॥ ४१ ॥

'मनुदस्रा'स्तु कौजस्य बौधस्या'ष्टाष्टसागराः' ।
'कृताद्रिचन्द्रा' जैवस्य 'त्रिखाङ्का'श्च गुरोस्तथा ॥ ४२ ॥

शनिपातस्य भगणाः कल्पे 'यमरसर्तवः' ।
भगणाः पूर्वमेवाऽत्र प्रोक्ताः चन्द्रोच्चपातयोः ॥ ४३ ॥

(*SūSi.*, 1. 29-43)

—Sūryasiddhānta

In an Age (*yuga*), the revolutions of the Sun, Mercury, and Venus, and of the conjunctions (*śighra*) of Mars, Saturn, and Jupiter, moving eastward, are 43,20,000. (29)

Of the Moon, 57,75,33,36; of Mars, 22,96,832. (30)

Of Mercury's conjunction (*śighra*), 17,93,760; of Jupiter, 3,64,220. (31)

Of Venus's conjunction (*śighra*) 70,22,376; of Saturn, 1,46,568. (32)

Of the Moon's apsis (*ucca*) in an Age, 4,86,203; of its node (*pāta*), in the contrary direction, 2,32,234. (33)

Of the asterisms, 1,58,22,37,828. The number of risings of the asterisms, diminished by the number of the revolutions of each planet, gives, respectively, the number of risings of the planets in an Age. (34)

Kalpa revolutions

The intercalary months, the omitted lunar days, the sidereal, lunar, and civil days—these, multiplied by a thousand, are the number of revolutions, etc., in an aeon (*kalpa*). (39)

Kalpa revolutions of the Apses and Nodes

The revolutions of the Sun's apsis (*manda*), moving eastward, in an aeon, are three hundred and eighty-seven; of that of Mars, two hundred and four; of that of Mercury, three hundred and sixty-eight. (40)

Of that of Jupiter, nine hundred; of that of Venus, five hundred and thirty-five; of the apsis of Saturn, thirty-nine. (41a-c)

Further, the revolutions of the nodes, retrograde, are: Of that of Mars, two hundred and fourteen; of that of Mercury, four hundred and eighty eight; of that of Jupiter, one hundred and seventy-four; of that of Venus, nine hundred and three; (41d-42)

Of the node of Saturn, the revolutions in an aeon are six hundred and sixty-two; the revolutions of the Moon's apsis and node have been given here already. (43) (E. Burgess)[1]

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[1] For elucidatory notes, see *ŚiDhVr*: *BC*, II. 4-6.

[1] For notes see *SūSi*: *Burgess*, pp. 28-32.

—आर्यभटः २

13. 4. 4. कल्पे सूर्यादीनां भगणा 'घडफेननेननुनीनाः' ।
'मथथमगग्लभननुना' 'खखझतजोगीपनीनोनाः' ॥ ७ ॥
'कसधगसनमघचसिपा' 'बोचीभोरीकुधितहीराः' ।
'सीनररगसकघडठा' 'कढतीमोतीधानीनेनाः ॥ ८ ॥
रविचक्रसमा बुधसितभगणाश्चारेज्यसौरिशीघ्राणाम् ।
पाठोक्ता बुधसितयोः शीघ्रोच्चाख्या; मृदूच्चजान् वक्ष्ये ॥
सूर्यादीनां 'घुतपा' 'ढजहेकुनहेत्सभा' 'झंधा' 'गुडुघाः' ।
'जुडिना' 'चिमिढा' 'सेता', चन्द्रादिविलोमपातानाम् ॥
'फगफगपडिलेमोढा' 'रिझिना' 'मुरुघा' 'धता' 'धढसाः' ।
'तरना'; सप्तर्षीणां 'कुणिधुधिधुधिजा'; 'मसिहटमुधाः' ॥
अयनग्रहस्य भास्करभगणा 'यख'ताडितास्तरणिमासाः ।
रविशशिचक्रवियोगः शशिमासा वीनमासका अधिकाः ॥
(ABh. II, *Mahā*, 1.7-12)

—Āryabhaṭa II

In a *kalpa* the revolutions (*bhagaṇa*) of (the planets), beginning with the Sun are: (Sun) 4,320,000,000; (Moon) 57,753,334,000; (Mars) 2,296,831,000; (Conjunction of Mercury) 17,937,054,671; (Jupiter) 364,219,682; (Conjunction of Venus) 7,022,371,432; (Saturn) 146,569,000. (7-8)

The revolutions of Mercury and Venus are the same as the revolutions (*cakra*) of the Sun; (so are those) of the conjunctions (*śīghra*) of Mars, Jupiter and Saturn. (Those) of the conjunctions (*śīghrocca*) of Mercury and Venus are mentioned (above). (9)

Now I give (the revolutions of) the apsides (*mṛdūcca*) of the Sun etc. (They are:) (Sun) 461; (Moon) 488,108,674; (Mars) 299; (Mercury) 339; (Jupiter) 830; (Venus) 654; (Saturn) 76.

(The revolutions) of the nodes (*vilomapāta*) of (the planets) starting from the Moon are: (Moon) 232,313, 354; (Mars) 298; (Mercury) 524; (Jupiter) 96; (Venus) 947; (Saturn) 620. (10-11a)

(The revolutions) of the Great Bear (Saptarṣayaḥ) are 1,599,998; and 578,159 are (those) of the equinoxes (*ayanagraha*).[1] (11b)

The revolutions of the Sun, multiplied by 12, are the solar months (*taraṇimāsa*). The difference between the revolutions of the Sun and Moon are the lunar months (*śaśimāsa*). (These), diminished by the solar months (*inamāsaka*), are the intercalary (months) (*adhika*).[2] (12). (SRS)

[1] The equinoxes (*ayana*) are also reagrded here as a planet (*graha*); their annual motion is 173.4577″. This is too high a value. Sudhakara Dvivedi suggests (in his edition, contents page 3) the reading *masihaṭayudhāḥ* (578,119), which would give a better value for precession.

[2] Solar months in a *kalpa*: 4,320,000,000 × 12 = 51,840,000,000
Lunar months in a *kalpa*: 57,753,334,000—4,320,000,000 = 53,433,334,000.
Intercalary months in a *kalpa*:
53,433,334,000—51,840,000,000 = 1,593,334,000.

—वटेश्वरः

13. 4. 5. 'खाभ्रखाभ्रदशनाब्धयो' युगे
भार्गवेन्दुसुतसूर्यपर्यया: ।
शीघ्रतुङ्गभगणाः प्रकीर्तिताः
सूर्यसूनुसुरपूजितासृजाम् ॥ ११ ॥
शशिनो 'रसवह्निसुरेषुनग-
क्षितिभृद्वित्रया',स्तनयस्य भुवः ।
'गजपक्षगजाङ्गनवद्विभुजाः'
'खयमाक्षिकृततुगुणा'श्च गुरोः ॥ १२ ॥
रविजे'भरसानिलषण्मनवः'
शशिसूनुचलस्य रसाग्नियुताः ।
'नखखाद्रिगुणाङ्कनगक्षितयो
भृगपुत्रचलस्य बुधैर्गदिताः ॥ १३ ॥
'रसशैलगुणाक्षिभुजाभ्रनगाः'
'शशिखाश्विकरीभपयोनिधयः' ।
हिमगूच्चयुगर्क्षगणाः 'कृतपुं-
द्विभुजाग्निभुजाः' शशिपातभवाः ॥ १४ ॥
(Vaṭeśvara: *Vsi.*, 1.1.11-14)

—Vaṭeśvara

43,20,000 are stated to be the revolutions performed in a *yuga* by Venus, Mercury, and the Sun and also by the *Śīghroccas* of Saturn, Jupiter, and Mars. (11)

The revolutions of the Moon, as stated by the learned, are 5,77,53,336; of Mars, 22,96,828; of Jupiter 3,64,220; of Saturn, 1,46,568; of the *Śīghrocca* of Mercury, 1,79,37,020 *plus* 36 (i.e., 1,79,37,056); of the *Śīghrocca* of Venus, 70,22,376; of the Moon's apogee, 4,88,211; and of the Moon's node, 2,32,234.[1] (12-14). (KSS)

[1] The revolutions, as suggested by Roger Billard, were probably obtained by the application of the *Bīja* correction prescribed by Lalla, to the revolutions given by Āryabhaṭa I. See the table below.

	Āryabhaṭa-I's revolutions	Bīja correction	Corrected revolutions	Actual revolutions by Vaṭeśvara
Sun	4320000	Nil	4320000	4320000
Moon	57753336	−2	57753334	57753336
Moon's apogee	488219	−9.12	488210	488211
Moon's node	−232226	−7.68	−232234	−232234
Mars	2296824	+3.84	2296828	2296828
Mercury's Śighrocca	17937020	+33.60	17937054	17937056
Jupiter	364224	−3-76	364220	364220
Venus Śighrocca	7022388	−12.24	7022376	7022376
Saturn	146564	+1.6	146566	146568

Since the revolution of the Sun, Moon and the planets stated by Āryabhaṭa I were divisible by 4, so, Vaṭeśvara, in order to preserve this characteristic feature of Āryabhaṭa I's revolutions, increased the *Bīja*-corrected revolutions of the Moon, Mercury's *Śighrocca* and Saturn by 2. Similarly he added 1 to the revolutions of the Moon's apogee to make them odd and prime to the number of civil days is a *yuga* as in Āryabhaṭa. This explains the difference of the revolutions of Vaṭeśvara from the *Bīja*-corrected revolutions of Āryabhaṭa-I.

—भास्करः २

13.4.6. अर्कशुक्रबुधपर्यया विधे-
 रह्नि कोटिगुणिता 'रदाब्धयः' ।
एत एव शनिजीवभूभुवां
 कीर्त्तिताश्च गणकैश्चलोच्चजाः ॥ १ ॥
'खाभ्रखाभ्रगगनामरेन्द्रिय-
 क्ष्माधराद्रिविषया' हिमद्युतेः ।
'युग्मयुग्मशरनागलोचन-
 व्यालषण्नवयमाश्विनो'ऽसृजः ॥ २ ॥
'सिन्धुसिन्धुरनवाष्टगोऽङ्कषट्-
 त्र्यङ्कसप्तशशिनो' ज्ञशीघ्रजाः ।
'पञ्चपञ्चयुगषट्कलोचन-
 द्व्यब्धिषड्गुण'मिता गुरोर्मताः ॥ ३ ॥
'द्विनन्दवेदाङ्कगजाग्निलोचन-
 द्विशून्यशैलाः' सितशीघ्रपर्ययाः ।
'भुजङ्गनन्दद्विनगाङ्गबाणषट्-
 कृतेन्दवः' सूर्यसुतस्य पर्ययाः ॥ ४ ॥
'खाष्टाब्धयो' 'ऽष्टाक्षगजेषुदिग्द्विप-
 द्विपाब्धयो' 'द्व्यङ्कयमा' 'रदाग्नयः' ।
'शरेष्विभा' 'स्त्र्यक्षरसाः' 'कुसागराः'
 स्युः पूर्वगत्या तरणेर्मृदूच्चजाः ॥ ५ ॥
'गजाष्टिभर्गत्रिरदाश्विनः' 'कुभू-
 द्रसाश्विनः' 'कुद्विशराः' 'क्रमर्त्तवः' ।
'त्रिनन्दनागा' 'युगकुञ्जरेषवो'
 निशाकराद् व्यस्तगपातपर्ययाः ॥ ६ ॥

नाक्षत्रदिनानि

'खखेषुवेदषड्गुणाकृतीभभूतभूमयः' ।
शताहता भपश्चिमभ्रमा भवन्ति काहनि ॥ ७ ॥

रविचन्द्रदिनानि

विधिदिने दिनकृद्दिवसाः 'करे-
 न्द्रियशरेषुभुवो'ऽर्बुदसङ्गुणाः ।
'नवनवाङ्ककराभ्ररसेन्दवः'
 प्रयुतसङ्गुणिता विधुवासराः ॥ ८ ॥

भूदिनानि

भूदिनानि 'शरवेदभूपगो-
 सप्तसप्ततिथयो'ऽयुताहताः ।
भभ्रमास्तु भगणैर्विवर्जिता
 यस्य तस्य कुदिनानि तानि वा ॥ ९ ॥

अधिमासादिः

लक्षाहता 'देवनवेषुचन्द्राः'
 कल्पेऽधिमासाः कथिताः सुधीभिः ।
दिनक्षयास्तत्र सहस्रनिघ्नाः
 'खबाणबाणाश्व्यहिखेषुदस्राः' ॥ १० ॥
रवेः कोटिनिघ्नाः 'कृताष्टेन्दुबाणाः'
 'सुराग्न्यब्धिरामेषवो' लक्षनिघ्नाः ।
शशाङ्कस्य, मासाः पृथक् सूर्यमासै-
 र्विहीनास्तु कल्पेऽथ वा तेऽधिमासाः ॥ ११ ॥
अधिदिनैर्दिनकृद्दिनसञ्चयः
 सहित इन्दुदिनान्यथ तानि वा ।
विरहितानि च तानि दिनक्षयैः
 क्षितिदिनान्यत उत्क्रमतोऽपरम् ॥ १२ ॥

(Bhāskara II, *SiŚi*, 1.1.2.1-12)

—Bhāskara II

The number of sidereal revolutions of the Sun during a *kalpa* is 4,32,00,00,000. It is also the number of those of Mercury and Venus, and those of the *śighroccas* of the planets Mars, Jupiter and Saturn. (1)

The Moon makes 57,75,33,00,000 sidereal revolutions in a *kalpa*, Mars 2,29,68,28,522, Mercury's *śighrocca* 17,93,69,98,984, Jupiter 36,42,26,455, the *śighrocca* of Venus 7,02,23,89,492 and Saturn 14,65,67,298. The sidereal revolucions of the apogees of the Sun and the Moon and those of the aphelia of Mars, Mercury, Jupiter, Venus and Saturn in a *kalpa* are respectively, 480, 48,81,05,858, 292, 332, 855, 653, 41. (2-5)

The retrograde sidereal revolutions of the nodes of the orbits of Moon, Mars, Mercury, Jupiter, Venus and Saturn are, respectively, 23,23,11,168, 267, 521, 63, 893, 584. (6)

Nākṣatra days

The number of diurnal revolutions of the stars in a *kalpa*, is 15,82,23,64,50,000. (7)

Solar and Lunar days

The number of solar days in a *kalpa* is equal to 15,55,20,00,00,000 and of the lunar days (or *tithis*) is 16,02,99,90,00,000. (8)

Civil days

The number of civil days in a *kalpa* is equal to 15,77,91,64,50,000; the number of the dirunal revolutions of the stars *minus* the number of sidereal revolutions of any particular planet constitute the days of that particular planet with respect to the Earth. (9)

Intercalary months etc.

The number of *adhikamāsas* or intercalary months in a *kalpa* is equal to 1,59,33,00,000 and the number of *dinakṣayas* is 25,08,25,50,000. (10)

The number of solar months in a *kalpa* is 51,84,00,00,000; the number of lunar months is 53,43,33,00,000. The number of solar months being subtracted from the number of lunations, we have the number of *adhikamāsas*. The number of solar days together with the days of *adhika* months are equal to the lunar days (or the *tithis*); or, again, the lunar days minus

the *kṣayāhas* are equal to the number of civil days or the reverse will be had by a reverse process.[1] (11-12). (AS)

—सिद्धान्तदर्पणम्

13. 4. 7. कोटिघ्न-'रदत्रेदा:' प्राक् कल्पे सूर्यस्य पर्ययाः ।
'भूदन्तरददेवेषुसप्ताद्रचर्था' विधोः, क्षितेः ॥ २ ॥
'खखेषुगोगुणाष्टाश्वाग्न्यश्विद्वयष्टशरेन्दवः' ।
'सप्ताग्न्येकाश्विषण्णागरसाङ्काकृतयो'ऽसृजः ॥ ३ ॥
स्ववृत्तेऽ'र्थाश्विभूखार्काद्रिचग्न्यङ्कात्यष्टयो' विदुः ।
गुरो'रीशाङ्गखाङ्गैकवेदाङ्गशिखिनः', कवेः ॥ ४ ॥
'द्वीष्वर्थाभ्रभनेत्राक्षिखागा', लङ्कोदयाच्छनेः ।
'नृपाशाश्वेषुतर्केन्द्रा', भुवोऽन्ये वारपा ग्रहाः ॥ ५ ॥

—Siddhāntadarpaṇa

Commencing from a sunrise at Laṅkā, the number of eastward revolutions of the Sun in a *kalpa* is four hundred and thirty-two multiplied by ten million, i.e. 4,32,00,00,000

(that) of the Moon is	57,75,33,32,321;
(that) of the Earth	15,82,23,78,39,500;
(that) of Mars	2,29,68,62,137;
(that) of Mercury	17,93,71,20,175;
(that) of Jupiter	36,41,60,611;
(that) of Venus	7,02,22,70,552;
(that) of Saturn	14,65,71,016.

These are the numbers of revolutions of the planets in a *kalpa* along their respective orbits (from west) to east.

(Of these), each of the planets other then the Earth is the lord of a day of the week. (2-5)

मन्दोच्चपाताः

13. 5. 1. बुध-भृग-कुज-गुरु-शनि न-व-
रा-ष-ह गत्वांशकान् प्रथमपाताः ।
सवितुरमीषां च तथा
द्वा-ञखि-सा-ह्दा-ह्लच खिच्य मन्दोच्चम् ॥ ९ ॥

मन्दवृत्तानि

झार्धानि मन्दवृत्तं
शशिनश्छ, ग-छ-घ-ढ-छ-झ यथोक्तेभ्यः ।
झ-ग्ड-ग्ला-र्धं-द्ड तथा
शनि-गुरु-कुज-भृगु-बुधोच्चशीघ्रेभ्यः ॥ १० ॥
मन्दात् ङ-ख-द-ज-डा
वक्रिणां द्वितीये पदे चतुर्थे च ।
जा-ण-क्ल-छ्ल-झ्नोच्चा-
च्छीघ्रात्, गियिङश कुवायुकक्ष्यान्त्या ॥ ११ ॥
(Āryabhṭa I, *ABh.*, 1. 9-11)

[1] For explanation, see *SiŚi:AS*, pp. 18-28.

Ascending Nodes and Apogees

The ascending nodes of Mercury, Venus, Mars, Jupiter and Saturn having moved to 20°, 60°, 40°, 80° and 100° respectively (from the beginning of the Sign Aries) (occupy those positions); and the apogees of the Sun and the same planets (*viz.*, Mercury, Venus, Mars, Jupiter and Saturn) having moved to 78°, 210°, 90°, 118°, 180° and 236° respectively (from the beginning of the Sign Aries) (occupy those positions). (9)

Manda and Śīghra epicycles

The *manda* epicycles of the Moon, the Sun, Mercury, Venus, Mars, Jupiter and Saturn (in the first and third anomalistic quadrants) are, respectively, 7, 3, 7, 4, 14, 7 and 9 (degrees) each multiplied by 4½ (*i.e.* 31.5, 13.5, 31.5, 18, 63, 31.5 and 40.5 degrees, respectively); the *śīghra* epicycles of Saturn, Jupiter, Mars, Venus and Mercury (in the first and third anomalistic quadrants) are, respectively, 9, 16, 53, 59 and 31 (degrees) each multiplied by 4½ (*i.e.*, 40.5, 72, 238.5, 265.5 and 139.5 degrees, respectively). (10)

The *manda* epicycles of the retrograding planets (*viz.*, Mercury, Venus, Mars, Jupiter and Saturn) in the second and fourth anomalistic quadrants are, respectively, 5, 2, 18, 8 and 13 (degrees) each multiplied by 4½ (*i.e.*, 22.5, 9, 81, 36 and 58.5 degrees respectively); and the *śīghra* epicycles of Saturn, Jupiter, Mars, Venus, and Mercury (in the second and fourth anomalistic quadrants) are, respectively, 8, 15, 51, 57 and 29 (degrees) each multiplied by 4½ (*i.e.*, 36, 67.5, 229.5, 256.5 and 130.5 degrees, respectively). 3375 is the outermost circumference of the terrestrial wind. (11). (KSS)

मन्दोच्चपातपर्ययाः

13. 5. 2. तेषां 'त्रीष्वग्नयो' 'ऽष्टैकसुरार्काष्टाहिसागराः' ।
मन्दोच्चानां 'कृतेष्वश्वा', 'वेदताना' 'कुखर्तवः' ॥ ६ ॥
'द्वीभान्य'-'ब्धीषवो'-'ऽर्थाब्ध्यद्रयङ्काश्विरदाश्विनः' ।
'वेदाग्नीभा' 'द्विशून्याङ्का'-'स्तत्त्वेभा' 'ऋतुषट्स्वराः' ॥
प्रत्यग'श्वेष्वगा'श्चन्द्रात् पातानां; तद्ग्रहान्तरात् ।

परमविक्षेपकलाः

'धृत्य-ङ्ग-मूर्च्छना-ऽब्धी-श-नागाः' स्वांशाङ्घ्रयोऽर्धयोः ॥
क्षेपा मन्दोच्चवृत्तानां;

मन्दपरिधयः शीघ्रपरिधयश्च

स्वांशैस्तान्यर्धपञ्चमैः ।
'त्र्य-श्वा-ष्टी-न्द्रा-हि-रामा-शाः'
सूर्यात्; शैघ्राणि भूसुतात् ॥ ९ ॥
'त्र्यर्था' 'रूपगुणा' 'भूपा' 'गोबाणा' 'नव' तुङ्गतः ।
'द्वि-द्वये-क-द्वये-क'-हीनास्तेऽप्योजयुग्मपदादिषु ॥ १० ॥
(Nīlakaṇṭha, *SD*, 6-10)

(The numbers of revolutions) of the Higher Apses (*mandocca*) of these planets in a *kalpa* are:

(Sun)	353;
(Moon)	48,81,23,318;
(Mars)	754;
(Mercury)	494;
(Jupiter)	601;
(Venus)	272;
(Saturn)	54. (6-7a)

Revolutions of the ascending nodes

(The numbers of revolutions of the ascending nodes), which move backward (from the east) to the west, are, beginning with the Moon:

(Moon)	23,22,96,745;
(Mars)	834;
(Mercury)	902;
(Jupiter)	825;
(Venus)	766;
(Saturn)	757. (7b-8b)

Maximum latitudes

The (maximum) latitudes of the epicycles of the higher apses at their halves, depending on the distance of the planet from the nodes, are, in units of quarter degrees (i.e., 15):

(Moon)	$18(\times 15=270)$;
(Mars)	$6(\times 15=90)$;
(Mercury)	$21(\times 15=315)$;
(Jupiter)	$4(\times 15=60)$;
(Venus)	$11(\times 15=165)$;
(Saturn)	$8(\times 15=120)$. (8b-9a)

Epicycles of the equation of the apses

(The magnitude of the circumferences[1] of the epicycles of the equation of the apses) in terms of $4\frac{1}{2}$ parts of themselves (i.e., in terms of $4\frac{1}{2}$ degrees) are:

(Sun	$3(\times 4\frac{1}{2}=14^\circ)$;
(Moon)	$7(\times 4\frac{1}{2}=31\frac{1}{2}^\circ)$;
(Mars)	$16(\times 4\frac{1}{2}=72^\circ)$;
(Mercury)	$14(\times 4\frac{1}{2}=63^\circ)$;
(Jupiter)	$8(\times 4\frac{1}{2}=36^\circ)$;
(Venus)	$3(\times 4\frac{1}{2}=14^\circ)$;
(Saturn	$10(\times 4\frac{1}{2}=45^\circ)$.

[1] The magnitude of the circumference is given in proportion to that of the circumferences of the orbits of the respective planets which are taken to be 360°, the number of degrees in a circle.

Epicycles of the equation of conjuction

(The circumferences of) the epicycles of the equation of conjuction, beginning with Mars, are, in the odd quadrants:

(Mars)	$53(\times 4\frac{1}{2}=238\frac{1}{2}^\circ)$;
(Mercury)	$31(\times 4\frac{1}{2}=139\frac{1}{2}^\circ)$;
(Jupiter)	$16(\times 4\frac{1}{2}=72^\circ)$;
(Venus)	$59(\times 4\frac{1}{2}=265\frac{1}{2}^\circ)$;
(Saturn)	$9(\times 4\frac{1}{2}=40\frac{1}{2}^\circ)$.

These same epicycles reduced, respectively, by 2, 2, 1, 2, and 1 give the circumferences of the even quadrants.

(Mars)	$51(\times 4\frac{1}{2}=229\frac{1}{2}^\circ)$;
(Mercury)	$29(\times 4\frac{1}{2}=130\frac{1}{2}^\circ)$;
(Jupiter)	$15(\times 4\frac{1}{2}=67\frac{1}{2}^\circ)$;
(Venus)	$57(\times 4\frac{1}{2}=256\frac{1}{2}^\circ)$;
(Saturn)	$8(\times 4\frac{1}{2}=36^\circ)$.

रवि-चन्द्रमध्यम्

रविमध्यम्—सौरसिद्धान्तः

13. 6. 1. द्युगणेऽर्कोष्टशतघ्ने वि'पक्षवेदार्णवे'ऽर्कसिद्धान्ते ।
'स्वरखाश्विद्रिनवयमो'द्धृते क्रमाद्दिनदलेऽवन्त्याम् ॥१॥

(Varāha, *PS*, 9.1)

Mean Sun and Moon

(Mean Sun—Saurasiddhānta)

According to the *Saurasiddhanta*, to get the mean Sun in revolutions etc., multiply the days from Epoch by 800, deduct 442, and divide by 2,92,207. This is for Ujjain mean noon.[1] (1). (TSK)

—सौरसिद्धान्तः

13. 6. 2. नवशतसहस्रगुणिते 'स्वरैकपक्षाम्बरस्वरर्तू'ने ।
'षट्छून्येन्द्रियनववसुविषयजिनै' र्भाजिते चन्द्रः ॥ २ ॥
नवशतगुणिते दद्याद् 'रसविषयगुणाम्बरर्तुयमपक्षान् ।
'नववसुसप्ताष्टाम्बरनवाश्वि'भक्ते शशाङ्कोच्चम् ॥ ३ ॥
'शशिविषय'घ्नानीन्दोः
'क्ववर्काग्नि'हृतानि मण्डलानि ऋणम् ।
स्वोच्चे 'दिग्'घ्नानि घनं
'स्वररन्ध्रयमो'द्धृते विकलाः ॥ ४ ॥

(Varāha, *PS*, 9.2-4)

—Saurasiddhānta

Multiply the days by 9,00,000, deduct 6,70,217, and divide by 2,45,89,506. The approximate mean Moon in revolutions etc. is got. (2)

Multiply the Days by 900, add 22,60,356, and divide by 29,08,789. The approximate Moon's apogee in revolutions etc. is obtained. (3)

[1] For the rationale, see *PS*:*TSK*, 9.1.

Multiply the revolutions of the mean Moon by 51, and divide by 3121. The resulting seconds are to be subtracted to get the exact mean Moon. Multiply the revolutions of apogee by 10 and divide by 297. The resulting seconds are to be added to get the exact apogee.[1] (4). (TSK)

—लल्ल:

13. 6. 3. निशाकराब्दैरधिवत्सरैर्हता-
वहर्गणौ कुद्युहृतौ फलान्तरम् ।
रविः पृथ'ग्विश्व'हृतो'ऽर्क'सङ्गुणै-
र्युतोऽधिवर्षग्रहमण्डलैः शशी ।। २० ।।

युगाधिमासैरवमैः स्वशेषके
हरेत् फलं तत्तु पृथग्दिनादिकम् ।
षडभ्रदिग्भिः प्रथमं हृतं गृहा-
द्यथेतरत् स्वार्कलवोनितं कलाः ।। २१ ।।

रविर्मधोर्मासदिनानि भांशका
द्वितीययुक्ताः प्रथमोनिताः पृथक् ।
युतो रविघ्नैस्तिथिभिर्लवैः शशी
कलासु तत्त्वघ्नफलार्धसंयुतः ।। २२ ।।

शशाङ्कमासैरधिशेषके हृते
ऋणं धनं चावमशेषके क्वहैः ।
धनाख्ययाताहयुतिः पृथक् त्रिभू-
हतावृणावर्कविधू लवादिकौ ।। २३ ।।

अहर्गणाद् भाहहतात् क्वहोद्धृतात्
फलं च चक्राद्यदिवागणं रविः ।
शशाङ्कमासोद्भवखेटमण्डलैः
पृथग्युतोऽसौ मृगलाञ्छनो भवेत् ।। २४ ।।

(Lalla, *SiDhVr.*, 1.20-24)

—Lalla

When the *ahargaṇa* is severally multiplied by the lunar years and the number of intercalary years (in a *yuga*), and each product divided by the number of civil days (in a *yuga*), and the difference of the two results taken, the final result is the mean longitude of the Sun.

13 times the mean longitude of the Sun added to 12 times the number of intercalary years during the *ahargaṇa* gives the mean longitude of the Moon. (20)

Divide the *adhimāsaśeṣa* (the remainder coresponding to the intercalary months obtained while calculating the *ahargaṇa*) and the *avamaśeṣa*, (i.e., the remainder corresponding to the omitted *tithis* obtained while calculating the *ahargaṇa*), by the respective intercalary months and the omitted *tithis* in a *yuga*). (The first quotient) gives solar days, etc. and (the second) *tithis*, etc. The first divided by 1006 is in terms of Signs, etc.; the second diminished by its twelfth part is in minutes, etc. (21)

The number of months from *Caitra* (elapsed in the current year) is equivalent to the same number of Signs, and the number of *tithis* (elapsed since the last *amāvasyā*) is equivalent to the same number of degrees. These Signs, etc. when decreased by the Signs etc. obtained previously and added to the minutes thus obtained (verse 21), give, in terms of Signs, degrees and minutes, the mean longitude of the Sun. To the longitude add degrees equal to 12 times the *tithis* and also 25/2 times the minutes obtained above. The result is the mean longitude of the Moon. (22)

The *adhimāsaśeṣa* divided by the number of lunar months (in a *yuga*) yields a subtractive result. The *avamaśeṣa* when divided by the number of civil days gives an additive result. The sum of the number of *tithis* elapsed (since the *amāvāsyā* in Caitra) and the second result, when diminished by the first, give (also) the mean longitude of the Sun in degrees, etc. The same multiplied by 13 and diminished by the first, gives the mean longitude of the Moon in degrees, etc. (23)

The *ahargaṇa* multiplied by the number of sidereal days and divided by the number of civil days (in a *yuga*) gives the corresponding number of sidereal days. This, diminished by the *ahargaṇa*, results in the mean longitude of the Sun (in revolutions etc.).

When to this longitude is added the excess of the Moon's longitude calculated from the number of lunar months (in a *yuga*) the Sun is the mean longitude of the Moon. (24). (BC)

विनाहर्गणं रविचन्द्रमध्यमः—भास्करः १

13. 6. 4. विना द्युराशेरपि चन्द्रभास्करौ
प्रकुर्वतो वा विधिरेष कथ्यते ।
समासु मासीकृतविग्रहासु ये
व्यतीतमासा विनियोज्य तान् पुनः ।। १३ ।।

'खराम'निघ्नान् दिवसेषु योजयेद्
गतेषु मासस्य ततोऽधिमासकैः ।
निहत्य सर्वं विभजेत सर्वदा
युगार्कमासैर्दिवसत्वमागतैः ।। १४ ।।

भवन्ति लब्धास्त्वधिमासकाः पुन-
स्ततोऽपनीयाशु च भागहारकम् ।
भजेत शेषं शशिमाससङ्ख्यया
ततोंऽशलिप्ताविकलाः सतत्पराः ।। १५ ।।

ततोऽधिमासान् प्रणिहत्य 'खाग्नि'भि-
र्नियोज्य सम्यग्गतवासरैः क्रमात् ।
युगावमघ्नान् शशिवासरैर्हरेत्
तमत्र शेषं प्रवदन्ति चाह्निकम् ।। १६ ।।

[1] For worked out examples, see *PS:TSK*, 9.2-4.

हत्वाऽधिमासैरवमस्य शेषं
छित्त्वा धराया दिवसैः प्रलब्धम् ।
संयोज्य नित्यं त्वधिमासशेषे
कार्यं पुनस्तत्करणैर्यथोक्तम् ।। १७ ।।
युगप्रसिद्धैर्धरणीदिनैर्हरे-
न्निहत्य षष्ट्यावमशेषमाह्निकम् ।
कला विलिप्ताः क्रमशः सतत्परा-
स्त्वतीतमासा दिवसा गृहांशकाः ।। १८ ।।
त्रयोदशघ्नादपि 'रूपताडिता-
द्विशोधयेद्यत्त्वधिमासशेषजम् ।
निशाकरार्कौ गणकैः प्रकीर्तितौ
भटप्रणीताविति बुद्धिमत्तमैः ।। १९ ।।

(Bhāskara I, *MBh.*, 1.13-19)

Mean Sun without Days from Epoch

For one (desirous of) calculating the mean longitudes of the Moon and the Sun without the use of the *ahargaṇa*, the following method is stated:

Reduce the years (elapsed since the beginning of Kaliyuga to months, and add to them the elapsed months (of the current year). Then multiply that (sum) by 30, and add the product to the number of (lunar) days elapsed since the beginning of the current month. Multiply that (sum) by the number of intercalary months (in a *yuga*) and divided by the number of solar months in a *yuga* reduced to days. (13-14)

The quotient denotes the number of intercalary months (elapsed). Delete (or rub out) the divisor and divide the remainder (called *adhimāsaśeṣa*, i.e., the residue of the intercalary months) by the number of lunar months (in a *yuga*): thus are obtained degrees, minutes, seconds, and thirds. (15)

Then multiply the (complete) intercalary months elapsed by 30 and to the product add the number of solar days elapsed since the beginning of Kaliyuga); then multiply that (sum) by the number of omitted lunar days in a *yuga* and divide by the number of lunar days (in a *yuga*): the remainder obtained is the *avamaśeṣa*, (i.e., the residue of the omitted lunar days) called *āhnika*. (16)

Then multiply the *avamaśeṣa* by the number of intercalary months (in a *yuga*) and divide by the number of civil days (in a *yuga*). Add the resulting quotient to the *adhimāsaśeṣa* and then apply the process stated above (i.e., divide by the number of lunar months in a *yuga*: the result is in degrees, minutes, etc. This is the total *adhimāsaśeṣa*). (17)

Next multiply the *avamaśeṣa* called *āhnika* by 60 and divide by the number of civil days in a *yuga*: the result is in minutes, seconds, and thirds respectively. The number of months elapsed (since the beginning of Caitra) are to be taken as Signs, and the number of lunar days elapsed (of the current month) as degrees. (The sum of these Signs and degrees and the minutes, seconds, etc. corresponding to the *avamaśeṣa* is the *grahatanu*). (18)

From thirteen times and from one time that (*grahatanu*) severally subtract the degrees, minutes, etc. corresponding to the (total) *adhimāsaśeṣa*: the remainders (thus obtained) are stated by the wise astronomers to be the mean longitudes of the Moon and Sun, (respectively), conforming to the teachings of (Ārya)bhaṭa.[1] (19). (KSS)

—महासिद्धान्तः

13. 6. 5. 'स्कुधिमुगनिब्धिधटणफा'
कल्यादौ द्युगण एष कलिजयुतः ।
इष्टो वा चक्रहतो
भूदिनभक्तो ग्रहो भगणात् ।। २५ ।।
'प्ल'घ्ने गणे 'ध्नग'हृते-
ऽवाप्तांशोनो गणो रविर्दिवसैः ।
'खगभणथै' लिप्तार्णं
स्वं च विलिप्ता 'झथीरमद'वर्षैः ।। २६ ।।

(ABh. II, *Mahā*. 1. 25-26)

—Mahāsiddhānta

The *ahargaṇa* at the dawn of Kali is 719,530,399,152. This increased by that (i.e., *ahargaṇa*) in Kali (gives the *ahargaṇa* upto the) desired (day). Multiply it by the

[1] The process described in the above rule is not in proper sequence. The direction given in verse 15 ought to have been after verse 17. Stated in proper sequence, the rule would be:

Reduce the years (elapsed since the beginning of Kaliyuga) to months, and add to them the elapsed months (of the current year). Then multiply the sum by 30, and add the product to the number of (lunar) days elapsed since the beginning of the current month. Multiply that sum by the number of intercalary months (in a *yuga*) and divide by the number of solar months in a *yuga* reduced to days: the quotient denotes the number of intercalary months (elapsed). (The remainder is the *adhimāsaśeṣa*). Multiply the (complete) intercalary months (thus obtained) by 30 and to the product add the number of solar days (elapsed since the beginning of Kaliyuga): then multiply that (sum) by the number of omitted lunar days in a *yuga* and divide by the number of lunar days (in a *yuga*); the remainder obtained is (the *avamaśeṣa* called *āhnika*). Then multiply the *avamaśeṣa* (called *āhnika* by the number of intercalary months (in a *yuga*) and divide by the number of civil days (in a *yuga*). Add the resulting quotient to the *adhimāsaśeṣa* and divide the sum by the number of lunar months in a *yuga*: this gives degrees, etc. (This is the total *adhimāsaśeṣa*.) Next multiply (again) the *avamaśeṣa* called *āhnika* by 60 and divide by the number of civil days in a *yuga*: the result is in minutes, seconds, and thirds, etc. The number of months elapsed (since the beginning of Caitra) are to be taken as Signs and the number of lunar days elapsed (of the current month) as degrees. (The sum of these Signs and degrees, and the minutes, second, etc. corresponding to the *avamaśeṣa* is the *grahatanu*). From thirteen times and from one time of that (*grahatanu*) severally subtract the degrees, minutes, etc. corresponding to the (total) *adhimāsaśeṣa*: the remainders (thus obtained) are stated by the wise astronomers to be the mean longitudes of the Moon and the Sun (respectively) conforming to the teachings of (Ārya)bhaṭa.

For the rationale and formulae, see *MBh*: *KSS*, pp. 11-15.

revolutions of any (planet) and divide by the civil days (in a *kalpa*. The result is) the position of that planet in revolutions. (25)

Multiply the *ahargaṇa* by 13 and divide by 903. The result in degrees, (when) subtracted from the *ahargaṇa*, (gives the position of) the Sun (in degrees)[1].

For every 23,457 days (of the *ahargaṇa*) subtract a minute from and for every 97,258 years (elapsed) add a second to (the position obtained above)[2]. (26). (SRS)

—भास्करः २

13. 6. 6. कोटचाहतैर्यद्भवभैरवाप्तं
न्यूनाहशेषे विहृते कलाद्यम् ।
तत् स्याद्धनाख्यं तरणेर्विधोस्तात्
'त्रिभू'हतं स्वे'षुगुणां'शयुक् स्वम् ॥ ६ ॥
चैत्रादियातास्तिथयः पृथक्स्था
'विश्वै'र्हताः सूर्यविधू लवाद्यौ ।
तौ चाधिशेषाच्छशिमासलब्ध्या
हीनौ युतौ स्वस्वधनाह्वयाभ्याम् ॥ ७ ॥
(*Bhāskara* II, *SiŚi.*, 1.1.3.6-7)

—Bhāskara II

Computation of the positions of the Sun and the Moon from the *adhimāsa-śeṣa* and *avama-śeṣa*. The *avama-śeṣa* divided by 27,11,00,00,000 is termed an additive constant in minutes of arc to the Sun's position; the same *avama-śeṣa* multiplied by 13 and divided by 35 is termed such an additive constant to the position of the Moon. (6)

Construe that the Sun's position is given by as many degrees as there are elapsed *tithis* after the beginning of Caitra and that the Moon's position is given by thirteen times the same. Let these positions of the Sun and the Moon be diminished by a number of degrees equal to what is obtained by dividing the *adhimāsa-śeṣa* by the number of lunations in a *kalpa*. Then add the respective additive constants to the positions of the Sun and the Moon so obtained. The results will be the positions of the Mean Sun and the Mean Moon.[3] (7). (AS)

—करणरत्नम्

13. 6. 7. द्युगणे 'गुणयम'रहिते द्विष्ठे त्यक्त्वाऽवमानि तान्युपरि ।
अधरे त्रिगुणं त्यक्त्वा नष्टं हित्वा च नष्टशेषं च ॥ ९ ॥
'नवनन्दरसै'र्लब्धं दत्वोपरि 'खरसगुण'विशुद्धेंऽशाः ।
रविमध्यममुदयगतं 'मङ्गलगिरय'स्तदुच्चांशाः ॥ १० ॥
अवमदिवसस्य शेषं स्व'शरयमां'शान्वितं कला योज्याः ।
भानौ चात्र द्वादशगुणतिथयोंऽशास्तु चन्द्रः स्यात् ॥११॥
(Deva, *KR*, 1.9-11)

—Karaṇaratna

Subtract 23 from the *Ahargaṇa* and set down the remainder in two places (one below the other). From the upper number subtract the *avama* days (elapsed since the epoch). From the lower number subtract 9, the *avama* days as also the *avamaśeṣa*, then divide by 699, and then add the quotient obtained to the upper number. (If it is greater than 360), subtract 360 from it. The resulting quantity is the mean longitude of the Sun at sunrise (at *Laṅkā*), in terms of degrees.

The longitude of the Sun's apogee is 78°.[1] (9-10)

To the Sun's mean longitude add degrees equal to 12 times the complete *tithis* (elapsed) and minutes equal to *avamaśeṣa* plus *avamaśeṣa* divided by 25. Thus is obtained the mean longitude of the Moon.[2] (11). (KSS)

रविचन्द्रस्फुटः

—वासिष्ठसिद्धान्तः

13. 7. 1. 'कृत'गुण'मृतु'युतमेक-
'तुमनु'हृतं षड्यमेन्दुभिर्विभजेत् ।
'शशि-ख-ख-ख-यम-कृत-
स्वर-नव-नव-वसु-षट्क-विषयोनैः ॥ १ ॥
(Varāha, *PS*, 2.1)

True Sun and Moon

True Sun—Vāsiṣṭhasiddhānta

Multiply the Days from Epoch by 4 and add 6. Divide this by 1461 and take the remainder. Take from this, successively, the quantity 126 reduced by 1, 0, 0, 0, 2, 4, 7, 9, 6, 5 (i.e., the twelve quantities 125, 126, 126, 126, 124, 122, 119, 117, 117, 118, 120, 121). (The Sun's *rāśis*, Meṣa etc. are successively got.) (1)

चन्द्रस्फुटः—वासिष्ठः

13. 7. 2. 'रसगुणनवेन्दु'युक्ते 'शशिगुणखगुणो'द्धृते घना द्युगणे ।
शेषे नवभिर्गुणिते गतयो 'ऽष्टजिनैः' पदं शेषम् ॥ २ ॥

घनषोडशहृतशेषं प्रोज्झ्याथस्त्रिगुणितं चतुर्भक्तम् ।
भादिकला द्विगुणधनाः 'शशिमुनिनवयमा'श्च राश्याद्याः ॥

'विषयधृतयो' गतिघ्ना गतिकाष्ठांशोनिताः कलाः प्रोक्ताः ।
'वेदार्काः' पदसंख्या गत्यर्धं धनमृणं परतः ॥ ४ ॥

गत्यर्धे भगणार्धं देयं लिप्ताचतुष्कसंयुक्तम् ।
शेषपदसमाश्चांशाः तत्र धनर्णात् फलं देयम् ॥ ५ ॥

व्येकपद'मिन्द्रिय'घ्नं 'कृतनवदश'संयुतं वियुक्तं च ।
'मनुवेदयमे'भ्यः पदगुणे त्रिषष्टचोद्धृते लिप्ताः ॥ ६ ॥
(Varāha, *PS*, 2. 2-6)

[1] For the rationale, see *Mahā.* : *SRS*, pt.II, p. 12

[2] For the rationale, see *Mahā.* : *SRS*. pt. II. pp. 13-14.

[3] For the rationale, see *SiŚi* : *AS*, pp. 34-37.

[1] For rationale, see *KR*: *KSS*, pp. 6-8.

[2] For rationale see *KR*: *KSS*, p. 8.

True Moon—Vāsiṣṭha

Add 1936 to the Days from Epoch, and divide the sum by 3031. The quotient are called *ghanas.* Multiply the remainder by 9 and divide by 248. The quotient are called *gatis* and the remainder are called *padas.* (2)

Divide the *ghanas* out by 16 and take remainder alone. Multiply this by 3, divide by 4 and take the result as *rāśi* etc. Subtract this from 12 *rāśis* and take the remainder. Add to this minutes equal to twice the total *ghanas.* Add also $1^{r}\ 7^{\circ}\ 29^{\circ}$. (The mean Moon at the end of the *ghanas* is got.) (3)

Multiply the *gatis* by 185, subtract a tenth of the *gatis* and add these also, taken as minutes. (The Mean Moon at the end of the *gatis* is got.) If the number of *padas* is less than 124, they are called plus-*padas.* If 124 or more, 124 *padas* are taken and set apart as a half-*gati.* The remaining *padas* are called minus-*padas.* (The three technical terms here, half-*gati*, plus-*pada* and minus-*pada* are for use in verses 5 and 6 below). (4)

If a half-*gati* has been obtained, for the sake of that half-*gati*, add *rāśis*, etc. 6-0-4. Add also degrees equal in number to the plus-*padas* or minus-*padas.* Using the plus-*padas* or minus-*padas*, respectively, in the two following formulae, find the value, which is in minutes and add that also. The True Moon is got. (5)

Deduct one from the plus-*padas* or minus-*padas* and multiply by 5. If plus-*pada*, add the product to 1094, multiply this sum by the plus-*pada*, and divide by 63. These are the minutes to be added. If minus-*pada*, subtract the product from 2414, multiply the remainder by the minus-*padas* and divide by 63. These are the minutes to be added. (6). (TSK)

रविस्फुटः—पौलिशः

13. 7. 3. 'खार्क'घ्ने'ऽग्निहुताशन'मपास्य 'रूपाग्निवसुहुताशकृतैः' ।
हृत्वा क्रमाद्दिनेशो मध्यः केन्द्रं सविंशांशम् ॥ १ ॥

'एकादशाष्टषट्करूपोना सप्ततिः खयुक्ता' च ।
'नवषट्कमुत्कृतिश्च' क्षयः कलाः केन्द्रराशिसमाः ॥ २ ॥

'दशषट्काष्टकसप्ततिसप्ततिरेकाधिका' च नवषटकम् ।
पञ्चकृतिश्चोपचयो मध्यमसूर्यस्फुटो भवति ॥ ३ ॥

(Varāha, *PS*, 3. 1-3)

True Sun—Pauliśa

Multiply the days from Epoch by 120, deduct 33 and divide by 43,831. The Mean Sun in revolutions, *rāśis* etc. is obtained. Add 20° to this mean Sun. What is called *kendram* is got. (1)

For the first six *rāśis* of *kendra* there are the following six quantities: 11, 48, 69, 70, 54 and 26, all deductive and in minutes. For the next six *rāśis* are the following: 10, 48, 70, 71, 54 and 25, all additive and in minutes. If these are taken one after another according to *rāśis* of the *kendra* gone and applied to the mean Sun, it becomes True sun.[1] (2-3). (TSK)

रविस्फुटः—रोमकः

13. 7. 4. रोमकसूर्यो द्युगणात् 'खतिथि'घ्नात् 'पञ्चकर्तु'परिहीणात् ।
'सप्ताष्टकसप्तकृतेन्द्रियो'द्धृतान्मध्यमः क्रमशः ॥ १ ॥

रविशशिनोः स्फुटकरणं स्वकेन्द्रभवनार्धसम्मितैः खण्डैः ।
व्युत्क्रमशश्च पुनस्तैर्मिथुनदलं शोध्यतेऽर्कस्य ॥ २ ॥

'तिथिमनुदशकृत'सहिता 'रसमनु'भिश्च विंशतिर्हीना ।
'धृतिविषयो'ना 'द्विदशाष्टिधृति'षु वृद्धिः कलाविकलाः ॥

(Varāha, *PS*, 8. 1-3)

True Sun—Romaka

According to the Romaka, the mean Sun in revolutions etc. is obtained by multiplying the Days from epoch by 150, deducting 65 from the product, and dividing by 54,787. (1)

Both the Sun and the Moon are to be made true by intervals of the equation of the centre for half-Signs of the respective mean anomalies given for the first three Signs. For the next three Signs they are to be taken in the reverse order. This is repeated for the next six Signs. In the case of the Sun, the anomaly is got by deducting *rāśi* 2-15-0 from the mean Sun. (2)

The minutes of intervals for the Sun, are 20+15, 20+14, 20+10, 20+4, 20−6 and 20−14, from which seconds 18, 5, are to be subtracted, and 2, 10, 16 and 18 are to be added in the given order.[2] (3). (TSK)

चन्द्रस्फुटः—रोमकः

13. 7. 5. 'खखरूपाष्टगुण'घ्नात् 'कृताष्टनवखैक'वर्जिताद् द्युगणात् ।
'त्रिविषयनवखकृताशा'परिशुद्धान्मध्यशीतांशुः ॥ ४ ॥

'शून्यैकैका'भ्यस्तात् 'नवशून्यरसा'न्विताद् दिनसमूहात् ।
'रूपत्रिखगुण'भक्तात् केन्द्रं शशिनोऽस्तगमवन्त्याम् ॥५॥

'मनु' 'भव' 'यम' सहितोंऽशो
'वसु-होत्रा' वर्जिते 'धृति'-'कृती' च ।
'विषयकृति'-'रष्टषट्कं'
'नव'-'तिथिरहितौ 'खचन्द्रेण' ॥ ६ ॥

(Varāha, *PS*, 8. 4-6)

[1] For worked out example, see PS: TSK 3. 1-3.

[2] For elucidation, see *PS*: *TSK*, 8. 1-2.

True Moon—Romaka

The mean Moon in revolutions etc. is got by multiplying the 'days' by 38,100, subtracting 10,984, and dividing by 10,40,953. (4)

The mean anomaly in revolutions etc. is obtained by multiplying the days by 110, adding 609, and dividing by 3031, the result being for sunset at Ujjain. (5)

For the half-Signs of anomaly the intervals of equation of the centre are: $1^\circ+14'+25''$, $1^\circ+11'+48''$, $1^\circ+2'-9''$, $48'-15''$, $48'-18'-0''$, and $48'-18'-20'-1''$ (*i.e.*, $1^\circ\,14'\,25''$, $1^\circ\,11'\,48''$, $1^\circ\,1'\,51''$, $47'\,45''$, $30'\,0''$, $9'\,59''$. (6).[1] (TSK)

रविचन्द्रयोः स्फुटः—सौरः

13. 7. 6. अंशाशीत्या हीनोऽर्कः केन्द्रं स्वोच्चवर्जितश्चन्द्रः ।
तज्ज्यार्कस्य 'मनु'घ्नी 'रूपाग्नि'गुणा शशाङ्कस्य ॥ ७ ॥
'व्योमरसानल'भक्ते तच्चापं द्विस्थितं स्वकेन्द्रवशात् ।
प्रथमे चक्रस्यार्धे क्षयश्चयः पश्चिमे भागे ॥ ८ ॥
सौर्यं स्थापितचापं तद्भुक्तिघ्नं 'खखाष्टियम'भक्तम् ।
प्रथमवदर्के कार्यं चन्द्रे च दिवाकरवशेन ॥ ९ ॥

(Varāha, *PS*. 9. 7-9)

True Sun and Moon—Saura

The mean longitude of the Sun minus 80° is called the Sun's (mean) anomaly. The mean Moon minus its apogee is its (mean) anomaly. Multiply the sine of the anomaly of the sun by 14, and that of the Moon by 31. (7)

Divide each by 360, and find their arcs. Put the Sun's arc in two places, for subsequent use. The arc of each is to be deducted from its mean longitude if its anomaly is less than six *rāśis*, and added if more than six *rāśis*. (The true Sun and Moon at Ujjain mean Moon is got.) (8)

Multiply the Sun's arc, kept aside in one place, by the Sun's true daily motion (in minutes) and that kept in the other place by the Moon's true daily motion (in minutes). Divide each by 21,600. Add or subtract the resulting minutes in the respective true longitudes found, according as the Sun's arc was first added or subtracted. (The true Sun and Moon at Ujjain true noon is obtained.)[2] (9). (TSK)

रविचन्द्रस्फुटः—भास्करः १ (महाभास्करीयम्)

13. 7. 7. बाहुकोटी क्रमात् केन्द्रे कोटिबाहू गतागते ।
तयोर्गुणफले प्राग्वत् कर्णार्थे परिकीर्तिते ॥ ८ ॥
आद्ये पदे चतुर्थे च व्यासार्धे कोटिसाधनम् ।
क्षिप्यते शोध्यते चैव शेषयोः कोटिका भवेत् ॥ ९ ॥
तद्बाहुवर्गयोगस्य मूलं कर्णः प्रकीर्तितः ।
बाहुकोटिफलाभ्यस्ते कर्णे व्यासार्धभाजिते ॥ १० ॥
भुजाकोटिफले स्यातां ताभ्यां कर्णश्च पूर्ववत् ।
भूयः पूर्वफलाभ्यस्ते कर्णे त्रिज्याविभाजिते ॥ ११ ॥
एवं पुनः पुनः कुर्यात् कर्णः पूर्वोक्तकर्मणा ।
यावत्तुल्यो भवेत् कर्णः पूर्वोक्तविधिनाऽमुना ॥ १२ ॥

(Bhāskara I, *MBh*., 4.8-12)

True Sun and Moon—Bhāskara I (MBh.)

The portions (of the mean anomalistic quadrant) traversed and to be traversed (by a planet) are called *bāhu* and *koṭi* or *koṭi* and *bāhu*, according as the mean anomalistic quadrant (occupied by the planet) is odd or even. The *bāhuphala* and *koṭiphala* are obtained as before for the determination of the hypotenuse (i.e., the distance of the planet). (8)

True distance in minutes of the Sun or Moon

(When the Sun or Moon is) in the first or fourth (mean anomalistic) quadrant, add the *koṭiphala* to the radius: (when) in the remaining (quadrants), subtract that from the radius: the resulting sum or difference is the upright. The square root of the sum of the squares of that and the *bāhuphala* is called the hypotenuse. Multiply that hypotenuse (severally) by the *bāhuphala* and *koṭiphala* and divide (each product) by the radius: the results are (again) the *bāhuphala* and *koṭiphala*. From them obtain the hypotenuse (again) as before. Again multiply this hypotenuse (severally) by the initial *bāhuphala* and *koṭiphala* and divide (each product) by the radius. In this way, processing as above, obtain the hypotenuse again and again until two successive values of the hypotenuse agree (to minutes). (Thus is obtained the nearest approximation to the true distance in minutes of the Sun or Moon).[1] (9-12). (KSS)

रविचन्द्रस्फुटः—भास्करः (लघुभास्करीयम्)

13. 7. 8a. मध्यमं पद्मिनीबन्धोः केन्द्रमुच्चेन वर्जितम् ।
पदं राशित्रयं तत्र भुजाकोटी गतागते ॥ १ ॥
ओजे युग्मे क्रमाज्ज्ञेये कोटिबाहू इति स्थितिः ॥ २ ॥

(Bhāskara I, *LBh*., 2. 1-2a)

True Sun and Moon—Bhāskara I (LBh.)

Sun's anomaly, bhujā and koṭi

The mean longitude of the Sun diminished by the longitude of the (Sun's) apogee is (called) the (Sun's

[1] For the working, see *PS*:*TSK*, 8. 4-6.

[2] For the rationales and calculation, see *PS*:*TSK*, 9. 7-9.

[1] For the rationale, see *MBh*: *KSS*, pp. 116-21.

mean) anomaly. There (in that anomaly) three Signs form a quadrant. In the odd quadrant, the arc traversed and the arc to be traversed are known as *bhujā* (of *bāhu*) and *koṭi* (respectively); in the even quadrant, (they are known as) *koṭi* and *bhujā* (or *bāhu*) respectively. This is the position. (1-2a)

13. 7. 8b. लिप्तीकृत्य धनुर्भागैर्जीवाः कल्प्या भुजेतराः ॥ २ ॥
वर्तमानाहतं शेषं धनुषाप्तं विनिक्षिपेत् ।
(Bhāskara, *LBh.*, 2. 2b-3a)

R sines of the bhujā and koṭi

After converting the *bhujā* and the other (i.e., the *koṭi*) into minutes of arc and dividing by 225, (in each case), take (the sum of) as many R sine-differences as the quotient. Then multiply the remainder (in each case) by the current (i.e., next) R sine-difference and divide by 225 and add the result (to the corresponding sum of the R sine-differences obtained above). (The sums thus obtained are the R sines of the *bhujā* and the *koṭi*). (2b-3a)

13. 7. 8c. ते परिध्याहतेऽशीत्या लब्धे कोटिभुजाफले ॥ ३ ॥
भुजाफलं धनर्णं स्यात् केन्द्रजूकक्रियादिके ।
(Bhāskara I, *LBh.*, 2. 3b-4a)

Bhujāphala and koṭipala

They (i.e., the R sines of the *bhujā* and the *koṭi*) multiplied by the (planet's tabulated) epicycle should be divided by 80: the results are (known as) *bhujāphala* and *koṭiphala*. (3b)

Bhujāphala correction

The *bhujāphala* is additive or subtractive according as the (mean) anomaly is in the half-orbit commencing with the sign Libra or in that commencing with the sign Aries. (4a)

13. 7. 8d. भुजाफलहते भोगे चक्रलिप्ताप्तमेव च ॥ ४ ॥
भुजाफलस्य षड्भागस्तिग्मांशोर्वा विलिप्तिकाः ।
त्रिरभ्यस्ता द्व्यशीत्याप्ता लिप्तिकाद्या निशाकृतः ॥५॥
(Bhāskara, *LBh.*, 2.4b-5)

Bhujāntara correction

So also is applied (the *bhujāntara* correction) which is obtained by multiplying the (mean daily) motion of the planet by the (Sun's) *bhujāphala* and dividing by the number of minutes of arc in a circle (i.e., 21600). (4b)

One-sixth of the (Sun's) *bhujāphala* is, in seconds of arc, (the *bhujāntara* correction) for the Sun; that for the Moon is obtained in minutes of arc etc. by multiplying (the Sun's *bhujāphala*) by 3 and dividing by 82. (5)

मन्दकर्णः

13. 7. 8e. कोटिसाधनयुक्तोनं व्यासार्धं मृगकर्कितः ।
तद्बाहुवर्गसंयोगमूलं कर्णः फलाहतः ॥ ६ ॥
व्यासार्धाप्तफलावृत्त्या कर्णः कार्योऽविशेषितः ।
शीतांशोरप्ययं ज्ञेयो विधिः कर्णाविशेषणे ॥ ७ ॥
(Bhāskara I, *LBh.*, 2. 6-7)

True distances of Sun and Moon

Increase or diminish the radius by the (Sun's) *koṭiphala* (according as the mean Sun is) in the half-orbit commencing with the anomalistic Sign Capricorn or in that commencing with Cancer. The square root of the sum of the squares of that and the (Sun's) *bāhuphala* is the (first approximation to the Sun's) distance. (Severally) multiply that by the (Sun's) *bāhuphala* and *koṭiphala* and divide (each product) by the radius: (the results are again the Sun's *bāhuphala* and *koṭiphala*). (Making use of them calculate the Sun's distance afresh: thus is obtained the second approximation to the Sun's distance. (Repeat this process again and again and thus) by the method of successive approximations, obtain the nearest approximation to the Sun's (true) distance. For the Moon, too, this is to be regarded as the method for finding the nearest approximation to the true distance. (6-7).

कर्णभुक्तिः

13. 7. 8f. व्यासार्धसङ्गुणा भुक्तिर्मध्या कर्णेन लभ्यते ।
स्फुटभुक्तिः सहस्रांशोः, शीतांशोरप्ययं विधिः ॥ ८ ॥

रविस्फुटभुक्तिः

अन्त्यमौर्वीहतां भुक्तिं मध्यमां धनुषा हरेत् ।
लब्धं स्ववृत्तसंक्षुण्णं छित्वाऽशीत्या विशोधयेत् ॥ ९ ॥
मकरादिस्थिते केन्द्रे कर्कटादौ तु योजयेत् ।
मध्यभुक्तौ सहस्रांशोः स्फुटभुक्तिरुदाहृता ॥ १० ॥
(Bhāskara I, *LBh.*, 2. 8-10)

Sun's true daily motion

Multiply the mean daily motion (of the Sun) by the radius and divide (the product) by the (Sun's true) distance (in minutes): the result is the Sun's true daily motion (known as *karṇabhukti* or *karṇasphuṭabhukti*). For the Moon, too, this is the method. (8)

Divide by 225 the (Sun's) mean daily motion as multiplied by the current R sine-difference. Multiplying the result (thus obtained) by its (tabulated) epicycle and dividing by 80, subtract that from the Sun's mean daily motion if the (Sun's) anomaly is in the half-orbit commencing with Capricorn and add that to the same if (the Sun's anomaly is) in the half-orbit commencing with Cancer. (The sum or difference

thus obtained) is known as the (Sun's) true daily motion. (9-10)

चन्द्रस्फुटभुक्तिः

13. 7. 8g. उत्क्रमक्रमतो ग्राह्याः पदयोरोजयुग्मयोः ।
वर्तमानगुणादिन्दोः केन्द्रभुक्तेः कलावशात् ।। ११ ।।

आद्यन्तधनुषोर्ज्ञेयं फलं त्रैराशिकक्रमात् ।
गतगन्तव्यधनुषी केन्द्रभुक्तेर्विशोधयेत् ।। १२ ।।

इत्थमाप्तगुणं हत्वा वृत्तेनाशीतिसंहृतम् ।
प्राग्वत् क्षयोदयाविन्दोर्मध्ये भोगे स्फुटो मतः ।। १३ ।।

अभिन्नरूपता भुक्तेश्चापभागविचारिणः ।
रवेरिन्दोश्च जीवानामूनभावाद्यसंभवात् ।। १४ ।।

एवमालोच्यमानेयं जीवाभुक्तिर्विशीर्यते ।
कर्णभुक्तिः स्फुटाह्नोर्वा विश्लेषः स्फुटयोर्द्वयोः ।।१५।।

(Bhāskara I, *LBh.*, 2. 11-15)

Moon's true daily motion

From the (mean daily) motion of the (Moon's) mean anomaly subtract the preceding or succeeding arc (of the current element of the arc, i.e., the elementary arc[1] containing the Moon) (according as the Moon is in the odd or even anomalistic quadrant). (Then) take (the tabulated R sine-differences) on the basis of the (residue in) minutes of the (mean daily) motion of the Moon's mean anomaly, starting from the current R sine-difference reversely and directly in the odd and even anomalistic quadrants respectively. The results (i.e., the R sine-differences) corresponding to the fractions of the first and last elementary arcs should be determined by proportion (and added to the sum of the previous R sine-differences). The R sine-difference (corresponding to the daily motion of the Moon's mean anomaly) thus obtained multiplied by the (Moon's tabulated) epicycle and divided by 80 should be subtracted from or added to the Moon's mean daily motion as before (in the case of the Sun, i.e., according as the Moon's anomaly is in the half-orbit commencing with the Sign Capricorn or in that commencing with the Sign Cancer). This is known as (the Moon's) true (daily motion). (11-13)

Defects of the jīvābhukti

(According to the rules stated above), whilst the Sun or the Moon moves in the (same) element of arc, there is no change in the rate of motion because (the current R sine-difference being fixed throughout that element) the R sine-difference does not decrease or increase: when viewed in this way, this *jīvābhukti* is defective. (14-15a)

Author's view on true daily motion

The *karṇabhukti* or the difference between the true (longitudes) for two consecutive days is the true (daily) motion. (15b). (KSS)

लल्लः—स्फुटः भुक्तिश्च लघुज्याप्रकारेण

13. 7. 9. केन्द्रज्ये स्वगुणेन सूर्यशशिनोः क्षुण्णे 'यम'घ्ने हृते
'शैलै'स्ते कलिकादिके निजफले ताभ्यां स्फुटावुक्तवत् ।
भोग्यं खण्डमिनस्य 'कामदहनै'र्भक्तं गतेः स्यात् फलं
चन्द्रस्य त्रिगुणं 'स्वरां'शरहितं प्राग्वद्धनर्णं गतौ ।। ४ ।।

स्पष्टीकृतेन गुणकेन हताः कुजज्ञ-
जीवास्फुजिद्दिनकरात्मजमन्दजीवाः ।
द्विघ्ना'स्तुरङ्गम'हृताः स्वफलानि लिप्ता-
स्ताः पूर्ववद् धनमृणं खचरेषु कुर्यात् ।। ५ ।।

प्राग्वच्च शीघ्रगुणकेन परिस्फुटेन
हत्वा हरेच्च 'खगजै'र्भुजकोटिजीवे ।
शीघ्रादिकर्म विदधीत ततो यथोक्तं
चापीयमुक्तवदनेन खगाः स्फुटाः स्युः ।। ६ ।।

(Lalla, *ŚiDhVṛ.*, 13. 4-6)

Lalla: True long. and motion—Laghujyā method

(In the *Laghujyā* method), multiply the R sine of the mean anomaly of the Sun or Moon by its *mandaguṇaka*. Then multiply each product by 2 and divide by 7. The results in minutes etc. are their respective *mandaphalas*. Hence find their true longitudes as before.

Divide the Sun's *bhogyakhaṇḍa* by 11. The result is the Sun's *mandagatiphala* or correction to be given to its mean motion.

Multiply the Moon's *bhogyakhaṇḍa* by 3 and subtract from the product 1/7 of the *bhogyakhaṇḍa*. The result is the correction to be given to the Moon's motion.

These corrections should be added to or subtracted from the respective mean motions of the Sun and Moon as instructed before. (4)

Multiply the R sines of the mean anomalies or the *mandakendras* of Mars, Mercury, Jupiter, Venus and Saturn by their respective corrected *mandaguṇakas*. Then multiply each product by 2 and divide by 7. The results in minutes are the respective *mandaphalas* of these planets. These should be added to or subtracted from their respective mean longitudes as explained before. (5)

As before, multiply the R sine and R cosine of the *śīghrakendra* of a planet by its correct *śīghraguṇaka* and divide the product by 80. Hence find the R sine of

[1] The twenty-four divisions of a quadrant, each equal to 225′, the R sine differences of which have been tabulated by Āryabhaṭa I, are called "elements of arc", or "elementary arcs".

śighraphala, etc. as said before. And hence the corresponding arc. Thus the true longitudes of the planets are calculated.[1] (6). (BC)

रविचन्द्रस्फुटः—आर्यभटार्धरात्रिकपक्षः

13. 7. 10. पञ्चगुणाः सप्तरसाः शरनन्दाः षोडशेन्दवो गोऽर्काः ।
कृतगुणचन्द्रा शश्यर्धलिप्तिकाः पिण्डकाः सवितुः ।।१६।।
शशिनः सप्तकमुनयो वसुमनवो नवनखा रसेषुयमाः ।
रसवसुयमलाः षण्णवयमा, ग्रहः केन्द्रमुच्चोनः ।। १७ ।।
विषमे भुक्तस्य, समे भोग्यस्य स्वफलमृणधनं मध्ये ।
भांशोऽर्कफलस्येन्दोः षड्राश्यूनाधिके केन्द्रे ।। १८ ।।
पञ्चदशकेन विभजेद् भानुमतो भोग्यमानकं पिण्डम् ।
शशिनोऽगगुणं वसुभिः क्षयधनधनहानयः स्वगतौ ।।१९।।
गतिभोग्यखण्डकवधाल्लब्धं नवभिः शतै रवीन्दुफलम् ।
प्राग्वच्छुक्रादीनां क्षयधनधनहानयः स्वगतौ ।। २० ।।

ब्रह्मगुप्तकृतः शोधः—भुजान्तरम्

'द्विकृतां'शोनं रविफलमिन्दो'र्दस्रेषु'भागयुतम् ।
अर्कफलभुक्तिघाताद् भगणकलाप्तं भुजान्तरं रविवत् ।।
(Brahmagupta, *KK*, 1.1.16-20; 2.1.5)

True Sun and Moon—ABh. Midnight System

35′, 67′, 95′, 116′, 129′ and 134′ are the *mandaphalas* of the Sun for every half Sign of the *mandakendra*; that is, when the *mandakendras* are respectively 15°, 30°, 45°, 60°, 75° and 90°. (16)

77′, 148′, 209′, 256′, 286′ and 296′ are the *mandaphalas* of the Moon.

When the longitude of a planet's *mandocca* is deducted from its mean longitude, the remainder is its *manda kendra*. (17)

When the *mandakendra* is in an odd quadrant, the *mandaphala* should be calculated from the arc of the quadrant passed over, and when it is in an even quadrant, from the arc to be passed over. This *mandaphala* should be added to or subtracted from the mean longitude of the planet, according as the *mandakendra* is greater or less than 6 Signs.

A further correction for the Moon is 1/27 of the *mandaphala* of the Sun applied positively or negatively as in the case of the Sun. (18)

One should divide the Sun's *bhogyamānapiṇḍaka*, (the difference between the *bhuktamandaphala* and the *bhogyamandaphala*) by 15, and 7 times that of the Moon by 8. The results are their *mandagatiphalas*, respectively. These should be applied to their respective mean motions negatively, positively, positively or negatively, according as the *mandakendra* is in the first, second, third or fourth quadrant. (The results are the corrected motions of the Sun and the Moon, respectively). (19)

When the daily motion of the *mandakendra* of the Sun or Moon is multiplied by the *bhogyakhaṇḍa* (that is, *bhogyamānapiṇḍaka*), and the product divided by 900, the result is the *mandagatiphala* of the Sun or Moon accordingly (and is to be applied as explained in the previous verse).

In the same manner, the *mandagatigphalas* of Venus and other planets may be calculated and applied to their mean motions negatively, positively, positively or negatively (according as the *mandakendra* is in the first, second, third or fourth quadrant. The results are the corrected motions of the sun, moon and the planets).[1] (20)

Emendation by Brahmagupta: *Bhujāntara*

The *mandaphalas* of the Sun (calculated according to *KK* I.1.16) should be decreased by 1/42 part, and those of the Moon (calculated according to *KK*, I.1.17) should be increased by 1/52 part.

Multiply the daily motion of any planet by the Sun's *mandaphala* and divide the product by 21,600. The result is *bhujāntara*, which should be added to the longitude of the planet, if the *mandaphala* is added to the Sun's longitude; and subtracted, if the *mandaphala* is subtracted. This gives the longitude of the planet corrected by *bhujāntara*. (5). (BC)

रविस्फुटः—सूर्यसिद्धान्तः

13. 7. 11. एवं विषुवति छाया स्वदेशे या दिनार्धजा ।। १२ ।।
दक्षिणोत्तरयोरेव सा तत्र विषुवत्प्रभा ।
शङ्कुच्छायाहते त्रिज्ये पृथक्तत्कर्णभाजिते ।। १३ ।।
लम्बाक्षज्ये तयोश्चापे लम्बाक्षौ दक्षिणौ सदा ।
मध्यच्छाया भुजस्तेन गुणिता त्रिभमौर्विका ।। १४ ।।
तत्कर्णाप्तधनुर्लिप्ता नतास्ता दक्षिणे भुजे ।
उत्तराश्चोत्तरे याम्यास्तत्सूर्यक्रान्तिलिप्तिकाः ।। १५ ।।
दिग्भेदे मिश्रिताः साम्ये विश्लिष्टाश्चाक्षलिप्तिकाः ।
तज्ज्याक्षज्याथ तद्वर्गं प्रोज्झ्य त्रिज्याकृतेः पदम् ।।१६।।
लम्बज्याक्षगुणोऽर्कघ्नः पलभाप्तोऽवलम्बकः ।
स्वाक्षार्कनतभागानां दिक्साम्येऽन्तरमन्यथा ।। १७ ।।
दिग्भेदेऽपक्रमः शेषः तस्य ज्या त्रिज्यया हता ।
परमापक्रमज्याप्तचापं मेषादिगे रविः ।। १८ ।।

[1] For the rationale, see *ŚiDhVṛ. BC*, II. 212-17.

11-*

[1] For the rationale and formulae involved, see *KK*: *BC*, I. App. vii. pp. 222-95.

कर्क्यादौ प्रोज्झ्य चक्रार्धात्तुलादौ भार्धसंयुतात् ।
मृगादौ प्रोज्झ्य भगणात् मध्याह्नार्कस्फुटो भवेत् ।।

(*Sū.Si.*, 3. 12b-19)

True Sun—Sūryasiddhānta

In like manner, the equatorial shadow which is cast at midday at one's place of observation upon the north and south line of the dial—that is the equinoctial shadow (*viṣuvatprabhā*) of that place. (12b-13a)

The Radius, multiplied, respectively, by gnomon and shadow, and divided by the equinoctial hypotenuse gives the sines of co-latitude (*lamba*) and of latitude (*akṣa*); the corresponding arcs are co-latitude and latitude, always south. (13b-14a)

The midday shadow is the base (*bhuja*); if radius be multiplied by that and the product divided by the corresponding hypotenuse, the result, converted to arc, is the Sun's zenith-distance (*nata*), in minutes; this, when the base is south, is north, and when the base is north, is south. (14b-15)

Of the Sun's zenith-distance and its declination, in minutes, take the sum, when their direction is different, and the difference, when it is the same; the result is the latitude, in minutes. From this find the sine of latitude; subtract its square from the square of radius, and the square-root of the remainder is the sine of co-latitude. (15-16a)

The sine of latitude, multiplied by twelve, and divided by the sine of co-latitude, gives the equinoctial shadow. (16b)

The difference of the latitude of the place of observation and the Sun's meridian zenith-distance in degrees (*natabhāgas*), if their direction be the same, or their sum if their direction be different, is the Sun's declination. (17)

If the sine of this latter be multiplied by radius and divided by the sine of greatest declination, the result, converted to arc will be the Sun's longitude, if it is in the quadrant commencing with Aries. (18)

If in that commencing with Cancer, subtract from a half-circle; if in that commencing with Libra, add a half-circle; if in that commencing with Capricorn, subtract from a circle; the result, in each case, is the true (*sphuṭa*) longitude of the Sun at midday.[1] (19) (E. Burgess)

[1] For elucidation, see *Sū.Si*: *Burgess*, pp. 121-24.

रविचन्द्रस्फुटः—ग्रहलाघवम्

भुजाकोट्यानयनम्

13. 7. 12. दोस्त्रिभोनं त्रिभोर्ध्वं विशेष्यं रसै-
श्चक्रतोऽङ्काधिकं स्याद् भुजोनं त्रिभम् ।
कोटिरेकैकं त्रित्रिभैः स्यात् पदं
सूर्यमन्दोच्च'मष्टाद्रयों'ऽशा भवेत् ।। १ ।।

रविचन्द्रस्फुटः

मन्दोच्चं ग्रहवर्जितं निगदितं केन्द्रं तदाख्यं बुधैः
केन्द्रे स्यात् स्वमृणं फलं क्रियतुलाद्येऽथो विधेयं रवेः ।
केन्द्रं तद्भुजभागखेचरलवोनघ्ना 'नखा'स्ते पृथक्
तद्'गोंशोननगेषुभिः' परिहृतास्तेंऽशादिकं स्यात् फलम्।।

चन्द्रमन्दफलानयनम्

विधोः केन्द्रदोर्भागषष्ठोननिघ्नाः
'खरामाः' पृथक् त'न्नखां'शोनितैश्च ।
'रसाक्षै'र्हृतास्ते लवाद्यं फलं स्याद्
रवीन्दू स्फुटौ संस्कृतौ स्तश्च ताभ्याम् ।। ३ ।।

रविचन्द्रयोः गतिफलानयनम्

केन्द्रस्य कोटिलव'खाश्वि'लवोननिघ्ना
'रुद्रा' रवे'स्त्रिकु'हृताः शशिनो द्विनिघ्नाः ।
स्वा'ङ्गां'शकेन सहिताश्च गतौ धनर्णं
केन्द्रे कुलीरमृगषट्कगते स्फुटा सा ।। ४ ।।

(Gaṇeśa, *Gh.* 2.1-4)

True Sun and Moon—Grahalāghava

Bhuja and Koṭi. If the mean position of a planet is less than 3 *rāśis* (90°), then that itself its *bhuja*; if it is more than 90°, then take the difference between it and 180°; if it is greater the 180°, subtract 180 from it; if greater than 270° subtract from 360° (i.e., 12 *rāśis, cakra*).

The difference between 90° and *bhuja* is the *koṭi*. The position of Sun's apogee is 78 degrees (2 *rāśis*, 18 degrees). (1)

True positions of Sun and Moon. From the *mandocca* subtract the position of the planet. The result gives *mandakendra*. The *mandaphala* is to be added if the Sun is in the six *rāśis* from Meṣa; this is to be subtracted if it is in the six *rāśis* from Tulā.

Mandaphala. Convert the Sun's position into *bhuja* in degrees. Divide it by 9 and add 20 degrees to the quotient. Multiply this by the quotient. Keep this separate, (say x). Find $57° - x/9 = y$. Convert x and y into seconds and find x/y. That in degrees, minutes and seconds gives the *mandaphala*. (2)

Mandaphala of Moon. Convert the position of Moon into *bhuja* in degrees. Divide by 6 and subtract the quotient from 30°. Multiply the balance by the

quotient (say x). Find $56^\circ - x/20 = y$, and x/y. The result gives Moon's *mandaphala*.

When the corrections for *mandaphala* are carried out, the Sun and the Moon have their true positions. (3)

Daily motion of Sun and Moon. From the central positions of Sun and Moon find the *koṭi*. (First calculate *bhuja* and take $90^\circ - bhuja$ for *koṭi*), in degrees. Divide by 20. Subtract the quotient from 11 and multiply the balance by the quotient. If this product (say x) is divided by 13, we get the daily motion of the Sun. Multiply x by 2. Add $x/6$. (Find $2x + x/6$). This gives the daily motion of the Moon.

For six *rāśis* from *karkaṭa* this *gatiphala* is to be added to *mandakendra*; it should be subtracted for six *rāśis* from Makara. (4). (VSN)

रविस्फुटः—वाक्यकरणम्

13. 7. 13. 'सेना'भक्तावशेषितम् ॥ ३ ॥
'श्रीर्गुणा'दिध्रुवं विद्याद्, भान्तं वाक्यैस्तु दृश्यते ।

श्रीर्गुणमित्रेति वाक्यानि

श्रीर्गुणमित्रा भूर्विधिपक्षा स्त्रीरतिशूरा भोगवराते ।
भावचरोरिः तेन वशत्वं लोकजभीतिः स्थूलहयोऽयम् ॥
अंगधिगारः स्तम्भितनाभिः नित्यशशीशो यागमयोऽयम् ।
तावुरुपूर्वं संक्रमवाक्यं तत्क्रमयोज्यं पादवशेन ॥ ४a ॥
वर्षस्य वर्तमानस्य विनाड्याद्यं गतं दिनम् ॥ ४ ॥
विलिप्तादि रवि,र्वाक्यकलाहीनोंऽशकान्त्यकः ।
नीतिः खण्डनयांशोना गतिः, पूर्वेऽधिके युता ॥ ५ ॥

भूपादिवाक्यानि

भूपज्ञ रागज्ञ वशेन दित्सुना
मुनीड्य गांगेय गतस्य वर्धकः ।
वारास्त्र भूमीन्द्र विहार पुण्यगो
मालाङ्ग देशाङ्ग तथाम्बु युद्धगः ॥
लीनाश्व कम्पाश्व शुकाभ तापवान्
प्रियाश्व तुन्नाश्व जलाम्बु तुन्दगः ।
वासाङ्ग पीताम्बु सवेग भोगगो
श्रीरङ्ग पुण्याङ्ग लुनाङ्ग सिन्धुराट् ॥
शुद्धास्त्र ताळेन्द्र कुनील धेनुगो
रुद्राङ्ग लिप्ताः दिवसैर्नयाऽऽहृतैः ॥ ५a ॥

(*VK*, 1. 3b-5b)

True Sun—Vākyakaraṇa

Divide the Kali days to the end of the true year by 7. The remainder is the *dhruva*, to which should be added each of the mnemonics beginning with *Śrīrguṇamitra* given at the rate of a foot (of 5 letters) for each, to get the time and week-day of the succeeding Saṅkramaṇas, Ṛṣabha etc. (3-4)

Śrir guṇamitra mnemonics

The mnemonics in days etc. are: 2-55-32; 6-19-44; 2-56-22; 6-24-34; 2-26-44; 4-54-6; 6-48-13; 1-18-37; 2-39-30; 4-6-46; 5-55-10; 1-15-31. (4-A)

To get the True Sun, as a first approximation take the *vināḍīs*, *nāḍīs* and days elapsed from the Meṣa-saṅkrāmaṇa as the Sun etc. Deduct the minutes of arc corresponding to the *vināḍīs* etc. (taking the minutes from the mnemonics *Bhūpajña* etc. which are given for 10, 20, 30 etc. days from Meṣa-saṅkramaṇa). The True Sun in degrees, minutes and seconds is got. (4b-5a)

Bhūpa mnemonics

The deductive minutes given by the mnemonics are: (1) 14, (2) 32, (3) 54, (4) 78, (5) 105, (6) 133, (7) 163, (8) 194, (9) 224, (10) 254, (11) 284, (12) 311, (13) 335, (14) 358, (15) 376, (16) 391, (17) 403, (18) 411, (19) 415, (20) 416, (21) 412, (22) 406, (23) 398, (24) 386, (25) 374, (26) 361, (27) 347, (28) 334, (29) 322, (30) 311, (31) 303, (32) 297, (33) 295, (34) 296, (35) 301, (36) 309, (37) 322. (5-A)

(Deducting 0 from 32, 32 from 54, etc., get the intervals in minutes. Divide this by 10, and deduct from 60 minutes if the succeeding mnemonic is greater than the previous.) If the succeeding is less add to 60 minutes. The daily True Motion in minutes is got, during the corresponding ten-day interval.[1] (5b). (TSK-KVS)

चन्द्रस्फुटः—वाक्यकरणम्

13. 7. 14. 'वेदेद्धज्ञानतुष्टो'नदिवसाद् 'रसगैरिकैः' ।
'कालानलै''र्देवरै'श्च शिष्टं वाक्यं शशाङ्कजम् ॥ ९ ॥
'गीर्नः श्रेया'दिकं ज्ञेयं, क्रमाल्लब्धफलैस्त्रिभिः ।
'नित्यदेवसुखाधीनं' 'यज्ञयोगसुनायकम्' ॥ १० ॥
'तनुर्विश्वसुखं' हत्वा 'सेनानानारिनाथनम्' ।
तद्योगे निक्षिपेच्छुद्धं विकलाद्यं ध्रुवं विधोः ॥ ११ ॥
फलद्वयाभ्यामन्त्याभ्यां हतौ 'जनखलौ' क्रमात् ।
तयोर्विश्लेषविघटी धनं, पूर्वेऽधिके क्षयः ॥ १२ ॥
'पुण्यलोक'स्पष्टगत्योरन्तरांशास्तु तद्गुणाः ।
विकलाः स्युः स्फुटाच्चन्द्राद् गुणवर्धनशोधने ॥ १३ ॥
स्फुटाधिके 'पुण्यलोके' विपरीतमृणं धनम् ।

(*VK*, 1. 9-14a)

True Moon—Vākyakaraṇa

Deduct 16,00,984 from the Kali days. Divide the remainder by 12,372. Divide the remainder after this

[1] For worked out example see *VK*: *TSK-KVS*, p. 253.

division by 3031. Divide the remainder of this by 248. The remainder of this is the number of the mnemonics to be taken from the Moon's mnemonic-tables beginning with *Gīrnaḥ, śreyaḥ*.[1] Multiply $9^r\ 27°\ 48'\ 10''$ by the first quotient, $11^r\ 7°\ 31'\ 1''$ by the second quotient, and $0^r\ 27°\ 44'\ 6''$ by the third quotient; add them up and add $7^r\ 2°\ 0'\ 7''$. This sum is the Moon's *dhruva*. Add to this the value of the mnemonic from the table. The Uncorrected True Moon is got. (9-11)

Multiply the second quotient by 8 and deduct this from the third quotient multiplied by 32. The result are *plus-vināḍis*. If the third quotient is zero, the product of the second quotient by 8 alone should be taken and treated as *minus-vināḍis*. Deduct 13° 11′ from the true daily motion in degrees, and multiply the degrees etc. by the *plus* or *minus-vināḍis*. The result are plus or minus seconds of arc etc., and these should be applied to the Uncorrected true Moon. If the daily motion is less than 13° 11′, then the defect is to be multiplied by the plus or minus *vināḍis* and the resulting seconds should be taken as *minus* or *plus* seconds, respectively, and applied to the Uncorrected True Moon. The True Moon is got.[2] (12-14a). (TSK-KVS)

गीर्नः श्रेयादि चन्द्रवाक्यानि वररुचिकृतानि

Moon sentences 'gīrnaḥ śreyaḥ' etc. of vararuci

Days	*Vākya*	*r*	*o*	*′*
1	गीर्नः श्रेयः	0	12	3
	धेनवः श्रीः	0	24	9
	रुद्रस्तु नम्यः	1	6	22
	भवो हि याज्यः	1	18	44
5	धन्येयं नारी	2	1	19
	धनवान् पुत्रः	2	14	9
	गृह्या सुरा राज्ञा	2	27	13
	बालेन कुलम्	3	10	33
	धनुर्भिः खलैः	3	24	9
10	दश सूनवः	4	7	58
	होमस्य स्रुवः	4	21	58
	दीनास्ते नृणाम्	5	6	8
	मुखं नारीणाम्	5	20	25
	भवभग्नास्ते	6	4	44
15	श्रीर्निधीयते	6	19	2
	शं किल नाथः	7	3	15
	श्रेष्ठा सा कथा	7	17	22
	सौख्यस्यानन्दः	8	1	17
	ध्यानं मान्यं हि	8	15	1
20	धीरो हि राजा	8	28	29
	श्रुत्वास्य युद्धम्	9	11	42
	अभवच्छ्राद्धम्	9	24	40
	गोरसो ननु स्यात्	10	7	23
	द्रुमा धन्या नये	10	19	52
25	इष्टं राज्ञः कुर्यात्	11	2	10
	धन्या विद्येयं स्यात्	11	14	19
	त्वं रक्षा राज्यस्य	11	26	24
	क्षेत्रजः	0	8	26
	नीले नेत्रे	0	20	30
30	जलं प्राज्ञाय	1	2	38
	शशी वन्द्यः स्यात्	1	14	55
	गोरसप्रियः	1	27	23
	वनानि यत्र	2	10	4
	अन्नं गोत्रश्रीः	2	23	0
35	रुष्टास्ते नागाः	3	6	12
	धिगन्धः किल	3	19	39
	पुरोगा अभीः	4	3	21
	मान्यः स कविः	4	17	15
	अरिष्टनाशम्	5	1	20
40	बालो मे केशः	5	15	33
	कुशधारिणः	5	29	51
	इष्टिर्विद्यते	6	41	10
	स राजा प्रीतः	6	28	27
	सुगुप्रायोऽसौ	7	12	37
45	धिगस्तु ह्रासः	7	26	39
	अङ्गानि यदा	8	10	30
	सेनावान् राजा	8	24	7
	धीराः सन्नद्धाः	9	7	29
	शालीनं प्रधानम्	9	20	35
50	क्षीरं गोर्नो नयेत्	10	3	26
	रत्नचयो नृपः	10	16	2
	ताः प्रजाः प्राज्ञाः स्युः	10	28	26
	अश्वानां को योग्यः	11	10	40
	तद्वैरं प्रियायाः	11	22	46
55	धवस्त्वम्	0	4	49
	ग्रामस्तस्य	0	16	52
	जन्मजरा	0	28	58
	इष्टका कार्या	1	11	10
	कुलगुरुः स्यात्	1	23	31
60	मुनिस्तु उग्रः	2	6	5
	प्रमोदकरः	2	18	52
	शशाङ्कानुगः	3	1	55
	वक्ष्यामि कालम्	3	15	14
	संभेदः खलैः	3	28	47

[1] For this table, known as *Vararuci-vākyāni* by Kerala-Vararuci, see below.

[2] For worked out example see, *VK:TSK-KVS*, p. 254.

Days	*Vākya*	*r*	*o*	*'*
65	शीलप्रियस्त्वम्	4	12	35
	वेलातरवः	4	26	34
	विभिन्नं कर्म	5	10	44
	धर्मवान् रामः	5	24	59
	दिग्व्याळो नास्ति	6	9	18
70	ते बाला भ्रान्ताः	6	23	36
	कामासन्नः सः	7	7	51
	होमं पुत्रार्थम्	7	21	58
	मणिर्मानदः	8	5	55
	नाविद्धः पादे	8	19	40
75	उत्पलं निधिः	9	3	10
	शूद्रस्तु योद्धा	9	16	25
	विरुद्धं स्त्रीधनम्	9	29	24
	हीनप्रायो नटः	10	12	8
	धिगश्वः खिन्नोऽयम्	10	24	39
80	दिशतु नः पथ्यम्	11	6	58
	जनोऽन्धः पापकः	11	19	8
	गृह्या स्यात्	0	1	13
	मान्यं लोके	0	13	15
	धन्यः शरैः	0	25	19
85	सुखी स नित्यम्	1	7	27
	लाभो धान्यस्य	1	19	43
	अङ्कुरं नीरे	2	2	10
	धावद्वैद्योऽत्र	2	14	49
	गत्वा सुराष्ट्रम्	2	27	43
90	गमनकालम्	3	10	53
	दयावान् रोगी	3	24	18
	होमस्थानं वनम्	4	7	58
	श्रीमान् पुत्रो वा	4	21	52
	तन्मम नाम	5	5	56
95	दानानां क्रमः	5	20	8
	क्षेत्रवानस्तु	6	4	26
	शम्भुर्जयति	6	18	45
	रत्नाङ्गनार्था	7	3	2
	लक्ष्योऽसौ पार्थः	7	17	13
100	सापत्यनिन्दा	8	1	17
	जनो मान्यो हि	8	15	8
	स वादी राजा	8	28	47
	आकारो युद्धम्	9	12	10
	दास्यामि श्राद्धम्	9	25	18
105	कार्यहानिर्नार्या	10	8	11
	दम्भान्नरा नष्टाः	10	20	48
	विकलानां कार्याः	11	3	14
	हरणं पाद्यस्य	11	15	28
	तुला संप्रत्यया	11	27	36

Days	*Vākya*	*r*	*o*	*'*
110	धिगन्धः	0	9	39
	कविः पुत्रः	0	21	41
	तत्त्वाङ्गनेयम्	1	3	46
	जीर्णो मे कायः	1	15	58
	दया हरस्य	1	28	18
115	अशनपरः	2	10	50
	तालुलेखोऽत्र	2	23	36
	सङ्गतो नागः	3	6	37
	विशुद्धो योगी	3	19	54
	ताराङ्गं नभः	4	3	26
120	प्रियार्थं कविः	4	17	12
	पापोऽयं निशि	5	1	11
	धन्यो मान्योंऽशे	5	15	19
	भोगार्धं रामा	5	29	34
	रामा गीयते	6	13	52
125	अत्याहारस्तु	6	28	10
	शारीरकोऽसौ	7	12	25
	लोलचक्रस्थः	7	26	33
	प्रागनिष्पदम्	8	10	32
	दिव्यवान् राजा	8	24	18
130	अंशार्थिनोर्धीः	9	7	50
	सेनायाः क्रोधः	9	21	7
	दानं भानोर्नष्टम्	10	4	8
	भूमिस्तस्य नित्यम्	10	16	54
	चक्रार्धं प्राज्ञाय	10	29	26
135	ता भार्याः पापोऽयम्	11	11	46
	दिशोऽम्बराण्यस्य	11	23	58
	ग्लौर्नास्ति	0	6	3
	मीनजेयम्	0	18	5
	दानानि नित्यम्	1	0	8
140	तपः श्रेयः स्यात्	1	12	16
	अम्बुभिरिष्टैः	1	24	30
	क्षमास्तु नरैः	2	6	56
	लोलधीः पुत्रः	2	19	33
	ते रौद्रा नागाः	3	2	26
145	विलोमकुलम्	3	15	34
	स मन्दो रागी	3	28	57
	तैलप्रियस्त्वम्	4	12	36
	साम्प्रतं रविः	4	26	17
	कुलानां कर्म	5	10	31
150	श्रुत्वा स्वराणि	5	24	42
	धर्मो दानं तु	6	8	59
	दूष्यं गोत्रं ते	6	23	18
	तुलार्थिनोऽर्थी	7	7	36
	जित्वास्य रथः	7	21	48

Days	*Vākya*	*r*	*o*	*'*
155	श्रमणो निन्दा	8	5	52
	षड्विधान्याहुः	1	19	46
	तत्र गोर्निधिः	9	3	26
	केशास्ते काळाः	9	16	51
	यानानि नो नयेत्	10	0	1
160	शिशिरे पानीयम्	10	12	55
	भोगमात्रं नित्यम्	10	25	34
	यूना दानं पथ्यम्	11	8	1
	सत्येन श्रेयः स्यात्	11	20	17
	मुखे श्रीः	0	2	25
165	धारावृष्टिः	0	14	29
	पलितं राज्ञः	0	26	31
	तैलजा नार्यः	1	8	36
	ताभिर्नराः स्युः	1	20	46
	मीनलग्नेऽत्र	2	3	5
170	तालुमध्ये श्रीः	2	15	36
	नोग्रा दारा राज्ञः	2	28	20
	धन्यः स्यात् कालः	3	11	19
	वर्गे त्वं खलैः	3	24	34
	श्वानो दीनो वा	4	8	4
175	धवाः कारवः	4	21	49
	क्षोभः शनैः शनैः	5	5	46
	गोशुद्धिकामः	5	19	53
	दीनो वो ज्ञातिः	6	4	8
	तत्र दीयते	6	18	26
180	शोभा राज्ञः सेना	7	2	45
	आज्ञा साध्या सा	7	17	0
	नटस्यानन्दः	8	1	10
	धनेशोऽयं जनः	8	15	9
	स मन्दो ह्रदः	8	28	57
185	नागरो युद्धः	9	12	30
	धीवशः क्रोधः	9	25	49
	श्रमो दीनो नित्यम्	10	8	52
	धूली स्याद्राज्ञोऽयम्	10	21	39
	बाह्यवने योग्यम्	11	4	13
190	विगतपापोऽयम्	11	16	34
	तावदत्र कार्यः	11	28	46
	ग्रामो नष्टः	0	10	52
	शशी रात्रौ	0	22	55
	दुःशुभा नष्टाः	1	4	58
195	भानुः सद्यः स्यात्	1	17	4
	दयार्धं श्रेयः	1	29	18
	प्रभायाः पुत्रः	2	11	42
	हर्यश्वः श्रेष्ठः	2	24	18
	धनुः सेनाङ्गम्	3	7	9

Days	*Vākya*	*r*	*o*	*'*
200	शाक्यज्ञो रागी	3	20	15
	सलिलं नवम्	4	3	37
	वैद्यः स कविः	4	17	14
	मेनका नाम	5	1	5
	सेना मध्यमा	5	15	7
205	संयुद्धक्रमः	5	29	17
	स्वर्गलोकोऽस्ति	6	13	34
	गुणार्थी रतिः	6	27	53
	काव्यप्रियोऽसौ	7	12	11
	भद्रतरोऽर्थी	7	26	24
210	धू राज्ञः पादे	8	10	29
	गुरुर्वरदः	8	24	23
	मानदो निधिः	9	8	5
	रङ्गस्य श्रद्धा	9	21	32
	स्वभावो ज्ञानस्य	10	4	44
215	अवस्थेयं नार्याः	10	17	40
	पुत्रो ज्ञानाढ्योऽयम्	11	0	21
	धवः श्रेयः पथ्यम्	11	12	49
	तेन शरैः पटुः	11	25	6
	वैद्योऽसौ	0	7	14
220	हयो धन्यः	0	19	18
	अप्रियो नये	1	1	20
	शास्त्रबाह्योऽयम्	1	13	25
	भोगमात्रस्य	1	25	34
	ग्रामार्थी नरः	2	7	52
225	यात्रान्नं श्रेष्ठम्	2	20	21
	भिन्नाङ्गो नागः	3	3	4
	प्रज्ञातो योगी	3	16	2
	मुख्यो धीरो लीनः	3	29	15
	गावः प्रिया वः	4	12	43
230	सुरतन्त्रिभिः	4	26	27
	त्रिराज्ञाङ्कुशः	5	10	22
	धाराभिः श्रमः	5	24	29
	त्रिभिर्हानिस्ते	6	8	42
	अनङ्गाश्रिता	6	23	0
235	धन्यः स नाथः	7	7	19
	तिलस्य रसः	7	21	36
	तव मानदः	8	5	46
	षड्विधं पदम्	8	19	46
	मङ्गलं नीळम्	9	3	35
240	योग्यः संयुद्धे	9	17	11
	योगो ज्ञानिनः स्यात्	10	0	31
	शैलालयो नम्यः	10	13	35
	मन्त्रितं प्राज्ञाय	10	26	25
	अनिधानं कपेः	11	9	0

Days	Vākya	r	o	′
245	श्रोत्रियः प्रियस्य	11	21	22
	मङ्गलम्	0	3	35
	कवेः शक्यम्	0	15	41
248	भवेत् सुखम्	0	27	44

Method for the verification of the *vākyas*:

'भवेत्सुख'स्य राशीनामर्धं वाक्यं तु मध्यमम् ।
आदिवाक्यमुपान्त्यं च भवतीति 'भवेत् सुखम्' ॥

यत्राप्यक्षरसन्देहस्तत्र संस्थाप्य 'देवरम्'(248) ।
त्यजेत्तद्गतवाक्यानि, शिष्टं शोध्यं 'भवेत् सुखात्' ॥

"Six rāśis *plus* half '*Bhavet sukham*' (0ʳ 27° 44′) is the middle, *i.e.* 124th *vākya*. The first *vākya*, '*Gīrnaḥ śreyaḥ*' (0ʳ 12° 3′) *plus* the penultimate *vākya* '*Kaveḥ śakyam*' (0ʳ 15° 41′) is '*Bhavet sukham.*' Therefore whenever there is a doubt regarding the letters of any *vākya*, deduct its serial number from 248 and take the *vākya* corresponding to the remainder; deduct this *vākya* from '*Bhavet sukham*'. (The doubtful *vākya* should agree with the result).

For the 248 'Moon sentences' correct to the seconds and the rationale of the verification, see K. V. Sarma, *Computation of True Moon*, (Hoshiarpur, 1973), pp. 46-59.

रविस्फुटः—पञ्चबोधः

13. 7. 15. याताश्च मासा दिवसाश्च नाड्यः
स्वाभीष्टकालावधिकाः क्रमेण ।
भवन्ति राश्यंशकलाश्च भानो-
'र्योग्या'दिवाक्यं च कलासु कुर्यात् ॥ २ ॥
(*Pañcabodha*, 2)

True Sun—Pañcabodha

(The number of) the elapsed (solar) (months counted from Meṣa), the days, and the *nāḍis* up to the chosen time, placed in sequential order, form, respectively, the *rāśis*, degrees and minutes of true Sun at that time. The correction enunciated in the chrongrams beginning with *yogya* shall have to be applied to the seconds.[1] (2)

चन्द्रस्फुटः—पञ्चबोधः

13. 7. 16. वाक्यध्रुवैक्यं ह्युदये स्फुटेन्दुः ॥ ५a ॥

वाक्यानयनम्

'अमितयवोत्सुक'हीनं
द्युगणं 'रसगैरिकैः' 'कुलीनाङ्गैः' ।
'देवेन्द्रै'रपि हृत्वा
तच्छिष्टं भवति वाक्यसंख्येन्दोः ॥ १२ ॥

ध्रुवः

'विविधं निजवसुरोधं'
'तापेनोह्यं कुलस्थनैपुण्यम्' ।
'धिगहरलघुसत्रोनं'
चैतान् हाराहृतैः फलैः क्रमशः ॥ १३ ॥
हत्वा तेषां योगः 'कौलटभूपालतनय'संयुक्तः ।
देशान्तरविघटीहत'रत्नप्राया'न्वितो ध्रुवो ज्ञेयः ॥ १४ ॥
(*Pañcabodha*, 5a, 12-14)

True Moon—Pañcabodha

The sum of the lunar chronogram and the *Dhruva* (for the derivation of which both, see below), is the true Moon at sunrise on the current day. (5a)

Lunar chronogram

Subtract from the current Kali day, the number 17,41,650, (being the cut off day of this *karaṇa* text), and divide the remainder successively by 12372, 3031 and 248. The final remainder gives the number of the lunar chronogram (vide Moon sentences table under 13.7.14, above) for the current day. (12)

Dhruva

With the three quotients (obtained during the previous operation) multiply, in order, the three terms 9ʳ 27° 48° 9″ 44″, 11ʳ 7° 31′ 10″ 16‴, and 0ʳ 27° 43′ 28″ 39‴. To the sum of the products obtained, add the term 1ʳ 6° 31′ 41″ 31‴. (The mean *dhruva* or the *dhruva* at the Prime meridian, is obtained.) (In order to reduce this to the particular place), the term 12° 2° is to be multiplied by the longitudinal correction in *vināḍikās* of the place, and the product added to the mean *dhruva*. (12-13). (KVS)

मध्यग्रहः—आनयनोपायाः

13. 8. 1. पर्ययाहर्गणाभ्यासो ह्रियते भूदिनैस्ततः ।
लभ्यन्ते पर्ययाः शेषा राशिभागकलादयः ॥ १५ ॥

[1] The seconds corrections *yogya* etc. are given below:

	Days of the month 1-8	9-16	17-24	25 to end
Meṣa	—11	—14	—16	—17
Ṛṣabha	—19	—21	—22	—24
Mithuna	—24	—25	—25	—24
Kaṭaka	—24	—23	—22	—21
Siṃha	—19	—17	—15	—13
Kanyā	—11	—8	— 6	— 5
Tulā	—1	+1	+3	+5
Vṛścika	+6	+8	+9	+10
Dhanu	+10	+11	+11	+11
Makara	+11	+9	+8	+7
Kumbha	+6	+4	+2	+0
Mīna	—2	—4	—7	—10

These corrections have to be applied negatively from the first day of Mīna, and positively from the 9th of Tulā, as indicated by the minus and plus signs prefixed to the numbers in the table. For days less than eight, the correction has to be calculated proportionately. The correction for any date beyond 8 is the sum of the correction for the previous 8-s of the month and the portion for the current 8. The proportion for the days beyond 24 has to be done on the basis of the number of the remaining days of the month.

'भास्करै'स्त्रिशता षष्ट्या सङ्गुणय्य पृथक् पृथक् ।
तेनैव भागहारेण लभ्यन्तेऽर्कोदयावधेः ॥ १६ ॥

विलिप्तान्ता ग्रहा मध्याः, शश्युच्चे राशयस्त्रयः ।
क्षिप्यन्ते षट् तमोमूर्तौ चक्रात् स च विशोध्यते ॥ १७ ॥

(Bhāskara I, *LBh.*, 1. 15-17)

Mean Planets

Principles of computation

Divide the product of the revolution-number of a planet and the *ahargaṇa* by the (number of) civil days (in a *yuga*); thus are obtained the (number of) revolutions (performed by that planet). From the (successive) remainders multiplied respectively by 12, 30, and 60 and divided by the same divisor (viz. the number of civil days in a *yuga*) are obtained the Signs, degrees, and minutes, etc. (of the mean longitude of that planet for mean at Laṅkā). (15-16)

To the (mean) longitude of the Moon's apogee (obtained by the above rule), add three Signs, and to that of the Moon's ascending node add six Signs, and subtract (the latter result) from a circle (i.e., from 360°). (17). (KSS)

13. 8. 2. अहर्गणे खेचरमण्डलाहते
क्वहोद्धृते मण्डलपूर्वको ग्रहः ।
कलीकृतेऽर्के ग्रहचक्रताडिते
कलाग्रहो वा रविचक्रभाजिते ॥ १७ ॥

'रसाद्रिबाणै'र्द्युगणेऽथवाहते
'नवाष्टरामाम्बरमूर्च्छनो'द्धृते ।
फलं सहस्रांशुसितेन्दुनन्दना-
श्चलोच्चमारार्कजशक्रमन्त्रिणाम् ॥ १८ ॥

'भुजङ्गनन्दाष्टवसुस्वरा'हते
'शराक्षिषड्बाणशरेन्दुलोचनैः' ।
हृते दिनानां निचयेऽथवा भवे-
न्मृगाङ्कमूर्तिर्भगणादिकं फलम् ॥ १९ ॥

(Lalla, *ŚiDhVṛ.*, 1. 17-19)

The *ahargaṇa* multiplied by the number of revolutions of a planet and divided by the number of civil days (in a *yuga*) gives the mean longitude of the planet in revolutions, etc.

Or, the mean longitude of the Sun expressed in minutes, multiplied by the number of revolutions of a planet and divided by the number of revolutions of the Sun (in a *yuga*), gives the mean longitude of the planet in minutes (at that time). (17)

Or, the *ahargaṇa* multiplied by 576 and divided by 2,10,389, gives the longitude of the Sun, Mercury or Venus and that of the *śighrocca* of Mars, Jupiter or Saturn. (18)

Of the *ahargaṇa* multiplied by 78,898 and divided by 21,55,625, gives the mean longitude of the Moon in revolutions.[1] (19). (BC)

—लल्लः, प्रकारान्तरम्

13. 8. 3. द्वयोर्द्वयोश्चक्रविशेषसङ्गुणा-
दहर्गणाद् भूदिवसैर्यदाप्यते ।
अनल्पगस्तद्रहितोऽल्पगो भवेद्
युतोऽल्पगस्तेन तयोरनल्पगः ॥ २५ ॥

प्रसिद्धलिप्ताहतसाध्यपर्यया:
प्रसिद्धचक्रापहृताः कलादिकः ।
भवेत् प्रसाध्यो मृदुतुङ्गकेन्द्रयोः
समागमो वा चलयोर्ग्रहान्तरम् ॥ २६ ॥

(Lalla, *ŚiDhVṛ.*, 1. 20-26)

—Lalla, Alternate method

Multiply the *ahargaṇa* by the difference of the number of revolutions of any two planets and divide by the number of civil days (in a *yuga*). Subtract the result from the greater longitude (of the two planets). The remainder is the lesser longitude. Again add the result to the lesser longitude and the sum is the greater longitude. (25)

The number of revolutions of the planet the longitude of which is required, when multiplied by the given longitude of another planet, expressed in minutes, and divided by the number of revolutions of the latter, gives its mean longitude in minutes.

The longitude of a planet is the sum of its apogee and mean anomaly. It is also equal to the difference of its *śīghrocca* and *śīghrakendra*.[2] (26). BC)

13. 8. 4. निजसावनदिनगुणितं द्युगणं
'क्लपुभिधुलटीरदमनेनैः' ।
विभजेदवाप्तराशिभि-
रूनोऽर्को भादिरिष्टखेटो वा ॥ २७ ॥

(ABh. II, *Mahā.*, 1. 27)

Multiply the *ahargaṇa* by (the number of) the sidereal days (of any planet in a *kalpa*) and divide by 131,493-128,500. Subtract the quotient in Signs from (the position of) the Sun (in Signs, and the difference is the position of) that given planet in Signs etc.[3] (27). (SRS)

[1] For the rationale, see *ŚiDhVṛ*: *BC*, II. 13-15.
[3] For elucidation and notes see *ŚiDhVṛ*; *BC*. II. 16-18.
[2] For the rationale, see *Mahā.*: *SRS*, pt. II. p. 14.

—भास्करः १

18. 8. 5. अम्बरोरुपरिधिर्विभाजितो
भूदिनैर्दिवसयोजनानि तैः ।
सङ्गुणय्य दिवसानथाहरेत्
कक्ष्यया भगणराशयः स्वया ।। २० ।।

(Bhāskara I, *MBh.*, 1.20)

—Bhāskara I

Divide the (*yojanas* of the) circumference of the sky by the number of civil days (in a *yuga*): the result is the number of *yojanas* traversed (by a planet) per day. By those (*yojanas*) multiply the *ahargaṇa* and then divide (the product) by the length (in *yojanas*) of the own orbit of the planet. From that are obtained the revolutions, Signs, etc. (of the mean longitude of the planet). (20). (KSS)

—भास्करः २

13. 8. 6. अहर्गणात् 'क्वक्षिनवाङ्क'निघ्ना-
'न्नवेन्दुवेदेषुहुताश'लब्ध्या ।
अहर्गणो 'गोऽक्षधृतीन्दु'निघ्नो
विवर्जितः स्युर्गतयोजनानि ।। ७ ।।

स्वया स्वया तानि पृथक् च कक्षया
हृतानि वा स्युर्भगणादिका ग्रहाः ।
ग्रहस्य कक्षैव हि तुङ्गपातयोः
पृथक् च कल्प्याऽत्र तदीयसिद्धये ।। ८ ।।

अर्कस्य कक्षैव सितज्ञयोः सा
ज्ञेया तयोरानयनार्थमेव ।
उक्ते तयोर्ये चलतुङ्गकक्षे
तत्रैव तौ च भ्रमतोऽर्कगत्या ।। ९ ।।

(Bhāskara II, *SiŚi.*, 1. 1. 4. 7-9)

—Bhāskara II

The *ahargaṇa* multiplied by 11,859 decreased by the quotient obtained by dividing the product of the *ahargaṇa* and 9921 by 35,419 gives the distance covered by a planet in *yojanas*. These *yojanas* divided by the circumference of the planet's orbit gives the fraction of a revolution and the integral number of revolutions made. (7-8a)

The orbit of the planet itself is no doubt the orbit of the *mandocca* (apogee with respect to the Sun and the Moon and aphelion with respect to the other star-planets (Mars, Mercury, Jupiter, Venus and Saturn) and of the node (point of intersection of the orbits of the star-planets with the ecliptic); but while computing the positions of these *mandoccas* and the *nodes,* by the method indicated above, their orbits are taken to differ from those of the planets, (because slow-moving planets will have longer orbits as per the assumption made, namely that the circumference of the universe divided by the number of the sidereal revolutions gives the length of the orbits). Similarly, the orbit of the Sun itself will be the orbit of Mercury and Venus, and the orbits of their *śīghroccas* are their real orbits wherein Mercury and Venus are taken to move with the velocity of the Sun. (8b-9)[1]. (AS)

मध्यग्रहानयनम्—सौरसिद्धान्तः

13. 9. 1. एष निशार्धेऽवन्त्यां ताराग्रहनिर्णयोऽर्कसिद्धान्ते ।
तत्रेन्दुपुत्रशुक्रौ तुल्यगतौ मध्यमार्केण ।। १ ।।

गुरुमध्यमः

जीवस्य शताभ्यस्तं 'द्वित्रियमाग्नित्रिसागरै'र्विभजेत् ।

कुजमध्यमः

द्युगणं कुजस्य 'चन्द्रा'हतं तु 'सप्ताष्टषड्'भक्तम् ।। २ ।।

शनिमध्यमः

सौरस्य सहस्रगुणा 'ऋतुरसशून्यर्तुषट्कमुनिखैकैः' ।
यल्लब्धं ते भगणाः शेषा मध्यग्रहाः क्रमेणैव ।। ३ ।।

संस्कारः

दश दश भगणे भगणे संशोध्यास्तत्पराः सुरेज्यस्य ।
'मनवः' कुजस्य देयाः, शनेश्च 'बाणाः' विशोध्यास्तु ।।४।।

राशिचतुष्टयमंशद्वयं कलाविंशतिर्वसुसमेताः ।
'नववेदा'श्च विलिप्ताः शनेर्धनं मध्यमस्यैव ।। ५ ।।

'अष्टौ' भागा लिप्ता 'ऋतवः' 'खपक्षौ' गुरौ विलिप्ताश्च ।
क्षेपः कुजस्य 'यमतिथिपञ्चत्रिंशच्च' राश्याद्याः ।। ६ ।।

बुधशीघ्रम्

शतगुणिते बुधशीघ्रं 'स्वरनवसप्ताष्ट'भाजिते क्रमशः ।
अत्रार्धपञ्चमाः तत्पराश्च भगणाहताः क्षेपः ।। ७ ।।

शुक्रशीघ्रम्

सितशीघ्रं दशगुणिते द्युगणे भक्ते 'स्वरार्णवाश्वियमैः' ।
अर्धैकादश देया विलिप्तिका भगणसंगुणिताः ।। ८ ।।

सिंहस्य 'वसुयमां'शाः 'स्वरेन्दवो' लिप्तिका ज्ञशीघ्रधनम् ।
शोध्याः सितस्य विकलाः 'शशिरसनवपक्षगुणदहनाः' ।।

(Varāha, *PS*, 16.1-9)

Mean planets—Computation

Saurasiddhānta

The following are the positions of the star-planets at midnight at Ujjain according to the *Arka* (i.e. *Saura*) *siddhānta*. For their computation, the mean Sun should be taken as mean Mercury and Venus. (1)

Mean Jupiter

For mean Jupiter multiply the days from epoch by 100 and divide by 4,33,232. Revolutions etc. are got. Deduct 10‴ per revolution. Add 8° 60′ 20‴ (viz., the mean at epoch), (which is called *kṣepa*). (A *bīja*

[1] For the rationale, see *SiŚi*: *AS*, pp. 50-52.

correction for the planets given by Varāhamihira is given in verses 10-11, below.) (2a)

Mean Mars

For mean Mars, multiply the days by 1 and 687. Revolutions etc. are got. Add 14″ per revolution. Add also 2^r 15° 35′ 0″, (the mean at epoch). (2b)

Mean Saturn

For mean Saturn, multiply the days by 1000 and divide by 1,07,66,066. Revolutions etc. are got. Deduct 5‴ per revolution. Add 4^r 2° 28′ 49″, (the mean at epoch). (3)[1]

Mean Mercury

For 'mean Mercury' (called *śīghra*), multiply the days by 100 and divide by 8797. Add $4\frac{1}{2}'''$ per revolution. Add 4^r 28° 17′ 0″, (the *śīghra* at epoch). (Add the *bīja* correction given in verses 10-11) (7)

Mean Venus

For mean Venus (called *śīghra*), multiply the days by 10, and divide by 2247. Revolutions etc. are got. Add $10\frac{1}{2}''$ per revolution. Add 8^r 27° 30′ 39″, (the *śīghra* at epoch). (Add also the *bīja* correction in verses 10-11). (8)

Twentyeight degrees in Siṁha and seventeen minutes (i.e. 4^r 28° 17′) are to be added to the *śīghra* of Mercury. From the *śīghra* of Venus 3,32,961 seconds (3^r 2° 29′ 21″) are to be deducted. (9). (TSK)

सौरोपरि वराहमिहिरकृतः संस्कारः

13. 9. 2. क्षेप्याः 'स्वरेन्दु'विकलाः प्रतिवर्षं मध्यमक्षितिजे ।
दश दश गुरोर्विशोध्याः, शनैश्चरे सार्धसप्तयुताः ।। १०।।

'पञ्चाब्धयो' विशोध्याः
सिते, बुधे 'खाश्विसप्त'युताः ।
'खखवेदेन्दु' विकलिकाः
शोध्याः स्युः सुरपूजितस्य मध्यात् स्युः ।। ११ ।।

(Varāha, *PS*, 13. 10-11)

Varāhamihira's correction to Saura

Add 17″ per year to mean Mars. Deduct 10″ per year from mean Jupiter. Add $7\frac{1}{2}''$ per year to mean Saturn. Subtract 45″ per year from (the *śīghra*) of Venus. Add 120″ per year to (the *śīghra*) of Mercury. In addition, subtract 1400″ (i.e. 23′ 20″) constant from Jupiter's mean. (10-11). (TSK)

—करणरत्नम्

13. 9. 3. द्विष्ठदिनं 'शिखिशिखि-
शरशिखिवसुरसपक्ष'भक्तमुपरियुतम् ।
'सार्धेन्दुकं' च 'नगवसु-
रसो'द्धृतं मण्डलाधारः ।। २ ।।

स्वा'ष्टिवसुमनु'हृताप्तं
स'कुनिधितिथि' युक्त्वाऽ'ग्निकरण'गुणितम् ।
दिनमथ 'खखनगरुद्रै'-
र्भक्तं भगणादिशशिजोच्चम् ।। ३ ।।

प्रतिदिन'मगशशिनिधितिथि'
हृतोनितं 'रुद्रपावकाक्षि'युतम् ।
'लोचनपावकपावक-
कृत'लब्धं मण्डलादिगुरुः ।। ४ ।।

'रसमनुनिधिनगशशि'हृत'मग्निशर'घ्नदिनगणे रहितम् ।
'खनगनगाष्ट'युता'न्निधिखरन्ध्ररुद्रै'श्च सितशीघ्रम् ।।५।।

स्वा'गेन्द्विषुरसरसशशि'लब्धोनदिनात् 'खभूतयमलाङ्गैः' ।
वित्र्यंशांशैश्च युता'द्रसरसाद्रिखेन्दुभिः' सौरिः ।। ६ ।।

(Deva, *KR*, 7.2-6)

—Karaṇaratna

Set down the *ahargaṇa* in two places, one below the other. Divide the lower number by 26,83,533, then add the (resulting) quotient as well as $1\frac{1}{2}$ to the upper number, and then divide that by 687: the result is the mean longitude of Mars in terms of revolutions etc. (2)

Mercury's śīghrocca

To 133 times the *ahargaṇa* add the *ahargaṇa* divided by 14,816 *plus* 1591, and then divide the (resulting) sum by 11,700: the result is the mean longitude of the *śīghrocca* of Mercury in terms of revolutions etc. (3)

Jupiter

Diminish the *ahargaṇa* by 1/15,917 of itself and add 2311, then divide that by 4332: the result is the mean longitude of Jupiter in terms of revolutions etc. (4)

Venus's śīghrocca

From 53 times the *ahargaṇa* subtract the *ahargaṇa* divided by 1,79,146 then add 8770, and then divide by 11,909; the result is the mean longitude of the *śīghrocca* of Venus, in terms of revolutions. (5)

Saturn

Subtract the *ahargaṇa* by 1/1,66,517 of itself, then add 6250 minus 1/3, and then divide by 10,766 : the result is the mean longitude of Saturn, in terms of revolutions.[1] (6). (KSS)

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[1] Translation of verses 4-6 and the corrections indicated therein have been incorporated in the translation of 2-3.

[1] For rationales and parallels, see *KR*: *KSS*, pp. 81-86.

मध्यग्रहशोधनम्

13. 10. 1. आर्यभटस्य च परीक्षापरत्वादेव सकलदेशकालयोः स्फुटार्थत्वम्, न पुनः तदुक्तभगणादिवैशिष्टचात् । अत इदमेव परीक्षासूत्रं सिद्धान्तान्तरेभ्योऽस्य गौरवमापादयति । . . . तस्माद् युक्तिपरत्वादार्यभटस्य न दूषणम्, प्रत्युत भूषणमेव, यतः संशयोत्थापनेन परीक्षां कारयति । अत एव परीक्षकाः संस्कारेण तदेव समीकुर्वन्ति ।

ते च बीजसंस्कारश्लोकाः नानाकर्तृकाः एकत्रैव प्रदर्शयितुं सूर्यदेवेन वासनाभाष्ये लिखिताः ।।

(Nīlakaṇṭha Somayāji, *JyM*, p. 5)

Correction of mean planets

The exactitude of the *Āryabhaṭīya* for all times and climes is on account of its advocating the practice of testing and verification and not on account of the revolution numbers enumerated in the work. Indeed, it is the rule for observation and correction given by it (*ABh.* 4.48) that confers on it the distinction which it enjoys amongst the Siddhāntas. Thus, its being entrenched on reasoning, instead of being a detraction, is a matter for compliment in that by causing a doubt Āryabhaṭa makes one test and verify and, thereby, enable observers to rectify (the system) by means of 'corrections', as necessary.

The several 'correction verses' enunciated by different astronomers have been noted by Sūryadeva (Yajvan) in his commentary *Vāsanā* (on the *Laghumānasa* of Muñjāla, *Upodghātaprakaraṇa*, 1-2), in order that they might all be available at one place.

स्फुटग्रहः

—आनयनोपायः

13. 11. 1. क्षयधनधनक्षयाः स्युर्मन्दोच्चाद् व्यत्ययेन शीघ्रोच्चात् ।
शनि-गुरु-कुजेषु मन्दार्धमृणं धनं भवति पूर्वे ।। २२ ।।
मन्दोच्चाच्छीघ्रोच्चादर्धमृणं धनं ग्रहेषु मन्देषु ।
मन्दोच्चात् स्फुटमध्याः शीघ्रोच्चाच्च स्फुटा ज्ञेयाः ।।
शीघ्रोच्चादर्धोनं कर्तव्यमृणं धनं स्वमन्दोच्चे ।
स्फुटमध्यौ तु भृगुबुधौ सिद्धान्मन्दात् स्फुटौ भवतः ।।

(Āryabhaṭa I, *ABh.*, 3. 22-24)

True Planets

Method of computation

The corrections from the apogee (for the four anomalistic quadrants) are respectively minus, plus, plus, and minus. Those from *śighrocca* are just the reverse. (22a)

In the case of (the superior planets) Saturn, Jupiter and Mars, first apply the *mandaphala* negatively or positively (as the case may be). (22b)

Apply half the *mandaphala* and half the *śighraphala* to the planet and to the planet's apogee negatively or positively (as the case may be). The mean planet (then) corrected for the *mandaphala* (calculated afresh from the new *mandakendra*) is called the true-mean planet and that (true-mean planet) corrected for the *śighraphala* (calculated afresh) is known as the true planet. (23)

(In the case of Mercury and Venus) apply half the *śighraphala* negatively or positively to the longitude of the planet's apogee (according as the *śighrakendra* is less than or greater than 180°). From the corrected longitude of the planet's apogee (calculate the *mandaphala* afresh and apply it to the mean longitude of the planet; then) are obtained the true-mean longitudes of Mercury and Venus. (The *śighraphalas*, calculated afresh, being applied to them), they become true (longitudes). (24). (KSS)

कुजगुरुशनिस्फुटः—भास्करः २

13. 11. 2. मन्दोच्चफलचापार्धं प्राग्वन्मध्ये धनक्षयौ ।
कृत्वा शीघ्रोच्चतः शोध्यं शीघ्रकेन्द्रं तदुच्यते ।। ३३ ।।
तस्माद् बाहुफलं हत्वा व्यासार्धेन विभज्यते ।
कर्णेनाप्तस्य चापार्धं धनर्णे मेषतौलितः ।। ३४ ।।
शोधयित्वा ततो मन्दं बाहोः कृत्स्नं फलं ततः ।
काष्ठितं मध्यमे कुर्यात् स्फुटमध्यः स उच्यते ।। ३५ ।।
शोधयित्वा तु तं शीघ्राच्छीघ्रन्यायागतं फलम् ।
चापितं सकलं कुर्यात् स्फुटमध्ये स्फुटो भवेत् ।। ३६ ।।
कुजार्कसुतसूरीणामेवं कर्म विधीयते ।

(Bhāskara I, *LBh.*, 2. 33-37)

True Mars, Jupiter, and Saturn—Bhāskara I

Having added half the *bāhuphala* due to the *mandocca* (apogee) to or subtracted that from the mean longitude of the planet as before, the result should be subtracted from (the longitude of) the *śighrocca*: that (difference) is called the *śighrakendra*. From that obtain the *bāhuphala*: (and) having multiplied that by the radius, divide (the product) by the (*śighra*)-*karṇa*. Half the arc corresponding to the result obtained should be added or subtracted according as the *śighrakendra* is in the half-orbit beginning with Aries or in that beginning with Libra. Then after subtracting (the longitude of) the *mandocca* from that (result), the entire *bāhuphala* (derived from that difference), reduced to arc, should be applied (as correction, positive or negative) to the mean longitude of the planet (according as the *mandakendra* is in the half-orbit beginning with the sign Libra or in that beginning with the sign Aries): this (result) is known as the true-mean longitude (of the planet). (33-35)

(Then) after subtracting the resulting quantity (viz. the true-mean longitude of the planet) from the *śīghrocca*, the entire correction obtained by the *śīghrocca* process, reduced to arc, should be applied (as correction, positive or negative), to the true-mean longitude of the planet (according as the *śīghrakendra* is in the half-orbit beginning with Aries or in that beginning with Libra): thus is obtained the true longitude of the planet. (36-37a). (KSS)

बुध-शुक्रयोः स्फुटः—भास्करः १

13. 11. 3. बुधभार्गवयोश्चाथ प्रक्रिया परिकीर्त्यते ।। ३७ ।।

प्रागेव चलकेन्द्रस्य फलचापदलं स्फुटम् ।
व्यस्तं स्वकीयमन्दोच्चे धनर्णं परिकल्पयेत् ।। ३८ ।।

तेन मन्देन यल्लब्धं सकलं तत् स्वमध्यमे ।
स्फुटमध्यश्चलोच्चेन संस्कृतः स स्फुटो ग्रहः ।। ३९ ।।

(Bhāskara I, *LBh.*, 2. 37-39)

True Mercury and Venus—Bhāskara I

The method used in the case of Mercury and Venus is being described now. First add or subtract half the arc corresponding to the *śīghraphala* in the reverse order (i.e., according as the *śīghrakendra* is in the half-orbit beginning with Libra or in that beginning with Aries) to or from its own *mandocca*. Whatever correction is (then) derived from that (corrected) *mandocca* should, as a whole, be applied as correction to the mean longitude of the planet: then is obtained the true-mean longitude (of the planet). That corrected for (the correction due to) the *śīghrocca* gives the true longitude (of the planet). (37b-39). (KSS)

ग्रहस्फुटः—भास्करः २

13. 11. 4. स्यात् संस्कृतो मन्दफलेन मध्यो
मन्दस्फुटोऽस्माच्चलकेन्द्रपूर्वम् ।। ३४ ।।

विधाय शैघ्र्येण फलेन चैवं
खेटः स्फुटः स्यादसकृत् फलाभ्याम् ।
दलीकृताभ्यां प्रथमं फलाभ्यां
ततोऽखिलाभ्यामसकृत् कुजस्तु ।। ३५ ।।

स्फुटौ रवीन्दू मृदुनैव वेद्यौ
शीघ्राख्यतुङ्गस्य तयोरभावात् ।

(Bhāskara II, *SiŚi.*, 1.2. 34c-36b)

True planets—Bhāskara II

The mean planet rectified for the equation of centre or *mandaphala* is called *Mandasphuṭa*. Then, subtracting the longitude of the *mandasphuṭa* from that of the respective *śīghrocca*, the result will be the *śīghra* anomaly, from which the *śīghraphala* is to be obtained. (Rectifying the *mandasphuṭa* for this second equation, namely *śīghraphala*, again obtaining therefrom the equation of centre effecting this in the original mean planet and again correcting for *śīghraphala* and repeating the process till a constant value is obtained, the true planet is obtained with respect to the star-planets other than Mars.) (34b-35a)

But with respect to Mars, let first the mean planet be corrected for half of the equation of centre. Then make half of the correction of *śīghraphala*. Take the resulting planet to be the mean planet and again finding the equation of centre, make this whole correction in the original mean planet. Again taking the resulting planet to be the *mandasphuṭa*, effect the entire *śīghraphala*. Then we have the true planet.[1] (35b-36b). (AS)

—करणरत्नम्

13. 11. 5. मन्दोच्चोनितमध्यज्या 'नगा'प्तलिप्तार्धम् ।। १३ ।।

वियुतयुतं गोलवशाद्
ग्रहमध्येनोनितं शीघ्रम् ।
तज्ज्या निजपरिधिगुणा
दोःकोटयोः 'खाष्ट'भाजिता फलदा ।। १४ ।।

कोटिफलं मध्यपदे त्रिशतै रहितं युतं परे ।
दोर्ज्याफलवर्गयुतात् तद्वर्गात् कीर्तितं पदं कर्णः ।। १५ ।।

त्रिशतघ्नं बाहुफलं कर्णनाप्तं धनुष्कार्यम् ।
तस्मान्मन्दोच्चफलं त्यक्त्वा सकलं फलं प्राग्वत् ।।१६ ।।

गोलवशादानीयाद्रहितं सहितं स्वमूलमध्यं तत् ।
स्फुटमध्यमसंज्ञं स्यात् तृतीयसंस्कारयुक्तं तत् ।। १७ ।।

तन्निजशीघ्रोच्चोनं स्फुटकेन्द्रं स्याज्ज्ययोः फलं प्राग्वत् ।
दोःकोट्योः पूर्वोक्तन्यायेनानीय कर्णं च ।। १८ ।।

आनीयात् कर्णफलं तस्मिन् सहितोनितं च तत्कार्यम् ।
उत्तरदक्षिणयोरपि कुजादयः स्युः स्फुटाः सकलाः ।।१९

(Deva, *KR*, 7. 13-19)

—Karaṇaratna

Diminish the mean longitude of the planet by the longitude of its apogee, and find the R sine thereof. Multiply that (R sine) by the (planet's true *manda*) epicycle and divide by 7: apply half of the resulting minutes negatively or positively depending on the (planet's) hemisphere (i.e., according as the planet is in the half orbit beginning with the anomalistic Sign Aries or in that beginning with anomalistic Sign Libra). (13ab-14ab)

Next diminish the longitude of the planet's *śīghrocca* by the (corrected) mean longitude of the planet, and find the R sines of the *bāhu* and *koṭi* thereof. Multiply the R sine of the *bāhu* as well as the R sine of the *koṭi* by

[1] For comments, see *SiŚi* : *AS*, pp. 154-55.

the planet's (true) *śīghra* epicycle and divide (each product) by 80: the results are the *bāhuphala* and *koṭiphala*. (14cd)

The *koṭiphala* should be subtracted from 300 or added to 300, according as the planet is in the second and third (*śīghra* anomalistic) quadrants or in the first and fourth (*śīghra* anomalistic) quadrants. Add the square of that (difference or sum) to the square of the *bāhuphala*, and take the square root (of the resulting sum): this (square root) is known as the (planet's) hypotenuse. (15)

Multiply the *bāhuphala* by 300 and divide by the (planet's) hypotenuse: and obtain the arc corresponding to the resulting R sine. (Apply half of the minutes in this arc to the corrected mean longitude of the planet, positively or negatively, according as the planet is in the half orbit beginning with the *śīghra* anomalistic Sign Aries or in that beginning with the *śīghra* anomalistic sign Libra). (16)

From that subtract the longitude of the planet's apogee, and then obtain the *mandoccaphala*, (i.e., the *bāhuphala* due to the planet's apogee), as before, and apply the whole of it to the original (uncorrected) mean longitude of the planet, depending upon the planet's hemisphere (i.e., negatively or positively, according as the planet is in the half orbit beginning with the anomalistic Sign Aries or in that beginning with the anomalistic Sign Libra). When the above-mentioned three corrections have been applied to the planet, it is called the 'true-mean planet'. (17)

Subtract the true-mean longitude of the planet from the longitude of its own *śīghrocca*: this is the true *śīghra* anomaly. From this (true *śīghra* anomaly) calculate, as before, the *bāhuphala* and the *koṭiphala*. Also find the hypotenuse by using the method stated above. Then obtaining the *karṇaphala*, apply (the whole of) it (to the true-mean longitude of the planet) positively or negatively, according as the planet is in the northern or southern hemisphere (i.e., in the half orbit beginning with the *śīghra* anomalistic Sign Aries or in that beginning with the *śīghra* anomalistic Sign Libra). This is how the true longitudes of all the planets, Mars etc., are obtained.[1] (18-19). (KSS)

—वाक्यकरणम्

13. 11. 6. इष्टं दिनं स्वस्वशोध्यहीनं स्वैर्मण्डलैर्हृतम् ।
वाक्यं शिष्टात्तदावृत्तौ वाक्योक्तदिवसैः क्रमात् ।। १ ।।

ज्ञेयं तच्छिष्टदिवसगतिलिप्तासमन्वितम् ।
वक्रे हीनं, ग्रहः स्पष्टः, कलादिध्रुवसंस्कृतः ।। २ ।।

कलात्मक'ग'वर्णस्पृग् ध्रुवान्तं नाडिकादिकम् ।
शोध्यं, व्यन्त्यं मण्डलं चान्त्याप्तमावर्तनं गतम् ।। ३ ।।

'वना'क्षरं तु शुक्रस्य चतुर्थं मण्डलध्रुवम् ।
संस्कारध्रुवयोरन्त्यवर्णो ह्रस्वो यदि क्षयः ।। ४ ।।

मण्डलानां ध्रुवं तत्तन्मण्डलाप्तफलाहृतम् ।
तच्छोध्यध्रुवयोर्भेदे विश्लेषस्त्वन्यथा युतिः ।। ५ ।।

अधिकस्य वशाच्छेषं स्वमृणं वा ध्रुवं भवेत् ।
बुधादीनां वर्ण एको वाक्यान्ते संस्कृतिर्भवेत् ।। ६ ।।

कुजस्य द्वौ भृगोस्त्वेतौ धनर्णध्रुवयोः क्रमात् ।
सर्वत्र संख्याभिहिता वर्णैः कटपयादिभिः ।। ७ ।।

भृगोः संस्कारवर्णस्तु 'टा'दिश्चेदर्धवर्जितः ।
संस्कारवर्णगुणिता ध्रुवांशाद्याः कलादयः ।। ८ ।।

ध्रुवसंस्कारयोः साम्ये योज्या वाक्येऽन्यथान्यथा ।
एवं तु संस्कृतं वाक्यं स्फुटमत्र प्रकीर्तितम् ।। ९ ।।

निरन्तरद्विवाक्योत्थस्फुटविश्लेषलिप्तिकाः ।
दिनान्तरहृता भुक्ति,र्वक्रा पूर्वेऽधिके तयोः ।। १० ।।

आद्यवाक्यं विमन्दोच्चं हृतं तद्दिवसैर्गतिः ।
ग्रहस्तत्रेष्टदिवसगतिर्मन्दोच्चसंयुता ।। ११ ।।

ग्रहमन्दोच्चाः

'जायाढ्य' 'नाकेन्द्र' 'नदेय' 'निर्धनं' ।
'तुङ्गाद्रि' मन्दोच्चलवाः कुजादितः ।। ११a ।।

(*VK*. 2.1-11A)

True planets—Vākyakaraṇa

From the Kali day for which the planet is desired, deduct the respective *śodhya* given in the tables. Divide the remainder by the respective *maṇḍalas* (any *maṇḍala* or *maṇḍalas* may be used, and any number of times) and note the quotients. Divide the remainder by the respective synodic cycle of days. The quotients are the cycles gone, and the remainder, the days in the next cycle. (i) Find the total *dhruva* in the manner to be given. (ii) Using the mnemonics of that cycle, corrected in the manner to be given, take values for the maximum number of days that can be done. (iii) Find the motion for the remaining days by interpolation. Add (i), (ii) and (iii), (iii being deductive if the motion is found to be retrograde.) The (unreduced) True planet is got. (1-2)

In each mnemonic line belonging to the tables giving the *śodhya* and *maṇḍalas*, the last three letters give the *dhruva* in minutes, and the others, first the *nāḍikās* and then the days. In the case of Venus, and that too in the 4th *maṇḍala* alone the last 4 letters give the *dhruva*. If the very last vowel is short, the *dhruva* is negative, otherwise positive. The same criterion of positive and negative applies to the correction-letter occurring in the mnemonics in the cycle-tables. In the *śodhya* etc. tables, the

[1] For the rationale, see *KR*: *KSS*, pp. 91-92.

nāḍīs and days given by the first mnemonic is *śōdhya*, the last is the cycle-days, and the rest the *maṇḍalas*. (3-4)

(It is given thus): Take the *dhruvas* of the *maṇḍalas* used for division, and multiply by the respective quotients. Add these and the *dhruva* of the *śōdhhya* algebraically having regard to the signs and find the total with its sign. This is the total *dhruva* to be used as (i) above. (5-6a)

(This gives ii): Among the mnemonic lines of the cycle-tables, in those of Mercury, Jupiter, and Saturn, the last is the correction-letter. In those of Mars, the last two letters are the correction-letters. In those of Venus, the last two letters are correction-letters, but each letter is a different correction, the first is to be used if the total *dhruva* is positive, and the second to be used if the total *dhruva* is negative. With regard to the correction letter of Venus, another point is to be noted. If the letter is one of the nine beginning from *ṭa*, the number indicated is to be reduced by half.

Incidentally it may be noted that all numbers used in this work, viz. the *Vākyakaraṇa*, are given in the *Kaṭapayādi* notation. The *dhruva* in degrees etc. multiplied by the correction-number gives the correction in minutes etc. This correction is positive if both the multiplier and the multiplicand have the same sign and negative otherwise, and this is to be added or subtracted respectively in the value given by the mnemonic line, and thus the correction is made for the sake of (ii). (6b-9)

(iii is done thus): Take the corrected value of the mnemonic used, and the next corrected value. Find their difference and divide by the interval in days. This is the daily motion, which is retrograde if the next value is less. The daily motion, multiplied by the days etc. remaining is the motion for those remaining days etc., to be used as (iii). (The mnemonic of the last day of a cycle is that of the 0 day of the next cycle. The mnemonic of the 0 day of the first cycle is the respective apogee, viz. 118° for Mars, 210° for Mercury, 180° for Jupiter, 90° for Venus and 236° for Saturn. These are to be corrected using the correction-letter of the last mnemonic of the last cycle of the respective planet). If the True planet or daily motion is required for a day which is less than the days pertaining to the first mnemonic of any cycle, the ideas within brackets are to be used in the calculation.[1] (10-11A). (TSK-KVS)

[1] In the instructions given some are for interpreting the mnemonic lines which consist of words, and converting them into numbers. Once these instructions are carried out and the tables are given in numbers, we may forget these instructions and give our attention to the remaining instructions which give the methods of computation. In the worked out examples, such prepared tables are presupposed. For the mnemonic tables see Appendix III, *Kujādi-pañcagraha-vākyāni* (pp. 135-249 of *VK*: TSK-KVS) and for worked out examples, see *op. cit.*, pp. 259-60.

स्फुटग्रहानयनम्—सौरसिद्धान्तः

शीघ्राणि

13. 12. 1. शीघ्राख्योऽर्कोऽन्येषां भौमादीनां तु परिधयो द्विगुणाः ।
'पञ्चत्रिशन्मनवोऽष्टयः स्वरास्त्रिशच्च' भागाः ॥ १ ॥

'रस'-'भव'-'वसु'-'देव'-'र्का'
विंशतिगुणिताः कुजस्य दशकोनाः ।
मन्दगतिनामभागाः
कुजबुधगुरुशुक्रसौराणाम् ॥ २ ॥

शीघ्रपरिधावथांशाः 'कृतगुणपक्ष'-'द्विवह्निशीतकराः' ।
'पक्षस्वराः' 'खषड्यमाः' 'खकृताः' कुजादीनाम् ॥ ३ ॥

स्फुटकर्म

शीघ्रान्मध्यमहीनाद् राशित्रितये गतैष्यदंशज्ये ।
भुजकोटी तत्परतः षड्भिः पतिते स एव विधिः ॥ ४ ॥
स्वपरिधिगुणिते भाज्ये 'खर्तुगुणै'स्ते विपरिणमते तच्च ।
कोटिफलं व्यासार्धे मृगकर्क्यादौ चयापचयम् ॥ ५ ॥
तद्भुजकृतियोगपदैर्भाज्यं भुजजं 'खसूर्य'घ्नम् ।
तच्चापार्धं मन्दे हानिधनं शीघ्रकेन्द्रवशात् ॥ ६ ॥
स्फुटयित्वैवं मन्दं मध्याच्च विशोध्य तस्य भुजम् ।
परिणम्य कार्मुकार्धं तन्मन्देनैव धनहानी ॥ ७ ॥
मध्यात् पुनर्विशोध्य तस्माद् बाहुर्नतस्य यच्चापम् ।
तन्मध्यमे क्षयधनं कर्तव्यं मन्दकेन्द्रवशात् ॥ ८ ॥
एवं स्फुटमध्याख्यं शीघ्रात् संशोध्य पूर्वविधिनैव ।
आदिवदाप्तं चापं स्फुटमध्याख्ये चयापचयम् ॥ ९ ॥

बुधशुक्रयोर्विशेषः

सर्वे स्फुटाः स्युरेवं, ज्ञस्य तु शीघ्राद्विहाय रविमन्दम् ।
रविपरिधिनतं बाहुं बुधेऽर्कवत् क्षयधने कुर्यात् ॥ १० ॥
शुक्रस्य सप्तषष्टिर्लिप्ताः शोध्याः स्फुटीकृतस्यैव ॥११a॥

(Varāha, *PS*, 17. 1-11a)

True planets—Saura

The epicycles

For the other planets (i.e. other than Mercury and Venus, viz. for Mars, Jupiter and Saturn), the Sun is their *śīghra*. The Epicycles of equation of the apsis of Mars etc. are twice 35°, 14°, 16°, 7° and 30°, (i.e. of Mars 70°, of Mercury 28°, of Jupiter 32°, of Venus 14° and of Saturn 60°). (1)

6, 11, 8, 4, 12, multiplied by 20 (Mars being less by 10) (i.e. 110°, 220°, 160°, 80° and 240° are the apogee positions of Mars, Mercury, Jupiter, Venus and Saturn. (2)

The degrees of epicycles of conjunction of Mars is 234, of Mercury 132, of Jupiter 72, of Venus 260, and of Saturn 40. (3)

Computation

Deduct the mean from the *śīghra*. If the remainder (called *śīghra-kendra*) is within 90°, sine *śīghra-kendra* is

called *bhuja*, and sine (90°—*śīghra-kendra*) is called *koṭi*., The rule is the same in the case of subtraction from 180°, (270° and 360°). (4)

The *bhuja* and *koṭi* must be multiplied by the planet's epicycle of conjunction and divided by 360. Thus transferred, they are called *bhuja*-result and *koṭi*-result pertaining to the equation of conjunction. If the *śīghra-madhya* is *mṛgādi* (i.e. from 270° to 90°), the *koṭi*-result is to be added to 120 (i.e. the R of *PS*) and if the *śīghra-madhya* is *karkyādi* (i.e. from 90° to 270°), the *koṭi*-result is to be subtracted from 120. Square this and add it to the square of the *bhuja*-result. Find its square root and by it divide (120 × *bhuja*-result). Find (arc minus sine) of this. Subtract half of this from the longitude of apsis if the *śīghra-kendra* is from 0° to 180° and add if from 180° to 360°. (5-6)

Thus, (half-)rectifying the apogee position, deduct it from the mean. (The result is to be used as the anomaly of the apsis.) Find the *bhuja* of the anomaly of apsis. (As before), get the transferred *bhuja*-result of the apsis. (This is sine equation of the centre). Find its arc-sine and add or subtract (appropriately) half this to or from the half-rectified longitude of apogee. (7)

Subtract this rectified apogee from the mean (and thus get the anomaly of apsis). Find its *bhuja*. (As before) calculate the *bhuja*-result (which is the equation of the centre.) Find its arc-sine and add or subtract (appropriately) the whole of this arc from the mean. The result is the rectified mean. (8)

Deduct the rectified mean from the *śīghra*. (The anomaly of conjunction is got). As before, calculate the arc-sine and add or subtract appropriately the result to the rectified mean. The (geo-centric) true planet is got. (9)

Difference in the case of Mercury and Venus

All star-planets are (geo-centrically) made true in the above manner. But in the case of Mercury, (in addition to what has been done above), subtract its apogee from the *śīghra* and using the Sun's epicycle, find the *bhuja*-result and apply it to mean Mercury, (which, of course, is the same as the Sun's), with the addition or subtraction done as the Sun's *bhuja* is additive or subtractive. (10)

In the case of Venus, subtract 67′, constant, from the desired mean. (11a). (TSK)

—भास्करः १

13. 12. 2. स्वमन्दकेन्द्रसंप्राप्तफलचापार्धमिष्यते ।
पदक्रमाद्यथा भानोः स्वमध्ये तद्विधीयते ।। ४० ।।
शीघ्रकेन्द्रफलाभ्यस्तं विष्कम्भार्धं विभज्यते ।
स्वकर्णेनाप्तचापार्धं कार्यं तस्मिन् विपर्ययात् ।। ४१ ।।
तस्मान्मन्दफलं कृत्स्नं कार्यमिष्टं स्वमध्यमे ।
एवं भौमार्किजीवानां विज्ञेयाः स्फुटमध्यमाः ।। ४२ ।।
तद्विहीनचलोत्पन्नफलचापेन संस्कृतः ।
स्फुटमध्यः स्फुटो ज्ञेयः शेषयोरुच्यते विधिः ।। ४३ ।।
शीघ्रन्यायाप्तचापार्धयुक्तहीनो विपर्ययात् ।
मन्दोच्चः स्फुटमध्यस्य कर्ता शीघ्रात् स्फुटं विदुः ।।४४।।
प्रतिमण्डलकर्मापि योज्यमत्र विपश्चिता ।
मन्दोच्चे पूर्ववत्कुर्याच्छीघ्रोच्चात्तद्विशोध्यते ।। ४५ ।।
तदेव केवलं शोध्यं चक्रार्धाच्छोध्य तच्चलात् ।
चक्रार्धसंयुतं चापं चक्राच्छुद्धं च शेषयेत् ।। ४६ ।।

मन्दकर्णः शीघ्रकर्णश्च

स्फुटवृत्तगुणां त्रिज्यां हृत्वाऽशीत्या स्वकोटितः ।
त्यक्त्वा पदेषु युक्त्वा वा कर्णः प्राग्वत् प्रसाध्यते ।। ४७ ।।
मन्दोच्चसिद्धतन्मध्यविश्लेषार्धसमन्वितः ।
मन्दसिद्धेऽधिके हीने रहितो मध्यमो ग्रहः ।। ४८ ।।
स शीघ्रोच्चात् पुनः साध्यः सिद्धयोरन्तरालजम् ।
अर्धीकृत्य सकृत्सिद्धे पूर्ववत् परिकल्पयेत् ।। ४९ ।।
एवं कृतस्य भूयोऽपि मन्दसिद्धिं समाचरेत् ।
मन्दसिद्धस्य तस्यायं विशेषोऽतोऽभिधास्यते ।। ५० ।।
द्विसिद्धमन्दसिद्धस्य द्विसिद्धस्य यदन्तरम् ।
प्राग्वत्तन्मध्यमे कृत्वा शीघ्रसिद्धः स्फुटो ग्रहः ।। ५१ ।।
एवमारार्किजीवानामाख्यातं प्रतिमण्डलम् ।
शेषयोरप्ययं सम्यगुच्यते यो विधिक्रमः ।। ५२ ।।

बुधशुक्रौ

शीघ्रोच्चसिद्धतन्मध्यविश्लेषार्धसमन्वितम् ।
मध्यान्न्यूनेऽधिके हीनं मन्दोच्चं संस्कृतं विदुः ।। ५३ ।।
बुधभृग्वोः पुनः साध्यं मन्दमेवं स्वकर्मणा ।
तेन सिद्धौ चलाद् भूयः स्फुटावेतौ प्रकीर्तितौ ।। ५४ ।।
कोटेरन्त्यफलं शोध्यं न शुध्येद् व्यत्ययस्तदा ।
कार्यः कर्णोऽसकृन्मान्दः सकृत्कर्णस्तु शीघ्रजः ।। ५५ ।।

वक्रगतिः तन्निवृत्तिश्च

स्फुटमध्यमान्तरदलं मध्यवशादृणं धनं चले कृत्वा ।
वक्रातिवक्रगमने विज्ञेये तन्निवृत्तिश्च ।। ५६ ।।
शीघ्रोच्चात् स्फुटखेचरो निपतितः शेषो यदा राशय-
श्चत्वारो यदि वक्रगत्यभिमुखः षट् चातिवक्रे स्थितः ।
अष्टौ चेत्कुटिलं जहाति विहगः पन्थानमाश्वेव स
त्वैष्यातीतविचारिणोर्विवरकं भुक्तिर्भवेदाह्निकी ।।५७।।
(Bhāskara I, *MBh*., 4. 40-57)

True planets—Computation (Bhāskara I)

Calculate half the arc corresponding to the (planet's) *mandakendraphala* and apply that to the (planet's) mean longitude depending on the quadrant (of the planet's *kendra*) as in the case of the Sun. (40)

(Then calculate the *śīghrakendraphala*). Multiply the radius by the *śīghrakendraphala* and divide (the product) by the (planet's) *śīghrakarṇa*; then reduce that to arc. Apply half of that arc to the longitude obtained above, reversely, (i.e., add when the *śīghrakendra* is in the half orbit beginning with the Sign Aries and subtract when the *śīghrakendra* is in the half orbit beginning with (Libra). (41)

Therefrom calculate (the arc corresponding to) the *manda-(kendra)phala* and apply the whole of that to the mean longitude of the planet. Thus are obtained the true-mean longitudes of Mars, Saturn and Jupiter. (42)

The true-mean longitude corrected for the arc derived from the *śīghrakendraphala*, (literally, the arc corresponding to the result derived from the longitude of the *śīghrocca* minus the true-mean longitude of the planet) is to be known as the true longitude. The method (to be used) for the remaining planets (i.e., Mercury and Venus) is now being told. (43)

The longitude of the (planet's) *mandocca* (i.e., apogee) reversely increased or decreased by half the arc derived from the *śīghrakendraphala* determines the true-mean longitude (of the planet). And that (true-mean longitude) corrected for the arc derived from the *śīghra-(kendraphala)* is known as the true longitude. (44)

The wise (astronomer) should apply the eccentric theory here (i.e., in the case of the planets Mars, etc.) also. (Under this theory the *mandocca* and *śīghrocca* operations are as follows:)

To the longitude of the *mandocca* ('apogee'), apply (the *spaṣṭabhuja* due to the *mandakendra*, as a positive correction) in the manner prescribed above (in stanza 22). From the longitude of the *śīghrocca* subtract the *spaṣṭa-bhuja* (due to the *śīghrakendra*) (as follows):

(When the *śīghrakendra* is) in the first and second quadrants, subtract from the longitude of the *śīghrocca* the *spaṣṭa-bhuja* itself and that subtracted from half a circle (i.e., 180°) respectively; (when the *śīghrakendra* is) in the remaining quadrants (i.e., third and fourth), subtract that (*spaṣṭa-bhuja*) as increased by half a circle and that (*spaṣṭa-bhuja*) subtracted from a circle, respectively. (45-46)

Mandakarṇa and Śīghrakarṇa

Multiply the radius by the (planet's) corrected epicycle and then divide (the product) by 80; then subtract the quotient from or add that to the R sine of the corresponding *koṭi* (due to the *kendra*) in accordance with the quadrant (of the *kendra*): and then calculate the (planet's) *karṇa* as before. (47)

Add half the difference between the (mean) planet corrected by the *mandocca* operation and the mean planet to or subtract that from the mean planet according as the (mean) planet as corrected for the *mandocca* operation is greater or less (than the mean planet). (The planet thus obtained is called the once-corrected planet.) (48)

Then correct it by the *śīghrocca* operation. (The planet thus obtained is called the twice-corrected planet). Then find the difference between the two planets thus obtained (i.e., the once-corrected and twice-corrected planets); divide that by two; and apply it to the once-corrected planet, as before. (49)

Whatever is thus obtained should again be corrected by the *mandocca* operation. Next calculate the difference between the twice-corrected planet, as corrected by the *mandocca* operation, and that (twice-corrected planet). Apply whatever be the difference between the twice-corrected planet as corrected by the *mandocca* operation and the twice-corrected planet to the mean longitude of the planet, as before. That (i.e., the resulting longitude) corrected by the *śīghrocca* operation is the true longitude of the planet. (50-51)

Thus has been stated the method for finding (the true longitudes of) Mars, Saturn, and Jupiter under the eccentric theory. Now is described the procedure to be adopted in the case of the remaining planets (viz. Mercury and Venus). (52)

Mercury and Venus

(First of all obtain the mean planet as corrected by the *śīghrocca* operation). Then add half the difference between the mean planet corrected by the *śīghrocca* operation and the mean planet to or subtract that from the planet's *mandocca*, according as the mean planet corrected by the *śīghrocca* operation is less or greater (than the mean planet). Thus is obtained the true *mandocca*. Then find out, by the method under the eccentric theory,[1] the correction due to the true *mandocca* for Mercury as well as for Venus. The mean longitudes of Mercury and Venus each corrected for that and thereafter for the correction due to the *śīghrocca* are known as true longitudes of the planets.[2] (53-54)

Further instructions relating to mandakarṇa and śīghrakarṇa

When the R sine of the greatest correction (*antyaphala*) is to be subtracted from the R sine of the *koṭi* (due to the *kendra*), but subtraction is not possible, then subtract reversely (i.e., the latter from the former).[3] Determine the *mandakarṇa* by the method of successive approximations (as in the case of the Sun or Moon) and the *śīghra-*

[1] The method is to find the difference between (i) the mean planet corrected by the *mandocca* operation and (ii) the mean planet.

[2] For the explanation and rationale of the several matters involved, see *MBh*: *KSS*, pp. 136-45.

[3] Reference is to the rule given in stanza 47.

karṇa by a single application of the process (as taught in stanza 47). (55)

Direct and retrograde motions of a planet

Having applied to the longitude of the *śīghrocca* half the difference between the true and mean longitudes (of a planet) positively or negatively, depending upon (whether) the mean longitude (of the planet is greater or less than the true longitude), determine whether the motion of the planet is *vakra* or *ativakra* or whether it is the end of the *vakra* motion. (56)

The true longitude of the planet having been subtracted from the longitude of the (corrected) *śīghrocca*, when the difference is 4 signs, the planet is about to take up *vakra* (retrograde) motion; when 6 signs, it is in *ativakra* (maximum retrograde) motion; and when 8 signs, it soon abandons the regressive path.

Rate of motion

The difference between the true longitudes of a planet computed for (sunrise on) the day to elapse (i.e., today) and for (sunrise on) the day elapsed (i.e., yesterday) is the (true) daily motion (of the planet for the day elapsed). (57). (KSS)

—ग्रहमन्दोच्चशीघ्रोच्चज्याः

13. 12. 3a. 'वस्वीशा' 'दशबाहवो'-'ऽम्बरधृतिः'
'खाङ्का' 'रसत्र्यश्विनो'
मन्दांशा, 'मनु-शैल-शैल-युग-गो'-
सङ्ख्याः स्वमान्दा गुणाः ।
शैघ्र्या 'रामशराः' 'शशाङ्कदहना'
'भूपा'-'स्त्रिवर्गेषवो'
'नन्दाश्च' क्षितिजज्ञजीवभृगुज-
च्छायासुतानां क्रमात् ॥ १ ॥
'वेद-ाक्षी-न्दु-यमा-ब्धि'भिर्मृदुभवां
दोर्ज्यां क्रमेणाहतां
व्यासार्धेन, भजेद् गुणाः फलयुता
हीनौ ज्ञभृग्वोः स्फुटाः ।
'द्वि-द्वी-न्दु-द्वि-कु'-भिर्हतां चलभवां
दोर्ज्यां भजेत् त्रिज्यया
सर्वे शीघ्रभवाः फलेन रहिताः
स्पष्टाः स्युरेवं गुणाः ॥ २ ॥

(Lalla, *ŚiDhVṛ.*, 3. 1-2)

—Apogees and Anomalies

The apogees of Mars, Mercury, Jupiter, Venus and Saturn are, respectively, 118°, 210°, 180°, 90°, and 236°.

The (circumferences of their *manda* epicycles divided by $4\frac{1}{2}$ or) *mandaguṇakas* are 14°, 7°, 7°, 4° and 9°, respectively.

The (circumference of their *śīghra* epicycles divided by $4\frac{1}{2}$ or) *śīghraguṇakas* are, respectively, 53°, 31°, 16°, 59° and 9°. (1)

Multiply the R sines of the anomalies (*mandakendras*) of Mars, Mercury, Jupiter, Venus and Saturn, respectively, by 4, 2, 1, 2 and 4 and divide each product by the radius (3438). The results added to the *mandaguṇakas* (see above) of Mars, Jupiter and Saturn, and subtracted from those of Mercury and Venus, make them (viz., *mandaguṇakas*) more correct.

Multiply the R sines of the *śīghrakendras* of (the above planets), respectively, by 2, 2, 1, 2 and 1 and divide each product by the radius. The results subtracted from their respective *śīghraguṇakas* (see above), make them more correct. (2)

—मन्दशीघ्रकर्णयोरानयनम्

13. 12. 3b. दोर्ज्यावर्गविवर्जितत्रिभवन-
ज्यावर्गमूलं भवेत्
कोटिज्या, भुजभागवर्जितनव-
त्यंशोत्थजीवाथवा ।
स्पष्टस्वस्वगुणाहते 'खवसुभि'-
र्दोःकोटिजीवे हृते
स्यातां दोःफलकोटिसंज्ञकफले
ताभ्यां श्रुतिं साधयेत् ॥ ३ ॥

(Lalla, *ŚiDhVṛ.*, 3-3)

—Manda and Śīghra hypotenuse

The square root of the difference of the squares of the R sine of 90° and R sine of an arc, is the R cosine (*koṭijyā*) of the arc. Or, the R sine of the difference of 90° and an arc is the R cosine (*koṭijyā*) of the arc.

When the R sine and R cosine of the *mandakendra* and the *śīghrakendra* of a planet are multiplied by the planet's corrected *mandaguṇaka* and *śīghraguṇaka*, respectively, and divided by 80, the results are, respectively, called *dohphala* and *koṭiphala*. From these the hypotenuse or *hṛti* (or *karṇa*) should be calculated. (3).

—ग्रहस्फुटानयनम्

13. 12. 3c. मन्दोच्चभागरहितग्रहबाहुमौर्व्या
संसाध्य बाहुफलमस्य धनुर्दलेन ।
संस्कृत्य मध्यममृणं स्वमवेत्य केन्द्रात्
संशोधयेच्च तदनष्टमतश्चलोच्चात् ॥ ४ ॥

शेषं भवेत्तु चलकेन्द्रमतो भुजज्यां
कोटयाह्वयां च विदधीत तयोः फले च ।
कोटीफलेन रहिता सहिता त्रिभज्या
कार्या कुलीरमकरादिगते स्वकेन्द्रे ॥ ५ ॥

तद्वर्गबाहुफलवर्गसमासमूलं
कर्णो भवेद् भुजफलं गुणितं त्रिमौर्व्या ।
कर्णोद्धृतं कृतधनुः फलमाशुसंज्ञं
स्यात्तद्दलं स्वमथवार्णमनष्टसंज्ञे ॥ ६ ॥

कार्ये क्रियाद्यथ तुलाद्यवगम्य केन्द्रं
प्राग्वत्ततो मृदुफलं सकलं विधेयम् ।
मध्ये पुनश्चलफलेन ततोऽखिलेन
प्राग्वत् सुसंस्कृततनुः स्फुटतामुपैति ।। ७ ।।

शीघ्रोद्भवेन दलितेन फलेन पूर्वं
संस्कृत्य वा ग्रहमतो विदधीत मान्दम् ।
तेनाखिलेन सकलेन च शीघ्रजेन
प्राग्वत् स्फुटो भवति संस्कृतिभाग् ग्रहः सः ।। ८ ।।

केचिद् वदन्ति बुधशुक्रपरिस्फुटत्वं
मध्यान्मृदूच्चरहितान्मृदुना फलेन ।
शीघ्रोच्चमध्यरहिताच्चलसंज्ञितेन
संसाधितेन सकलेन सकृद् विदध्यात् ।। ९ ।।

भानोः फलेन परमेण दलीकृतेन
स्पष्टे भृगुर्विरहितोऽतिपरिस्फुटः स्यात् ।
सूर्योच्चवर्जितशशाङ्कजशीघ्रतुङ्गा-
ज्जातेन भास्करफलेन बुधोऽर्कवच्च ।। १० ।।

(Lalla, *ŚiDhVṛ.*, 3. 4-10)

—Computation of True planets

Subtract the longitude in degrees of the apogee of a planet from its mean longitude. (The remainder is the mean anomaly. It should be reduced to the) first quadrant. Find its R sine and hence the *doḥphala* (as given above). Find the corresponding arc, (which is the *mandaphala*). Add or subtract half of it to or from the mean longitude of the planet, according as the mean anomaly is (greater or less than 180°). (The result is the longitude of the planet after the first correction) or *anaṣṭa*.

Subtract (*anaṣṭa*) from the longitude of the planet's *śīghrocca*. The remainder is called *śīghrakendra*. Find its R sine and R cosine and hence the *doḥphala* and *koṭiphala* (as above). Add or subtract the *koṭiphala* to or from the radius, according as the *śīghrakendra* is in the first and fourth or second and third quadrants. (The result is called *sphuṭakoṭi*). The hypotenuse or *karṇa* is the square root of the sum of the squares of the *doḥphala* and *sphuṭakoṭi*.

Multiply the *doḥphala* by the radius and divide by the hypotenuse. The arc corresponding to the result as R sine is called *śīghraphala*. Add or subtract half of the *śīghraphala* to or from the once-corrected longitude of the planet (*anaṣṭa*), according as the *śīghrakendra* is in the first and second quadrants or in the third and fourth. (The result is the longitude of planet after the second correction).

From the twice-corrected longitude calculate the *mandaphala* as before and apply the whole of it to the *mean* longitude. (From the longitude of the planet thus corrected) calculate the *śīghraphala* as before and apply the whole of it to the (thrice-corrected longitude of) the planet. The result is the true longitude of the planet. (4-7)

Or, (first calculate the *śīghraphala* from the planet), and apply half of it to the longitude of the planet. Then (calculate the *mandaphala* from the corrected longitude) and apply half of it to the corrected longitude. Then, as before, calculate and apply the whole of the *mandaphala* and *śīghraphala*. Thus corrected, the planet's true longitude is obtained. (8)

Some say that the true longitudes of Mercury and Venus (should be obtained in the following manner): The *mandaphala* (calculated) from the difference between the mean planet and its apogee, should be first applied to the planet. Then the *śīghraphala* (calculated) from the difference between the planet and its *śighrocca*, should be applied to the corrected planet. (The results are correct Mercury and Venus). (9)

When the true longitude of Venus (as calculated above is diminished by half the maximum *mandaphala* of the Sun, the longitude is correct.

Subtract the Sun's apogee from the *śīghrocca* of Mercury. From the remainder calculate the *mandaphala* using the Sun's equations. Apply it to the (twice corrected) Mercury in the same manner as it would be applied to the Sun. (The result is more correct.) (10) (BC)

ज्याभिर्विना ग्रहस्फुटः

13. 13. 1. जीवाशकलैर्द्युसदां स्पष्टीकरणं मयेरितं विधिवत् ।
अधुना विनैव मौर्वीशकलैर्वक्ष्ये स्फुटीकरणम् ।। १ ।।

भुजांशैः पिण्डाख्यराशिः

चक्रार्धांशा भुजांशैर्विरहितनिहता-
स्तद्विहीनैर्विभक्ताः
'खव्योमेष्वभ्रवेदैः' 'सलिलनिधि'हताः
पिण्डराशिः प्रदिष्टः ।
षड्भांशघ्ना भुजांशा निजकृतिरहिता-
स्तत्तुरीयांशहीनैः
भक्ताः स्यात् पिण्डराशि'र्विशिखनयनभू-
व्योमशीतांशुभि'र्वा ।। २ ।।

भुजकोटिज्ये मन्दशीघ्रफले च

परफलगुणनिघ्ना स्यात् फलज्या त्रिमौर्व्या
भवति हि भुजजीवा चैवमन्त्याहतेऽपि ।
मृदुफलमिह साध्यं प्रोक्तवद् बाहुभागैः
त्वरफलमपि चैवं बाहुकोटयंशकैः स्वैः ।। ३ ।।

ज्यातश्चापः

त्रिभवनगुणयुक्तो ज्यातुरीयोऽत्र हारो
'विशिखरविखचन्द्रै'स्ताडितायास्तु मौर्व्याः ।

'खखविशिखखवेदै'राहता वेष्टजीवा
त्रिभगुणकृतिखातज्यासमासेन भक्ता ।। ४ ।।

फलहीना नवतिकृतिस्तन्मूलेन च वर्जिता नवतिः ।
शेषं धनुरथवा यत् त्रिज्याखण्डैर्विनैव फलम् ।। ५ ।।

(Vaṭeśvara, *VSi.*, 2. 4.1-5)

Correction without R sine table

Correction of the planets with the help of the (tabular) R sines has been duly described by me. I shall now describe that correction without the use of the (tabular) R sines. (1)

Piṇḍarāśi or sine of bhujā

Diminish and multiply the degrees of half a circle (i.e., 180) by the degrees of the *bhujā*. Divide that by 40,500 minus that; and then multiply (the quotient) by 4. The result is called the *piṇḍarāśi*.

Or, multiply the degrees of the *bhujā* by 180 and diminish that (product) by the square of the degrees of the *bhujā*. Divide that by 10,125 minus one-fourth of that (difference). Then (too) is obtained the *piṇḍarāśi*.[1] (2)

Mandaphala and śīghraphala

(The *piṇḍarāśi*) when multiplied by the R sine of the maximum correction (i.e., the radius of the *manda* epicycle) gives the R sine of the correction (i.e., *mandaphala*) and when multiplied by the radius, gives the R sine of the *bhujā*; similarly, when multiplied by the other numbers.

One should calculate the *mandaphala* with the help of the degrees of the *bāhu* in the manner stated above; and also the *śīghraphala* with the help of the degrees of its own *bāhu* and *koṭi*, in the same way. (3)

Arc from R sine

Here, take one-fourth of the R sine plus the radius as the divisor of the R sine multiplied by 10,125; or, divide the given R sine multiplied by 40,500 by the sum of the R sine and the product of the radius and 4. (4)

Subtract the quotient (thus obtained) from the square of 90 (i.e., from 8100); and subtract the square root of that from 90. Whatever is obtained as the remainder is the arc or the correction (as the case may be), derived without taking recourse to the (tabular) R sines.[1] (5). (KSS)

राहु—रोमकः

13. 14. 1. 'व्यष्टक'गुणिते दद्याद्
'रसर्तुयमषट्कपञ्चकान्' राहोः ।
'भवरूपाग्न्यष्टि'हृते
क्रमात् झषान्तोच्च्युते वक्त्रम् ।। ८ ।।

(Varāha, *PS*, 8.8)

Moon's Node—Romaka

Multiply the days from epoch by 24, add 56,266, and divide by 1,63,111. Subtract the revolutions etc. obtained, from the end of Pisces, (*i.e.*, from any whole number of revolutions). The Head of Rāhu (Ascending Node) is obtained.[2] (8). (TSK)

राहुः—पौलिशः

13. 14. 2. अष्टगुणे दिनराशौ रूपेन्द्रियशीतरश्मिभिर्भक्ते ।
लब्धा राहोरंशाः भगणसमाश्च क्षिपेल्लिप्ताः ।। २८ ।।

वृश्चिकभागा राहोः षड्विंशतिरेकलिप्तिकालुप्ताः ।
आदिरतः प्रोह्य मुखं षड्राशियुतं तु पुच्छाख्यम् ।।२९।।

(Varāha, *PS*, 3. 28-29)

Moon's Node—Pauliśa

28-29. Multiply the days from epoch by 8 and divide by 151. Rāhu's motion is got in degrees etc. Add minutes equal to revolutions. The motion becomes exact. Deduct the motion from $7^{r}\ 25^{\circ}\ 59'$. The remainder is Rahu's Head (what is called Dragon's Head, a popular name for the Ascending Node). Add 6 *rāśis* to Rāhu's Head; Rāhu's Tail (Dragon's Tail or Descending Node) is got. (28-29). (TSK)

राहुः—सौरः

13. 14. 3. त्रिघनशतघ्ने 'नवकैकपक्षरामेन्दुदहनरस'सहिते ।
'स्वरयमवसुभूतार्णवगुणधृति'भक्ते क्रमाद् राहोः ।।५।।

चक्रात् पतितं वक्त्रं षड्राशियुतं तु पुच्छाख्यम् ।
नवतिविवरस्य लिप्ता विक्षेपः सप्ततिर्द्विशती ।। ६ ।।

(Varāha, *PS*, 9. 5-6)

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[1] The *piṇḍarāśi* is the sine of the *bhujā*. Let θ be the degrees of the *bhujā*. Then, according to the above rule,

$$\sin\theta = \frac{4(180-\theta)\theta}{40500-(180-\theta)\theta}, \text{ or } \frac{180\,\theta-\theta^2}{10125-\dfrac{180\,\theta-\theta^2}{4}}.$$

For the rationale of these formulae, the reader is referred to MBh: *KSS*, vii. 17-19.

[1] That is,

$$\theta = 90 - \sqrt{(90^2 - Q)},$$

$$\text{where } Q = \frac{10125.\ R\sin\theta}{R\sin\theta/4+R}, \quad \text{or} \quad \frac{40500.\ R\sin\theta}{R\sin\theta+4R}$$

This formula may be easily derived from the previous rule.

[2] The following is instructed to be done:—

(i) Revolutions etc. = (days × 24 + 56,266) ÷ 1,63,111.

(ii) Head of Rahu = rā. 12-0-0 — Revolutions etc. omitting the full revolutions. For worked out examples, see *PS*:*TSK*, 8.8.

Moon's Node—Saura

Multiply the Days from Epoch by 2700, add 63,13-219, and divide by 1,83,45,827. Revolutions etc. are obtained, to be used in getting Rāhu. (5)

This deducted from twelve *rāśis* is the Rāhu-head (*i.e.*, Ascending Node of the Moon). Rāhu-head *plus* six *rāśis* is the Rāhu-tail (*i.e.*, Descending Node). At the (maximum) distance of 90° from Rāhu (the node, the Moon's latitude is 270 minutes (*i.e.*, this is the maximum latitude).[1] (6). (TSK)

राहुः—वाक्यकरणम्

13. 14. 4. 'स्तुतिज्ञानेन तोष्यो'नं दिनं 'धन'हृतं, हृतम् ।। १७ ।।
'धनजग्धतटै'-र्लब्धहीनं 'क्रुद्धसुतै'-र्हृतम् ।
राशय'स्ततमैः' शेषा'न्नील'-'नीति'-गुणात् क्रमात् ।।
भागाद्यं, चक्रतः सर्वं शोध्यं, शिष्टं तमस्स्फुटम् ।
(*VK*, 1. 17b-19a)

Moon's Node—Vākyakaraṇa

Deduct 16,00,066 from the Kali days. From the remainder deduct days etc. got by multiplying the remainder by 9 and dividing by 1,69,809 (? 1,69,804). Divide out the remaining days etc. by 6792 and take the remainder alone in days etc. The quotient got by dividing this by 566, are *rāśis*; the quotient of dividing by 566, what is left over, multiplied by 30, gives degrees, the quotient of dividing by 566, what is still left over multiplied by 60, gives minutes, and so on. Deduct these *rāśis* etc. got, from 12ʳ. Rāhu is obtained, (no distinction being made between True and Mean Rāhu.)[2] (17b-19a). (TSK-KVS)

ग्रहध्रुवाः—कल्यादौ न निरंशत्वं ग्रहाणाम्

13. 15. 1. कल्यादौ न निरंशत्वं भगणादेर्द्युचारिणाम् ।
गतिभेदात्तु, दृक्सिद्धास्तत्रैषां स्युर्ध्रुवास्ततः ।। ७८४ ।।
(Śaṅkara, Com. *Yuktidīpikā* on *Tantrasaṅgraha*, 1.35)

Zero positions at Epoch

Planets not at zero at Zero Kali

At Zero Kali, the revolutions of the planets do not have zero positions, on account of changes in their rates of motion. Hence their zero positions at Zero Kali have to be computed on the basis of observations (from time to time).[3] (784). (KVS)

13. 15. 3. ग्रहाणामथ सिद्धानां दृग्भेदो दृश्यतेऽधुना ।
दृग्भेदहेतुः कोऽत्र स्यादस्माभिरिति चिन्त्यते ।। १ ।।
अव्यवस्था तु खेटानां भुक्तेरेव हि युज्यते ।
शैघ्र्यं मान्द्यं तथा कल्प्यं क्रमादेव गतेस्ततः ।। २ ।।
कालतो गतिभेदाच्च सिद्धान्ता बहुधा कृताः ।
ब्रह्माद्यैरित्यतः सिद्धं भेदोऽतस्तेषु युज्यते ।। ३ ।।
तत्तत्काले गतिवशादनुमानेन कल्पिताः ।
भगणास्तैर्निजनिजे सिद्धान्ते सिद्धमित्यपि ।। ४ ।।
दृश्यमानोऽधुना तेषां दृग्भेदः सृष्टिकालतः ।
ऊर्ध्वकालेन सञ्जात इति कल्प्यं बुधैरतः ।। ५ ।।
कल्यादौ जातसंस्कारो ध्रुवरूपेण कथ्यते ।
अत्रैषामनुपातेन तैः कल्प्यश्चोर्ध्वकालतः ।। ६ ।।
भानौ योज्या 'बाण'कला विकला'श्चेषुवह्नयः' ।
लिप्ताद्वयं विधौ शोध्यं विलिप्ताश्च 'नगाग्नयः' ।। ७ ।।
तुङ्गे योज्याश्चतुर्भागाः 'सप्तचन्द्र'कला अपि ।
विकला 'विश्व'तुल्याश्च विकलान्तास्त्रयस्त्वमी ।। ८ ।।
पञ्च लिप्ताः कुजे शोध्या, बुधे 'वेदां'शकास्तथा ।
शोध्याः 'शराब्धि'लिप्ताश्च गुरौ भागत्रयं तथा ।। ९ ।।
शोध्यं भृगौ 'वेद'भागाः शोध्या लिप्ता 'जिनै'र्मिताः ।
शनौ योज्या 'वेद'भागाः सप्तलिप्तास्तथैव च ।।१०।।
योज्यं भागद्वयं पातमध्ये लिप्ताः 'शराग्नयः' ।
दृक्साम्यसिद्धये कुर्यात् कल्यादौ ध्रुवका इति ।। ११ ।।
ग्रहभुक्त्यंघ्रिसंयुक्ता ग्रहा औदयिका अमी ।
सृष्टिकालेन संस्कारो गर्गेणापि प्रदर्शितः ।। १२ ।।
(Parameśvara: Com. on *Sū.Si*, 1.66)

True longitude correction

At present, difference is seen in (the longitudes of) planets as calculated (according to the older texts) from their observed positions. We shall discuss here as to what could be the reason for this difference in the observed positions. (1)

Here, indefiniteness has to be ascribed only to the rates of motion of the planets. Hence slowness or fastness has to be suitably applied to the rates of motion. (2)

Different *Siddhānta*-texts have been composed by Brahmā and others which differ in the time (when they were composed) and in the rates of motion stated therein. For this reason, it is only proper that appropriate changes are effected in the rates of motion (given in these texts). (3)

The number of revolutions (of the planets during a period) given in each of these texts have been calculated on the basis of contemporary rates of motion and by logical reasoning. (4)

The difference in planetary positions as observed now from what it was at the time of Creation has to be

[1] For the working see *PS*: *TSK*, 9.5-6.

[2] For worked out example, see *VK*: *TSK-KVS*, p. 256.

[3] It is worth noting that the difference in revolution numbers of the planets and the different zero corrections for Kali, found especially in the later Karaṇa texts stem from this practice.

constructed by the wise as to have accumulated during the interim period. (5)

Here-in-below is indicated, in the form of constants, the correction to be applied to planetary positions at the beginning of Kali. For the subsequent periods, this correction has to be calculated in proportion to these constants. (6)

Sun: 5′ 35″; +Moon:—2′ 37″; Moon's node:+4° 17′ 13″. These three are correct to the seconds. (7-8)

Mars:−5′; Mercury:−4°; Jupiter:−3° 45′; Venus: −4° 24′; Saturn:+4° 7′; Rāhu:+2°35′. These zero corrections have to be applied to the mean positions of the planets (as calculated according to the *Sūrya-siddhānta*) at the beginning of Kali, to get their observed positions (at Zero Kali). (9-11)

These planetary positions are for sunrise as arrived at by corrections applied to quarter rates of motion. Corrections for the time of Creation have been indicated even by sage Garga. (12) (KVS)

कृतयुगान्ते ग्रहस्थितिः—सूर्यसिद्धान्तः

13. 15. 3. अस्मिन् कृतयुगस्यान्ते सर्वे मध्यगता ग्रहाः ।
ऋते तु पातमन्दोच्चान्मेषादौ तुल्यतामिताः ॥ ५६ ॥
मकरादौ शशाङ्कोच्चं तत्पातश्च तुलादिगः ।
निरंशत्वं गता नान्ये ते नोक्ता मन्दचारिणः ॥ ५७ ॥
(*Sū.Si.*, 1. 56-57)

Zero positions at the end of Kṛta Age

Now, at the end of the Golden Age, (*Kṛta-yuga*), all the planets, by their mean motion—excepting, however, their nodes and apsides (*mandocca*)—are in conjunction in the first of Aries. (56)

The Moon's Apsis (*ucca*) is in the first of Capricorn and its Node is in the first of Libra; and the rest, which have been stated above to have a slow motion—their position cannot be expressed in whole Signs. (57). (Burgess)[1]

कल्यादिध्रुवाः—भास्करः २

13. 15. 4. कलिगतादथ वा दिनसञ्चयो
दिनपतिर्भृगुजप्रभृतिस्तदा ।
कलिमुखध्रुवकेण समन्वितो
भवति तद्द्युगणोद्भवखेचरः ॥ १८ ॥
'खाद्रिरामाग्नयः' 'क्वग्निरामाङ्कका'
'वेदवेदाङ्कचन्द्रा' विलिप्ताः क्रमात् ।
'षड्रसाङ्गाब्धयो'-'ऽङ्गाभ्रवेदाब्धयो'
'वेदषट्काभ्रभूपाभ्रभू'संमिताः ॥ १९ ॥
'वेदचन्द्रद्विवेदाब्धिनागाः' 'कर-
द्व्यब्धिवेदाब्धिशैला' भवेयुः कुजात् ।
द्वापरान्तध्रुवाश्चक्रशुद्धास्तथा
सूर्यतुङ्गेन्दुतुङ्गेन्दुपातोद्भवाः ॥ २० ॥
(Bhāskara II; *ŚiSi*, 1. 1. 3. 18-20)

Planets at Zero Kali—Bhāskara II

(The first day of) the *ahargaṇa* from Kali is (a Friday) with Venus as the Lord of day. The mean planetary position (on any day in any *ahargaṇa* is found by adding to the calculated mean positions (of the planets) the zero positions of the planets at the beginning of Kali known as *dhruvakas*.

The figures in *vikalās* (i.e. seconds) given below when subtracted from *cakra* (i.e. 360°), indicate the zero positions of the planets, the Sun's apogee, Lunar apogee and the Ascending Node at the end of the Dvāpara Age (i.e. at the beginning of the Kali Age): Mars, 3370; Mercury, 9331; Jupiter, 1944; Venus, 4666; Saturn, 4406; Solar Apogee, 1016064; Lunar apogee, 844214), and Ascending Node 744422.[1] (19-20). (KVS)

—दृग्गणितम् (परमेश्वरः)

13. 15. 5. विलिप्तादिध्रुवं भानोः 'खरनारी', विधोस्तथा ॥ २२ ॥
'प्राज्ञे मायाङ्गि', तुङ्गस्य 'चित्रभावशशरारिनुत्' ।
एते योज्याः, कुजादीनां ध्रुवं लिप्तादि कथ्यते ॥ २३ ।
धनं भौमस्य 'गानानि', बुधस्यर्णं 'भवाङ्गना' ।
ऋणं गुरोर् 'धर्मराज्ञी', शुक्रस्यर्णं 'ननावनम्' ॥ १४ ॥
योज्यं शनेर् 'जनाम्भो', ऽह्येर्योज्यं 'तुङ्गारिनूतनम्' ॥
(Parameśvara, *Dṛggaṇita*, 2. 12a-15a)

—Dṛggaṇita (Parameśvara)

The zero correction (for the longitudes of the planets at the beginning of Kali) are: Sun: 20′ 22″; Moon: 3° 15′ 2″; Moon's Apsis: 2^r 25° 44′ 26″; these are additive. Mars: 3′, additive; Mercury: 3° 44′, negative; Jupiter: 2° 59′, negative; Venus: 4°, negative; Saturn: 4° 8′, additive; and Node: 6^r 2° 36′. (KVS)

—तन्त्रसंग्रहः (नीलकण्ठः)

13. 15. 6 'षड्वेदेष्वब्धिवेदा'स्तु विलिप्तादिध्रुवो विधोः ।
'प्राणात्यष्टचङ्कनेत्राग्नि'तुल्यं चन्द्रोच्चमध्यमम् ॥ ३५ ॥
'सप्तसागरशैलेन्दुभवा' लिप्तादयोऽसृजः ।
'षट्त्रि'शल्लिप्तिका शोध्या विदो, जीवे तु योजयेत् ॥
'पङ्क्त्यर्क'तुल्यलिप्तादि सितो राशिः षडंशकाः ।
'विश्व'तुल्याः कलाश्च स्वं 'नखात्यष्टिभवाः' शनेः ॥३७॥
पाते तु मण्डलाच्छुद्धे 'नखाकृतिरसा' अपि ॥ ३८ ॥
(Nīlakaṇṭha: *TS.*, 1. 35-38a)

[1] For notes, see *SūSi*: *Burgess*, pp. 41-43.)

[1] The Mean Sun and Moon are at the zero point itself. Converting the figures given above into higher units, the zero positions are: Mars 11^r 29° 3′ 50″; Mercury, 11^r 27° 24′ 29″; Jupiter, 11^r 29° 27; 36″; Venus, 11^r 28° 42′ 14″; Saturn 11^r 28° 46′ 34; Solar apogee 2^r 17° 45′ 36″; Lunar apogee 4^r 5° 29′ 46″; Ascending lunar Node 5^r 3° 12′ 58″.

—Tantrasaṅgraha (Nīlakaṇṭha)

The mean zero positions, correct up to seconds (*viliptā*), at the commencement of Kali, of the Moon is 4° 45′ 46″. That of the mean of Moon's apogee is 3^r 29° 17′ 5″. Those correct to the minutes, of Mars is 11^r 17° 47′; of Mercury, 36′, negative; of Jupiter, positive, 12° 10′; of Venus, positive, 1^r 6° 13′; of Saturn, 11^r 17° 20′. To the Ascending Node subtracted from the *maṇḍala* (i.e. 12^r) should be added 6^r 22° 20′. (35-38). (VSN)

—स्फुटनिर्णयः (अच्युतपिषारटि)

13. 15. 7. 'तुला किरीटैरविनून' मिन्दुः,
'चेलार्धनूत्ना मृदुरङ्गनो'च्चः ।
'त्रिकालवेदी पटुरेष' राहुः,
'चलापराधानुजपापिका'ऽऽरः ॥ १६ ॥

'दृढादरः प्राज्यनरः पुटु'र्वित्
'धोराशिशोणः कुशिको नु' जीवः ।
'ज्ञानेन नीतोऽनशनाय' शुक्रो
'नूनं दिवि श्रोत्रनयाकृपा'ऽऽर्किः ॥ १७ ॥

कल्यादिभूताः खलु तत्पराद्याः
ध्रुवा इमे सूक्ष्मतरा ग्रहाणाम् ।
कल्यादिजं संक्रमणध्रुवं तु
'न निश्चलश्छात्मकमित्दुशोभाः' ॥ १८ ॥

(*Sphuṭanirṇayasaṅgraha*, 1. 16-18)

—Sphuṭanirṇaya (Acyuta Piṣāraṭi)

(The following) are the zero positions of the planets at the beginning of Kali, and are very accurate, correct to the fourths (*tatparā-s*): Moon: 0^r 4° 21′ 21″ 36‴; Moon' apogee: 3^r 28° 50′ 9″ 36‴; Node: 6^r 21° 18′ 43″ 12‴; Mars: 11^r 18° 9′ 21″ 36‴ Mercury: 11^r 20° 12′ 28″ 48‴; Jupiter: 0^r 15° 15′ 55″ 24‴; Venus: 1^r 5° 6′ 0″ 0‴; Saturn: 11^r 10° 22′ 48″ 0‴. The Sun's entry into Sign Aries is at 4^r 58° 51′ 57″ 36‴ 0⁗ (17-18). (KVS)

स्फुटनिर्णयतुल्यः

13. 15. 8. 'भानुः सुनम्यो ननु नून'मर्कः
'सेनाजयं तालवनेन' चन्द्रः ॥ ७ ॥

'शुकार्थमुग्धानुजपापिका'ऽऽरो
'विद्यार्थिनो सेव्यनरः पुटु'र्वित् ।
'लग्नेषु मणिक्यमयो नु' जीवः
'पत्नी धवस्थानमिनस्य' शुक्रः ॥ ८ ॥

'सम्पन्नशारीरनयाकृपा'ऽर्किः
'पापार्थयत्ना मुहुरङ्गनो'च्चः ।
'चण्डालगेहे कविरस्ति' राहु-
रिमे हि कल्याद्युदयध्रुवाख्याः ॥ ९ ॥

(*Sphuṭanirṇaya-tulya-grahamadhyamānayanam*, 7b-9)

—Sphuṭanirṇaya-based

The zero positions of the planets at the commencement of Kali are; Sun: 0^r 0° 1′ 7″ 4‴; Moon: 0^r 4° 36′ 18″ 7‴; Mars: 11^r 18° 9′ 57″ 15‴; Mercury: 11^r 20° 17′ 7″ 14‴; Jupiter: 0^r 15° 15′ 56″ 3‴; Venus: 1^r 5° 7′ 49″ 1‴; Saturn: 11^r 10° 22′ 50″ 17‴; Apsis: 3^r 28° 50′ 17″ 11‴; Node: 6^r 24° 18′ 39″ 236‴. (7-9). (KVS)

—सद्रत्नमाला (शङ्करवर्मा)

13. 15. 9. कल्यब्दनिघ्न-'मकुटोद्भवकृष्णतालं'
गुर्वक्षरादि 'भयलेशदनक्र'हीनम् ।
कल्यादितो दिनगणोऽच्छदिनाद् द्वयोने
सप्ताप्तशेष इनसंक्रमणध्रुवोऽस्मिन् ॥ ६ ॥

(Śaṅkara Varman, *Sadratnamālā*, 3.6)

—Sadratnamālā (Śaṅkara Varman)

Multiply (the measure of the mean day) from *gurvakṣaras*, being 365 days, 15 *nāḍikās*, 31 *vināḍikās* and 15 *gurvakṣaras*, by the Kali years (expired) and subtract from the product 2 days, 8 *nāḍikās*, 53 *vināḍikās* and 14 *gurvakṣaras*. The result would be the days from the beginning of Kali, counted from a Friday. If 2 is subtracted from this number and the number divided by 7, the remainder would give the day from Sunday and the accurate time of the entry of the Sun into Sign Aries of the current year. (KVS)

कल्यादिमन्दोच्चध्रुवाः

13. 15. 10. 'ज्ञानी वरिष्ठो नु मनोजपुत्रो'
'न निश्चलो बालहरिः सुनामः' ।
'अनन्तगुः पुत्रधुगर्थनाथो'
'नूत्नाचले यत्र शिवः पुराणः' ॥ १९ ॥

'नूत्नोत्र कृष्णः शिशुरूपरौद्रे'
'ज्ञानाच्चलाधीनजनोऽत्र नन्दः' ।
इमे हि कल्यादिभवा मृदूच्चाः
क्रमेण सूर्याच्छशिनस्तु तुङ्गः ॥ २० ॥

(*Sphuṭanirṇayasaṅgraha*, 1. 20)

Apses at Zero Kali

(The following) are the zero positions of the apses (of the planets) at the commencement of Kali: Sun: 2^r 18° 5′ 6″ 24‴ 0″″; Mars: 4^r 7° 28′ 33″ 36‴ 0″″; Mercury: 7^r 7° 39′ 21″ 36‴ 0″″; Jupiter: 5^r 21° 45′ 21″ 36‴; 0″″; Venus: 2^r 21°′25′ 55″ 12‴ 0″″; Saturn: 8^r 2° 8′ 9″ 36‴ 0″″; The apse of the Moon is its Apogee. (19-20). (KVS)

कल्यादिपातध्रुवाः

13. 15. 11. 'आनन्दभावेन विना नृपेढ्यो'
'ज्ञानेन्द्रियेशो मिथुनाकरज्ञः' ।
'ज्ञानेश्वरश्चारुशिरो नरेन्द्रो'
'ज्ञानं क्रियाङ्गोद्भवमग्निनिष्ठम्' ।

'अग्निप्रकाशो मलयाद्रिनागः'
कल्यादिपाताः क्रमशो हि भौमात् ॥ २१ ॥
(*Sphuṭanirṇaya-saṅgraha*, 1. 21)

Nodes at Zero Kali

The zero positions of the Nodes of Mars and other (planets) at the commencement of Kali are: 1ʳ 11° 4′ 4″ 48‴ 0″″; Mercury: 0ʳ 21° 7′ 55″ 12‴ 0″″; Jupiter: 0ʳ 20° 25′ 26″ 24‴ 0″″; Venus: 2ʳ 0° 54′ 43″ 12‴ 0″″; Saturn: 3ʳ 2° 13′ 55″ 12‴ 0″″. (21) (KVS)

खण्डध्रुवाः—(खण्डः १७,६७,०००)

13. 15. 12. 'अनूनसिद्धिसौख्यो'नात् बुधवाराद्यहर्गणः ।
'धिक् प्रेतराज्यमशुभं कालस्याज्ञार्थिनं' रविः ॥ १ ॥
'अप्रजेनाभिभोगो वै स्त्रीणां दुःखाय' चन्द्रमाः ।
'दुष्टभावा भोगचिह्ना नष्टसत्याङ्गनो'च्चकः ॥ २ ॥
'धीवरो रसनद्धात्मा रत्नरूपेण' पन्नगः ।
'मर्त्यो दानफलार्थी स्यात् सभार्यो यौवने' कुजः ॥ ३ ॥
'गूढप्रज्ञो गुरुं नत्वा शिष्टार्थी पावनो' बुधः ।
'जन्मना मृगराजेन्द्रश्चण्डाग्निनयनो' गुरुः ॥ ४ ॥
'धन्यो रविं सकृन्नत्वा मृष्टाङ्गो ननु' भार्गवः ।
'चन्द्राङ्गभागको भानुः फलभद्रासनः' शनिः ॥ ५ ॥
सुसूक्ष्मा भास्करादीनां ध्रुवा दृक्पक्षजा इमे ।
प्रतत्परादधः स्थानादारभ्य समुदीरिताः ।
स्फुटनिर्णयनाम्नैते करणेन विनिर्मिताः ॥ ६ ॥
(Puradahanapura-dvija)

Zero positions for Kali 17, 97,000

These are the very accurate zero positions, correct up to the fifths (*pratatparā*) of the Sun (and other planets according to the Dṛk school (of Kerala), computed according to the *Sphuṭanirṇaya* (Tantra of Acyuta Piṣāraṭi) for Kali 17,97,000:

Sun	7ʳ	1°	41′	45″	58‴	28″″	29″‴
Moon	1	28	52	43	44	8	20
Apsis	3	17	10	6	34	44	18
Node	5	12	2	59	7	22	49
Mars	4	11	47	17	32	8	15
Mercury	4	15	15	40	23	2	43
Jupiter	1	0	36	28	23	50	58
Venus	0	3	15	40	17	42	19
Sat.	7	24	32	4	13	43	26

(1-6). (KVS)

भटाब्द (शकाब्द) संस्कारः—१

13. 16. 1. 'वाग्भावो'नाच्छकाब्दाद् 'धन-शत-लय-'हा-
'न्मन्द-वैलक्ष्य-रागैः'
प्राप्ताभिर्लिप्तिकाभिर्विरहित तनव
श्चन्द्रतत्तुङ्गपाताः ।
'शोभा-नीरूढ-संविद्-गणक-नर'हृता-
'न्मागरा'प्ताः कुजाद्याः
संयुक्ता ज्ञारसौराः सुरगुरु भृगुजौ
वर्जितौ भानुवर्ज्यम् ॥
(Haridatta, *Mahāmārganibandhana*)

Bhaṭābda or Śakābda correction—I

The mean longitudes of the Moon, its apogee and ascending Node should (respectively) be diminished by the minutes of arc which are obtained by diminishing the (elapsed) years of the Śaka era by 444, (then severally, multiplying (that difference) by 9, 65, and 13 and dividing them by 85, 134, and 32 (respectively). (Severally) multiplying (the same difference) by 45, 420, 47, 153, and 20 (respectively) and dividing (all of them) by 235 are obtained (the corrections in minutes of arc) for Mars, etc. (The corrections) for (the *śīghrocca* of) Venus should be subtracted (from their mean longitudes). The Sun is to be excluded (from this correction). (KSS)

भटाब्द (शकाब्द) संस्कारः—२

13. 16. 2. करणाब्दं 'गिरिरसशशि'सहितं प्रालेयदीधिते'स्तत्त्वैः' ।
उच्चस्य 'वेदरुद्रै'र्भुजङ्गराजस्य 'रसरन्ध्रैः' ॥ १६ ॥
भूसूनोः 'शरवेदै'र्बुधशीघ्रस्या'म्बराश्विवारिधि'भिः ।
'मुनिवेदै'रिन्द्रगुरोः सितशीघ्रस्य 'त्रिबाणैकैः' ॥१७॥
सौरस्य 'नखै'र्गुणये'च्छराग्नियुग्मै'र्भजेच्च ता लिप्ताः ।
कुजशनिशशितनयेषु क्षेप्याः शेषेषु संशोध्याः ॥ १८ ॥
(Deva, *KR*, 1. 16-18)

Bhaṭābda or Śakābda correction—II

To the *karaṇābda* (i.e. to the years elapsed since the epoch of the present karaṇa work) add 167. Multiply the sum obtained, by 25 in the case of the Moon; by 114 in the case of the Moon's apogee; by 96 in the case of the Moon's ascending node; by 45 in the case of Mars; by 420 in the case of Mercury's *śighrocca*; by 47 in the case of Jupiter; by 153 in the case of Venus's *Śīghrocca* and by 20 in the case of Saturn; and divide each product by 235. The resulting quotients, treated as minutes of arc, should be added to the mean longitude in the case of Mars, Saturn, and Mercury's *śīghrocca* and subtracted from the mean longitude in the case of the remaining planets.[1] (16-18). (KSS)

भटाब्दसंस्कारोपरि संस्कारः

13. 16. 3. आचार्योदितखेटेषु संस्कारः क्रियते बुधैः ।
'शकाब्दा'ख्यः स चान्यत्र 'वाग्भावे'त्यादिनोदितः ॥ १ ।

[1] The correction stated above is well known as the Śakābda correction. This correction is supposed to have been zero in the beginning of the *Śaka* year 444 (=A.D. 522), and thereafter to have increased uniformly at the rate given. See also *KR:KSS*, p. 13.

तत्रेन्दोः शाकजा लिप्ताः स्वपञ्चांशेन वर्जिताः ।
ग्राह्या, राहोर्द्वादशांशहीनास्तुङ्गस्य केवलाः ।
विशेषोऽयं दृष्टिसाम्यसिद्धये क्रियतेऽधुना ।। २ ।।

(Parameśvara, *Māhābhāskarīya-Bhāṣya-vyākhyā* on 5.77, edn. pp. 321-22)

A Correction to the Bhaṭābda correction

A correction by name *Śakabda-Śaṃskara*, otherwise called *Vāgbhāva-saṃskāra* is applied by astronomers to the (mean positions) of planets as computed according to the system of Ācārya (Āryabhaṭa). (1)

There, the minutes of the Śakābda correction of the Moon should be taken one-fifth less and that of Rāhu's correction, one-twelfth less. The correction for the Higher Apsis can be taken as it is.

This special correction is done for (making computed positions) accord with observed positions. (2). (KVS)

मनुयुगसंस्कारः

13. 17. 1. 'वस्वे-के-षु-युग-'घ्नं मनुयुगमर्कादिमध्यमचतुर्णाम् ।
धनमृणमृणमृणमथ 'कृति'गुणितं 'चक्रे-श-भै'र्लब्धम् ।। २०
भौमाङ्गिरश्शनीनां देयमृणं देय'मब्धि-नन्द'हृते ।
सितबुधयोर्हेयं देयं सप्तहृतं बुधस्योक्तम् ।। २१ ।।

(Deva, *KR*, 1. 20-21)

Manuyuga correction

Multiply the number of *yugas* elapsed since the beginning of the (current) *Manu* by 8, 1, 5 and 4, respectively, and (treating the products as minutes) apply them additively, subtractively, subtractively, and subtractively to the mean longitudes of the four planets beginning with the Sun, (i.e. the Sun, the Moon, the Moon's apogee and the Moon's ascending node) in their respective order. Again, multiply the same (number of *yugas*) by 20 and divide (severally) by 12, 11, and 27 respectively and (treating the quotients as minutes) apply them additively, subtractively and additively to the mean longitudes of Mars, Jupiter and Saturn, respectively. Divide (the same product of the number of *yugas* elapsed and 20) by 4 and 9, respectively, and (treating the quotients as minutes) apply them subtractively and additively to the mean longitudes of the *śighroccas* of Venus and Mercury, respectively. In the case of the *śighrocca* of Mercury, multiplication by 7 is also prescribed.[1] (20-21). (KSS)

[1] This Manuyuga correction is found to be discussed also by the Kerala astronomer Śaṅkaranārāyaṇa (A.D. 869) in his commentary on the *Laghu-Bhāskarīya* (2.22) of Bhāskara I. Śaṅkaranārayaṇa tells us that some people in his time ascribed this correction to Āryabhaṭa I. See also *KR*: *KSS*, p. 16.

कल्पसंस्कारः १

13. 18. 1. कल्पारम्भगतान् चतुर्युगगणान्
गोपादकेनाधिकान्
'लङ्का-क्षेत्र-दशास्य-ताम्रभुज-सद्-
धाम-क्लम'घ्नान् हरेत् ।
भागाद्याप्तफलं क्रमाद्'वितनुजै'-
श्चन्द्राद् बुधारार्कियुक्
त्वन्योनं तु भटोपदेशजमिदं
कात् पञ्चसिद्धान्तिनाम् ।। १९ ।।

(Deva, *KR*, 1.19)

Kalpa correction—I

Multiply the number of *caturyugas* elapsed since the beginning of the (current) *Kalpa*, as increased by 3/4,. severally by 13, 26, 158, 26, 84, 7, 59, and 53 and divide (each product) by 8064: the quotients obtained, in degrees etc., are the (*Kalpa*) corrections for the Moon etc. in the respective order. The corrections for Mercury's Śīghrocca, Mars and Saturn are additive and those for the other planets are subtractive. This is based on the instruction of (Ārya)bhaṭa and is applicable to all the five *siddhāntas* beginning with the *Brahma-siddhānta*.[1] (19) (KSS)

कल्पसंस्कारः—२

13. 18. 2. 'खाभ्रखार्कै'र्हृताः कल्पयाताः
समाः शेषकं भागहारात् पृथक् पातयेत् ।
यत्तयोरल्पकं तद्द्विशत्याभजे-
ल्लिप्तिकाद्यं फलं तत् 'त्रिभिः' 'सायकैः' ।। ७ ।।
'पञ्चभिः' 'पञ्चभूभिः' 'कराभ्यां'
भानुचन्द्रेज्यशुक्रेन्दुतुङ्गेष्वृणम् ।
'इन्दु'ना 'दस्रबाणैः' 'करा'भ्यां
'कृतै'र्भौमसौम्येन्दुपातार्किषु स्वं क्रमात् ।। ८ ।।

(Bhāskara II, *SiŚi*, 1. 1. 7.7-8)

Kalpa correction—II

The number of years from the beginning of the *Kalpa* divided by 12,000, the remainder, or the difference of the divisor and the remainder, whichever is less, is to be divided by 200. The quotient in minutes of arc, multiplied by 3, 5, 5, 15, 2, respectively, is a negative correction in the positions of the Sun, Moon, Jupiter, Venus and the lunar apogee and multiplied by 1, 52, 2 and 4 gives the positive correction in the positions of

[1] The *Kalpa* correction has not been found to occur in any other work so far. However, it has been referred to by the Kerala astronomer Parameśvara (A.D. 1431). In his commentary on the *Laghu-Bhāskarīya* (1.37) of Bhāskara I, he writes: "There are five corrections which are to be performed on the basis of time elapsed (since some particular epoch). They are: Two *Bhaṭābda* corrections, *Nibandhokta* correction, *Kalpa* correction and *Manuyuga* correction. One should perform that one which makes calculations tally with observation." See also *KR*: *KSS*, p. 15.

Mars, Mercury, the lunar Node and Saturn, respectively.[1] (7-8). (AS)

ग्रहस्फुटानां संस्कारः

13. 19. 1. चरार्धदेशान्तरदोर्विवराणां विनाडिकाः ।। १४ ।।
योज्याः साम्येऽन्यथा भेदे स्वमृणं वा यथाधिकम् ।
तद्गुणाः स्फुटभुक्त्यंशा विकलास्ताः स्फुटग्रहे ।। १५ ।।
धनभूते गुणे योज्यास्त्याज्यास्तस्मिन् क्षये सति ।
(*VK*, 1. 14b-16a)

Corrections for True planets

Add up algebraically: (i) half the *vināḍīs* of difference of day-time from 30 *nāḍis*, these *vināḍīs* being positive if the day-time is less (*Cara*), (ii) the *vināḍīs* of difference in sunrise due to difference in longitude, (*Deśāntara*), and (iii) the *vināḍīs* of the part of the Equation of Time due to the Sun's Equation of the Centre (*Bhujāntara*).[2] Multiply the True Daily motion of the Sun etc. in degrees by the *vināḍīs* got, and take them as seconds etc. of arc. Add or subtract these seconds in respect of the True Sun etc. according as the *vināḍīs* got are positive or negative. (14b-16a)

चरसंस्कारः

—चरम्—पौलिशः

13. 20. 1. 'विंशतिरष्टिस्सार्धा पादोनास्सप्त' चाजपूर्वाणाम् ।
विषुवच्छायागुणिताः क्रमोत्क्रमाच्चरविनाड्योऽर्धे ।।१०।।

अहर्मानम्

मेषादिषु तदुपचितैः कर्कटकाद्येषु तदपचयमितैः ।
दिनवृद्धिस्साध्येत क्षयस्तुलाद्येषु कर्कटकात् ।। ११ ।।
सागरहिमगिरिपरिधौ स्पष्टमिदं चरविनाडिकाकर्म ।
अन्यत्रापि यथैतत् स्पष्टं तच्छेद्यके वक्ष्ये ।। १२ ।।
(Varāha, *PS*, 3. 10-12)

Oblique ascension (Pauliśa)

Multiply the constants, 20, $16\frac{1}{2}$ and $6\frac{3}{4}$ by the equinoctial shadow. The results are oblique ascensional differences (*cara-khaṇḍas*) in *vināḍīs*, first in the given order, then in the reverse order for the first six months (solar) and again the given and reverse orders for the second half of the ecliptic, i.e. the second six months, (i.e., the differences are for the solar months Meṣa etc. in *vināḍīs* (20, $16\frac{1}{2}$, $6\frac{3}{4}$, $6\frac{3}{4}$, $16\frac{1}{2}$, 20, 20, $16\frac{1}{2}$, $6\frac{3}{4}$, $6\frac{3}{4}$, $16\frac{1}{2}$, 20) equinoctial shadow.) (10)

[1] For a discussion, see *SiŚi.*:*AS*, pp. 92-99.

[2] This should be applied to all celestial bodies, viz. the Sun, Moon, Rāhu and the planets.

Strictly speaking, the whole of the Equation of Time, including the part due to the Reduction to the Equator, should be used here. But Hindu astronomers before Śrīpati's time were not aware of this, and Bhāskara I has not given this. So this does not find a place in the present work also.

Day-time (Pauliśa)

To find the day-time in Meṣa, Vṛsabha and Mithuna, add the *cara* differences one by one, in the order given, to 30 *nāḍikās* and in the next three, subtract in the reverse order. In the next three *rāśis*, Tulā, etc., again subtract from 30 nāḍīs in the given order, and for Makara etc., add in the reverse order. This will give the day-time fairly accurately for places in Northern (whole) India. I shall give the method to find the day-time accurately in other places (in the fourth chapter) when dealing with spherical astronomy.[1] (11-12). (TSK)

—करणरत्नम्

13. 20. 2. 'कृत'गुणविषुवच्छाया-
व्यङ्गुलयो 'रवि-शरेन्दु-रसशिखि'भिः ।
लब्धाश्चरजविनाड्य
स्त्रिषु राशिष्वर्कशशिभुजयोः ।। ३४ ।।
(Deva, *KR*, 1. 34)

—Karaṇaratna

Multiply the *vyaṅgulas* of the equinoctial midday shadow by 4 and divide the product severally by 12, 15 and 36: the quotients obtained are twice the ascensional differences, in *vināḍīs*, of the (first) three signs of the ecliptic (lit. relating to the longitude of the Sun and the Moon).[2] (3a). (KSS)

—भास्करः १

क्षितिज्या चरज्या च

13. 20. 3. तद्वर्गव्यासकृत्योर्यद्विश्लेषस्य पदं भवेत् ।
स्वाहोरात्रार्धविष्कम्भः पलज्येष्टापमाहता ।। १७ ।।
क्षितिज्या लम्बकेनाप्ता व्यासार्धेनाहता हृता ।
स्वाहोरात्रेण यल्लब्धं चरजीवार्धमिष्यते ।। १८ ।।

चरसंस्कारः

तच्चापलिप्तिकाः प्राणाः स्फुटभक्त्या समाहताः ।
'खखषड्घन'भागेन लभ्यन्ते लिप्तिकादयः ।। १९ ।।
उदग्गोलोदये शोध्या देया याम्ये विवस्वति ।
व्यत्ययोऽस्तस्थिते कार्यं न मध्याह्नार्धरात्रयोः ।। २० ।।
उदग्गोले द्विरभ्यस्तैश्चीयतेऽह्नश्चरासुभिः ।
निशाऽपचीयते तत्र गोले याम्ये विपर्ययः ।। २१ ।।
चरप्राणै रवेर्हत्वा स्फुटभुक्तिं निशाकृतः ।
अहोरात्रासुभिश्छित्वा यत् फलं लिप्तिकादि तत् ।।२३।।

[1] *Cara* is the difference of day light from 30 *nāḍikās* being the average length of day and night. *Carārdha* is half of this, and the Sun rises later or earlier by this amount according as the day light is less or greater. This is given by the modern equation, $\sin h = \tan \phi . \tan \delta$, where ϕ is the latitude of the place, δ, is the declination of the Sun, and h is *carārdha* converted into degrees at 1° per 10 *vinadis*.

See also below 15. 23. 1-9.

[2] For the rationale, see *KR*: *KSS*, p. 24.

धनक्षयौ स्फुटे चन्द्रे भास्करस्य वशात् सदा ।
आदित्यकर्मणा तुल्यं शेषमिन्दोर्विधीयते ॥ २४ ॥

Bhāskara I, *LBh.* 2. (17-21; 23-24)

—Bhāskara I

Earthsine and the Ascensional difference

Whatever be the square root of the difference between the squares of that (i.e., of the R sine of the Sun's declination) and of the radius is the (Sun's) day-radius. The R sine of the latitude multiplied by the R sine of the (Sun's) declination and divided by the R sine of the co-latitude is (known as) the (Sun's) Earthsine. This is multiplied by the radius and divided by the (Sun's) day-radius: whatever is obtained is called the R sine of the (Sun's) ascensional difference. (17-18)

Correction for Cara

The minutes of arc in the arc of that (Sun's ascensional difference) are known as *prāṇa* (or *asu*). On multiplying them by the (Sun's) true daily motion and dividing by 21600 are obtained the minutes, etc., (of the Sun's motion corresponding to its ascensional difference). (In order to obtain the Sun's true longitude) at sunrise (for the local place), these (minutes, etc.) should be subtracted (from the Sun's true longitude at sunrise for the local equatorial place) provided the Sun is in the northern hemisphere (i.e., to the north of the equator) and added if the Sun is in the southern (hemisphere). In the case of sunset, (the law of correction is) the reverse. In the case of midday or midnight, this (correction) should not be performed. (19-20)

Lengths of day and night

(When the Sun is) in the northern hemisphere, the day increases and the night decreases by twice the *asus* of the (Sun's) ascensional difference. (When the Sun is) in the southern hemisphere, the contrary is the case. (21).

Other corrections for the Moon

The result in minutes of arc, etc., which is obtained on multiplying the true daily motion of the Moon by the *asus* of the Sun's ascensional difference and dividing (that product) by the number of *asus* in a day and night (i.e., by 21600) should always be added to or subtracted from the true longitude of the Moon (for true sunrise at the local equatorial place) according to (the position of) the Sun. The remaining (*bhujāphala*) correction for the Moon is applied (to the Moon's longitude corrected for the longitude and *bhujāntara* correction) in the same manner as in the case of the Sun. (23-24). (KSS)

—मेषवृषमिथुनानां चरः

13. 20. 4. राश्यन्तापक्रमैः कार्याः पूर्ववत्तच्चरासवः ।
पूर्वशुद्धः क्रमात्ते स्युर्मेषगोवल्लकीभृताम् ॥ ४ ॥
'शून्याद्रिरसरूपणि' 'भूतरन्ध्रमुनीन्दवः' ।
'पञ्चाग्निरन्ध्रशशिनो' मेषादीनां निरक्षजाः ॥ ५ ॥
चरप्राणाः क्रमाच्छोध्या दीयन्ते व्युत्क्रमेण ते ।
स्वदेशभोदया मेषाद् व्यत्ययेन तुलादितः ॥ ६ ॥

(Bhāskara I, *LBh.*, 2. 4-6)

Asc. Difference of Aries, Taurus and Gemini

From the declinations of the last points of the (first three) Signs should be obtained, as before, their ascensional differences in terms of *asus*. When (each of them is) diminished by the preceding (ascensional difference, if any), they become (the *asus* of ascensional difference) for Aries, Taurus, and Gemini, respectively. (4)

1670, 1795 and 1935 are (in *asus*) the times of rising of (the first three tropical Signs) Aries etc., at Laṅkā. (5)

Risings at local place

(From the above times of rising of Aries, Taurus, and Gemini at Laṅkā should be subtracted the *asus* of their (own) ascensional differences, in order, and (then) (to the same times of rising of Aries, Taurus, and Gemini at Laṅkā) they should be added in the reverse order: the results (in order) are the times (in *asus*) of rising at the local place of the tropical signs beginning with Aries, and (the same results) in the reverse order (are for those) beginning with Libra. (6). (KSS)

—वाक्यकरणम्

13. 20. 5. 'नर'-'स्तोयं'-'विगांशं सत्'-पलभागुणिता गुणाः ।
चरे भुजस्य राशीनां, जूकमेषाद् धनक्षयौ ॥ ७ ॥
'नीलं' चरविनाडीभिर्व्यत्यस्तं संस्कृतं दिनम् ।
रात्रिर्दिनोन'नीतिः' स्यात् तयोरर्धे तयोर्दले ॥ ८ ॥
'दासश्री'-'धींधरा'-'गोत्रगा' मेषादि विनाडिकाः ।
ऊना गुणार्धैः क्रमशो व्यस्ताव्यस्तैश्च संयुताः ॥ ९ ॥
स्वदेशराशयो मेषाद् व्यत्ययेन तुलादयः ।

(*VK.* 3.7-10a)

—Vākyakaraṇa

For each *rāśi* of the *bhujā* of the True Sun, there are the numbers 20, 16, and 6 2/3, respectively. These multiplied by the Equinoctial Shadow, give the respective *Cara* intervals in *vināḍis*. If the Sun is from 6^r to 12^r the *Cara* is positive, and if from 0^r to 6^r, it is negative. (7)

Day-light

The Cara-*vināḍīs* with its sign reversed and applied to 30 *nāḍikās* is the Day-time. The Day-time deducted from 60 *nāḍikās* is the Night-time. The middle of each is Mid-day and Mid-night, respectively. (8)

Rāśimāna: The Total Ascensional Difference

The Right Ascensional Differences for the three *rāśis* (*sāyana*), Meṣa etc., are 278, 299 and 323 *vināḍis*, respectively. Take half the *cara* in the given order and deduct

them for the first 3 *rāśis*, i.e. Meṣa, Ṛsabha and Mithuna; add taking them both in the reverse order to Kaṭaka, Siṁha and Kanyā; add taking them both in the given order to Tulā, Vṛścika and Dhanus; and deduct taking them both in the reverse order from Makara, Kumbha and Mīna. (9-10a). (TSK-KVS)

देशान्तरसंस्कार::

देशान्तरम्—करणरत्नम्

13. 21. 1. गणितप्रक्रियाप्राप्तप्रत्यक्षीकृतकालयोः ।
विश्लेषो ग्रहणे यः स्यात् कालो देशान्तरस्य सः ।। २८ ।।
(Deva, *KR*, 1-28

Longitudinal correction

The difference between the computed and observed times of an eclipse is the longitude (of the place) in terms of time.[1] (28). (KSS)

—भास्करः २

13. 21. 2. यत्र रेखापुरे स्वाक्षतुल्यः पल-
स्तन्निजस्थानमध्यस्थितैर्योजनैः ।
खेटभुक्तिर्हता स्पष्टभूवेष्टने-
नोद्धृता प्रागृणं स्वं तु पश्चाद् ग्रहे ।। ३ ।।

प्राग्भूविभागे गणितोत्थकाला-
दनन्तरं प्रग्रहणं विधोः स्यात् ।
आदौ हि पश्चाद्विवरे तयोर्या
भवन्ति देशान्तरनाडिकास्ताः ।। ४ ।।

तद्घ्नं स्फुटं षष्टिहृतं कुवृत्तं
भवन्ति देशान्तरयोजनानि ।
घटीगुणा षष्टिहृता द्युभुक्तिः
स्वर्णं ग्रहे चोक्तवदेव कार्यम् ।। ५ ।।

अर्कोदयादूर्ध्वमधश्च नाभिः
प्राच्यां प्रतीच्यां दिनपप्रवृत्तिः ।
ऊर्ध्वं तथाधश्चरनाडिकाभी
रवावुदग्दक्षिणगोलयाते ।। ६ ।।
(Bhāskara II, *SiŚi.*, 1.1.7.3-6)

—Bhāskara II

The distance between two places on the same latitude multiplied by the daily motion of a planet and divided by the rectified circumference is a correction subtractive for places in the east and additive in the west of the primary meridian, since the planetary positions obtained (by computation are for the primary meridian). (3)

The eclipse of the Moon occurs at a place situated on the east of the primary meridian later than on the primary meridian and vice versa. The time in between the two moments is the *Deśāntara* expressed in time. The distance of *Deśāntara*, i.e., the distance of the locality from the primary meridian measured along a parallel to the terrestrial equator or *nirakṣa-rekhā*, is obtained by multiplying the rectified circumference, by the *Deśāntara* measured as above in *ghaṭīs* and dividing by 60. Also, the above time in *ghaṭīs* multiplied by the planets' daily motion and divided by 60, gives the correction in arc in the computed mean planetary motion.[1] (4-6). (AS)

—आर्यभटार्धरात्रिकपक्षः

13. 21. 3. उज्जयिनीयाम्योत्तररेखायाः प्रागृणं धनं पश्चात् ।
देशान्तरभुक्तिवधात् 'खखाष्टवेदैः' कलाद्याप्तम् ।।१५।।
(Brahmagupta, *KK*, 1. 1. 15)

Multiply the mean daily motion of a planet (in minutes) by the difference in longitude between the observer's station and Ujjayinī expressed in *yojanas*. Divide the product by 4800. The result in minutes, etc., should be subtracted from the calculated longitude of the planet, if the station is to be the east of the meridian of Ujjayinī and added if it is to the west. (15) (BC)

—भास्करः १

13. 21. 4. समरेखास्वदेशाक्षविश्लेषान्तरसङ्गुणम् ।
वृत्तं स्वदेशजो भूमेर्बाहुश्चक्रांशकोद्धृतम् ।। २५ ।।
कर्णः स्वदेशतस्तिर्यक् समरेखावधेः स्थितः ।
तद्बाहुवर्गविश्लेषमूलं देशान्तरं स्मृतम् ।। २६ ।।
इत्याहुः केचिदाचार्या नैवमित्यपरे जगुः ।
स्थूलत्वात् कर्णसङ्ख्याया वक्रत्वात् परिधेर्भुवः ।। २७ ।।
मध्यच्छायादिनार्धोत्थतिग्मरश्म्योर्यदन्तरम् ।
न तत्पलस्य तुल्यत्वात् समपूर्वापराशयोः ।। २८ ।।
(Bhāskara I, *LBh.*, 1.25-28)

—Bhāskara I

Distance of the local place from the prime meridian

The circumference of the Earth multiplied by the difference between the latitudes of (a place on) the prime meridian and the local place and divided by the number of degrees in a circle (i.e., by 360) gives the *bāhu* (i.e., the base of the longitude triangle) due to the local place. The oblique distance from that local place to (the place on) the prime meridian is the hypotenuse (of the triangle). The square root of the difference between the squares of that (hypotenuse) and the *bāhu* is said to be the longitude (in *yojanas* of that place). (25-26)

Some learned scholars say like as above; others say that it is not so, because of (i) the grossness of the hypotenuse, and (ii) the sphericity of the Earth. (27)

[1] *Deśāntara* is the well-known difference in sunrise due to the difference in longitude, sunrise being earlier as we go East. The computed time is the local time for the place lying at the intersection of the prime meridian and the local circle of latitude, while the observed time is the local time for the local place. The difference between the two is obviously the longitude in time for the local place. It may be mentioned that in Hindu astronomy time is measured from sunrise.

[1] For explanation, see *SiŚi*: *AS*, pp. 89-92.

(It has been said that) the difference between (the longitude of) the Sun derived from the midday shadow (of the gnomon at the local place) and that calculated for the middle of the day (without the application of the longitude correction) (gives the longitude correction for the Sun). But that is not so, as to the east and west of a place on the prime meridian (i.e., on the same parallel of latitude) the latitude (and therefore the shadow of the gnomon) remains the same.[1] (28). (KSS)

—लल्लः

13. 21. 5. क्रमेण लङ्कोज्जयिनीहिमाचल-
प्रबद्धरेखाविषयेषु मध्यमाः ।
भवन्ति पूर्वापरपत्तनेष्वमी
ततश्च देशान्तरकर्मसंस्कृताः ।। ४२ ।।
'खखामरा' योजनवेष्टनं भुवो
'नभः शराभ्रक्षितयो'ऽस्य विस्तृतिः ।
दिवाकरघ्नं पलकर्णभाजितं
स्फुटं महीगोलकवेष्टनं भवेत् ।। ४३ ।।
कुमध्यरेखाविषयं स्वपत्तना-
दवस्थितं तिर्यगवेत्य योजनम् ।
स्वकीयतत्रत्यपलांशकान्तरं
'खखामर'घ्नं विभजेत् 'खषड्गुणैः' ।। ४४ ।।
पलकृतिरथ यावद्वर्गतो योजनानां
पदमृजु निजधामक्ष्मार्धयोरन्तरं स्यात् ।
ग्रहगतिहतमेतत् स्पष्टभूवृत्तभक्तं
धनमृणमपरैन्द्र्योर्लिप्तिकादि क्ष्मार्धात् ।। ४५ ।।
(Lalla, *ŚiDhVṛ.*, 1. 42-45)

—Lalla

The mean longitude (of the planets calculated above) are for places on the meridian line passing through Laṅkā, Ujjayinī and Himālaya. When, to these longitudes are applied corrections for difference in terrestrial longitudes, the results are longitudes for places east or west (of the meridian line). (42)

The (mean) circumference of the Earth is 3300 *yojanas*. Its diameter is 1050 *yojanas*. The (mean) circumference multiplied by 12 and divided by the hypotenuse of the equinoctial shadow (*palakarṇa*) of a place, gives the corrected circumference of the earth (at that place). (43)

Ascertain the shortest distance in *yojanas* between a place on the meridian line (passing through Laṅkā) and the observer's station. Multiply the degrees in the difference of latitudes of these two places by 3300 and divide by 360.

Square the result. Subtract it from the square of the *yojanas* (found above). Find the square root. This is the distance (in *yojanas*) between the meridian lines passing through the two places.

Multiply it by the mean daily motion of a planet and divide by the corrected circumference of the earth. The result in minutes should be added (to the mean longitude of a planet) for places to the west of the meridian line of Laṅkā and subtracted for places to the east.[1] (44-45) (BC)

—करणरत्नम्

13. 21. 6. देशान्तरघटीक्षुण्णा मध्या भुक्तिः खचारिणाम् ।
षष्ट्या भक्तं ऋणं प्राच्यां रेखायाः पश्चिमे धनम् ।।२७।।
(Deva, *KR*, 1.27)

—Karaṇaratna

The mean daily motion of a planet multiplied by the longitude (of the place) in terms of *ghaṭīs* and divided by 60 should be subtracted (from the longitude of the planet (if the place is) to the east of the prime meridian and added (to the longitude of the planet) if the place is to the west of the prime meridian. (27). (KSS)

—पौलिशसिद्धान्तः

13. 21. 7. यवनान्तजनाड्यः सप्तावन्त्यां त्रिभागसंयुक्ताः ।
वाराणस्यां त्रिकृतिः साधनमन्यत्र वक्ष्यामि ।। १३ ।।
त्रिकृतिघ्नात् 'खवसु'हृताद्
योजनपिण्डात् स्वताडिताज्जह्यात् ।
अक्षद्वयविवरकृति
मूल्याः षट्कोद्धृता नाड्यः ।। १४ ।।
(Varāha, *PS*. 3.13-14)

—Pauliśasiddhānta

The correction to the time of the longitude of Yavanapura to get the time of the longitude of Ujjainī is 7 *nāḍikās*, 20 *vināḍikās*, and that of Vāranāsī is 9 *nāḍikās*. How to find the correction for other longitudes will be given (in the next verse).[2] (13)

[1] This rule has also been criticised by Śrīpati, who says:

"Whatever is obtained here as the difference between the longitudes of the Sun derived from the midday shadow (of the gnomon) and that obtained by calculation (for midday, without the application of the longitude correction), when multiplied by the (local) circumference of the Earth and divided by the (Sun's daily) motion gives the *yojanas* of the longitude (i.e., the distance in *yojanas* of the local place from the prime meridian). This is gross on account of the small change in the Sun's declination."

[1] For the rationale, see *ŚiDhVṛ*: *BC*, II., 28-33.

[2] "What is given here is the difference in time due to difference in longitude alone from Yavanapura of Ujjainī and Vāranāsī, *Deśāntara-nāḍīs*, being the difference in time of the occurrence of any event due to longitude, the occurrence being earlier by this time if the place is East, and later if West. Actually the time difference due to longitude for Ujjainī from Yavanapura is *nāḍīs* 7-38, and for Vāranāsī, *nāḍīs* 8-50. The Greenwich East longitude of Yavanapura, Ujjainī and Vāranāsī are 30°, 75° 50′ and 83°. From this we can find the actual time difference: (75° 50′—30°)/6 *nāḍīs*=7-38.; (83°—30°)/6 *nāḍīs*=8-50). But considering the difficulty faced by the ancients, in doing this, for want of facilities, the achievement of the Siddhānta is commendable."

Take the distance in *yojanas* between the two places between which the time difference for longitude has to be found. Multiply this by 9 and divide by 80. (The result is their distance in degrees.) Square the result. From this deduct the squre of the difference in latitude between the two places. Find the square root of the remainder. (This is the East-West difference in degrees). This divided by 6 is the time difference in *nāḍikās*. (14). (TKS)

भुजान्तरसंस्कारः

भुजाफलः—पौलिशः

13. 22. 1. पदमेकोनं 'पञ्चाष्टक'घ्नमेकर्तुपक्षविषयेभ्यः ।
प्रोज्झ्य पदघ्नं छिन्द्या'न्नवयममुनि'भिः कला इन्दोः ॥
(Varāha, *PS*, 3.5)

Equation of time due to the unequal motion of the Sun on the ecliptic

Equation of the Centre (Pauliśa)

Reduce the plus or minus *padas* by one and multiply by 40. Subtract this from 5261. Multiply the result by the *padas* and divide by 729. The resulting minutes are the equation of the centre of the Moon. (5).[1] (TSK)

—भास्करः १

रवेः भुजाफलम्

13. 22. 2a. स्वपरिध्याहृतेऽशीत्या लब्धं क्षयधनं फलम् ॥ ४ ॥
केन्द्रात्पदविभागेन क्षयधनधनक्षयाः ।
देशान्तरकृते सूर्ये कुर्यात्तन्मध्यमे सदा ॥ ५ ॥
केन्द्रे क्रियादिके चाथ फलं बाहोर्विशोधयेत् ।
तुलादिके तु तन्नित्यं देयं स्फुटदिदृक्षुणा ॥ ६ ॥
क्रमोत्क्रमफलाभ्यस्ता मध्या बाहुफलेन वा ।
भुक्तिश्चक्रकलालब्धं पूर्ववत्तत्प्रकल्पयेत् ॥ ७ ॥
(Bhāskara I, *MBh.*, 4.4b-7)

—Bhāskara I

Sun's Equation of the centre

The R sines and R versed-sines (of the parts of the Sun's mean anomaly lying in the odd and even quadrants respectively) should be (severally) multiplied by the (Sun's) own epicycle and divided by 80: the resulting quantities should be subtracted and added (in the manner prescribed below). (4b)

The resulting quantities due to the first, second, third and fourth anomalistic quadrants should always be respectively subtracted from, added to, added to, and subtracted from the Sun's mean longitude corrected for the (local) longitude. (5)

Alternative rule

Or, (find the *bāhuphala* and) subtract the *bāhuphala* when the (Sun's mean) anomaly is in the half-orbit beginning with Aries; and add that when (the Sun's mean anomaly is) in the half-orbit beginning with Libra. This correction should always be performed by one who seeks the true longitude (of the Sun). (6)

Correction for the Sun's equation of time

Multiply the mean daily motion (of the Sun) by the (Sun's) equation (of the centre derived from the R sines and R versed-sines of the parts of the Sun's mean anomaly lying in the odd and even quadrants respectively), or by the (Sun's) *bāhuphala* (i.e., the Sun's equation of the centre derived from the *bāhu*) and then divide the product by the number of minutes in a circle (i.e., by 21600); apply that (as correction, positive or negative, to the Sun's mean longitude corrected for the local longitude and for the Sun's equation of the centre) as before.[1] (7). (KSS)

—चन्द्रे विशेषः

13. 22. 2b. सूर्यबाहुहता भुक्तिर्मध्या चक्रकलाहृता ।
भास्करस्य वशात् क्षेपः शुद्धिर्वापि निशाकृतः ॥ २९ ॥

शेषं विवस्वता तुल्यं कर्म चन्द्रस्य कीर्तितम् ।
भास्वद्भुजाफलेनैव शेषाणां तु प्रकल्पयेत् ॥ ३० ॥
(Bhāskara I, *MBh.*, 4.29-30)

—Correction for the Moon

Multiply the (Moon's) mean daily motion by the Sun's equation of the centre and then divide (the product) by the number of minutes in a circle (i.e., by 21,600): (the result is the *bhujāntara* correction for the Moon). Add it to or subtract it from the Moon's (mean) longitude (corrected for the longitude of the local place) in the same way as in the case of the Sun.

All remaining corrections for the Moon are prescribed as in the case of the Sun.

(The *bhujāntara* correction) for the remaining planets also is calculated from the Sun's equation of the centre. (29-30). (KSS)

[1] *Bhujāntara* is the part of the Equation of time due to the unequal motion of the Sun on the ecliptic, and therefore is the time corresponding to the difference in longitude between the mean and the true Sun. Sunrise will be later if the true Sun if greater. This is given in *vināḍis*.

In the Pauliśa rule enunciated here, the formula is: The equation of the centre$=\{(5261-40\ (pada-1)\}\ pada/729$, where *pada* is any *pada*, plus or minus, without distinction.) (TSK)

[1] For explanation and rationale, see *MBh*: *KSS*, pp. 110-15.

—करणरत्नम्

13. 22. 3. स्वोच्चोनसूर्यशशिनो बाहुज्या 'खमनु'भिश्च षष्ट्या च ।
लब्धा भागाः शोध्या हृद्युत्तरतो दक्षिणे देयाः ।। २५ ।।

भवतः स्फुटौ रवीन्दू भगणाप्तं रविभुजाफलं शशिनि ।
रविवत् . . . . ।। २६ ।।

(Deva, *KR*, 1.25-26)

—Karaṇaratna

Equation of the centre for the Sun and the Moon

Diminish the mean longitude of the Sun or the Moon by that of its own apogee, and obtain the R sine of the *bāhu* thereof. In the case of the Sun, divide that by 140; and in the case of the Moon, divide that by 60. The resulting degrees should be subtracted from the (respective) mean longitude (of the Sun or the Moon) or added to that, according as the Sun or the Moon is in the northern part (of the anomalistic sphere) or in the southern part. Thus are obtained the true longitudes of the Sun and the Moon. (25-26a)

Bhujāntara correction for the Moon

The Sun's equation of the centre (*bhujāphala*) divided by 27 (gives the Moon's *bhujāvivara* correction). This should be applied to the Moon's longitude in the same way as the Sun's equation of the centre is applied (to the Sun's longitude). (26b). (KSS)

—लल्लः

13. 22. 4. गतिं ग्रहस्यार्कफलोत्थलिप्तिका-
हतां हरेत् 'खाम्बरभूपलोचनैः' ।
फलं कलाद्यत्र भवेत्तदर्कवद्
विधेयमर्कादिखगेष्वृणं धनम् ।। १६ ।।

(Lalla, *ŚiDhVr.*, 2.16)

—Lalla

The mean daily motion of a planet multiplied by the Sun's equation of the centre in minutes (*mandaphala*) and divided by 21,600 gives, in minutes, etc., the correction for the difference between the Sun's mean and true longitudes or *bhujāntaraphala*. It should be added to or subtracted from the longitude of the planet, according as the equation of the centre is added to or subtracted from the Sun's longitude. (16)

उदयान्तरसंस्कारः

13. 23. 1. युक्तायनांशस्य तु मध्यमस्य
भुक्तासवोऽर्कस्य निरक्षदेशे ।
मेषादिभुक्तोदयसंयुता ये
यश्चायनांशान्वितमध्यभानोः ।। ६२ ।।

लिप्तागणस्तद्विवरेण निघ्नी
गतिर्ग्रहस्य द्युनिशासु भक्ता ।
स्वर्णं ग्रहे चेदसवोऽधिकोनाः
इदं ग्रहाणामुदयान्तराख्यम् ।। ६३ ।।

(Bhāskara II, *SiŚi.*, 1.2. 62-63)

Correction owing to the Equation of Time due to Obliquity

The difference in the minutes of arc in the longitude of the *sāyana* mean Sun and the *asus* in its right ascension, multiplied by the daily motion of the planet and divided by 21,659 is the result to be added or subtracted from the planet's longitude according as the *asus* of the Sun's right ascension are greater or less than the minutes of arc of the Sun's longitude. This is what is called *Udayāntara* correction or correction owing to the Equation of Time due to Obliquity.[1] (62-63) (AS)

ग्रहगतयः

13. 24. 1. वक्रानुवक्रा विकला मन्दा मन्दतरा समा ।
तथा शीघ्रतरा शीघ्रा ग्रहाणामष्टधा गतिः ।। १२ ।।

तत्रातिशीघ्रा शीघ्राख्या मन्दा मन्दतरा समा ।
ऋज्वीति पञ्चधा ज्ञेया याऽन्या वक्रादिका मता ।। १३ ।।

(*SūSi.*, 2.12-13)

Types of planetary motion

The motion of the planets is of eight kinds: retrograde (*vakrā*), somewhat retrograde (*anuvakrā*), transverse (*kuṭilā*), slow (*mandā*), very slow (*mandatarā*), even (*samā*), also vey swift (*śīghratarā*), and swift (*śīghrā*). (12)

Of these, the very swift (*atiśīghrā*), that called swift, the slow, the very slow, the even—all these five are forms of the motion called direct (*ṛju*); the 'somewhat retrograde' is retrograde. (13). (Burgess)

—वटेश्वरसिद्धान्तः

13. 24. 2. स्फुटमध्यखेचरान्तरं दलितं मध्यखगात् स्फुटेऽल्पके ।
स्वमृणं महति स्फुटोनिते स्वचलेऽस्मिन् भवनेषु खेचरः ।।

ग्रहाणामष्टधा गतिः

अतिशीघ्रगतिः शीघ्रो निसर्गकस्तदनु भावयोराद्ये ।
मन्दोऽपरेऽतिमन्दो वक्री भे तदनु भेऽतिवक्रगः ।। ७ ।।

चक्रच्युतेऽपि चास्मिन् ग्रहचारश्चैष एव निर्दिष्टः ।
चक्रच्युतस्य मन्दा ग्रहस्य भुक्तिरकुटिलसंज्ञा ।। ८ ।।

वक्रानुवक्रगत्योरारम्भे ग्रहाणां शीघ्रकेन्द्रभगणाः

'रामाष्टिभिः' 'क्षितिभृतः' चलकेन्द्रभागैः
वक्रीन्दुजो'ऽक्षमनुभिः', गुरु'रङ्गसूर्यैः' ।
शुक्रः 'शरर्तुशशिभिः', शनि'रग्निरुद्रैः'
चक्रच्युतैरकुटिलाः कथितास्त्वमीभिः ।। ९ ।।

[1] In contrast with the *bhujāntara* correction to be effected in the planetary positions owing to the Equation of Time due to '*Eccentricity*', the present *Udayāntara* correction is to be effected for the Equation of Time due to '*Obliquity*'. This correction has been identified for the first time by Śrīpati, (vide *SiŚe.*, 11-1-2), and later followed by Bhāskara II. For a detailed exposition and rationale, see *SiŚi.*: *AS*, pp. 204-06.

गहाणां वक्रगति-ऋजुगतिदिनानि

'पञ्चर्तव:', 'कुदस्रा', 'बाहुशिखा', 'द्वीषवो', 'द्विगुणचन्द्रा:' ।
वक्रदिनान्युर्वीजान्निरंशदिनशोधितान्यृजूनि स्युः ।। १० ।।

गहाणां निरंशदिनानि

'खाष्टनगा', 'रसरुद्रा', 'नवनवरामा' 'पयोधिधीपवना:' ।
'वसुशैलगुणा' क्रमशो भौमादीनां निरंशनिशाः ।। ११ ।।

ग्रहाणां पूर्वोदये पश्चिमास्ते वा शीघ्रकेन्द्रांशाः

'धीयमलै',-'स्त्रिखपक्षै'-,
'विश्वै',-'स्त्रिमतीन्दुभि,'-'नंगशशाङ्कैः' ।
दृश्यैः प्रागपरायां
च्युतैर्भचक्रादिमेऽदृश्याः ।। १२ ।।

बुधशुक्रयोः पश्चिमास्ते शीघ्रकेन्द्रांशाः

विपरीतदिशोरेवं ज्ञसितौ तानै,र्जिनै,श्चलैर्भागैः ।
एष्यातीतकलाभ्यः स्वकेन्द्रभुक्त्या दिनानि स्युः ।। १३ ।।
(Vaṭeśvara, *VSi.*, 2. 4. 6-13)

—Vaṭeśvarasiddhānta

When the true longitude of the planet is less than the mean longitude of the planet, add one-half of the difference between the true and mean longitudes of the planet, to the longitude of the planet's *śīghrocca* as diminished by the true longitude of the planet; and when the true longitude of the planet is greater (than the mean longitude of the planet), subtract the same (one-half of the difference between the true and mean longitudes of the planet) from the longitude of the planet's *śīghrocca* as diminished by the true longitude of the planet). (The result is the planet's corrected *śīghrakendra*). (6)

Eight types of planetary motion

In the (successive) Signs of this corrected *śīghrakendra*, the planet is 'very fast' (in the first Sign); 'fast' (in the second Sign) and 'natural or mean' (in the third Sign); in the two halves of the next (i.e., fourth) Sign, it is 'slow' in the first half and 'very slow' in the other half; in the next (i.e., fifth) Sign it is 'retrograde'; and in the next (i.e., sixth) Sign, it is 'very retrograde'. (7)

In the (six) Signs obtained by subtracting the corrected *śīghrakendra* from a circle (i.e., 360°), the planet is said to have the same motion. But when the corrected *śīghrakendra* is subtracted from a circle, the '(very) slow' motion is designated as 're-retrograde or direct' motion. (8)

Śīghra anomalies for retrograde and direct motions

Mars becomes retrograde when its *śīghrakendra* is 163°; Mercury, when its *śīghrakendra* is 145°; Jupiter, when its *śīghrakendra* is 126°; Venus, when its *śīghrakendra* is 165°; and Saturn, when its *śīghrakendra* is 113°.

They become direct when their *śīghrakendras* become 360°—163°; 360°—145°; 360°—126°; 360°—165°; and 360°—113° (i.e., 197°, 215°, 234°, 195° and 247°), respectively. (9)

Periods of retrograde and direct motion

The civil days (of duration) of retrograde motion for the planets, beginning with Mars, are 65, 21, 112, 52 and 132, (respectively).

These, subtracted from the days of their synodic periods, are the days (of duration) of their direct motion. (10)

Synodic periods of planets

780, 116, 399, 584, and 378 are, in days, the synodic periods of the planets, Mars etc., in their respective order. (11)

Śīghra anomalies of planets

(The planets, Mars etc.) rise (heliacally) in the east when their *śīghrakendras* amount to 28, 203, 13, 183 and 17 degrees, respectively; they set heliacally in the west when their *śīghrakendras* amount to 360—28; 360—203; 360—13; 360—183; and 360—17 degrees (i.e., 332, 157, 347, 177 and 343 degrees) respectively. (12)

Śīghra anomalies of Mercury and Venus

In the same way, Mercury and Venus rise in the opposite direction (i.e., in the west) when their *śīghrakendras* are 49 and 24 degrees respectively.

The (number of) days to elapse or elapsed are obtained from the minutes to elapse or elapsed with the help of the motion of the *śīghrakendra* of the planet. (13). (KSS)

ग्रहयोजनगतिः

—आर्यभटः

13. 25. 1. षष्ट्या सूर्याब्दानां प्रपूरयन्ति ग्रहा भपरिणाहम् ।
दिव्येन नभःपरिधिं समं भ्रमन्तः स्वकक्ष्यासु ।। १२ ।।
मण्डलमल्पमधस्तात् कालेनाल्पेन पूरयति चन्द्रः ।
उपरिष्टात् सर्वेषां महच्च महता शनैश्चारी ।। १३ ।।
(Āryabhaṭa I, *ABh.*, 3.12-13)

Linear motion in yojanas: Āryabhaṭa I

The planets moving with equal linear velocity in their own orbits complete (a distance equal to) the circumference of the sphere of the asterisms in a period of 60 solar years, and (a distance equal to) the circumference of the sphere of sky in a *yuga*.[1] (12)

[1] That is, a planet moves through a distance of 17,32,60,008 *yojanas* in 60 solar years and a distance of 1,24,74,72,05,76,000 *yojanas* in 43,20,000 solar years. Since there are 1,57,79,17,500 days in 43,20,000 solar years, it follows that the mean daily motion of a planet, according to Āryabhaṭa, is

$$\frac{1{,}24{,}74{,}72{,}05{,}76{,}000}{1{,}57{,}79{,}17{,}500} \text{ yojanas}$$

or 7905.8 *yojanas* approx.

The Moon completes its lowest and smallest orbit in the shortest time; Saturn completes its highest and largest orbit in the longest time. (13). (KSS)

—भास्करः २

13. 25. 2. कल्पोद्भवैः क्षितिदिनैर्गगनस्य कक्षा
भक्ता भवेद्दिनगतिर्गगनेचरस्य ।
पादोन'गोऽक्षधृतिभू'मितयोजनानि
खेटा व्रजन्त्यनुदिनं निजवर्त्मनीमे ॥ ६ ॥

(Bhāskara II, *SiŚi*, 1. 1.4-6)

—Bhāskara II

The circumference of the universe divided by the number of days in the *kalpa*, gives the daily spatial motion of a planet. The planets move thus a distance of $11{,}858\frac{3}{4}$ *yojanas* in a day. (6). (AS)

—सिद्धान्तदर्पणम्

13. 25. 3. ग्रहयोजनभुक्तिः स्याद् दशघ्नेन्दोः कलागतिः ॥ १३b ॥

(Nīlakaṇṭha, *SiDar.*, 13b)

—Siddhāntadarpaṇa

The Moon's daily motion in minutes multiplied by ten will give the daily linear velocity of the planets (in *yojanas*) (which is the same for all planets including the Sun and the Moon, being 7906 *yojanas*). (13b). (KVS)

रविचारः

—लल्लः

13. 26. 1. तुलादिधन्वन्तमुपैति भास्कर-
स्त्रिभिस्तु मासैर्मिथुनान्तमप्यजात् ।
रवेर्जिनांशान्तसमा परोन्नति-
र्दितेः सुतानामदितेश्च वासरे ॥ ११ ॥

भवेत् खमध्यं क्षितिजाद् गृहत्रयं
फलं निरक्षे तिथयोऽस्य नाडिकाः ।
यतोऽत एषां परमेऽपमे रवेः
परोन्नतिः स्यात् स्वदिने क्षणद्वयम् ॥ १२ ॥

ततो हि सङ्क्रान्तिवशेन च त्रिभिः
क्रमेण मासैः क्षितिजं समाप्नुयात् ।
गतोऽथ यत्र प्रथमं च दृक्पथं
पुनश्च तद्वामभियात् प्रदर्शनम् ॥ १३ ॥

सामार्धमाद्यं विबुधा दिवाकरं
द्वितीयमीक्षन्ति दलं सुरद्विषः ।
दिनप्रमाणं मनुजाः स्ववासरे
सितं च पक्षं पितरः शशिस्थिताः ॥ १४ ॥

(Lalla, *ŚiDhVṛ.*, 18. 11-14)

Sun's motion

—Lalla

The Sun moves from the first point of Aries to the last point of Gemini in three months; and again from the first point of Libra to the last point of Sagittarius in three months.

The greatest altitude of the Sun is 24° from the horizon during the day of the gods or demons. (11)

The zenith is 90° from the horizon. This is equivalent to 15 *ghaṭikās* along the equator. So, when the Sun has its greatest declination of 24° and reaches its greatest altitude, it is seen by gods or demons for 2 *kṣaṇas* (or 4 *ghaṭikās*) of their day. (12)

After that, on account of its motion along the ecliptic, the Sun, after three months, again reaches the horizon gradually. (If the gods saw it) rising (when it was on the horizon), (the demons will now see it) rising, and vice versa. (13)

The gods see the Sun during the first half of the solar year; their enemies, the demons, see it during the second half. The manes, who live (in the upper half) of the Moon, see the Sun for one half of a lunar month and the men during their own day. (14). (BC)

चन्द्रचारः

13. 27. 1. सितेऽधिके पक्ष इहोच्यते सितो
यतोऽसितश्चाप्यसिते विधोस्ततः ।
तदीयसन्ध्यासमयोऽष्टमीद्वयं
निशाद्युमध्ये सितकृष्णपर्वणी ॥ १५ ॥

शशाङ्कबिम्बोपरि सूर्यमण्डलं
यतो भवेत् पर्वणि कृष्णपक्षजे ।
अतः स्वमूर्ध्नोर्ध्वगतं प्रपश्यतां
भवेत् पितृणां द्युदलं नृणां तथा ॥ १६ ॥

यथा यथा सूर्यतलं त्यजेच्छशी
तथा तथैषां नतिमेति भास्करः ।
गृहत्रयेणान्तरितोऽष्टमीदले
कुजस्थितो वा समुपैति दर्शनम् ॥ १७ ॥

शशाङ्कबिम्बान्तरितोऽर्धरात्रगो
गृहैश्च षड्भिः सितपक्षपर्वणि ।
इनान्तिकं गच्छति पक्षमध्यगे
विधौ व्रजेद् दर्शनमुन्नतिं ततः ॥ १८ ॥

(Lalla, *ŚiDhVṛ.*, 18. 15-18)

Moon's motion

In this world, the bright half of the lunar month is that during which the illuminated portion of the Moon increases and the dark half is that during which the dark portion increases. So, to the manes, the morning twilight is the eighth day of the Moon in the dark half of the lunar month and the evening twilight is the eighth day of the Moon in the light half of the lunar month. (15)

Since on the day of the new moon the Sun's disc is just above that of the Moon, to the manes the Sun appears to be on the zenith. This then is their midday as it is in the case of men (when the Sun is on their zenith). (16)

As the Moon recedes from its position below the Sun, to the manes, the Sun appears to decline. Then, on the middle of the eighth day when the Moon is at a distance of 90° from the Sun, it appears to be on their horizon or very near it. (17)

On the full moon day, the Sun is hidden by the Moon's disc which is at a distance of 6 Signs from it. It is then midnight for the manes. Again, on the eighth day, when the Moon approaches the Sun, the latter is visible (to the manes). Then, gradually it rises above their horizon. (18). (BC)

रविचन्द्रगतिः

—रविमध्यगतिः (पौलिशः)

13. 28. 1. 'गुण(?यम)-शिखि-गुणा-ऽग्नि-यम-शशि-
वियुता सैका-सरूप-रूपै-का'।
'खै-का'-वियुता च भानोः
षष्टिर्भुक्तिः क्रमादेवम् ॥ १७ ॥

(Varāha, *PS*, 3.17)

Daily motion of Sun and Moon

Mean Sun (Pauliśa)

The daily motion of the Sun in minutes during each of the twelve months, Meṣa etc. is 57 (58?), 57, 57, 57, 58, 59, 61, 61, 61, 61, 60, 59. (17). (TSK)

चन्द्रमध्यगतिः (रोमकः)

13. 28. 2. 'खनवनगाः' शशिभुक्तिः 'कृतवसुमुनयः' शशाङ्ककेन्द्रस्य।
यातस्फुटान्तरे दिवसभुक्तिरागामिकी नैशी ॥ ७ ॥

(Varāha, *PS*, 8.7)

Moon's mean daily motion (Romaka)

The daily motion of the mean Moon is 790′, and that of the mean anomaly, 784′. For work relating to the day-time, the true daily motion is the difference between the true Moon of the taken day and of the previous day. For work relating to the night time, the true daily motion is the difference between the true Moon of the taken day and of the next day.[1] (7) (TSK)

चन्द्रस्फुटगतिः (पौलिशः)

13. 28. 3. विनवात् पदाद्दशघ्नात् सप्तांशः 'साश्विखाम्बरो' भुक्तिः।
गत्यर्धान्ताच्छोध्यो लिप्ताभ्यो 'नवमुनिवसुभ्यः' ॥ ४ ॥

(Varāha, *PS*, 3.4)

[1] For the calculation, see *PS*: *TSK*, 8.7.

Moon's true motion (Pauliśa)

If the *padas* obtained by ch. 2, verse 2 are plus-*padas*, (i.e., in the first half-*gati*) subtract 9 from the *padas*, multiply by 10 and divide by 7. Add the result to 702. The Moon's daily true motion in minutes is obtained. In the second half-*gati*, i.e., if the *padas* are minus-*padas*, deduct 9, multiply by 10 and divide by 7 and subtract the result from 879. The resulting minutes are the daily true motion of the Moon. (4). (TSK)

रविचन्द्रस्फुटगतिः

रविशशिनोः मध्यभुक्तिः—सौरसिद्धान्तः

13. 28. 4. नवतिः सप्तशतीन्दोः सचतुस्त्रिंशद्विलिप्तिका भुक्तिः।
षष्टिर्व्येका विकलाष्टकं च मध्या सहस्रांशोः ॥ ११ ॥

चन्द्रकेन्द्रभुक्तिः

सप्तकला विन्यंशाश्चन्द्रोच्चस्येन्दुभुक्तिरनयोना।
केन्द्रस्य परिज्ञेया स्फुटभुक्तिश्चानया कार्या ॥ १२ ॥

रविचन्द्रस्फुटभुक्तिः

केन्द्रान्तरज्या गुणिता 'तिथि'वर्गेणोद्धृता च परिणाम्या।
तत्कार्मुकं क्षयचयौ भुक्तौ मृगकर्कटाद्येषु ॥ १३ ॥

तत्कालभुक्तिरेषाऽऽहोरात्रिकी शशिविशेषात्।
व्यासार्धहता भुक्तिः स्फुटभुक्तिहृता स्फुटः कर्णः ॥ १४ ॥

(Varāha, *PS*, 9.11-14)

Sun and Moon: True daily motion

Mean motion of the Sun and the Moon—Saura-siddhānta

The mean daily motion of the Moon is 790′ 34″, and that of the Sun is 59′ 8″. (11)

Motion of Moon's anomaly

The daily motion of the Moon's apogee is $6\frac{2}{3}$ minutes. The Moon's mean daily motion less the motion of the apogee is the daily motion of the Moon's (mean) anomaly. The true daily motion is to be found using this motion of anomaly. (12)

True motion of the Sun and the Moon

The daily motion of anomaly should be multiplied by the current sine-interval and divided by 225. This should be reduced to the epicycle, (i.e., multiplied by the degrees of epicycle and divided by 360°). The change in sine of the equation of centre is to be calculated. Its arc should be subtracted from the mean daily motion, if the anomaly falls within *rāśis* 9 to 3, and added if it falls within *rāśis* 3 to 9. (13)

This is the true motion per day, for the moment (for which the anomaly is taken). The true daily motion in the case of the Moon is obtained by subtracting the previous day's true Moon from the given day's true Moon. The daily mean motion, multiplied by 120′

and divided by the momentary motion per day is the radius vector at the moment.[1] (14) (TSK)

—लल्लः

13. 28. 5. स्वभोग्यखण्डं 'क्षितिखेन्दुभि'र्हृतं
रवेर्विधो'र्दिग्'गुणितं 'सुरो'द्धृतम् ।
तदूनयुक्ते भवतः स्फुटे गती
क्रमात् स्वकेन्द्रे मृगकर्कटादिके ।। १५ ।।
(Lalla, *ŚiDhVṛ.*, 2. 15)

—Lalla

Divide the *bhogyakhaṇḍa* of the Sun by 101 and multiply that of the Moon by 10 and divide by 33. (The results called *mandagatiphalas* or corrections to be given to the mean motions) should be applied to their respective mean motions negatively, if the respective mean anomalies are within six Signs beginning with Capricorn, and positively if they are within six Signs beginning with Cancer; (that is, negatively, positively, positively or negatively, according as their mean anomalies are in the first, second, third or fourth quadrant). Thus are obtained their true motions. (15). (BC)

—करणरत्नम्

13. 28. 6. भानो'र्नवशर'लिप्ता
मध्यगतिः 'शशिनवस्वराः' शशिनः ।
अथ वर्तमानजीवा
स्वत्र्यंशयुताः कलाश्चेन्दोः ।। ३१ ।।
कृतयामभक्ता भानोर्मध्यमपदयोः स्वमध्यगतौ ।
क्षिप्त्वा त्यक्त्वाऽऽद्यन्ते स्फुटभुक्तिर्भवति रविशशिनोः ।।
(Deva, *KR*, 1.31-32)

—Karaṇaratna

Mean motion

The mean daily motion of the Sun is 59 minutes: of the Moon, 791 minutes. (31a)

True motions of the Sun and the Moon

The current R sine-difference increased by one-third of itself, gives the minutes of the motion-correction for the Moon; the same (current R sine-difference) divided by 24, is for the Sun. This being added to the mean daily motion in the second and third (anomalistic) quadrants and subtracted from the mean daily motion in the first and fourth (anomalistic) quadrants, yields the true daily motion in the case of the Sun and the Moon.[2] (31b-32). (KSS)

[1] For the calculations, see *PS*: *TSK*, 9. 11-14.

[2] For the rationale, see *KR*: *KSS*, pp. 22-23.

रविस्फुटगतिः—पञ्चबोधः

13. 28. 7. 'नीते' च तत्तद्दिवसाष्टकोक्त-
'योग्या'दिवाक्या 'दिन'भक्तलिप्ताः ।
कुर्यात्, तदर्कस्फुटभुक्तिरेषा
स्याद् 'यज्ञ'-'रत्ना'दिवशाद् धनर्णम् ।। ३ ।।
(*Pañcabodha*, 3)

Sun's true motion—Pañcabodha

To 60 (being the average length of the day in seconds) apply the correction in seconds given by the chronograms *yogya* etc., (see table under 13.7.15, above), divided by 8, positively from *yajña*, (being the chronogram for Tulā 9th), and negatively from *ratna*, (being the chronogram for Mīna 1st). The result will be the true rate of motion for the current day. (3). (KVS)

चन्द्रफुटगतिः—पञ्चबोधः

13. 28. 8. अभीष्टपूर्वापरवाक्यभेद-
स्यार्धं गतिः स्यादुदये हिमांशोः ।
अभीष्टवाक्यान्निजपूर्ववाक्ये
त्यक्ते तु शिष्टा गतिरस्तकाले ।। ४ ।।
(*Pañcabodha*, 4)

Moon's true motion—Pañcabodha

The difference between the lunar chronogram of the current day (vide the chronograms *gīr naḥ śreyaḥ* etc., given in the table under 13.7.14, above) and that of the previous day will give the rate of true motion of the Moon at sunrise on the current day. The true motion at sunset of the current day is given by the difference between the chronogram of the current day and the next day. (4). (KVS)

ग्रहादीनां मध्यगतयः—लल्लः

13. 29. 1. दिवाकरादेर्गतिलिप्तिकाः क्रमा-
'न्नवेषवः' 'खाङ्कनगाः' 'क्षमाग्नयः' ।
'शिलीमुखाम्भोनिधिबाहवः' 'शरा'-
स्ततः 'षडङ्का' 'द्वितयं' 'रसा' 'गुणाः ।। ४० ।।
क्रमाद् विलिप्ता 'वसवः' 'शरानलाः'
'षडश्विनो''ऽयुग्मगुणा' 'नभो' 'गजाः' ।
'नभः' 'कुवेदाः' 'कुभुवो' दिवात्यये
गतेर्ग्रहाः स्युः स्वगतेर्दलोनिताः ।। ४१ ।।
(Lalla, *ŚiDhVṛ.*, 1. 40-41)

Mean daily motions of planets

The daily motions of the Sun (Moon, Mars, śīghrocca of Mercury, Jupiter, śīghrocca of Venus, Saturn and Moon's apogee and node) are, respectively, 59′ 8″, 790′ 35″, 31′ 26″, 245′ 32″, 5′, 96′ 8″, 2′ 0″, 6′ 41″ and 3′ 11″.

The mean longitudes of planets, (as calculated above), when diminished by half of their respective daily motions, give the longitudes at the previous sunset. (40-41). (BC)

—करणरत्नम्

13. 29. 2. 'शशिगुण'-'रसकृतलोचन'-
'विषया'-'ऽङ्गच्छिद्र'-'बाहवो' ऽलिप्ताः ।
भौमादिमध्यगतयः
सितबुधयोः सा तु शीघ्रगतिः ॥ ५ ॥

प्रथमाध्याये कथिता रविचद्रमसोः, क्रमेण भुक्तिः स्यात् ।
इन्दूच्चस्य च राहोर्भुक्ति'र्मुनयो'-'ऽग्नय'-श्चेति ॥ ६ ॥
(Deva, *KR*, 8. 5-6)

—Karaṇaratna

31′, 246′, 5′, 96′ and 2′, (respectively), are the mean daily motions of Mars etc. In the case of Mercury and Venus, the motions pertain to their *śīghrocca*. (5)

The mean daily motions of the Sun and the Moon are the same as stated in the first chapter. Of the Moon's apogee and the Moon's ascending node, the mean daily motions are 7′ and 3′ respectively. (6). (KSS)

—ग्रहलाघवम्

13. 29. 3. 'गोऽक्षा गजा' रविगतिः, शशिनो'ऽभ्रगोऽश्वाः ।
पञ्चाग्नयो'ऽथ, 'षडिलाब्धय' उच्चभुक्तिः ॥१४॥

राहो'स्त्रयं कुशशिनो',ऽसृज 'इन्दुरामा-
स्तर्काश्विनो', ज्ञचलकेन्द्रजवो'ऽर्यहिक्ष्माः' ।
लिप्ता 'जिना' विकलिकाश्च, गुरोः 'शरः खं'
शुक्राशुकेन्द्रगति'रद्रिगुणाः' शनेर्द्वे ॥ १५ ॥
(Ganesa, *GL*. 1.14-15)

—Grahalāghava

The daily motion of the planets are: Sun: 59′ 8″; Moon: 790′ 35″; Moon's apogee: 6′ 45″; Rāhu: 3′ 11″; Mars: 31′ 26″; Mercury (*kendra*) 186′ 24″; Jupiter: 5′ 0″; Venus (*śīghrakendra*) 37′ 0″; Saturn 2′ 0″. (14-15)

ग्रहगतिस्फुटः

—भास्करः २

13. 30. 1. दिनान्तरस्पष्टखगान्तरं स्याद्
गतिः स्फुटा तत्समयान्तराले ॥ ३६ ॥

कोटीफलघ्नी मृदुकेन्द्रभुक्ति-
स्त्रिज्योद्धृता कर्किमृगादिकेन्द्रे ।
तया युतोना ग्रहमध्यभुक्ति-
स्तात्कालिकी मन्दपरिस्फुटा स्यात् ॥ ३७ ॥

[1] For com. and rationale, see *G.L:RCP*, II, pp. 27-29.

चन्द्रगतिः

समापतिथ्यन्तसमीपचालनं
विधोस्तु तत्कालजयैव युज्यते ।
सुदूरसंचालनमाद्यया यतः
प्रतिक्षणं सा न समा महत्वतः ॥ ३८ ॥

फलांश'खाङ्का'न्तरशिञ्जिनीघ्नी
द्राक्केन्द्रभुक्तिः श्रुतिहृद्विशोध्या ।
स्वशीघ्रभुक्तेः स्फुटखेटभुक्तिः
शेषं च वक्रा विपरीतशुद्धौ ॥ ३९ ॥
(Bhāskara II, *SiŚi.*, 1.2.36b-39)

True motion of planets

—Bhāskara II

The true daily motion of the planet is the excess of the longitude of the true planet of the next day over that of the true planet of the previous day. (36)

Mandasphuṭagati

The *koṭiphala* being multiplied by the daily motion of the *manda* mean anomaly and divided by the radius, and the result being added to or subtracted from the mean motion, gives what is called *mandasphuṭagati*. (37)

Moon's motion

In the case of the Moon, for obtaining the true Moon for a particular moment and its daily motion for the day, the ending moment of the *tithi* near at hand is to be computed with that daily motion, and the method of successive approximations is to be used to rectify the ending moment. In the case of the ending moment of the *tithi* being sufficiently far away, it does not matter even if the above daily motion is applied to get the approximate ending moment. Inasmuch as the Moon's daily motion is great and varies from moment to moment, the motion at the moment is to be used. (38)

Śighragatiphala

$$\delta^1 - \frac{R \sin (90 - E_2)\ \delta^m}{K} = \delta^s,$$ where δ^1 is the daily motion of the *śīghroccha*, E_2 = *śīghraphala*, δ^m is the daily motion in the *śīghra* mean anomaly, K = *śīghrakarṇa* and δ^s is the true motion of the planet. If δ^s is negative, the planet is retrograde.[1] (39). (AS)

—करणरत्नम्

13. 30. 2. स्वस्वस्फुटवृत्तगुणां मध्यगतिं मन्दखण्डजीवाघ्नाम् ।
'खषडिन्दुसागरै'स्तां विभजेल्लब्धं फलं योज्यम् ॥ ७ ॥

मध्यपदे स्वे भोगे ह्याद्यन्तपदे फलं तु तत् त्याज्यम् ।
सा मध्यस्फुटभुक्तिः स्वशीघ्रभुक्त्यूनिता भवति ॥ ८ ॥

[1] For the rationale involved, see *SiŚi*: *AS*, pp. 155-61.

शीघ्रफलभोगवर्गाद् 'रामाक्ष्या'प्तफलैर्युतं कर्णः ।
तत्त्रिज्याविवरकलाहता भवेच्छीघ्रभुक्त्यूना ॥ ९ ॥
यद्विभजेच्छ्रवणेन प्राप्तफलं योजयेत् स्वमन्दगतौ ।
कर्णेऽधिके त्रिशत्या ऊने कर्णे तु तत्त्याज्यम् ॥ १० ॥
वक्रसमये ग्रहाणां स्फुटगतिविषये विपर्ययो भवति ।
उभयत्रापि धनर्णे व्यस्तेऽन्यत् सर्वमेवं स्यात् ॥ ११ ॥
कथितैवं स्फुटभुक्तिर्भौमादेः ग्रहगणितविद्भिः ।
मध्यगतिरेव राहोरुच्चस्य च कथ्यते तद्वत् ॥ १२ ॥

(Deva, *KR*, 8. 7-12)

—Karaṇaratna

Multiply the mean daily motion (of the planet) by its own true (*manda*) epicycle as well as by the (current) R sine-difference of *manda* anomaly and divide by 4160.[1] The resulting quantity should be added to its own mean daily motion (when the planet is) in the middle (anomalistic) quadrants; (when the planet is) in the first and last (anomalistic) quardants, that quantity should be subtracted (from the mean daily motion). This (sum or difference) is the planet's true-mean daily motion. (7)

Subtract it from the daily motion of its *śīghrocca*: (the result is known as the *śīghra-kendragati*). Divide the square of the *śīghraphala-gati* by 23 and add it to (or subtract it from) the (planet's) hypotenuse (as the case may be): (the result is the true hypotenuse). By the minutes of difference of that (true hypotenuse) and the radius, multiply the *śīghrakendra-gati* and divide that by the (true) hypotenuse. Add the resulting quantity to the true-mean daily motion, provided the hypotenuse is greater than the radius viz., 300; if the hypotenuse is smaller than the radius, subtract that (from the true-mean daily motion): the result is the true daily motion of the planet. (7-10)

True motion of a retrograde planet

When the planet is retrograde, there is difference in the procedure for finding its true daily motion. In both the places (where addition and subtraction have been prescribed above), the process of addition and subtraction should be reversed: the rest should be taken as it is. (11)

This is how the learned scholars of astronomy state the method for finding the true daily motion of Mars, etc.

In the case of the Moon's ascending Node, the mean daily motion itself is the true daily motion. The same is also said (to be true) for the Moon's apogee.[2] (12). (KSS)

[1] The correct number is 4190.

[2] The above rule is generally the same as that found to occur in the *Sūrya-siddhānta* and the *Vṛddha-vāsiṣṭha-siddhānta*. The correction for the hypotenuse prescribed in the above rule, however, has no counterpart in any other known work on Hindu astronomy.

मन्दशीघ्रस्फुटभुक्तिः

13. 31. 1. ज्याखण्डकेन गुणिता मृदुकेन्द्रजेन
भुक्तिर्ग्रहस्य 'शरयुग्मयमै'र्विभक्ता ।
क्षुण्णा स्फुटेन गुणकेन हृता 'खनागै'-
र्लिप्ता गतेः फलमृणं धनमुक्तवच्च ॥ ११ ॥
तद्वर्जिता स्वचलतुङ्गगतिः स्वभोग्य-
खण्डाहता 'शरयमाक्षि'हृता हता च ।
स्वेन स्फुटेन गुणकेन 'खनाग'भक्ता
त्रिज्याहता श्रुतिहृताशुफलं गतेः स्यात् ॥ १२ ॥
मन्दस्फुटा ग्रहगतिः स्फुटतामुपैति
युक्तोनिता विरहिता सहितामुना च ।
शीघ्राभिधाननिजकेन्द्रपदक्रमेण
वक्रा गतिर्भवति चेदृणतो विशुद्धा ॥ १३ ॥
'बाणाब्धिभिः' 'शशिगुणैः' 'खयमैः' 'खबाणै'-
'रङ्गै''र्लवै'स्त्रिगृहमाद्यपदं युतं स्यात् ।
ऊनं तृतीयमिति केन्द्रपदोक्तलक्ष्म
बुध्वा गतौ चलफलं स्वमृणं विधेयम् ॥ १४ ॥

(Lalla, *ŚiDhVṛ.*, 3. 11-14)

Manda and Śīghra motion

The mean motion of a planet multiplied by the *bhogyakhaṇḍa* resulting from its mean anomaly and also by the corrected *mandaguṇaka* and divided by 225 and 80 gives in minutes the correction to motion or *mandagatiphala*. It is to be applied to the mean motion as explained before. (The result is the motion corrected once). (11)

Subtract it (viz., corrected motion) from the motion of the *śīghrocca* of the planet. The remainder multiplied by the *bhogyakhaṇḍa* resulting from the *śīghrakendra*, the radius and the corrected *śīghraguṇaka* and divided by 225, 80 and the hypotenuse, gives the second correction of *śīghragatiphala*. (12)

When it is applied to the once-corrected motion, (i) positively, (ii) negatively, (iii) negatively and (iv) positively, according as the *śīghrakendra* is in the first, second, third or fourth *pāda* (see the next verse), the result is the true motion. (13)

When the result is negative, the motion is said to be retrograde.

The first *pāda* extends from 0° to 90° *plus* 45°, 31°, 20°, 50° and 6° for Mars, Mercury, Jupiter, Venus and Saturn respectively. The third *pāda* extends from 180° to 270° *minus* 45°, 31°, 20°, 50° and 6°, respectively, for these planets. This definition of *pāda* should be kept in view while applying the *śīghragatiphala* positively or negatively to the motion.[1] (14). (BC)

[1] For notes, see *ŚiDhVṛ.*, *BC*, II. 53-55.

ग्रहाणां वक्रगतिः

13. 32. 1. दूरस्थिताच्च शीघ्रोच्चाद् ग्रहाः शिथिलरश्मिभिः ।
सव्येतराकृष्टतनुर्भवेद् वक्रगतिस्तदा ॥ ५१ ॥
महत्त्वाच्छीघ्रपरिधेः सप्तमे भृगुभूसुतौ ।
अष्टमे जीवशशिजौ नवमे तु शनैश्चरः ॥ ५२ ॥
'कृतर्तुचन्द्रै''र्वेदेन्द्रैः' 'शून्यत्र्येकै''र्गुणाष्टिभिः' ।
'शररुद्रै'श्चतुर्थांशकेन्द्रांशैर्भूसुतादयः ॥ ५३ ॥
भवन्ति वक्रिणस्तैस्तैः स्वैस्वैश्चक्राद्विशोधितैः ।
अवशिष्टांशतुल्यैः स्वैः केन्द्रैरुज्झन्ति वक्रताम् ॥ ५४ ॥
(*SūSi.*, 2. 51-54)

Retrograde motion of planets

When at a great distance from its conjunction (*śighrocca*), a planet, having its substance drawn to the left and right by slack cords, comes then to have a retrograde motion. (51)

Mars and the rest, when their degrees of commutation (*kendra*), in the fourth process, are respectively 164, 144, 130, 163 and 115, become retrograde (*vakrin*). And when their respective commutations are equal to the number of degrees remaining after subtracting those numbers, in each case, from a whole circle, they cease retrogradation. (52-53)

In accordance with the greatness of their epicycles of the conjunction (*śighraparidhi*), Venus and Mars cease retrograding in the seventh Sign, Jupiter and Mercury in the eighth, Saturn in the ninth. (54). (Burgess)

13. 32. 2. श्वस्तनेऽद्यतनाच्छुद्धे वक्रभोगः प्रकीर्तितः ।
विपरीतविशेषोत्थश्चारभोगस्तयोः स्फुटः ॥ ४१ ॥
(Bhāskara I, *LBh*, 2.41)

(Whatever is obtained from) subtracting the longitude (of a planet) for tomorrow from the longitude for today (when it is possible), is called the retrograde motion (of the planet for the day) and whatever results on performing the subtraction reversely gives the direct motion (of the planet for the day). (41). (KSS)

—भास्करः २

13. 32. 3. द्राक्केन्द्रभागै'स्त्रिनृपैः' 'शरेन्द्रै'-
'स्तत्त्वेन्दुभिः' 'पञ्चनृपै'-'स्त्रिरुद्रैः' ।
स्याद्वक्रता भूमिसुतादिकाना-
मवक्रता तद्रहितैश्च भांशैः ॥ ४२ ॥
(Bhāskara II, *SiŚi.*, 1. 2.41)

—Bhāskara II

The planets Mars, Mercury, Jupiter, Venus and Saturn will be retrograde when the *śighra* anomaly assumes values 163, 145, 125, 165 and 113 respectively, and the direct motion again ensues at (360—163), (360—145), (360—125), (360—165) and (360—113), respectively.[1] (42). (AS)

—लल्लः

13. 32. 4. 'गुणनृपतिभि''र्बाणाब्ध्येकैः' 'शराक्षिनिशाकरैः'
'शररसकुभि''र्विश्वक्ष्माभि'र्लवैश्चलकेन्द्रजैः ।
भवति नियतं वक्रारम्भः कुजादिनभःसदां
पुनरपि भवेद्वक्रत्यागश्च्युतैस्तु भमण्डलात् ॥ २० ॥
'रसरसाः' क्रमशः 'शशिबाहवो'
'यमनिशाकरशीतमरीचयः' ।
'यमशरा' 'युगपावकभूमयो'-
ऽनृजुगतेर्दिवसाः कथिताः कुजात् ॥ २१ ॥
'वसुयमैः' 'शरपूर्णविलोचनै'-
'र्मनुभि''रग्निभुजङ्गनिशाकरैः' ।
'खनयनै'रुदयो दिशि वज्रिणो
भवति यश्चलकेन्द्रभवैर्लवैः ॥ २२ ॥
'गगनषट्कहुताश'परिच्युतैः
ककुभि पाशभृतोऽस्तमयो भवेत् ।
'शशिशरै''स्त्रियमै'र्बुधशुक्रयो-
र्जलपतेरुदयो दिशि जायते ॥ २३ ॥
निगदितः पतितैश्च 'भमण्डलाद्'
दिशि सहस्रदृशोऽस्तमयो भवेत् ।
'यमगुणै'र्दिवसैः 'शशिभूधरै'-
र्भवति चाभ्युदयोऽस्तगयोस्तयोः ॥ २४ ॥
'व्योमार्काः' 'क्षितिपा' 'नभो हुतभुजो' 'नागा'स्तथा 'षड्गुणा'
वारुण्यां क्रमशः स्युरस्तदिवसा भौमादिकानां दिनैः ।
'आकाशाङ्गरसै''स्तुरङ्गदहनै''र्नासत्यशैलानलै'
'रूपाक्षाक्षि'भि'रश्विसागरगुणै'रस्तं प्रयान्त्युद्गताः ॥
(Lalla, *ŚiDhVṛ.*, 3. 20-25)

—Lalla

The retrograde motion of the planets beginning with Mars (viz., Mars, Mercury, Jupiter, Venus and Saturn) commences when their *śighrakendras* are, respectively, 163°, 145°, 125°, 165°, and 113°. When the *śighrakendras* are, respectively, 360° *minus* each of these values, their retrograde motion ceases. (20)

It is specified that the retrograde motion of Mars, (Mercury, Jupiter, Venus and Saturn) lasts for 66, 21, 112, 52 and 134 days, respectively. (21)

When the *śighrakendras* (of Mars, Mercury, Jupiter, Venus and Saturn) are, respectively, 28°, 205°, 14°, 183° and 20°, they rise in the east. When their *śighrakendras* are 360° *minus* these values, respectively, they set in the west.

[1] For the formula, see *SiŚi:AS*, p.167.

When their *śighrakendras* are, respectively, 51° and 23°, Mercury and Venus (when direct) rise in the west. When they are 360° *minus* these values, respectively, they set in the east. (22-24a)

Mercury and Venus, once set, rise again 32 and 71 days, respectively, after setting. (24b)

For 120, 16, 30, 8 and 36 days, respectively, Mars, etc. remain invisible in the west. They set 660, 37, 372, 251 and 342 days, respectively, after rising. (25). (BC)

पराशरसिद्धान्तः

13. 33. 1. कलिसंज्ञे युगपादे पाराशर्यं मतं प्रशस्तमतः ।
वक्ष्ये तदहं तन्मम मततुल्यं मध्यमान्यत्र ।। १ ।।

एतत् सिद्धान्तद्वयमीषद् याते कलौ युगे जातम् ।
स्वस्थाने दृक्साम्ये तत्तन्मतेन खेटाः स्फुटाः कार्याः ।।२।।

कल्पभगणाः

नात्र मते सृष्टचब्दाः शेषं कल्पादिकं प्राग्वत् ।
कल्पेऽत्राधिकमासा 'यमधीलूलागघामपणाः' ।। ३ ।।

'रमनिजखुभितमघणना' न्यूनाहा मेदिनीदिवसाः ।
'कुमसीसोधीपोसामुसिनेननिना'ऽथ चक्राणि ।। ४ ।।

सूर्यादीनां 'भेलीफेनीनेनीननीनीनाः' ।
'मसिसमिगडबढमुकिमा' 'रेरेधातीदडीगनगेसाः' ।। ५ ।।

'पथिधभसनिममढसघा' 'गीतीघेखिटिधधीमेघाः' ।
'सोनीखिरिडेसेरेकोढाहा' 'पढितणेसुपीजपगाः' ।। ६ ।।

तुङ्गानां 'धेदोना' 'भुजिजेकोनीघचीलोभाः' ।
'गुरूसा' 'गुणिता' 'धद्रा' 'मरता' 'मेढा'ऽथ पातानाम् ।।

रजनीकरपूर्वाणां 'खबखबपडिखेगमा' 'रघुणाः' ।
'तोघेहा' 'खेधेना' 'जूझेला' 'तीगना' क्रमशः ।। ८ ।।

सप्तर्षीणां 'कणधझझुझिजा' 'मुदयसिनधा'ऽयनकाख्यस्य ।
त्रैराशिकेन साध्यं द्युगणाद्यखिलं तु कल्पगतात् ।। ९ ।।

मध्यग्रहाः

'घुमतिस' गुणितं 'केननननेनै'र्विभजेद् यदत्र तस्य स्यात् ।
लब्धं ध्रुवकः कलिजः कलिगततो वाखिलं साध्यम् ।।

द्युगणं वा 'कननुनुनै'र्विभजेद् गुणकस्तदाहता भगणाः ।
'कुमुसथधुटिथमस'हृता ध्रुवकाः सक्षेपकाः परं प्राग्वत् ।।

'र'घ्नगणोऽधः 'कगधै' 'खगननकाषैः' क्रमाद् भक्तः ।
फलयुतिहीने द्युगणे भागाद्यर्को भवेच्च वर्षौघात् ।। १२ ।।

'भ'हतात् 'समकलनै' आप्ताद्विलिप्तिकाहीनः ।
'कभ'निघ्ने दिनवृन्दे स्व'कस'लवोने लवादिरिन्दुः स्यात् ।।

'म'घ्नगणाद् 'वरहसरै' आप्तांशोनस्तु वर्षगणात् ।
'घै' निघ्नाद् 'रकमसलै' आप्तविलिप्तागणेनाढ्यः ।।

द्विष्ठो द्युगणो विभक्तो 'धै' 'भनक्नै' फलांशसंयोगात् ।
तुङ्गं स्यादब्दगणात् 'तभतै' आप्ताद् विलिप्तोनम् ।।

'भ'घ्नगणो द्विः 'कनसीधीधै' 'सेतै' क्रमाद् धृतोंऽशैक्यम् ।
पातः स्यात् 'क्नू'घ्नाब्दाद् 'गमकध'भक्ताप्तविकलाढ्यम्।।

'सरनचगघधमभरकेधा' कल्यादौ द्युगण एषः ।
प्राग्वत् कर्तव्यमखिलं द्युसदां मध्यादिकं सुधिया ।।१७।।

भानि जघन्यबृहत्समसंज्ञानि स्युः स्वनामफलदानि ।
संक्रमविधूदयादौ तत्सिद्धयै सूक्ष्मभानयनम् ।। १८ ।।

(ĀBh. II, *Mahā.*, 2. 1-18)

Parāśara-siddhānta

In this quarter of the *yuga*, called Kali, the theory of Parāśara is generally accepted. Therefore, I enunciate it, which is similar to my (Āryabhaṭa II) theory except in the mean (motions). (1)

This pair of *Siddhāntas*, (i.e., Parāśara's and mine), originated after some (part) of Kali era had elapsed. The (positions of the) planets are to be corrected at one's own place for the coincidence of the observed (and computed results) (*dṛksāmya*) according to this or that theory. (2)

Revolutions in a kalpa

In this theory (of Parāśara) there are no *sṛṣṭi*-years. The rest concerning the *kalpa* (and its divisions) etc. is as before. Here (in Parāśara's theory) the intercalary months in a *kalpa* are 1,593,334,515. (3)

The intercalary days are 25,082,465,450 and the civil days are 1,577,917,570,000. Now, the revolutions (in a *kalpa*) of the Sun etc. are: (Sun) 4,320,000,000; (Moon) 57,753,334,515; (Mars) 2,296,833,037; (Conjunction of Mercury) 17,937,055,474; (Jupiter) 364,219,954; (Conjunction of Venus) 7,022,372,148; (Saturn) 146,571,813. (4-6)

(The revolutions) of the apsides (*tuṅga*) are: (Sun) 480; (Moon) 488,104,634; (Mars) 327; (Mercury) 356; (Jupiter) 982; (Venus) 526; and (Saturn) 54.

Now, (those) of the nodes of (the planets), beginning with the Moon, are: (Moon) 232,313,235; (Mars) 245; (Mercury) 648; (Jupiter) 190; (Venus) 893; and (Saturn) 630. (7-8)

(The revolutions) of the Great Bear are: 1,599,998 and (those) of the (planet) called equinox (*ayanākhya*), are 581,709. All others like the *dyugaṇa* (in Kali) etc. are to be derived from (the number of years) elapsed in this *kalpa* by the rule of three. (9)

Mean planets

Multiply (the number of revolutions of any planet in a *kalpa*) by 4567 and divide by 10,000. The quotient is its position at the beginning of Kali (*kalija-dhruvaka*).

From (the *dyugaṇa*) in Kali, all (the mean positions at a given day) can be obtained.[1] (10)

Divide the *dyugaṇa* (in Kali) by 10,000; (the quotient is called) *guṇaka*. Multiply by it the revolutions (of a planet in a *kalpa*) and divide by 157,791,757. (The quotient is called) *dhruvaka*, which is to be increased by *kṣepaka*. The rest is the same as before.[2] (11)

(Set) twice the number (of the remaining days) down. Divide it by 139 and 230,016 separately. Subtract the sum of the quotients from the number of (the remaining) days. (A). Multiply the years (elapsed in Kali) by 4 and divide by 75,130. Subtract the quotient in seconds (from A. The result is the *kṣepaka* of) the Sun in degrees etc.[3] (12-13a).

Multiply the (remaining) sum of days by 14 and diminish (this product) by the 17th part of itself (A). Multiply the number (of the remaining days) by 5 and divide by 42,872. Subtract the quotient in degrees from A. (B). Multiply the number of years (elapsed in Kali) by 4 and divide by 21,566. Add the quotient in seconds (to B. The result is the *kṣepaka* of) the Moon in degrees etc.[4] (13c-d-14)

Set the number of (the remaining) days at two places, and divide by 9 and 4010. Take the sum of the quotients in degrees (A). Divide the number of years (elapsed in Kali) by 646, and subtract the quotient in seconds (from A. The difference is the *kṣepaka* of) the apsis of the Moon.[5] (15)

(Take) the number (of the remaining days), multiplied by 4, twice. Divide it by 10,799 and 76; add (the quotients in) degrees. (A). Multiply the years (elapsed in Kali) by 10 and divide by 3519; add the quotient in seconds (to A. The sum is the *kṣepaka* of) the Node of the Moon.[1] (16)

The *dyugaṇa* at the beginning of the present Kali is 720,634,954,219. All (other) mean (positions) etc. of the planets are to be calculated as before by a wise (man). (17)

The asterisms are designated as inferior (*jaghanya*) or superior (*bṛhat*) or even (*sama*)[2]; and they bestow rewards according to their own designations at the time of rise of the Moon in the asterism (concerned) (*saṃkramavidhūdaya*). In order to determine that (time) a minute calculation of the asterisms is to be made. (18). (SRS)

आर्यभटीयार्धरात्रिकपक्षः

—भास्करः १

13. 34. 1. निबन्धः कर्मणां प्रोक्तो योऽसावौदयिको विधिः ।
अर्धरात्रे त्वयं सर्वो यो विशेषः स कथ्यते ।। २१ ।।

भूदिनादौ भेदः

त्रिशती भूदिने क्षेप्या ह्यवमेभ्यो विशोध्यते ।
ज्ञगुर्वोर्भगणेभ्योऽपि विंशतिश्च ततोऽब्धयः ।। २२ ।।

भूरविचन्द्रव्यासाः

'अष्टिश्शतगुणा' व्यासो योजनानां भुवो रवेः ।
'खाष्टाब्ध्यङ्गानि' शीतांशोः 'शून्यवस्वब्धय'स्तथा ।।२३

रविचन्द्रकर्णौ

'वस्विन्द्रियगुणच्छिद्रवस्वङ्गानि' विभावसोः ।
'अङ्गाङ्गेष्वेकभूतानि' चन्द्रकर्णः प्रकीर्तितः ।। २४ ।।

मन्दोच्चाः

'अष्टिरष्टौ जिना रुद्रा विंशति'र्द्व्यधिकाः क्रमात् ।
दशघ्ना गुरुशुक्रार्किभौमज्ञांशाः स्वमन्दजाः ।। २५ ।।

मन्दशीघ्रवृत्तानि

मन्दवृत्तानि द्वात्रिंश'न्मनवः' षष्टिरेव च ।
'खाद्रयो' 'वसुदस्राः' स्युः शीघ्रवृत्तान्यथ क्रमात् ।

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[1] The number of years elapsed up to the beginning of the present Kali, without the *sṛṣṭi*-years, are 1,972,944,000. The number of revolutions, made by a planet during this period, are

$$\frac{1{,}972{,}944{,}000 \times \text{revolutions in a } kalpa}{4{,}320{,}000{,}000} = \frac{4567 \times \text{rev.}}{10000}$$

The fraction thereof is the position of the planet at the beginning of Kali. (*Mahā*, 1.19)

[2] After determining the number of civil days elapsed in Kali, the change in the mean positions of planets during this period is calculated in two successive steps. First, the biggest multiple of 10,000 is taken off from the number of civil days, and the change in position for this number of days, which is called here *dhruvaka*, is found in the following way:

Dyugaṇa is divided by 10,000, and the quotient is called *guṇaka*. Then, *dyugaṇa* = 10,000 × *guṇaka* × remainder

Then *dhruvaka* = *guṇaka* × 1000 × revolutions of the planet in a *kalpa* ÷ civil days in a *kalpa*

$$= \frac{\text{rev.} \times gu\d{n}aka \times 10{,}000}{1{,}577{,}917{,}570{,}000} = \frac{\text{rev.} \times gu\d{n}aka}{157{,}791{,}757}$$

The change in position during the remaining days, which is called *kṣepaka*, is calculated for each planet in the following verses. The sum of all the three quantities, *kalijadhruvaka*, *dhruvaka* and *kṣepaka*, is the mean position of the planet on a given day.

[3] Let the number of the remaining days be x and the number of years elapsed in Kali be y. Then the *kṣepaka* of the Sun= $x^\circ - (2x/139)^\circ - (2x/230016)^\circ - (4y/75{,}130)''$

[4] *Kṣepaka* of the Moon= $14x^\circ - (14x/17)^\circ - (5x/42782)^\circ + (4y/21566)''$

[5] *Kṣepaka* of the apsis of the Moon= $(x/9)^\circ + (x/4010)^\circ - (y/646)''$

[1] *Kṣepaka* of the Node of the Moon $= (4x/10799)^\circ + (4x/76)^\circ + (10y/3519)''$. The proof for these equations is similar to that in I. 43-47.

[2] This classification of the asterisms into 'inferior', 'superior' and 'even' is based on an ancient tradition ascribed by Bhāskarācārya II to Pauliśa, Vasiṣṭha, Garga etc. According to their teachings, the ecliptic is not divided into equal parts of lunar mansions, but into six smaller, six greater and fifteen even lunar mansions. Thus Āśleṣā, Ārdrā, Svātī, Bharaṇī, Jyeṣṭhā and Śatabhiṣāj are small each consisting of 395′17″. Viśākhā, Punarvasu, Rohiṇī, Uttaraphālguṇī, Uttarāṣāḍhā and Uttarabhādrapada are great, each consisting of 1185′ 52″, and the remaining fifteen are even, each consisting of 790′ 35″. The 22nd lunar mansion, Abhijit consists of 254′ 21″.

'द्व्यद्रय:' 'खाङ्गनेत्राणि' 'खाब्धयोऽब्ध्यग्निदस्रका:' ।
'द्व्यग्नीन्दवो' रवेर्मन्दं शुक्रवद् वृत्तमेव च ।। २७ ।।
एकत्रिंशत्क्षपाभर्तुरर्धरात्रे विधीयते ।

मन्दशीघ्रपाता:

पातभागाश्च विज्ञेया: पण्डितै: परिकल्पिता: ।। २८ ।।
मन्दशीघ्रोच्चयो: क्षेप्यं चक्रार्धं बुधशुक्रयो: ।
राशित्रयं तु शेषाणां पात्यते पातसिद्धये ।। २९ ।।
शुक्रार्किदेवपूज्यानां भागौ द्वावेव संयुतौ ।
मन्दपाताच्च शीघ्रोच्चात् सार्धांशस्तु कुजज्ञयो: ।। ३० ।।
विबुधानां च सर्वेषां शीघ्रपाता: प्रकीर्तिता: ।

विक्षेपानयनम्

शोधयित्वा क्रमात् पातान् विक्षेपांशान् प्रसाधयेत् ।।३१।।
योगविश्लेषनिष्पत्तिरेकानेकस्वदिग्वशात् ।
विक्षेप: स स्फुटो ज्ञेयो ग्रहस्यैकस्य कीर्तित: ।। ३२ ।।
अन्यस्याप्येवमेव स्याच्छेषा: प्रागुक्तकल्पना: ।
एतत्सर्वं समासेन तन्त्रान्तरमुदाहृतम् ।। ३३ ।।

स्फुटमध्यग्रहा:

शीघ्रमन्दोच्चचापार्धसंस्कृतात् स्वीयमन्दत: ।
स्फुटमध्यग्रहा: सर्वे विशेष: परिकीर्तित: ।। ३४ ।।

आकाशकक्ष्या ग्रहकक्ष्याश्च

'वेदाश्विराम'गुणितान्ययुताहतानि
चन्द्रस्य शून्यरहितान्यथ मण्डलानि ।
स्वै: स्वैर्हृतानि भगणै: क्रमशो ग्रहाणां
कक्ष्या भवन्ति खलु योजनमानदृष्टया ।। ३५ ।।

(Bhāskara I, 7. 21-35)

Midnight day-reckoning of Āryabhaṭa I —Bhāskara I

The astronomical processes which have been set forth above come under the sunrise day-reckoning. In the midnight day-reckoning too, all this is found to occur; the difference that exists is being stated (below). (21)

Civil days, Omitted lunar days, Rev. of Mercury and Jupiter

(To get the corresponding elements of the midnight day-reckoning, add 300 to the number of civil days (in a *yuga*) and subtract the same (number) from the number of omitted lunar days (in a *yuga*); and from the revolution-numbers of (the *śīghrocca* of) Mercury and Jupiter subtract 20 and 4, respectively. (22)

Diameters of the Earth, the Sun, and the Moon

(In the midnight day-reckoning) the diameter of the Earth is (stated to be) 1600 *yojanas*); of the Sun, 6480 (*yojanas*); and of the Moon, 480 (*yojanas*). (23)

Mean distances of the Sun and the Moon

The (mean) distance of the Sun is stated to be 6,89,358 (*yojanas*) and of the Moon, 51,566 (*yojanas*). (24)

Longitudes of the apogees of the planets

160, 80, 240, 110 and 220 are in degrees the longitudes of the apogees of Jupiter, Venus, Saturn, Mars and Mercury respectively. (25)

Manda and śīghra epicycles

The *manda* epicycles (of the same planets) are 32, 14, 60, 70, and 28 (degrees) respectively; and the *śīghra* epicycles are 72, 260, 40, 234, and 132 (degrees) respectively. The Sun's apogee and epicycle are the same as those of Venus (i.e., 80° and 14° respectively). The Moon's epicycle in the midnight day-reckoning is stated to be 31 (degrees). (26-28a)

Manda and Śīghra pātas

(The following directions for) the degrees of the (*manda* and *śīghra*) *pātas* of the planets as devised (under the midnight day-reckoning) should be noted carefully by learned scholars. (28b)

Add 180° to the longitudes of the *mandoccas* (apogees) and *śīghroccas* (perigees) of Mercury and Venus, and subtract 3 Signs from the *mandoccas* and *śīghroccas* of the remaining planets. Then are obtained the longitudes of the *manda* and *śīghra pātas* of the planets. (Also) add 2 degrees to the longitudes of the *manda pātas* and *śīghroccas* of Venus, Saturn, and Jupiter; and $1\frac{1}{2}$ degrees to those of Mars and Mercury. (It should be noted that) the *śīghrapātas* have been stated for all the planets excepting Mercury. (Mercury does not have a *śīghra-pāta*).[1]

(That is to say, the longitudes of the *manda pātas* of Mars, Mercury, Jupiter, Venus, and Saturn are 21.5°, 41.5°, 72°, 262°, and 152°, respectively; and the longitudes of the *śīghra pātas* of Mars, Jupiter, Venus, and Saturn are (*śīghrocca* — 88°.5, *śīghrocca* — 88°), (*śīghrocca*+182°) and (*śīghrocca* — 88°), respectively). (28b-31a)

Celestial latitude of a planet

(From the longitude of a planet severally) subtract the longitudes of its (*manda* and *śīghra*) *pātas* and therefrom calculate (as usual) the corresponding celestial latitudes of that planet. Add them or take their difference according as they are of like or unlike directions. Then is obtained the true celestial latitude of that particular planet. The true celestial latitude of any other planet is also obtained in the same way. The remaining (astronomical) determinations are the same as stated before. This all in brief is the difference of this other *tantra* (embodying the midnight day-reckoning of Āryabhaṭa I). (31b-33)

[1] This passage represents rather old ideas of Hindu astronomy. The conception of *manda* and *śīghra pātas* does not occur in any other work. Our translation agrees with the interpretations given in the various commentaries. The translation given by P.C. Sengupta in the introduction to his *Khaṇḍa-khādyaka* appears to be wrong.

Longitude of the true-mean planet

Apply half the *śīghraphala* and (then) half the *mandaphala* to the longitude of the planet's own *mandocca* (reversely). From the resulting longitude of the planet's *mandocca*, calculate (the *mandaphala* and apply it to the mean longitude of the planet: the resulting longitude of planet is stated to be) the true-mean longitude of the planet. This is stated to be another difference (of the midnight day reckoning). (34)

Circle of the sky and Orbits of the planets

Multiply the revolutions of the Moon (in a *yuga*) by 32,40,000 and then discard the zero in the unit's place: (this is the length of the circle of the sky in terms of *yojanas*). (Severally) divide that by the revolutions of the planets (in a *yuga*): thus are obtained the lengths of the orbits of the respective planets in terms of *yojanas*.[1] (35). (KSS)

—खण्डखाद्यक:

—रविचन्द्रयोर्मध्यम्

13. 34. 2a. तिसृभि: शनिर्ज्ञशीघ्रं द्वाविंशत्या कुजोऽधिको द्वाभ्याम् ।
चतसृभिरधिको जीवोऽर्धरात्रिकार्यभटमध्यसमा: ।। ७ ।।

'खखवसु'गुणिताद्' वसुगुणकृता'धिकान्'मुनिनखद्विनन्दयमै:' ।
भगणाद्यर्कबुधसिता: शीघ्रोच्चं कुजगुरुशनीनाम् ।। ८ ।।

रविगुणतिथिभागयुत: पृथग'र्के'स्त्रिगुणितादवमशेषात् ।
'त्र्यगशशि'लब्धांशयुतोऽर्धरात्रिको मध्यमश्चन्द्र: ।। ९ ।।

'खखरस'गुणितात् स'शरां'-
श'मुनीन्दुकृता'धिकात् 'त्रिनवगुणाष्टि'हृतात् ।
भगणादिर्वा हिमांशु-
'र्नवयमतानै:' कलरहित: ।। १० ।।

'द्विनवरस'भक्तमवमावशेषमाप्तदिनादि तत्सहितात् ।
अधिमासशेषकाच्च त्रिंशद्गुणिताद् 'ऋतुखदिग्भि:' ।।

मासदिनप्रथमैक्यं पृथक् त्रयोदशगुणं द्वितीयोनौ ।
द्वावप्येवं मध्यौ राश्याद्यावर्कचन्द्रौ वा ।। १२ ।।

भागाशीतिरिनोच्चं शशिन: पादोन'कृतशरकृतो'नात् ।
भगणादि 'द्वित्रिरदै''र्वसुनवयमनवगुणै:' सकलम् ।। १३ ।।

'द्व्यगगुण'हीनाद् द्युगणाल्लब्धं भगणादि 'शरनवागरसै:' ।
'षडिषुरसाब्धीन्दुशरै:' सांशं चक्राच्च्युतं पात: ।। १४ ।।

(Brahmagupta, *KK*, 1.1. 7-14)

—Mean Sun and Moon—Khaṇḍakhādyaka

The mean longitude of Saturn (I.2.5) decreased by 3″, the *śīghrocca* of Mercury (I.2.2.) decreased by 22″, the mean latitude of Mars (I.2.3) increased by 4″ are equal to the respective mean longitudes of the planets at midnight, as calaulated by Āryabhaṭa I (in his Ārdharātrika system). (7)

Multiply the *ahargaṇa* (as calculated above) by 800. Add 438 to the product. Divide the sum by 2,92,207. The result in revolutions etc. is the mean longitude of the Sun, Mercury or Venus and of the *śīghrocca* of Mars, Jupiter or Saturn. (8)

Add to the mean longitude of the Sun the number of degrees equal to 12 times the number of *tithis* elapsed since the last *amāvasyā*. Add to the sum the number of degrees, etc., obtained from dividing 3 times the *avamaśeṣa* by 173. The result is the mean longitude of the Moon at midnight. (9)

Multiply the *ahargaṇa* by 600. Add $417\frac{1}{5}$ to the product. Divide the sum by 16,393. Subtract from the result the number of minutes obtained by dividing the *ahargaṇa* by 4929. (10)

The result is also the mean longitude of the Moon in terms of revolutions, etc.

Divide the *avamaseṣa* by 692. The result is in terms of *dinas*, *ghaṭikās*, etc. (When degrees, minutes etc., are, respectively, substituted for *dinas*, *ghaṭikās*, etc., in the result, it is called Prathamā). Add the result to the *adhimāsaśeṣa*. Multiply the sum by 30 and divide the product by 1006. (When degrees, minutes etc., are respectively, substituted for *dinas*, *ghaṭikās*, etc., it is called Dvītīya). Then add the months (*cāndramāsas* elapsed since the light half of Caitra considered as *sauramāsas*), days (*tithis* elapsed since the last *amāvasyā* considered as *sauradinas*) and the Prathama. (The sum is in terms of Signs etc.) Subtract the Dvītiya from this sum. The result is the mean longitude of the Sun in signs, etc. Subtract the Dvītiya again from 13 times the above sum and the result is the mean longitude of the Moon in Signs, etc. (11-12)

The *mandocca* of the Sun is 80° and that of the Moon is calculated as follows: Subtract $453\frac{3}{4}$ from the *ahargaṇa*. Divide the remainder by 3232. Add to the result the minutes obtained by dividing the *ahargaṇa* by 39,298. The result is the Moon's *mondocca* in revolutions etc. (13)

Deduct 372 from the *ahargaṇa* Divide the remainder by 6795. The result is in revolutions etc. Divide again the *ahargaṇa* by 5,14,656. The result is in degrees. Add both the results and substract the sum from 360°. The remainder is the longitude of the *pāta* (Node) of the Moon.[1] (14). (BC)

[1] From stanzas 20 and 35 it is evident that one *yojana* of the sunrise day-reckoning is one and a half times that of the midnight day-reckoning.

[1] For the rationale and formulae involved, see *KK:BC* I. 96-100.

—मध्यग्रहाः

13. 34. 2b. पादोन'रसनवोदधि'हीनाद् भगणाद्यहर्गणाद् भौमः ।
'सप्ताष्टरसैः' सकलो 'नवतत्त्वाब्धिस्वरशशाङ्कैः' ॥१॥

बुधशीघ्रं शतगुणिताद्
द्युगणाद् भगणादि 'शशिधृतियमो'नात् ।
'सप्तनवस्वरवसुभिः'
'कृतखमनुनगैः' कलाभ्यधिकम् ॥ २ ॥

पञ्चांशोन'गुणेशद्व्यूनाद्' द्युगणाद् 'रदत्रिकृत'लब्धम् ।
भगणादि गुरुः 'शशियमरसयमषोडशभि'रंशोनम् ॥३॥

पादयुता'गगुणो'नाद् दशगुणितान्मण्डलादि सितशीघ्रम् ।
'मुनिजिनयमलैः' सांशं व्य'र्कनगात्' 'त्रिकृतखनगागैः' ।।

सार्धे'न्दुनवजिनो'नाद्
भगणादि शनिः 'षड्ऋत्वगखचन्द्रैः' ।
'खशराब्धिशून्यवसुभि'-
र्लिप्तोनोऽङ्गारकादीनाम् ॥ ५ ॥

मन्दांशा दशगुणिता
'रुद्रा' 'द्वियमा'श्च 'षोडशाष्टजिनाः' ।
(Brahmagupta, *KK*, 1. 2. 1-5a)

—Mean planets

When the *ahargaṇa* less $495\frac{3}{4}$ is divided by 687, the result is in terms of revolutions etc. This, together with the minutes obtained from dividing the *ahargaṇa* again by 1,74,259, gives the mean longitude of Mars. (1)

When the *ahargaṇa* multiplied by 100 is reduced by 2181 and divided by 8797, the result is in terms of revolutions etc. This, together with the minutes obtained from dividing the *ahargaṇa* again by 71,404, gives the longitude of the *śīghrocca* of Mercury. (2)

When the *ahargaṇa* less $2112\frac{4}{5}$ is divided by 4332, the result is in terms of revolutions etc. This, lessened by the degrees obtained from dividing the *ahargaṇa* again by 1,62,621, gives the mean longitude of Jupiter. (3)

When the *ahargaṇa* less $37\frac{1}{4}$ is multiplied by 10 and divided by 2247, the result is in terms of revolutions etc. This, together with the degrees obtained from dividing the *ahargaṇa* less 712 again by 77,043, gives the longitude of the *śīghrocca* of Venus. (4)

When the *ahargaṇa* less $2491\frac{1}{2}$ is divided by 10,766, the result is in terms of revolutions etc. This, diminished by the minutes obtained from dividing the *ahargaṇa* again by 80,450 gives the mean longitude of Saturn. (5)

11, 22, 16, 8 and 24, each multiplied by 10, give, respectively, in degrees, the *mandoccas* of the planets beginning with Mars. (6a). (BC)

—मध्यग्रहाणां ब्रह्मगुप्तकृतः शोधः

13. 34. 2c. भानुमतो मन्दोच्चं
राशिद्वयमंशकाश्च सप्तदश ॥ १ ॥

द्युगणात् 'खरुद्र'गुणिताद्
'भवशर'युक्ता'च्छशित्रिखाग्नि'हृतात् ।
भगणादि फलं शोध्यं
मध्यमचन्द्राच्छशाङ्कोच्चम् ॥ २ ॥

सार्ध'कृतेषुगुणो'नादहर्गणाद् 'द्विनवमुनिरसै'र्भक्तात् ।
यन्मण्डलादि लब्धं चक्रात् संशोध्य तत्पातः ॥ ३ ॥
(Brahmagupta, *KK*, 2.1.1b-3)

—Mean Planets—Emend. by Brahmagupta

The more correct *mandocca* of the Sun is 2 signs 17° (and not 2 signs 20° as given in *KK*. I.1.13).

Multiply the *ahargaṇa* by 110. Add 511 to the product. Divide the sum by 3031. The result in terms of revolutions etc., when subtracted from the mean longitude of the Moon gives its *mandocca* (which is more correct than that given in *KK*. I 1.13). (1b-2)

Subtract $354\frac{1}{2}$ from the *ahargaṇa*. Divide the remainder by 6792. The result in terms of revolutions etc., when subtracted from 360°, gives the longitude of the Moon's *pāta* (which is more correct than that given in *KK*. I.1.14). (3). (BC)

—स्फुटग्रहाः

13. 34. 2d. शुक्रस्य सूर्यवत् फलमिन्दुसुतस्य द्विसङ्गुणितम् ॥ ६ ॥
भौमस्य पञ्चगुणितं गुरोः 'स्वरां'शेन संयुतं द्विगुणम् ।
सौरे'र्मनु'भागयुतं चतुर्गुणं शीघ्रकेन्द्रांशैः ॥ ७ ॥

भौमोऽष्टयमै 'रुद्रान्' भुक्त्वा पूर्वोदितो 'रदै''रर्कान्' ।
'खगुणै'र्दश 'रूपगुणैः' सप्तांशान् 'मनुभिरर्धांशान् ॥८॥

धनमृण'मग्निशशाङ्कै'स्त्री'नष्टया' भास्करानतो वक्री ।
नवभिस्त्रयोदश 'नगै'र्द्वादश सार्धान् विलोमोऽतः ॥९॥

'एकेषु'भिस्त्रयोदश
भुक्त्वाभ्युदितोऽपरेण सोमसुतः ।
'अष्टाग्नि'भिः 'स्वरा'-
'ञ्छशिगुणै'श्च सार्धमेकं स्वमृणमस्मात् ॥ १० ॥

षड्विंशत्या 'विषयान्'
वक्री नवभिर्दलाधिकं त्रितयम् ।
अपरेऽस्तमितस्तत्त्वै-
स्त्रयोदशांशान् विलोमोऽतः ॥ ११ ॥

'मनु'भिः सत्र्यंशौ द्वौ
भुक्त्वा प्रागुद्गतः 'खवेदैः' षट् ।
षट्कृत्या त्रीन् 'धृत्या'
दशलिप्ता धनमृणोक्तिरतः ॥ १२ ॥

'द्विकयमलै'रध्यर्धं
वक्री 'मनु'भिर्द्वयं 'नखै'श्चतुरः ।
अष्टया चतुरो भागान्
विपरीतमतः पुनर्जीवः ॥ १३ ॥

भुक्त्वोदितो 'जिनै'र्दश
पश्चा'न्नन्दाग्निभि'र्भृगुसुतोऽष्टिम् ।
त्रिगुणैः सूर्यान् त्रिधनेन
सप्त 'धृत्या' सपादांशम् ॥ १४ ॥

स्वमृणं त्र्येकैश्चतुरः
पादाभ्यधिकान् भवैर्दिशो वक्री ।
'सूर्यैजिना'नदृश्यस्त्रिभिरष्टौ
प्राग्विलोमोऽस्तः ॥ १५ ॥

विंशत्या द्वौ भुक्त्वा
प्रागुदितो 'रसहुताशनै'स्त्रितयम् ।
'खयमै'रंशकमेकं
'नखै'स्त्रिभागांशकं स्वमृणम् ॥ १६ ॥

विंशत्यांशत्र्यंशं
वक्री सप्तदशभिः शनिर्भागम् ।
द्वाविंशत्या द्वितयं
'तत्त्वै'स्त्रितयं विलोमोऽस्तः ॥ १७ ॥

शीघ्रफलार्धमनष्टे
मन्दफलार्धं च मन्दशीघ्रफले ।
सकले मध्ये स्पष्टः
शीघ्रं मध्योनकं केन्द्रम् ॥ १८ ॥

(Brahmagupta, *KK*, I. 2. 6-18)

—True Planets

The *mandaphala* of Venus is the same as that of the Sun. The *mandaphala* of the son of the Moon, (that is, of Mercury), is twice that of the Sun. The *mandaphala* of Mars is 5 times that of the Sun. The *mandaphala* of Jupiter is $1\frac{1}{7}$ times that of the Sun and doubled. The *mandaphala* of Saturn is $1\frac{1}{14}$ times that of the sun and quadrupled. (6b-7)

Mars has a *śīghraphala* (ŚP) of 11° corresponding to a *śīghrakendra* (ŚK) of 28°, when it rises in the east. For the next ŚK of 32°, it has a ŚP of 12°; for the next 30°, it is 10°; for the next 31°, one of 7° more; and for the next 14°, one of $\frac{1}{2}$° more. All these ŚPs are positive. Then the ŚPs are negative. For the next ŚK of 13°, the ŚP decreases by 3°.[1] and for the next 16° by 12°. Then the motion is retrograde. For the next ŚK of 9°, the ŚP decreases by 13°, and for the next 7° by $12\frac{1}{2}$°. Then Mars has the same ŚP in the reverse order. (8-9)

[1] For the positive, 'more' is used. For the negative, 'less' or 'decreased' is used.

Mercury has a ŚP of 13° corresponding to a ŚK of 51° and rises in the west. Then for the next ŚK of 38°, it has a ŚP of 7° more, and for the next 31° one of $1\frac{1}{2}$° more. All these ŚPs are positive. Then they become negative. For the next ŚK of 26°, the ŚP decreases by 5°. Then the motion is retrograde. For the next ŚK of 9°, the ŚP decreases by $3\frac{1}{2}$°. Then Mercury sets in the west. For the next ŚK of 25°, the ŚP decreases by 13°. After this, Mercury has the same ŚP in the reverse order. (10-11)

When Jupiter has a ŚP of 2 1/3° corresponding to a ŚK of 14°, it rises in the east. For the next ŚK of 40°, it has a ŚP of 6° more; for the next 36°, one of 3° more; and for the next 18°, one of 10° more. These ŚPs are positive. Then they become negative. For the next ŚK of 22°, the ŚP decreases by $1\frac{1}{2}$°. The motion is then retrograde. For the next ŚK of 14°, the ŚP is 2° less; for the next 20°, 4° less and for the next 16°, 4° less. The ŚPs then repeat in the reverse order. (12-13)

When the son of Bhṛgu, (that is Venus), has a ŚP of 10° corresponding to a ŚK of 24°, it rises in the west. For the next ŚK of 39°, it has a ŚP of 16° more; for the next 33°, one of 12° more; for the next 27°, one of 7° more; and for the next 18°, one of $1\frac{1}{4}$° more. These ŚPs are positive. After this, they are negative. For the next ŚK of 13°, the ŚP decreases by $4\frac{1}{4}$°; and for the next 11°, by 10°. Then the motion is retrograde. For the next ŚK of 12°, the ŚP is 24° less. Venus then sets in the west. For the next ŚK of 3°, the ŚP is 8° less. The ŚPs then repeat in the reverse order. (14-15)

When Saturn has a ŚP of 2° corresponding to a ŚK of 20°, it rises in the east. For the next ŚK of 36°, it has a ŚP of 3° more; for the next 20°, one of 1° more, and for the next 20°, one of 1/3° more. These ŚPs are positive. Then they are negative. For the next ŚK of 20°, it has a ŚP of 1/3° less. Then the motion is retrograde. For the next ŚK of 17°, the ŚP is 1° less; for the next 22°, 2° less and for the next 25°, 3° less. Then the ŚPs repeat in the reverse order. (16-17)

Calculate the ŚP from the mean longitude of a planet. If it is positive, add half of it to the mean longitude; if negative, subtract half of it from the mean longitude. The result is the longitude of the planet corrected once. Use this result to calculate the *mandaphala*. Add half of the *mandaphala* to the result, if the *mandakendra* is greater than 6 Signs; subtract, if less than 6 Signs. The reuslt is the longitude of the planet corrected twice. Use this result to calculate again the *mandaphala*. Add it to the given mean longitude of the planet, if the *mandakendra* is greater than 6 Signs; subtract, if less. The result is the longitude of the planet corrected thrice or *manda-sphuṭa* planet. From this calculate the ŚP. Add it to

the *mandasphuṭa* planet, if the ŚP is positive; subtract, if negative. The result is the true longitude of the planet.

When the mean longitude of a planet is subtracted from its *śīghrocca*, the remainder is its *śīghrakendra*. (18)

—ग्रहस्फुटः—ब्रह्मगुप्तकृतः शोधः

13. 34. 2e. सप्तदशांशैरधिकं भौम-
स्योच्चं गुरोर्दशभिरंशैः ।
सितशीघ्रात् 'कृतमुनयो'
लिप्ताः शोध्याः शनेः फलं मन्दम् ॥
पञ्चांशोनं शैघ्र्यं षोडश-
भागाधिकं बुधस्य फलम् ॥ १ ॥
भुक्तगतिफलांशगुणा
भोग्यगतिर्भुक्तगतिहृता लब्धम्
भुक्तगतेः फलभागा-
स्तद्भोग्यफलान्तरार्धहतम् ॥ २ ॥
विकलं भोग्यगतिहृतं
लब्धेनोनाधिकं फलैक्यार्धम् ।
भोग्यफलादधिकोनं
तद्भोग्यफलं स्फुटं भवति ॥ ३ ॥

(Brahmagupta, *KK*, 2. 2.1-3)

True Planets: Em. by Brahmagupta

The *mandoccas* of Mars and Jupiter (as given in KK I.2.6) should respectively, be increased by 17° and 10°. The *śīghrocca* of Venus should be decreased by 74′. The *mandaphala* of Saturn should be decreased by its $\frac{1}{5}$. The *śīghraphala* of Mercury should be increased by its $\frac{1}{16}$. (The results would then be more correct). (1)

Multiply the *bhogyagati* by the *bhuktagatiphalāṁśa* and divide the *bhuktagati*. The result is the corrected *bhuktagatiphalāṁśa*. Find half the difference of the *bhogyagatiphalāṁśa* and the corrected *bhuktagatiphalāṁśa*. Multiply it by the *vikala* and divide by the *bhogyagati*. Add or subtract the result to or from half the sum of the *bhogyagatiphalāṁśa* and the corrected *bhuktagatiphalāṁśa*, according as the half sum is less or greater than the *bhogyagatiphalāṁśa*. The result is the *sphuṭabhogyagatiphala* or correct tabular difference of the *śīghraphala* to be passed over (and hence the correct *śīghraphala*). (2-3).[1] (BC)

—स्फुटभुक्तिः

13 34. 2f. कार्यैवं स्फुटभुक्तिस्तृतीयमन्दस्फुटोनशीघ्रगतिः ।
गतयेयकलाच्छेदो दिनानि गतिभुक्तभोग्यानि ॥ १९ ॥

(Brahmagupta, *KK*, 1.2-19)

True Motion

The motion of a planet at any given time should be calculated in the same manner (i.e., using methods adopted for True planets).

Subtract the *mandasphuṭa* motion of a planet (that is, the mean daily motion thrice corrected) from the motion of its *śīghrocca*. The number of minutes in the arcs of *śīghrakendra* passed and to be passed, at the time when the planet has direct or retrograde motion or rises or sets, should be divided by the above remainder. The result will be the number of days passed and to be passed as regards any of the above phenomena. (19). (BC)

[1]For the rationale and worked out examples, see *KK:BC*, I.146-48.

14. अयनचलनम् – PRECESSION OF THE EQUINOXES

विषुवत् अयनं च

14. 1. 1. यथा वै पुरुष एवं विषुवांस्तस्य यथा दक्षिणोऽर्धं एवं पूर्वार्धं विषुवतो, यथोत्तरोऽर्धं एवमुत्तरोऽर्धं विषुवतः ।

(*Ait. Brāhmaṇa*, 4.22)

Solstices and Equinoxes

The solstice is like a man; the first half of the solstice is like the right half of a man; the second half of the solstice is like the left half.

अयनांशसद्भावः

14. 2. 1. आश्लेषार्द्धाद्दक्षिणमुत्तरमयनं रवेर्धनिष्ठाद्यम् ।
नूनं कदाचिदासीद् येनोक्तं पूर्वशास्त्रेषु ।। १ ।।

साम्प्रतमयनं सवितुः कर्कटकाद्यं मृगादितश्चान्यत् ।
उक्ताभावो विकृतिः प्रत्यक्षपरीक्षणैर्व्यक्तिः ।। २ ।।

दूरस्थचिह्नवेधादुदयेऽस्तमयेऽपि वा सहस्रांशोः ।
छायाप्रवेशनिर्गमचिह्नैर्वा मण्डले महति ।। ३ ।।

(Varāha, *Br.Saṃ.*, 3. 1-3)

Cognition of Precession

There was indeed a time when the Sun's southerly course began from the middle of the star *Āśleṣā* and the northerly one from the commencement of the star *Dhaniṣṭhā*. For, it has been stated so in ancient works. (1)

At present, the southerly course of the Sun starts from the beginning of Cancer and the other from the initial point of Sign Capricorn. The actual fact which goes against the old statement can be verified by direct observation. The Sun's change of course can be detected by marking every day the position of a distant object either at sunrise or sunset, or by watching and marking the entry and exit of the shadow of the gnomon planted at the centre of a big circle drawn on the ground. (2-3). (M.R. Bhat)

अयनांशहेतुः

14. 3. 1. त्रिंशत्कृत्या युगे भांशैश्चक्रं प्राक्परिलम्बते ।
तद्गुणाद् भूदिनैर्भक्ताद् द्युगणाद्यदवाप्यते ।। ६ ।।

तद्दोस्त्रिघ्ना दशाप्तांशा विज्ञेया अयनाभिधाः ।
तत्संयुक्ताद् ग्रहात् क्रान्तिच्छायाचरदलादिकम् ।। १० ।।

स्फुटदृक्तुल्यतां गच्छेदयने विषुवद्द्वये ।
प्राक्चक्रं चलितं हीने छायार्कात्करणागते ।। ११ ।।

अन्तरांशैरथोद्धृत्य पश्चाच्छेषैस्तथाऽधिके ।। १२ ।।

(*Sū.Si.*, 3. 9-12a)

Cause of Precession

In an Age (*yuga*), the circle of the asterisms (*bha*) falls back eastward thirty score of revolutions. Of the result obtained after multiplying the sum of days (*dyugaṇa*) by this number, and dividing by the number of natural days in an Age, take the part which determines the sine, multiply it by three, and divide by ten; thus are found the degrees called those of the precession (*ayana*). From the longitude of a planet as corrected by these are to be calculated the declination, shadow, ascensional difference (*caradala*), etc. (9-10)

The circle, as thus corrected, accords with its observed place at the solstice (*ayana*) and at either equinox; it has moved eastward, when the longitude of the Sun, as obtained by calculation, is less than that derived from the shadow, by the number of degrees of the difference; then, turning back, it has moved westward by the amount of difference, when the calculated longitude is greater.[1] (11-12a). (Burgess)

14. 3. 2. 'भां'शैश्चलति तद्योगः प्राक्प्रतीच्योः पृथक् पृथक् ।
वृद्धिर्ह्रासश्च दिव्याब्दैः पञ्चभिश्च क्रमोत्क्रमात् ।।१७।।
कलिसन्ध्याष्टमांशे स्वशतांशाढ्ये गते ततः ।
धनुर्मिथुनयोर्मध्ये प्रायशस्त्वयने उभे ।। १८ ।।

(Nīlakaṇṭha, *SiDar.*, 17-18)

The conjunction (of the equinoxes) moves east and west by 27 degrees on each side. This increase and decrease (i.e., moving east and returning, then moving west and returning) occurs regularly, (each increase or decrease taking place) once in five divine years (i.e., once in 1800 ordinary years).[2] (17)

Position of the equinoxes at a specific date

Taking the time when one-eighth of the dawn[3] of the *Kali* age had passed and increasing this number by its one-hundredth part (we find) a moment when the two solstices were roughly in the middle of Sagittarius

[1] For elucidation, see *SūSi*: *Burgess*, pp. 114-21.

[2] One divine year is equal to 360 ordinary years. Thus, to complete one oscillation, both ways, it would take 4 × (5×360) =7200 years.

[3] The twelfth part of the total duration of a *yuga* in its beginning is its dawn (*sandhyā*) and the twelfth part at the end is the twilight (*sandhyāṃśa*).

(Dhanus) and of Gemini (Mithuna), respectively.[1] (18). (KVS)

अयनचलनशून्यकालः—आर्यभटीयप्रणयनकालः

14. 4. 1. षष्टयब्दानां षष्टिर्यदा व्यतीतास्त्रयश्च युगपादाः ।
त्र्यधिका विंशतिरब्दास्तदेह मम जन्मनोऽतीताः ॥१०॥

Āryabhaṭa I, *ABh.*, 3-10

सूर्यदेवयज्वनः व्याख्या

इह वर्तमानेऽष्टाविंशे युगे युगचतुर्थभागत्रयं षष्टयब्दानां षष्टिश्च यदा गताः, तदा मम जन्मनः प्रभृति त्र्यधिका विंशतिरब्दा गताः । वर्तमानयुगचतुर्थपादस्य कल्याख्यस्य षट्छताधिकसहस्रत्रयसम्मितेषु सूर्याब्देषु गतेषु त्रयोविंशतिवर्षेण मया शास्त्रं प्रणीतमित्यर्थः ।

किमनेन प्रयोजनम् ? उच्यते । अस्मिन् काले गीतिकोक्तभगणैस्त्रैराशिकेनानीता ग्रहोच्चपातमध्यमाः शुद्धाः । मकरादावुत्तरायणं कर्क्यादौ दक्षिणायनं च स्थितम् । इत उत्तरं ग्रहादिमध्यमेष्वयनद्वये व किञ्चित् सम्प्रदायसिद्धं क्षेपशोधनमस्तीति ज्ञापनम् । तच्छिष्येणे लल्लाचार्येण शिष्यधीवृद्धिदाख्ये महातन्त्रे (1. 1. 59-60) ग्रहमध्यमेषु तत्प्रभृति क्षेपशोधनोक्तेः ।

अयनक्षेपशोधनमपि सम्प्रदायविद्भिर्निबद्धेनार्याद्वयेनाह—
कल्यब्दात् 'खखषट्कृति'हीनाद् 'वसुशून्यनागशर'भक्तात् ।
शेषे 'द्विबाणशक्रैः' पदं भुजाब्दा द्विसंगुणिताः ॥
'शशिसूर्य'हृता लब्धं भागादिफलं भुजाफलवत् ।
ऋणधनमयनध्रुवयोः कुर्यात् ते दृक्समे भवतः ॥

इति । अतो ग्रहादिमध्येष्वयनद्वये च सम्प्रदायसिद्धं क्षेपशोधनं कर्तव्यम् ।

(*ABh.*, *Com.*, *Sūryadevayajvan*: *KVS*, pp. 93-94)

Time of zero precession: Āryabhaṭa's date of birth

When three quarter *yugas* (viz. Kṛta, Tretā and Dvāpara) had passed and also 3600 years (in the fourth, viz. Kali), then 23 years had passed after my birth. (10)

Sūryadevayajvan's Commentary

Here, in the current twentyeighth *yuga* when three *yuga* quarters and 3600 years had passed by, then twentythree years were over after my birth. That is, in the 3600th solar year of the current *yuga*-quarter, viz. Kali, this scientific treatise (viz. *Āryabhaṭīya*) has been composed by me who am twentythree years of age.

What is the purpose of this statement? I shall explain. At this point of time the mean positions of the apogees, nodes and planets would be exact when computed using the number of planetary revolutions enunciated in the *Gītikā* (section of the *Āryabhaṭīya*) (and so do not stand the need of any correction). Then the northern solstice was eaxctly at the beginning of Cancer and the southern solstice at the beginning of Capricorn, (the precession of the equinoxes then being zero). The above statement is to indicate that there is an additive or deductive correction, enunciated by tradition, to be done thenceforward, which addition and subtraction to be done to the mean planets, has been specified by his (i.e., Āryabhaṭa's) pupil Lalla-ācārya in his work entitled *Śiṣyadhīvṛddhida*. . .

The traditional additive-subtractive correction for the precession of the solstices, (and hence for the equinoxes as well), has also been set down as follows in two *āryā*-verses:

'Subtract 3600, (being the cut-off year), from the Kali year and divide by 5808. (Neglect the quotint and) divide the remainder by 1452. Treating the remainder as *bhuja*, multiply it by 2 and divide by 121, the result in degrees etc. is to be applied like the *bhujā-phala* to the longitudes of the planets and the solsticial points. By so doing, they would agree with observation.'

Hence the above additive-subtractive correction should be done to the mean positions of planets and of the solstices (and so also equinoxes). (KVS)

14. 4. 2. इत्यभावेऽयनांशानां कृतदृक्कर्मका ध्रुवाः ।
कथिताश्च स्फुटा बाणाः सुखार्थं पूर्वसूरिभिः ॥ १७ ॥
अयनांशवशादेषामन्यादृक्त्वं च जायते ।
शरज्या अस्फुटाः कार्याः स्फुटीकृतिविपर्ययात् ॥ १८ ॥
ताभिरायनदृक्कर्म मुहुर्व्यस्तं ध्रुवेष्वथ ।
अयनांशवशात् कार्यं तद्दृक्कर्म यथोदितम् ॥ १९ ॥
एवं स्युर्ध्रुवकाः स्पष्टाः शरज्याश्च ततः स्फुटाः ।
यथोक्तविधिना कार्यास्तच्चापानि स्फुटाः शराः ॥२०॥
ततो भग्रहयोगादि स्फुटं ज्ञेयं विजानता ।
इत्याधिक्येऽयनांशानामल्पत्वे त्वल्पमन्तरम् ॥ २१ ॥

(Bhāskara II, *SiŚi.*, 1. 11. 17-21)

Ancient astronomers happened to give a list of polar longitudes and polar latitudes at a time when there were no *ayanāṃśas*, i.e. when the zero-point of the ecliptic as taken by the Indian astronomers, namely Aśvini, coincided with the modern zero-point.[1]

[1] The date works out to the Kali year 4545 (A.D. 1444). For:

No. of years in a *catur-yuga*	=4,320,000 years
,, ,, Kali-age $\frac{1}{10} \times 4,32,0000$	=4,32,000 ,,
Kali-age dawn $\frac{1}{12} \times 43,20,000$	=36,000 years
One-eighth part of above $\frac{1}{8} \times 36,000$	=4500 years
One-hundredth part of the above $\frac{1}{100} \times 4500$	= 45 years
Date specified	4545 Kali

The author says in his commentary that he has stated this to specify the year of his birth.

[1] For the formula involved, see *SiŚi*: *AS*, pp. 519-20.

In fact, these polar longitudes and latitudes do change if there be *ayanāṃśas*. Here in this case, from the polar latitudes the celestial latitudes are to be computed in a reverse process. (18)

With the half of these celestial latitudes, the *āyana-dṛk-karma* is to be effected in the reverse proceess to obtain the celestial longitudes. After having obtained the correct celestial longitudes and celestial latitudes, and now, bringing the *ayanāṃśas* into the picture, compute the correct *dṛk-karma* and also rectify the celestial longitudes, to obtain the correct polar longitudes and polar latitudes to compute the moment of polar latitudinal conjunction. This case may be taken when the *ayanāṃśas* are large, otherwise, there will be a small difference (which is insignificant). (19-21). (AS)

अयनचलननिर्णय:

14. 5. 1. कल्यब्दात्तिथिषड्भिराप्तमयनं स्याद्राशिभागादिकं
तद्दो:क्रान्तिजलिप्तिका ऋणधनं स्याद् गोलतो भास्करे।
सौम्याद् दक्षिणतः क्रमाद् दिनदलच्छायाविधौ सर्वदा
नान्यस्मिन् कुमुदाधिपस्य च तथा मध्यप्रभाया विधौ ।।
(Deva, *KR*, 1. 3. 56.)

Computation of Precession

Divide the elapsed Kali years by 615: the result is the longitude of the *ayana-graha* (i.e., solstitial planet) in terms of signs, degrees, and so on. The declination, in terms of minutes, of that *ayana-graha* (is the motion of the equinoxes. This) should be subtracted from or added to the longitude of the Sun, according as the Sun is in the northern or southern hemisphere. This correction should always be applied while computing the midday shadow, but in no other computation. This correction should be applied in the case of the Moon too while computing the Moon's meridian shadow.[1] (36) (KSS)

14. 5. 2. अयनग्रहदो:क्रान्ति-
ज्याचापं केन्द्रत्वाद् धनर्णं स्यात् ।
अयनलवास्तत्संस्कृत-
खेटाद् आयनचरार्धपलजानि ।। १३ ।।
(ABh. II, *Mahā.*, 3. 13.)

[1] For the rationale, see *KR*: *KSS*, p. 25-26. Here, motion of the equinoxes $= \text{arc}\ \dfrac{\text{R sine } (\textit{ayana-graha}) \times \text{R sin } 24^\circ}{\text{R}}$, $\text{R} = 300'$, where *ayana-graha* $=$ Y/615 signs, and Y $=$ number of Kali years elapsed.

Deva is probably the first in the school of Āryabhaṭa I to have given a rule for finding the value of the precession of the equinoxes. He bases his rule on the assumption that the motion of the equinoxes is oscillatory, and that its rate is about 47″ per annum. The modern value is about 50″ per annum.

(Find out from) the base (of the longitude) of *ayanagraha* the sine of declination, (and from it the corresponding) arc. (This arc) is positive or negative according as the *kendra* (is positive or negative. The degrees of this arc) are called, the degrees of precession (*ayanalava*). From the (longitude of a) planet as corrected by these, the ecliptic deviation (*āyana*) and the ascensional difference in *palas* (*carārdhapalajāni*?) are to be calculated.[1] (13). (SRS)

14. 5. 3. 'गोऽगैकगुण'युक्शाकात् 'खाष्टवह्निनगै'र्हृतात् ।
भगणादे: क्रान्तिभागा ऋणस्वं सौम्यदक्षिणा: ।।

अयनांशा: प्रदातव्या लग्ने क्रान्तौ चरागमे ।
वित्रिभे सत्रिभे याते तथा दृक्कर्मपातयो: ।।
(Āmarāja's comm. on KK 3. 11, Q from Trivikrama's *KK-ṭika*, 3. 11)

Add 3179 to (the elapsed) Śaka years and divide the sum by 7380: the quotient gives (the longitude of the *ayana-graha* in terms of) revolutions etc. The corresponding declination should be applied (to the longitude of the planet) negatively or positively according as it is north or south. This motion of the solstice (or equinox) should always be applied while calculating the Sun's declination, the Sun's ascensional difference, longitudes of the horizon-ecliptic and meridian-ecliptic points and to the computations relating to the visibility of the planets and the *pāta*. (KVS)

14. 5. 4. अजतुलाक्षविवरज्या ।। २४ ।।
त्रिज्यागुणिता भक्ता परमापक्रान्तिजीवयाऽऽप्तधनु: ।
देयं ग्रहे यदा भा दक्षिणगोलादिगम्यभादूना ।। २५ ।।

महती मेषादिगतच्छायातस्त्वन्यथा शोध्यम् ।
पातेऽन्यथा विधेयं चापं त्रिप्रश्नकर्मविधौ ।। २६ ।।

षड्राश्यन्तरिताद् वा भानुमतोऽभीष्टकालिकात्साध्यम् ।
अयनचलनं स्वबुद्ध्या गणकेन हि चापचतुरेण ।। २७ ।।
(Vaṭeśvara, *VSi.*, 3. 2. 24-27)

Multiply the R sine of the difference between the latitudes due to the (*sāyana* and *nirayana*) *meṣādi* or *tulādi* by the radius and divide (the resulting product) by the R sine of the (Sun's) greatest declination. The arc corresponding to that (is the *ayana-calana*,

[1] The number of revolutions of *ayanagraha* in a *kalpa* is given in I.11-12. From these the positions of *ayanagrahv* at a given moment are to be calculated. (*tatsaṃskṛtakheṭāt āyanaṃ dṛkkarmādi carapalāni ca sādhyāni*: (Sudhākara Dvivedi).

The above correction is based on the assumption that the summer and winter solstices (and therefore the vernal and autumnal equinoxes) have an oscillatory motion of amplitude 24°, the period of one complete oscillation being 7380 years. The rate of motion of the equinoxes thus amounts to about 47″ per annum, the modern value being 50″ approximately.

which) should be added to the longitude of a planet provided the (midday) shadow of the gnomon at the time of (the Sun's next position at the *nirayana*) *tulādi* is greater than the (midday) shadow of the gnomon at the time of (the Sun's previous position at the *nirayana*) *meṣādi*, and subtracted in the contrary case. In the case of the Moon's ascending nodes, it is to be applied contrarily. This correction should be applied in all calculations pertaining to the *Tripraśna*, (Three Problems). The astronomer who is proficient in trigonometry (lit. 'the science of arc'), should calculate the *ayanacalana*, or when the Sun is six Signs distant from the *nirayana meṣādi* or *tulādi* or at any desired time, by the application of his own intellect. (24d-27). (KSS)

14. 5. 5. कल्यब्दे 'ज्ञानतुङ्गो'नः 'करकां'शोनितो हृतः ।
'नत्या'यनांशो 'देवां'शाच्छोध्या 'विप्रा'धिका यदि ।।

तत्संयुक्ताद् ग्रहात् क्रान्तिच्छायापातचराः स्फुटाः ।
(*VK*, 3. 1-2a)

Deduct 3600 from the Kali years gone, deduct from it the 121th part of itself and divide by 60. The total precession is obtained in degrees etc. If the precession so determined is more than 24°, deduct it from 48° to get (right) the precession.[1] (1)

In computing the *cara* (declinational ascensional difference, i.e. difference between day time and 30 *nāḍis*) the declination, the shadow, the mahāpāta etc., the Sun, the Moon and planets should be taken as true after adding the precession to them. (2a). (TSK-KVS)

14. 5. 6. विषुवत्क्रान्तिवलययोः सम्पातः क्रान्तिपातः स्यात् ।
तद्भगणाः सौरोक्ता व्यस्ता अयुतत्रयं कल्पे ।। १७ ।।

अयनचलनं यदुक्तं मुञ्जालाद्यैः स एवायम् ।
तत्पक्षे तद्भगणाः कल्पे 'गोङ्गर्तुनन्दगोचन्द्राः' ।। १८ ।।
(Bhāskara II, *SiŚi.*, 2.5. 17-18)

The point of intersection of the celestial equator and and the ecliptic is known as the *Kranti-pāta* (vernal equinox). As per the *Sūryāsiddhānta*, its revolutions are 3,00,000 retrograde during a Kalpa. The *ayanacalana* described by Muñjāla refers to the same, but the number of revolutions according to Muñjāla is 1,99,669 during a Kalpa.[2] (17-18). (AS)

इष्टकाले अयनांशानयनम्

14. 5. 7. 'गजाश्वनेत्र'रहिताः शकाब्दाः
'खसप्त'भक्ता अयनांशकाः स्युः ।
प्रत्यब्दजातायनजा गतिस्तु,
'रूपाक्ष'तुल्या विकलाः प्रदिष्टाः ।। २ ।।
(Jagannātha, *SiSam.*, pp. 6-7)

Precession at desired time

The Śaka years reduced by 278 and divided by 70 would give the total precession in degrees at any time. The rate of precession per annum is 51 seconds. (2). (KVS)

अयनचलनवेधप्रकारः

14. 6. 1. अयनचलनस्य मानज्ञानाय विलिख्यतेऽथ चोपायः ।
छायाप्रसाधितो यः सूर्यस्तत्कालगणितसिद्धश्च ।। ८५ ।।
योऽर्कस्तयोस्तु विवरं तत्काले प्रोच्यतेऽयनचलनमिति ।
भवति च यस्मिन् काले याम्योदग्गतिनिवृत्तिरर्कस्य ।। ८६
अयनान्तस्य च तत्कालेऽर्कस्य च विवरमयनचलनं स्यात् ।
शङ्कुं कृत्वात्युन्नतमचलस्तम्भादिकं प्रभां तस्य ।। ८७ ।।
ईक्षेत गमनकाले चागमकाले च गणितविद् भानोः ।
अर्कस्य गमनकाले यस्मिन् काले त्वभीष्टबिन्दुगतम् ।।
छायाग्रमागमेऽपि च तद्बिन्दुगतं यदा प्रभाग्रं स्यात् ।
तत्कालद्वयभवयोः स्फुटभान्वोर्योगदलसमोऽर्कः स्यात् ।।
यस्मिन् काले तस्मिन् काले हि गतिनिवृत्तिरर्कस्य ।
ऋणमथवा धनमिति च प्रकल्प्यते तद्धि गणितयुक्तिविदा ।।
(Parameśvara, *GD*, 4. 85-90)

Measuring precession with instruments

Now a method for knowing the amount of the precession of the equinoxes is given below. (85a)

The difference (in the longitude) of the Sun as determined by the shadow and as derived for that moment by calculation (from astronomical texts) is said to be the precession of the equinoxes at the time.

Or find the moment when the Sun is free from southward or northward motion. The difference between the Sun at that time and the solstice (i.e., the end of the *ayana*, according to astronomical texts) will be (the amount of) the precession of the equinoxes. (85b-87a)

The astronomer (*gaṇita-vid*) should take as the gnomon a very high pillar or the like, and observe its noon-

[1] The instruction to deduct from 48° is based on the oscillation theory propounded by most ancient Indian astronomers, and so is inaccurate.

[2] Calculated from this data, the yearly rate of motion of the equinoxes according to Muñjāla (year of writing A.D. 932) comes to 59.86″. During A.D. 932, the yearly rate of precession was 50.041″ (cf. Ball who says that the yearly rate of precesson was $50.2453 = .0002225\,t$, where t is the number of years from A.D. 1850). According to the Indian practice, adding the excess of the tropical year, viz. 9.76″ to the rate of precession, Muñjāla construed that the rate of precession was 59.86″, whereas the correct rate at that time, adding the said excess, was 59.8″ per year. What a correct estimate has Muñjāla made! It might also be noted that Muñjāla postulated secular precession and not liberation.

day shadow on its onward movement (north or south) and also on its backward movement. (87b-88a)

(He should observe) the movement when the end of the shadow passes a given point on the onward movement and the moment when the end of the shadow comes back to the same point. (88b-89a)

At the moment when the Sun is at (a position equal to) half the sum of the True Suns at the above two moments, the Sun will have been free from (northward or southward) motion. (89b-90a)

This (precession of the equinoxes) is determined either as additive or subtractive, by (the astronomer) versed in astronomical principles (*gaṇita-yukti-vid*).[1] (90b). (KVS)

[1] It is additive (*dhana*) when the longitude of the Sun derived from the shadow is greater than the value obtained by calculation from the texts.

15. शङ्कुच्छाया – GNOMONIC SHADOW

शङ्कुभेदाः

15. 1. 1. तले द्व्यङ्गुलविस्तारः समवृत्तो द्वादशोच्छ्रायः ।
सारदारुमयः शङ्कुर्द्वितीयो द्वादशाङ्गुलः ।। ३२ ।।

सूच्यग्रः स्थूलमूलोऽन्यस्तदुत्सेधस्तलाग्रयोः ।
सतिर्यङ्क्वेधसूच्योस्तु लम्बसूची स्फुटो नरः ।। ३३ ।।

तुल्याग्रस्तलवृत्तोऽन्यः शङ्कुः स्याद् द्वादशाङ्गुलः ।
या व्यक्ता शङ्कुभा यन्त्रात् सा व्यक्तैव नतप्रभा ।।३४।।

Gnomon—Types

(The first kind of gnomon is) two *aṅgulas* in diameter at the bottom, uniformly circular (*i.e.*, cylindrical), twelve *aṅgulas* in height, and made of strong wood.

The second kind of gnomon is twelve *aṅgulas* (in height), pointed at the top, and massive at the bottom (i.e., conical in shape). (Associated with it is) another true gnomon of the same height, mounted vertically on two (horizontal) nails fixed (to the previous gnomon) at the top and the bottom thereof. (32-33)

Another (third) kind of gnomon (which is more handy) is that having equal circles at the top and bottom (i.e., cylindrical) and of twelve *aṅgulas*.

Whatever shadow of the gnomon is seen to be cast by this instrument is indeed the projection of the (Sun's) zenith distance, *i.e.*, the R sine of the Sun's zenith distance. (34)

छायातो दिग्ज्ञानम्

15. 2. 1. वृत्ते समक्षितितलेऽन्तरसंस्थितस्य
तुल्याग्रमूलपरिधेः प्रगुणस्य शङ्कोः ।
भा द्वादशाङ्गुलमितेः प्रविशत्यपैति
यत्र क्रमाद् वरुणशक्रदिशौ मते ते ।। १ ।।

छायात्रयाग्रझषयोर्मुखपुच्छरज्ज्वो-
र्योगो यमस्य दिगुदग् द्युदलप्रभाग्रे ।
यत्र ध्रुवो धनपतेर्दिगसौ भवेद्वा
तन्मत्स्यपुच्छमुखरज्जुरसिद्धसिद्धौ ।। २ ।।

यो मत्स्यपुच्छमुखनिर्गतरज्जुयोग-
स्तस्मात् प्रभात्रितयचिह्नशिरोऽवगाहि ।
वृत्तं लिखेन्न विजहाति हि तस्य रेखां
छाया कुलस्थितिमिवामलवंशजा स्त्री ।। ३ ।।

(Lalla, *ŚiDhVṛ*. 4. 1-3)

Direction from gnomon shadow

When a gnomon 12 *aṅgulas* high and cylindrical in shape is placed vertically at the centre of a circle drawn on a smooth surface, the points (on the circumference of the circle) where the shadow of the gnomon comes inside the circle and goes outside it, are, respectively, known as the west and the east points. (1)

(Mark) the extremities of three shadows of the gnomon (on any day, two being the two above and the third, the extremity of the midday shadow). Draw two fish-figures (by means of intersecting arcs of circles drawn with the first and second points as centres and with a radius of any length and, again, with the second and third points as centres and a radius of the same length as before). Draw the two straight lines (passing through the points of intersection of these arcs) known, as mouths and tails of the fish-figures. Their point of intersection, the Sun being in the northern hemisphere, is the southern direction, the extremity of the midday shadow being to the north, that is, in the direction of the polar star. The point is the northern direction (the Sun being in the southern hemisphere and the midday shadow falling towards the south. The line joining the north or south points with the base of the gnomon is the north-south line). Then the straight line drawn at right angles to the north-south lines by means of a fish-fiigure (through the base of the gnomon) is the east-west line.

Describe the circle with the above point of intersection as the centre and passing through the three extremities of the shadows. The shadow (of the gnomon) never falls outside this circle, just as a woman of a noble family never abandons her family tradition.[1] (2-3). (BC)

मध्याह्नच्छाया

15. 3. 1. कर्कटकादिषु भुक्तं द्विगुणं माध्यन्दिनी भवेच्छाया ।
मकरादिषु चाप्येवं किन्त्वस्मिन्मण्डलाच्छोध्यम् ।। ९ ।।

मध्याह्नच्छायार्धं सत्रिभमर्कोऽयने भवेद्याम्ये ।
उदगयने संशोध्यं पञ्चदशभ्यो रविर्भवति ।। १० ।।

(Varāha, *PS*, 2. 9-10)

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[1] For an exposition, see *ŚiDhVṛ*: *BC*, II. pp. 58-64.

Midday shadow

When the Sun is in the six *rāśis* beginning with Karkaṭaka, the number of *rāśis* traversed from the beginning of Karkaṭaka multiplied by 2 is the midday shadow (of the twelve-digit gnomon) in *digits*. When the Sun is in the six *rāśis* beginning with Makara, find the distance in *rāśis* traversed by the Sun, from the beginning of Makara and multiply by 2. Subtract this from 12, to find the midday shadow. (9)

When the Sun is in its southward course, (i.e, in the six *rāśis* from Karkaṭaka) half the midday shadow plus three is the longitude of the Sun in *rāśis*. When in the northward course (in the six *rāśis* from Makara), half the noon-shadow subtracted from fifteen is the position of Sun in *rāśis*. (10). (TSK)

मध्याह्नच्छाया मध्याह्नकर्णश्च

15. 3. 2. भानुभिर्विनतभागशिञ्जिनी
सङ्गुणाथ भवनत्रयस्य च ।
उन्नताख्यलवजीवया भजेद्
भाश्रुती गगनमध्यगे रवौ ॥ १७ ॥

उत्तरेतरभषट्कगे रवौ
कुज्यया दिनगुणो युतोनितः ।
त्रिज्यका च चरखण्डजीवया
तत् क्रमाद् दिनदलान्त्यकान्त्यके ॥ १८ ॥

छेदहारकसमाह्वये च ते
हारकोऽथ गुणितो द्युजीवया ।
छेदकोऽथ भवति त्रिभज्यया
भाजितोऽयमथवान्यथा हृतिः ॥ १९ ॥

(Lalla, *ŚiDhVṛ*, 4. 17-19)

Midday shadow and hypotenuse

When the Sun is on the meridian, multiply the R sine of its zenith distance and the R sine of 90°, each by 12 and divide each product by the R sine of its meridian altitude. The quotients are, respectively, the midday shadow (of the gnomon) and the corresponding hypotenuse. (17)

The radius of the day-circle of the Sun increased or diminished by the earth-sine, and the radius increased or diminished by the R sine of the ascensional difference, according as the midday Sun is in the northern or southern hemisphere, give, respectively, the midday *antyā* and the *antyakā*. (18)

They are also called *cheda* and *hāraka*, respectively. When the *hāraka* is multiplied by the radius of the day-circle of the Sun and divided by the radius, the result is *cheda*. By the reverse process, (*cheda* is converted into *hāraka*). (19). (BC)

मध्याह्नच्छाया कर्णश्च—प्रकारान्तरम्

15. 3. 3. शङ्कुचापरहितत्रिभज्यका
दृग्गुणः, स विहृतोऽर्कसङ्गुणः ।
शङ्कुनैव भवति प्रभाऽथवा
तिग्मरोचिषि खमध्यसंस्थिते ॥ २१ ॥

युग्मचन्द्रविहृतेन शङ्कुना
भाजिता त्रिभवनस्य शिञ्जिनी ।
लब्धमङ्गुलमयं दिनार्धगे
कर्णमाहुरथ तिग्मदीधितौ ॥ २२ ॥

(Lalla, *ŚiDhVṛ*, 4. 21-22)

Midday gnomonic shadow

When the altitude of the Sun at midday or *unnatāṃśa* is subtracted from 90°, the R sine of the remaining arc is called *dṛgjyā* or R sine meridian zenith distance. This multiplied by 12 and divided by the R sine meridian altitude, gives the midday shadow. (21)

When the Sun is on the meridian, divide the R sine meridian altitude by 12, and then divide the R sine of 90° by this result. The final result is the hypotenuse of the midday shadow in *aṅgulas*. (So the wise) say. (22). (BC)

मध्याह्नच्छाया

15. 3. 4. अर्कापमाक्षधनुषोर्योगः साम्येऽन्यथान्तरम् ।
तस्य क्रमोत्क्रमगुणौ कार्यौ, क्रमगुणो हतः ॥ २१ ॥
'राज्ये'नोत्क्रमजीवोनत्रिज्ययाप्ता दिनार्धभा ।

(*VK*, 3. 21-26a)

The Midday shadow

Add the Sun's declination and the latitude of the place, if they are of the same direction (or Sign); deduct one from the other if they have different directions (or Sign). Find its sine, and also the co-versine (i.e., the combination of the segments taken in the reverse order). Multiply the sine by 12 and divide by 43 *minus* co-versine, (i.e. by the cosine). The midday gnomonic shadow is obtained. (21-22a)

विषुवच्छाया

15. 4. 1. भानौ गते क्रियतुलादिमहर्दले ये
छाये तयोर्युतिदलं विषुवत्प्रभा स्यात् ।
छायाकृतीनकृतियोगपदं तु कर्णः
कर्णेन वर्गविवरस्य पदं प्रभा स्यात् ॥ ४ ॥

(Lalla, *ŚiDhVṛ.*, 4. 4)

Equinoctial shadow

One-half of the sum of the lengths of the two midday shadows of the gnomon, when the Sun is at the first points of Aries and Libra, is the *palabhā* or 'equinoctial midday shadow'. The square root of the sum of the

squares of 12 (gnomon) and the shadow is the *palakarṇa* or the hypotenuse of the equinoctial midday shadow. The square root of the difference of the squares of the hypotenuse and 12 is the equinoctial midday shadow. (4). (BC)

विषुवच्छायातः स्वदेशाक्षो लम्बश्च

15. 5. 1. अक्षप्रभा रविहते पलकर्णभक्ते
त्रिज्ये फले पलगुणस्त्ववलम्बकश्च ।
तत्कार्मुके गगनषट्कहृते तदंशा
याम्यः पलो रविवशाद्विगपक्रमस्य ॥ ५ ॥
(Lalla, *ŚiDhVṛ.*, 4.. 5.)

Local latitude and colatitude from Eq. shadow

The radius severally multiplied by the equinoctial midday shadow and 12 (gnomon), and divided by the hypotenuse of the shadow, gives, respectively, the R sine of the latitude of the place and the R sine of the colatitude. Their arcs, divided by 60, give, respectively, in degrees, (the latitude and the colatitude). The latitude is always to the south. The direction of the declination of the Sun depends on its (i.e., the Sun's) position. (5). (BC)

15. 5. 2. विषुवद्दिनसममध्यच्छायावर्गात् स'वेदकृतरूपात्' ।
मूलेन शतं विंशं विषुवच्छायाहतं छिन्द्यात् ॥ २० ॥
लब्धं विषुवज्जीवा चापमतोऽक्षोऽथवैवमिष्टदिने ।
मेषाद्यपक्रमयुतस्तुलादिषु विवर्जितः स्वाक्षः ॥ २१ ॥
(Varāha, *PS*, 4. 19-22)

Measure the midday shadow on the day when the Sun is at the equinoxes (the equinoctial shadow). Square it, add 144, and find the square root. By this divide the product of the shadow multiplied by 120. The result is the sine of the latitude of the place, called *viṣuvajjyā*. Its arc is the latitude. Or, do this work on any other day and obtain the arc; If the Sun is in the six Signs from sāyana-Meṣa, i.e., if the Sun's declination is north, add the declination to the arc; the latitude is obtained. If the Sun is in the six Signs from sāyana-Tulā, i.e., if the declination is south, subtract the declination from the arc; the latitude is determined. (20-21). (TSK)

विषुवच्छायातः लम्बज्या, दिनव्यासश्च

15. 6. 1. विषुवज्ज्याऽऽयामार्धे वर्गविश्लेषमूलमवलम्बकः ।
क्रान्तित्रिज्याकृत्योरन्तरपदं द्विगुणं दिनव्यासः ॥ २३ ॥
अजवृषमिथुनापक्रमजीवाः षड्घ्नाः स्युर्वेदमुनिवसवः ।
त्र्यष्टकतिथिषट्काष्टकविकलाभ्यधिकाः परिज्ञेयाः ॥२४॥
(पञ्चत्रिंशत्) त्र्यष्टकसरूपधृतिसंयुता क्रमाद् द्विशती ।
पञ्चाष्टकतिथिविकलाधिकौ वृषान्त्यौ दिनव्यासः ॥
(Varāha., *PS*, 4. 23-25)

Sine co-latitude and day-diameter

Square the sine of latitude and deduct the result from the square of the radius. Its square root is the 'sine of co-latitude', (its arc being the 'co-latitude'). Square the sine of declination, deduct it from the square of the radius and find its root. Twice the result is the 'day-diameter'. (23)

The sines of the declinations of the points of the ecliptic ending Aries, Taurus and Gemini are 24′ 24″, 42′ 15″ and 48′ 48″. (24)

The respective day-diameters are, in terms of minutes: $200 + 35$, $200 + 24$, and $200 + 19$, with 40″ and 15″ added to the second and third, *i.e.*, the day-diameters are 235′, 204′ 40″ and 219′ 15‴.[1] (25). (TSK)

विषुवच्छायातः सममण्डलशङ्कुः

15. 7. 1. क्रान्तिज्यया विगुणितौ पृथगक्षकर्णौ
शङ्क्वङ्गुलैर्विषुवतः प्रभया च भक्तौ ।
अग्राप्तमाद्यमितरत् समवृत्तशङ्कु-
स्तद्वर्गयोगजपदं खलु तद्धृतिः स्यात् ॥ ६ ॥
(Lalla, *ŚiDhVṛ.*, 4. 6)

Samamaṇḍala-śaṅku from Eq. shadow

When the hypotenuse of the equinoctial midday shadow is multiplied by the R sine of the declination of the Sun and the product divided severally by the gnomon (of 12 *aṅgulas*) and by the equinoctial midday shadow, the results are, respectively, R sine of the amplitude of the Sun or R sine of the *agrā* or *agrajyā*, and R sine altitude of the Sun when on the prime vertical or *samaśaṅku*. The square root of the sum of the squares of the R sine *agrā* and *samaśaṅku* is the hypotenuse or *taddhṛti*. (6). (BC)

नतकालः इष्टकालच्छाया च

नतकालः

15. 8. 1. प्राक्कपालमथ पूर्वमम्बरं
पश्चिमं च तदुशन्ति पश्चिमम् ।
प्राचि यातमगतं तु पश्चिमे
कालमुन्नतमतोऽपरं नतम् ॥ २३ ॥
(Lalla, *ŚiDhVṛ*, 4.23)

Hour angle

That part of the sky which is to the east (of the observer's meridian) is the eastern hemisphere and the part to the west is the western hemisphere. So the wise say.

[1] For details see *PS* : *TSK*, 4. 23-25.

When the Sun is in the eastern hemisphere, the time passed since its rising is called *unnatakāla*, and the time to pass (till it reaches the meridian) is called the *natakāla* or hour angle.

(When the Sun is) in the western hemisphere, the time to pass before it sets) is called *unnatakāla*, and the time passed (since it crossed the meridian) is called the *natakāla* or hour angle. (23). (BC)

इष्टकालच्छाया

15. 8. 2. उन्नताच्चरदलेन वर्जितात्
संयुतादुदगसौम्यगोलयोः ।
शिञ्जिनी द्युगुणसङ्गुणा हृता
त्रिज्यया कुगुणयुक्तवर्जिता ।। २८ ।।

छेद इष्टसमये भवेत् ततः
शङ्कुभाश्रवणसिद्धिरुक्तवत् ।
व्यासखण्डकृतिशङ्कुवर्गयो-
र्दृग्गुणो भवति चान्तरात् पदम् ।। २५ ।।

(Lalla, *ŚiDhVṛ.*, 4. 23-25)

Shadow at any time

When the Sun is in the northern hemisphere, subtract the local ascensional difference from the time elapsed since sunrise or to elapse till the Sun sets, and add when the Sun is in the southern hemisphere. Find the R sine (of the result). Multiply it by the radius of the day-circle of the Sun and divide by the radius. The result increased or decreased by the earth-sine, (according as the Sun is in the northern or southern hemisphere), is the divisor or *cheda* at the given time or *iṣṭaccheda*. From this, one can find R sine altitude, shadow and the hypotenuse at any time, by the method given above.

The *dṛgjyā* or R sine zenith distance (at any time) is the square root of the difference of the squares of the radius and the R sine altitude. (24-25). (BC)

मध्याह्नशङ्कुः

15. 9. 1. लम्बकार्कसमशङ्क्वपक्रमैश्
छेदकः पृथगथाहतो हृतः ।
व्यासखण्डपलकर्णतद्धृती-
नाग्रकाभिरिह सन्ति शङ्कवः ।। २० ।।

(Lalla, *ŚiDhVṛ.*, 4. 20)

R sine of the midday altitude

Cheda multiplied severally by R sine of the co-altitude, 12, R sine of Sun's altitude when it is on the prime vertical, and by R sine of declination, and divided respectively by the radius, the hypotenuse of of the equinoctial midday shadow, *taddhṛti* and by R sine of the Sun's amplitude, gives, in each case, R sine of the Sun's midday altitude. (20). (BC)

शङ्कुच्छायातः रविमध्यम्

15. 10. 1. दिनमध्यच्छायार्कादुच्चविशुद्धाद् भूजाफलं यत् स्यात् ।
तत् क्षयधनविपरीतादविशेषविधे रवेर्मध्यम् ।। ५ ।।

(Bhāskara I, *MBh.*, 8. 5)

Mean Sun from midday shadow

Subtract the longitude of the Sun's apogee from the Sun's true longitude derived from the midday shadow (of the gnomon) and (then treating the remainder as the Sun's mean anomaly) calculate the Sun's equation of the centre. Apply that (equation of the centre) to the Sun's true longitude, contrarily to the usual law, for its subtraction and addition. (Treating this result as the mean longitude of the Sun, calculate the Sun's equation of the centre afresh and apply that to the Sun's true longitude as before.) Repeat the process again and again (until two successive results agree up to minutes). Thus is obtained the mean longitude of the Sun. (5). (KSS)

मध्याह्नच्छायातो रविस्फुटः

15. 11. 1. मध्यस्थे नभसः सहस्रकिरणे शङ्कोर्भवेद् या प्रभा
कल्प्या सैव पलप्रभोक्तविधिना साध्यस्तथेष्टः पलः ।
तत्स्वस्थानपलान्तरं यदपमस्तत् स्याद्युतिर्दक्षिणे
छायाग्रेऽस्य गुणेन भत्रयगुणः क्षुण्णः परक्रान्तिहृत् ।।

फलमिनभुजजीवा तद्धनुस्तिग्मरश्मिः
प्रथमचरण एवं वत्सरस्य प्रदिष्टः ।
हृतमथ भगणार्धाद्विद्धि भार्धेन युक्तं
भगणनिपतितं स्याच्चान्त्यपादेषु चापम् ।। ३८ ।।

(Lalla, *ŚiDhVṛ.*, 4. 37-38)

True Sun from midday shadow

When the Sun is on the meridian, assume the shadow of the gnomon at that time as the equinoctial midday shadow. Hence, find the latitude by the method given above. The difference between this latitude and the actual latitude of the place is the declination of the Sun. Their sum, however, would be the declination, if the shadow is to the south of the gnomon. The R sine of 90° multiplied by R sine of the declination and divided by R sine of the maximum declination, gives R sine of the longitude of the Sun. The corresponding arc is the longitude of the Sun.

Note that if it is in the first quadrant, that is, in the first quarter of the year, it is the longitude; if the arc is in the second quadrant, the remainder after subtracting it from six Signs, is the longitude; if in the third quadrant, 6 Signs should be added to it and the sum would be the longitude; and, if in the fourth, the remainder, after subtracting it from 12 Signs, is the longitude of the Sun. (37-38). (BC)

15. 11. 2. समममण्डलकर्णभाजिता
त्रिभजीवार्कगुणाऽक्षजीवया ।
निहता जिनजीवयोद्धृता
भुजजीवा सवितुर्धनू रविः ।। ४१ ।।

क्रान्तेर्जीवां श्रवणगुणितां लम्बमौर्व्या विभक्ता-
मक्षच्छायाविरहितयुतां गोलयोर्बाहुमाहुः ।
छायावर्गादपभुजकृतेर्यत्पदं तद्धि कोटिः
केन्द्राच्छायावलयजतले सा च पूर्वापरा स्यात् ।।४२।।

(Lalla, *ŚiDhVṛ.*, 4. 41-42)

The radius multiplied by 12 and by R sine of the latitude (of the observer's station) and divided by the hypotenuse of the shadow when the Sun is on the prime vertical, and also by R sine of 24°, gives R sine of the longitude of the Sun. The corresponding arc is its longitude. (41)

Multiply R sine of the Sun's declination by the hypotenuse of the shadow at any time and divide by R sine of the colatitude (of the observer's station). This quotient increased or diminished by the equinoctial midday shadow, according as the Sun is in the southern or northern hemisphere, is called *bāhu* or *bhujā*. The square root of the difference of the squares of the shadow and the *bhujā* is called *koṭi* or perpendicular. This line is the east-west line in the circle, the centre of which is the point of intersection of the lines indicating the directions (in the plane of the dial), and the radius is the shadow (of the gnomon at the centre). (41-42). (BC).

छायातः कालानयनम्

15. 12. 1. पलश्रवणताडिता त्रिभवनोत्थजीवाकृतिः
प्रभाश्रवणताडितद्युगुणभाजितात् तत्फलात् ।
चरासुगुणसंयुतादनुदगुत्तरे वर्जिताद्
धनुश्चरदलोनयुग्दिनगतावशेषासवः ।। ३१ ।।

फलाच्चरज्या न विशुद्धिमेति
यदा तदास्यां फलमेव जह्यात् ।
शेषस्य चापेन चरार्धमूनं
शेषं गतं वा दिवसस्य शेषम् ।। ३२ ।।

(Lalla, *ŚiDhVṛ.*, 4. 31-32)

Time from Shadow

Multiply the square of the radius by the hypotenuse of the equinoctial midday shadow and divide by the product of the hypotenuse of the shadow at the time required and the radius of the Sun's day-circle. Increase or diminish the quotient by R sine of the ascensional difference (according as the Sun is in the southern or northern hemisphere). Find the corresponding arc. This arc, increased or diminished by the ascensional difference (according as the Sun is in the northern or southern hemisphere) is the number of *asus* in the day passed since sunrise, (if it is before midday), or that to pass till sunset (if it is after midday). (31)

(If the above quotient is less than R sine of the ascensional difference), and, so the latter cannot be subtracted, subtract the former from the latter. Find the arc of the remainder and subtract it from the ascensional difference. The remainder is the day passed since sunrise, (if it is before midday), or that to pass till sunset, (if it is after midday). (32). (BC)

15. 12. 2. मध्यच्छायाशङ्क्वोः कृतियुतिमूलं त्रिभिः शतैर्गुणितम् ।
शङ्क्विष्टच्छायाकृतिसमासमूलोद्धृतं ज्या स्यात् ।।

जीवापिण्डाज्जीवां हित्वा हित्वा क्षिपेद् दशदशांशान् ।
दशगुणशेषं हृत्वा स्थितिज्यया योज्य इति काष्ठा ।।

सा रविशशिनोर्दिनदलगुणिता
'वियत्खजलधिविषय' विहृता ।
षष्टिहृता गतनाड्यो
दिवसविशुद्धाः स्युरपराह्णे ।। १३ ।।

(Deva, *KR*, 4. 11-13)

Find the square root of the sum of the squares of the midday shadow and the gnomon. Multiply that (square root) by 300 and divide by the square root of the sum of the squares of the gnomon and the given shadow: the result is the so-called 'R sine'. (11)

From this R sine subtract as many tabular R sine-differenees as possible and for each R sine-difference subtracted, take 10° and sum them up. The ultimate residue should be multiplied by 10 and divided by the current R sine-difference. The resulting degrees should also be added to the previous sum. The sum (thus obtained) gives (the degrees in) the arc (corresponding to the 'R sine'). (12)

That arc multiplied by (the *vināḍīs* in) half a day of the Sun or the Moon (as the case may be) and divided by 5400 and also by 60, gives the *nāḍīs* elapsed (in the forenoon) or the same *nāḍīs* subtracted from the *nāḍīs* in a day, give the *nāḍīs* to elapse in the afternoon.[1] (13). (KSS)

[1] That is, the desired time in *nāḍīs*

$$= \frac{\text{arc}\left[\dfrac{\sqrt{(\text{midday shadow})^2+(\text{gnomon})^2}\times 300}{\sqrt{\{(\text{given shadow})^2+(\text{gnomon})\}}}\right]\times \text{semi-duration of day in } vin\bar{a}\d{d}\bar{\imath}s}{5400\times 60}$$

This rule is just the reverse of the rule stated below in verses, 8-9, and can be easily deduced therefrom.

कालतः छाया

15. 13. 1. षड्गुणितेष्टविनाड्यः तदहर्विघटीहृतास्तु भवनाति ।
दोर्ज्या शङ्कुदिनार्धच्छायाकृतियोगमूलमपि ॥ ८ ॥
त्रिशतगुणे ज्याभजिते लब्धकृतौ शङ्कुवर्गरहितायाम् ।
मूलं षष्ट्या लब्धं छाया पूर्वापरकपाले ॥ ९ ॥
(Deva, *KR*, 4. 8-9)

Shadow from time

Multiply the given *vināḍīs* (elapsed since sunrise in the forenoon or to elapse before sunset in the afternoon) by 6, and divide by the *vināḍīs* in a day (i.e., by 1800); the result is the Signs corresponding to time elapsed since sunrise in the forenoon or to elapse before sunset in the afternoon (i.e., *unnatakāla*). Find the R sine of these signs (and set it down at some place to be used later). Now, find the square root of the sum of the squares of the gnomon and (the *vyaṅgulas* of) the midday shadow. Multiply this (square root) by 300 and divide by the R sine (kept at the other place, and diminish the square of the result by the square of the gnomon. The square root thereof divided by 60, gives the shadow (in terms of *aṅgulas*) in the eastern or western hemisphere (as the case may be).[1] (8-9). (KSS)

छायातोऽपक्रमः

15. 14. 1. छायाग्रपूर्वापररेखयोर्यत्
स्यादन्तरं तत् पलभावियोगः ।
सौम्ये युतिर्दक्षिणगे च लम्ब-
ज्या तद्वधः कर्णहृतोऽपमज्या ॥ ३९ ॥
(Lalla, *ŚiDhVṛ.*, 4. 39)

Declination from shadow

Find the distance between the extremity of the shadow at any time and the east-west line. Subtract it from the equinoctial midday shadow when the Sun is in the northern hemisphere and add when it is in the southern hemisphere. The result multiplied by R sine of the colatitude (of the observer's station) and divided by the hypotenuse of the shadow, gives R sine of the declination of the Sun. (39). (BC)

छायातो लग्नम् लग्नात् छाया च

15. 15. 1. द्वादशभिः सच्छायैर्मध्याह्नोनैर्भजेद् 'रसहुताशम्' ।
अपराह्णे चक्रार्धाद् विशोध्य सार्कं भवति लग्नम् ॥११॥
व्यर्के लग्ने लिप्ताः प्राक्पश्चात् शोधितास्तु चक्रार्धम् ।
कार्यश्छेदः 'शून्याम्बराष्टलवणोदषट्कानाम्' ॥ १२ ॥
लब्धं द्वादशहीनं मध्याह्नच्छायया समायुक्तम् ।
सा विज्ञेया छाया वासिष्ठसमाससिद्धान्ते ॥ १३ ॥
(Varāha, *PS*, 2. 11-13)

Lagna from Shadow and vice versa

Add 12 to the shadow (of the twelve-digit gnomon, measured in digits) at any time of the day, and deduct from it the midday shadow for the day. Divide 36 by this and take the result. This result taken as *rāśis*, plus the Sun in *rāśis* is the *lagna* at the moment, if it is forenoon. If afternoon, deduct this from the Sun plus six *rāśis* and the *lagna* is obtained.

The formulae, (a) for the forenoon and (b) afternoon, respectively, can be expressed thus: (a) Lagna=Sun+36/ (12 shadow—noon shadow): (b) Lagna=Sun+6—36/ (12 shadow—noon shadow). (11)

Deduct the Sun from the *lagna* and convert the remainder into minutes of arc, if forenoon. If afternoon, deduct the minutes from a half circle, i.e., from 10800 minutes and take these as minutes. Divide 64800 by the minutes so obtained. Add the result to the noon shadow of date, and deduct 12 from this. This is the shadow at the time of the given *lagna*. This is, according to the brief *Vāsiṣṭha Siddhānta*. (12-13). (TSK)

लग्नात् कालः

15. 16. 1. 'वसुभं' 'व्येकत्रिशतं' 'शिखियमदहनं' क्रमोत्क्रमान्न्यस्य ।
चरदलविरहितसहितं मेषादि तुलादि चोत्क्रमतः ॥ ६ ॥
अर्केन्द्वभुक्तलिप्ताः स्वोदयगुणिता 'नभोऽम्बरपुराणैः' ।
लब्धाः पुरोदययुताः प्राग्लग्नाल्लग्नकालः स्यात् ॥ ७ ॥
(Deva, *KR*, 4. 6-7)

Time from lagna

First write down the numbers 278, 299, and 323 in serial order and then the same number in the reverse order: (the six numbers thus written down denote the right ascensions, in terms of *vināḍīs*, of the six Signs beginning with Aries. The same numbers in the reverse order are the right ascensions, in terms of *vināḍīs*, of the six Signs beginning with Libra). Now diminish the numbers 278, 299, 323 by the ascensional differences, in terms of *vināḍīs*, of the Signs, Aries, Taurus and Gemini respectively, and write them down in the serial order; then increase the same numbers by the ascensional differences, in terms of *vināḍīs*, of the signs, Aries, Taurus and Gemini, and write them down in the reverse order. The six numbers thus written down are the oblique ascensions, in terms of *vināḍīs*, of the six Signs beginning with Aries; and the same six numbers in the reverse order are the oblique ascensions, in terms of *vināḍīs*, of the six Signs beginning with Libra. (6)

Multiply the minutes to be traversed of the Sign occupied by the Sun or the Moon by the oblique ascension of that Sign and divide by 1800. Add the resulting (*asus*) to the (*asus* of the) oblique ascensions of the (succeeding) Signs which have risen prior to the rising

[1] For the rationale, see *KR*: *KSS*, pp. 68-69.

point of ecliptic (*lagna*). This is how time is obtained from (the longitude of) the *lagna*.[1] (7). (KSS)

कालतो लग्नः

15. 17. 1. लग्नार्थं रविशशिनोरभुक्तलिप्ता निजोदयाभ्यस्ता ।
राशिकलाभिर्लब्धात् स्वेष्टविनाडीगणात् त्यक्त्वा ॥१४॥
सम्पूर्य वर्तमानं राशिमुपर्युद्गमात् त्यजेच्छेषात् ।
राशिस्तस्मिन्नेव क्षिपेत् तदा भवति लग्नं तत् ॥ १५ ॥
(*Deva*, *KR*, 4. 14-15)

Lagna from time

To find the longitude of the rising point of the ecliptic (proceed as follows): Multiply the untraversed portion of the Sign occupied by the Sun or the Moon by that Sign's own time for rising (at the local place) and divide by the number of minutes in a Sign (i.e., by 1800); then subtract the resulting (*vināḍīs*) from the given time, in terms of *vināḍīs*; then having completed the current sign, subtract from the residue (of the *vināḍīs*) the times of rising of as many succeeding Signs (or part of a Sign) as possible and add those Signs (and part of a Sign, if any) (to the completed Signs). Whatever is thus obtained is the longitude of the rising point of the ecliptic (called *lagna*). (14-15). (KSS)

कोणशङ्कुः

15. 18. 1. सूर्याग्राया इष्टसंयोजिताया
वर्गे त्रिज्यावर्गतो लोचनघ्ने ।
त्यक्ते मूलं यत् स शङ्कुर्विदिक्स्थ-
स्तत्राक्षाभासङ्गुणे सूर्यभक्ते ॥ ३४ ॥
लब्धमिष्टममुना सहिताग्रा
प्राग्वदेवमसकृन्नरसिद्धिः ।
दक्षिणे सवितुरुत्तरगोले-
ऽग्रज्ययेष्टवियुताद् विधिरेषः ॥ ३५ ॥
(Lalla, *ŚiDhVṛ.*, 4. 34-35)

R sine altitude in an intermediate direction

When the Sun is in the southern hemisphere and on a *koṇavṛtta* or a vertical circle in an intermediate direction, add its R sine amplitude or *agrajyā* to the assumed length of the *śaṅkutala* or the base of the altitude. Square the result, double it and subtract from the square of the radius. The square root of the remainder is the R sine altitude at that time or *koṇaśaṅku*. Multiply it by the equinoctial midday shadow (at the observer's station) and divide by 12. The result is a more correct (base). Add it to the R sine amplitude or *agrajyā*, and repeat the process till the R sine amplitude is fixed. When the Sun is in the northern hemisphere, R sine amplitude should be diminished by the assumed (base). This is the rule.[1] (34-35). (BC)

कोणशङ्कुच्छाया

15. 19. 1. यदा भवेत् क्रान्तिरुदक् पलाल्पा
विशेत् तदानीं समवृत्तमर्कः ।
तदुत्थशङ्कोरथ कोणजाद् वा
छाया च कर्णश्च यथेष्टशङ्कोः ॥ ३६ ॥
(Lalla, *ŚiDhVṛ.*, 4.36)

Shadow of R sine altitude in an intermediate direction

When the Sun's declination is to the north and is less than the latitude (of the observer's station), the Sun enters the prime vertical. From the then R sine altitude and from the R sine altitude when the Sun is on the *koṇavṛtta* or a vertical circle in an intermediate direction, the shadows and their hypotenuses can be calculated.[2] (36). (BC)

कोणच्छायातो रविस्फुटः

15. 20. 1. विदिक्छायावर्गाद्दलपदमुदक्स्थे नरतले
पलच्छायायुक्तं रहितमनुदक्स्थे भवति यत् ।
फलं तल्लम्बघ्नं श्रुतिहृतमिनापक्रमगुणो
भवेत् त्रिज्याघ्नोऽसौ जिनलवगुणाप्तो रविगुणः ॥
(Lalla, *ŚiDhVṛ.*, 4. 40)

True Sun from shadow in intermediate direction

Find the shadow from R sine of the altitude when the Sun is on the north-east or north-west vertical circle (*koṇaśaṅku*), square it and halve the result. Find its square root. If the *śaṅkutala* or the base is to the north, add the result to the equinoctial midday shadow (at the observer's station) and subtract, if it is to the south. The result multiplied by R sine of the colatitude (of the observers's station), and divided by the hypotenuse of the shadow gives R sine of the Sun's declination. This, multiplied by the radius and divided by R sine of 24°, gives R sine of the Sun's longitude.[3] (40). (BC)

शङ्कुच्छायाविधिः लेखनं च

15. 21. 1. कृतदिग्ग्रहणे वृत्ते रेखां पूर्वापरां यदा छाया ।
प्रविशति सम्यक्छङ्कोः सममण्डलगस्तदा सूर्यः ॥ ३८ ॥

अथा

इष्टक्रान्तिज्याघ्नव्यासशकललम्बकांशमुष्णांशुः ।
समपूर्वापररेखामतीत्य यात्यस्तमुदयं वा ॥ ३९ ॥

[1] There are 21600 *asus*, 3600 *vināḍīs* and 60 *nāḍīs* in a sidereal day, so that the *asus* divided by 6 give the *vināḍīs* and the *vināḍīs* divided by 60 give the *nāḍīs*. For the Tables, see *KR*: *KSS*, pp. 67-68.

[1] For demonstration, see *ŚiDhVṛ*: *BC*, II. pp. 87-91.

[2] For explanation and rationale, see *ŚiDhVṛ.*: *BC*, II, pp. 90-91.

[3] For a demonstration, see *ŚiDhVṛ.*: *BC*, II. 94-96.

अग्रया अक्षः

तेन हृता खार्कघ्नी क्रान्तिज्या लम्बकोऽस्य यच्चापम् ।
तेन नवतिर्विहीना यच्छेषं तेऽक्षभागाः स्युः ।। ४० ।।

इष्टकालच्छाया

तत्कालचरविनाडी द्विदशांशं द्विष्ठमजतुलाद्येषु ।
षड्घ्नीभ्यो नाडीभ्यो जह्यात् संयोजयेच्चापि ।। ४१ ।।

तज्ज्या स्थितज्याया संयुता विसंयोजिताऽजतुलाद्येषु ।
अविशोधने च जीवा षड्घ्नीनामेव कर्तव्या ।। ४२ ।।

एवं कृत्वा हन्यात् द्युव्यासेनावलम्बकघ्नेन ।
छिन्द्यात् 'खखाष्टवस्वश्वभिः' फलं शङ्कुलिप्ताख्यम्' ।।

तत्कृतिविनाकृतानां 'खखवेदसमुद्रशीतरश्मीनाम् ।
पदमर्कघ्नं शङ्क्वङ्गुलाऽऽख्यलिप्तोद्धृतं छाया ।।४४।।
(Varāha, *PS*, 4. 38-44)

Practical work on shadow diagrams

On a circle with the east-west line drawn, and the directions marked, (according to *PS* 4. 19), the time when the gnomonic shadow perfectly coincides with the east-west line, is the time of the Sun crossing the prime vertical. (38)

Agrā

Multiply the Sun's declination by the radius and divide by the sine of co-latitude, and find the sine (of the amplitude of the rising or setting point, called *agrā*). At a point distant by this amount from the west-east line, according to the declination north or south, the Sun rises or sets. (39)

Latitude from agrā

Multiply the sine declination by 120 and divide by the sine of amplitude. The sine of co-latitude is obtained. Find its arc in degrees. Deduct the degrees from 90. The remainder is the degrees of latitude. (40)

Shadow at desired time

To find the gnomonic shadow caused by the Sun at any time: Take the *cara* in *vināḍīs* and divide by 20. Degrees of half-*cara* are obtained. Place the degrees in two places. Convert the time from sunrise in *nāḍīs*, into degrees by multiplying by 6. From these degrees, deduct or add the half-*cara* degrees, according as the Sun is in the six Signs beginning with Meṣa, or in the six Signs beginning with Tulā, respectively. (41)

Find the sine of the resulting degrees and add to or subtract this from the sine of the half-*cara* kept apart in the second place, according as the Sun is in the 6 Signs Meṣa etc., or in the six Signs Tulā etc. The result is a sine. If the half-*cara* degrees cannot be deducted from the time converted into degrees, then simply find the sine of the degrees of sine, and take it for further work. (42)

Multiply this sine by the sine of co-latitude and the day-diameter, and divide by 28,800. The result is sine altitude of the Sun. (43)

Square this and deduct from 14,400. Take its square root, multiply this by twelve, and divide by sine altitude. The result is the length of the shadow of the twelve digit gnomon.[1] (44). (TSK)

विषुवच्छायासाधनविधयः

15. 21. 2. कार्यं स्थण्डिलमथवा वृत्तं जलसिद्धमस्तकं विपुलम् ।
भगणांशाङ्कितपरिधिं स्वस्कन्धसमुच्छ्रितं च सिद्धाशम् ।।

तस्यापरभागस्थो दिग्योगन्यस्तदृष्टिरुद्यन्तम् ।
पश्यति यत्र खरांशुं तद्भागज्या रवेरग्रा ।। १३ ।।

अग्रा द्वादशगुणिता क्रान्तिज्याभाजिता पलश्रवणः ।
श्रुतिशङ्क्वन्तरगुणितात्तद्योगान्मूलमक्षाभा ।। १४ ।।

क्रान्तिज्याग्राकृत्योर्विशेषमूलं द्युमण्डले कुज्या ।
द्वादशगुणिता कुज्या क्रान्तिज्याहृत् पलाभा वा ।। १५ ।।

सूर्याभिमुखी यष्टिर्धार्या तद्वत् त्रिभज्यया तुल्या ।
यद्वत् छायाभावः शङ्कुस्तल्लम्बकः प्रोक्तः ।। १६ ।।

तत्पूर्वापररेखाविवरं बाहुः नृयष्टिभातुल्यम् ।
दृग्ज्या कर्णो यष्टिः द्युदलभुजो दृग्ज्यया तुल्यः ।। १७ ।।

बाह्वग्रयोः समासो भिन्नदिशोरन्तरं तथैक[दिशोः ।
शङ्कुतलं शङ्कुगुणा] तज्ज्या लम्बकहृताऽक्षाभा ।।१८।।

सूर्यघ्ना वाऽक्षज्या लम्बज्याहृच्च पलभा वा ।। १९ ।।

द्युदलच्छायाभ्यस्ता सूर्याग्रा च स्वदृग्ज्यया भक्ता ।
फलयुतहीना भुजवद् द्युदलाभा [वा] पलच्छाया ।।२०।।

इष्टद्वययोर्भुजयोः साम्यं ककुभोर्वियोगसंयोगः ।
सूर्याहतो विभक्तः शङ्क्वोर्विवरेण वा पलच्छाया ।।

अन्योन्यकर्णनिघ्नौ श्रुतिविवरहृतौ प्रभाद्वयोर्बाहू ।
इष्टफलविवरयुतिः समान्यककुभोः पलच्छाया ।।२२।।

द्वादशगुणिता वाऽग्रा सममण्डलशङ्कुभाजिताऽक्षाभा ।
स्वधृतिः समशंकुहृता रविगुणिता वा पलश्रवणः ।।

त्रिज्या द्वादशगुणिता भक्ता लम्बज्ययाऽथवा कर्णः ।
समकर्णगुणा कुज्या पलजीवाहृत् पलाभा वा ।। २४ ।।

स्वदेशाक्षः

दिनदलदृग्ज्याचापं क्रान्त्या युतवर्जितं क्रियतुलादौ ।
अक्षो दक्षिणदृग्ज्याधनुषोना क्रान्तिरक्षं स्यात् ।। २५ ।।

शङ्कुं परिकल्प्य भुजं त्रिभुजेन विलोकयेत् ध्रुवमुदीच्याम् ।
यन्त्रेण दृष्टिभुजयोर्विवराग्रा वा पलच्छाया ।। २६ ।।

[1] For elucidation, see *PS*: *TSK*, 4.41-44.

उत्क्षिप्तैकाक्षिर्वा दक्षिणदिशि यस्य मस्तकासक्तम् ।
पश्यति पौष्णं शङ्कोस्तन्मूलदृगन्तरं वाऽक्षाभा ।। २७ ।।

त्रिज्याग्राकृतिवियुतेः पदं द्विनिघ्नमुदयास्तसूत्रं स्यात् ।
उदयास्तसूत्रतस्स्याच्छंक्वग्रप्ररोपिणी स्वधृतिः ।। २८ ।।

नृतलास्तोदयसूत्रान्तरं रविगुणं नृहृत्पलाभा वा ।
स्वधृतिर्वा सूर्यगुणा शङ्कुविभक्ता पलश्रवणः ।। २९ ।।

इष्टच्छायाभ्यस्तं नृतलं दृग्ज्योद्धृतं पलाभा वा ।
अथवेष्टकर्णगुणितं यष्टिविभक्तं [च] पलभा स्यात् ।। ३० ।।

अग्रेष्टाभागुणिता दृग्ज्याभक्ता वाग्रेष्टभाभुजयोः ।
एकान्यदिगुद्भवयोर्विव[रै]क्यं वा पलच्छाया ।। ३१ ।।

इष्टश्रुतिगुणिताऽग्रा त्रिज्याहृता [भाकर्णवृत्ताग्रा ।
इष्टश्रुतिगुणिता बाहुस्तथैव त्रिज्यया भक्ता] ।। ३२ ।।

लब्धेष्टभाभुजान्या पलप्रभा प्रोक्तवद्वा स्यात् ।। ३३ ।।

प्रोक्तपलाभाकृत्योर्विवरपदेनोदयास्तसूत्रं यत् ।
तच्छङ्कुभ्रमवृत्तान्तरं पलभेष्टभावृत्ते ।। ३४ ।।

उज्जयिनीयाम्योत्तरयोजनविषयाहृतेः [फलं यद्] वा ।
निजविषयादुत रेखाविषयनिरक्षान्तरं शराभ्यस्तम् ।। ३५ ।।

'रसकृत'हृत्पलभागाः 'शरगुणगगना'पहृदक्षाभा ।। ३६ ।।

(Vaṭeśvara, *VSi.*, 3.1. 12-36)

Determination of Equinoctial midday shadow

Method 1

One should build an earthen platform which should be large, circular, as high as one's shoulders, with surface levelled with water, with circumference graduated with Signs and degrees, and with well-ascertained cardinal points. (12)

Let a person, standing on the western side of that (platform) observe the rising Sun through the centre of the circle. Then the R sine of the degrees of that point of the circle where he sees the rising Sun is the Sun's *agrā*. (13)

The (Sun's) *agrā* multiplied by 12 and divided by the R sine of the (Sun's) declination is the hypotenuse of the equinoctial midday shadow (*palaśravaṇa* or *palakarṇa*). By the difference between the hypotenuse of the equinoctial midday shadow and the gnomon multiply their sum and take the square root (of the product); the result is the equinoctial midday shadow (*akṣābhā* or *palabhā*). (14)

Method 2

The square root of the difference between the squares of the R sine of the (Sun's) declination and the *agrā* is the earthsine (*kujyā*), which lies in the plane of the (Sun's) diurnal circle. The earthsine multiplied by 12 and divided by the R sine of the (Sun's) declination is also the equinoctial midday shadow (*palābhā* or *palabhā*). (15)

Method 3

One should hold a *yaṣṭi*, equal to the radius of the celestial sphere, pointing towards the Sun in such a way that it may not cast any shadow. Then, the perpendicular (dropped on the ground from the upper extremity of the *yaṣṭi*), which is called 'Upright', is the *śaṅku* (or R sine of the Sun's altitude). (16)

The distance between (the foot of) that (*śaṅku*) and the east-west line is (called) the *bāhu* or (base). The shadow of that *śaṅku-yaṣṭi* is equal to the R sine of the (Sun's) zenith distance. The *yaṣṭi* is the hypotenuse. The *bāhu* for the middle of the day is equal to the R sine of the Sun's (meridian) zenith distance. (17)

The sum or difference of the *bāhu* and the *agrā*, according as they are of unlike or like directions, is the *śaṅkutala*. That *śaṅkutala* multiplied by 12 and divided by the upright (R sine of the Sun's altitude) gives the equinoctial midday shadow. (18)

Method 4

The equinoctial midday shadow (*palabhā*) is also equal to the R sine of the latitude multiplied by 12 and divided by the R sine of colatitude. (19)

Method 5

Multiply the Sun's *agrā* by the midday shadow and divide by the R sine of the Sun's own (i.e., meridian) zenith distance. The resulting quotient when added to or subtracted from the midday shadow, in the same way as the *agrā* is added to or subtracted from the *bāhu*, also gives the equinoctial midday shadow. (20)

Method 6

Find the difference or sum of the two given *bhujās* (of shadow) according as they are of like or unlike directions. Multiply (the difference or sum thus obtained) by 12 and divide by the difference between the R sine of the Sun's altitude corresponding to the two *bhujās*: the result is the *aṅgulas* of the equinoctial midday shadow (*akṣabhā* or *palabhā*). (21)

Method 7

Multiply each of the two given *bhujās* of shadow by the hypotenuse of shadow corresponding to the other *bhujā*, and divide (both the products) by the difference between the two hypotenuses of shadow. The difference or sum of the two results, according as they are of like or unlike directions, is the equinoctial midday shadow. (22)

Method 8

Or the *agrā* multiplied by 12 and divided by the R sine of the Sun's prime veritcal altitude, gives the equinoctial midday shadow.

Also, the *taddhṛti* multiplied by 12 and divided by the R sine of the Sun's prime vertical altitude gives the hypotenuse of the equinoctial midday shadow. (23)

Method 9

Or, the hypotenuse of the equinoctial midday shadow is equal to the radius multiplied by 12 and divided by the R sine of co-latitude; and the equinoctial midday shadow is equal to the earth's sine multiplied by the hypotenuse of the prime vertical shadow and divided by the R sine of latitude. (24)

The Sun's zenith distance for midday increased or diminished by the Sun's declination according as the Sun is in the six Signs beginning with the first point of Aries or in the six Signs beginning with the first point of Libra, gives the latitude. (But when at midday the Sun is to the north of the zenith) the Sun's declination diminished by its northern zenith distance gives the latitude. (25)

Method 10

One should observe the pole star towards the north along the hypotenuse of the triangle-instrument, assuming its base to be equal to the gnomon; then the upright (of the triangle-instrument), which lies between the line of vision and the base, will be equal to the equinoctial midday shadow. (26)

Method 11

When a person, with one of his eyes raised up, observes towards the south the star Revatī as clinging to the tip a (vertical) gnomon, then the distance between the foot of the gnomon and the eye equals the equinoctial midday shadow. (27)

Method 12

The square root of the difference between the squares of the radius and the *agrā*, multiplied by 2, gives the length of the rising-setting line. The distance from the rising-setting line to the (upper extremity of the (great) gnomon is the *svadhṛti*. (28)

Method 13

The distance between the foot of the (great) gnomon and the rising-setting line, multiplied by 12 and divided by the (great) gnomon (i.e., the R sine of the Sun's altitude) is also the equinoctial midday shadow. And the *svadhṛti* multiplied by 12 and divided by the (great) gnomon, gives the hypotenuse of the equinoctial midday shadow. (29)

Methods 14 *and* 15

Or, the *śaṅkutala* multiplied by the given shadow of the gnomon and divided by the R sine of the Sun's zenith distance gives the equinoctial midday shadow; the same (*śaṅkutala*) multiplied by the hypotenuse of the given shadow and divided by the *yaṣṭi* (i.e., the radius) also gives the equinoctial midday shadow. (30)

Method 16

Multiply the *agrā* by the given shadow and divide by the R sine of the Sun's zenith distance: (the result is the *chāyākarṇāgrī agrā*). The difference or sum of that 'result' (viz. the *chāyākarṇāgrī agrā*) and the *bhujā* for the given shadow (*iṣṭabhujā or chāyākarṇāgrī bhujā*), according as they are of like or unlike directions, is the equinoctial midday shadow. (31)

Method 17

The *agrā* multiplied by the hypotenuse of the shadow and divided by the radius gives the *agrā* for the sphere of radius equal to the hypotenuse of the shadow. Similarly, the *bhujā* multiplied by the hypotenuse of the shadow and divided by the radius gives another *bhujā* which corresponds to the sphere of radius equal to the hypotenuse of shadow. From these (*chāyākarṇāgrī agrā* and *chāyākarṇāgrī bhujā*) the equinoctial midday shadow is obtained as before. (See vs. 31). (32-33)

Method 18

The square root of the difference between the squares of the 'result' stated above (in vs. 31) (i.e., *chāyākarṇāgrī agrā*) and the length of the shadow, gives (half the length of) the rising-setting line (in the shadow sphere). The distance between this rising-setting line and (the gnomon's position in) the circle forming the locus of the gnomon, is the equinoctial midday shadow in the shadow circle. (34)

Method 19

The result obtained by multiplying the distance (of the local place from the equator) along the meridian of Ujjayinī by 5, or the distance of the local equatorial place or the equator from the local place, as multiplied by 5, when divided by 46, gives the degrees of the (local) latitude, and when divided by 5×40 gives the equinoctial midday shadow (at the local place, in terms or *aṅgulas*).[1] (35-36). (KSS)

छायाभ्रमणपथः

15. 22. 1. ततो भुजौ व्यस्तदिशौ विधेयौ
केन्द्रात् प्रभाग्रे च भुजाग्रसक्ते ।
मध्यप्रभा चेष्टफलेतरा सा
याम्योत्तरा दिग्गणयोगचिह्नात् ॥ ४३ ॥
अग्रेषु चिह्नानि विधाय वृत्तै-
मिथोऽवगाहैर्लिखितैस्तु तेभ्यः ।

[1] For elucidations and observations on these methods, see *VSi*: *KSS*, Trans., pp. 276-92.

तिमी भवेतां मुखपुच्छसक्ते
रज्जू प्रसार्य त्वनयोर्युतिर्या ॥ ४४ ॥
ततश्च चिह्नत्रयसङ्गिवृत्तं
यल्लिख्यते भाग्रपरिभ्रमः सः ।
अतोऽन्यथा बाहुखमध्यभासां
या सा भवेच्छङ्कुपरिभ्रमः सः ॥ ४५ ॥
शङ्कोर्दिशां मध्यगतस्य भाग्रं
रेखां न जह्यात् खलु भाभ्रमेण ।
शङ्कुभ्रमेण भ्रमतश्च शङ्को-
श्छायाग्रमाशागणयोगचिह्नम् ॥ ४६ ॥
(Lalla, *ŚiDhVṛ*., 4. 43-46)

Path of the shadow

From the two ends of the shadow drawn through the centre (and meeting the circle), two *bhujās* must be drawn in directions opposite to their own. Then the midday shadow should be laid from the point of intersection of the directions along the north-south line in a direction opposite to the midday *bhujā*. The points of extremities of these three shadows should be marked, and hence get three intersecting arcs resembling two fish figures. Join the mouth and tail of each figure. With the point of intersection of the two lines thus formed, describe a circle passing through those three extremities. That is the path along which the extremity of the shadow moves.

The locus of the foot of the *śaṅku* is a circle drawn in the same way but the *bhujā* and the midday shadow are fixed in a manner contrary to the above. (43-45)

The extremity of the shadow of the gnomon placed at the centre of the dial, that is, at the point where the lines of the directions meet, should not go beyond the above circle, (viz., the locus), while the shadow moves.

The gnomon moves in such a manner that while moving the extremity of its shadow always passes through the point where the lines of the directions meet. (46). (BC)

राशिमानम्, चरः, लग्नं च

15. 23. 1. (राशिज्या) भापक्रमज्याकृतिविश्लेषमूलविस्तारात् ।
द्युव्यासहृताच्चापं दिग्घ्नं राश्युद्गमविनाड्यः ॥ २९ ॥
'वसुमुनिपक्षा' व्येकं शतत्रयं 'त्रिद्विकाग्नय'श्चाङ्कात् ।
परतस्त एव वामाः षड्त्क्रमात्ते तुलाद्यर्धे ॥ ३० ॥

चरदलराश्युदयः

चरदलकालक्षीणास्त्रयस्त्रयः संयुताः प्रतीपैस्तैः ।
उदयर्क्षतुल्यकालेन यान्ति तत्सप्तमाश्चास्तम् ॥ ३१ ॥

रवेरुन्नतकालः

इष्टोत्तरगोलापक्रमांशकज्यां खभास्कराभ्यस्ताम् ।
हृत्वाक्षजीवया तच्चापादुदयेन तत्कालः ॥ ३२ ॥
तस्मिन् दिनकृत् कुरुते सममण्डलसंश्रयं दिनाद्यर्धे ।
तावच्छेषे परतो न तुलादिषु विद्यते चैतत् ॥ ३३ ॥
(Varāha, *PS*, 4. 29-33)

Right ascensional difference and Lagna

Right ascensional difference

Square the sine of the longitude of a point on the ecliptic, and deduct from it the square of the sine of declination of the point. Find its root, multiply it by the diameter and divide by the the day-diameter. Find the arc of the resulting sine in degrees. Multiply the degrees by 10. The right ascension of the point is obtained in *vināḍīs*. Deducting the right ascension of the next *rāśi* from that of the previous one, the right ascensional differences of the *rāśis* are obtained. (29-30)

Rising signs

Take the difference of right ascension of three Signs at a time. From the first triplet subtract the difference of half-*caras* one by one, taken in the given oreder. Add the half-*cara* differences one by one taken in the reverse order to the second triplet. To the third triplet add the half-*cara* differences taken in the given order. From the fourth triplet subtract the half-*cara* differences one by one, in the reverse order. The *vināḍīs* of the rising Signs, called the ascensional differences, as seen from any place, are obtained. The seventh from the rising Signs sets during the same time as the Signs themselves rise. (31)

Time to reach prime vertical

When the Sun is within 6 Signs from Aries, i.e., when the Sun's declination is north, multiply the sine of the declination by 120′ and divide by the sine of the latitude, (the place being presumed to be north of the equator also). The sine of the Sun's altitude at prime vertical, (*sama-śaṅku*), is obtained. Find its arc. Treat this arc as part of the ecliptic, and find its right ascension in *vināḍīs*. This is the time taken by the Sun to reach the prime vertical in the forenoon, after crossing the *unmaṇḍala*, and the time remaining to reach it after reaching the prime vertical, in the afternoon. The Sun does not touch the prime vertical when it is in the six Signs beginning from Libra, *i.e.*, when the declination is south (as seen from places in the northern hemisphere).[1] (32-33). (TSK)

चरखण्डाः

15. 23. 2. सूर्यं क्रियोक्षमिथुनान्त्यलवं प्रकल्प्य
संसाधयेच्चरदलानि पृथक् क्रमेण ।
विश्लेषितान्यजवृषाह्वयवैणिकानां
तानि स्युरिष्टविषये चरखण्डकानि ॥ ७ ॥
(Lalla, *ŚiDhVṛ*., 4. 7)

[1] For elucidation, see *PS:TSK*, 4. 32-33.

Ascensional differences

Taking the Sun's longitude as 1, 2 and 3 Signs, (that is, assuming it at the end of Aries or Taurus or Gemini), calculate the three corresponding ascensional differences (at the observer's station). The first, the difference between the first two, and the difference between the third and second, are, respectively, the ascensional differences corresponding to the first, second and third Signs at the observer's station. (7). (BC)

लङ्कोदयप्राणाः

15. 23. 3. मेषस्य गोः पृथगतो मिथुनस्य मौर्व्या
क्षुण्णे गृहत्रयभवो द्युगुणो विभक्तः ।
स्वद्युज्यया फलधनूंषि विशेषितानि
लङ्कोदयासव इति प्रवदन्ति सन्तः ।। ८ ।।

ते चासवो गगनभूधरषट्कचन्द्राः
पञ्चाङ्कसप्तशशिनोऽक्षगुणाङ्कचन्द्राः ।
व्यस्तास्तथा निजचरार्धविहीनयुक्ताः
षण्णां क्रमात् स्वविषये पुनरुत्क्रमाच्च ।। ९ ।।
(Lalla, *ŚiDhVṛ.*, 4. 8-9)

Times of the R sine of the Signs at Laṅkā

Multiply the R sine of 30°, 60° and 90° by the R sine of 66° and divide each product by the corresponding radius of the day-circle. Find the arcs corresponding to the three quotients as R sines. The first quotient gives, in *asus*, the time of the rising of Aries in Laṅkā; the second diminished by the first gives that of Taurus; and the third diminished by the second that of Gemini. So say the wise. (8)

These three times, are, respectively, 1670, 1795 and 1935 *asus*. When these are diminished, respectively, by the three corresponding ascensional differences at the observer's station, the remainders are the times of rising of the first three Signs of the zodiac at the same place.

When the ascensional differences at the observer's station are written in the reverse order and are added to the three times (at Laṅkā), also written in the reverse order, the results are the times of rising of the next three Signs at the same place.

These six written in the reverse order are the times of rising of the last six Signs. (9). (BC)

तत्काललग्नम्

15. 23. 4. उद्गच्छतः परिमितिर्भवनस्य या स्या-
दस्तंगते जलपतेर्दिशि सास्तराशेः ।
कृत्वेष्टकालिकमिनं द्युगते विधेयं
काले विलग्नमथ भार्धयुतं रजन्याम् ।। १० ।।
(Lalla, *ŚiDhVṛ.*, 4. 10)

Rising point of the ecliptic

Whatever portion of a Sign of the zodiac has arisen in the east above the horizon, the same portion of the Sign (seventh from it), has set in the west.

When the orient ecliptic point or *lagna* for any time in the day is calculated, the true longitude of the Sun at that time is considered. But when the orient ecliptic point at any time in the night is calculated, the true longitude of the Sun is increased by 6 Signs. (10). (BC)

इष्टकाललग्नम्

15. 23. 5. भोग्यान् सहस्रकिरणेन गृहस्य भागान्
सन्ताडयेत् तदुदयेन हरेत् 'खरामैः' ।
लब्धं त्यजेदसुसमूहमभीप्सितेभ्यो-
ऽसुभ्यः क्षिपेद् दिनकरेऽपि च राशिभुक्तम् ।। ११ ।।

यावन्त एवमुदया निपतन्त्यसुभ्यो
राशीन् क्षिपेत् तदनु तावत एव सूर्ये ।
शेषात् 'खराम'गुणितादविशुद्धलब्धं
भागादिकं च भवतीष्टविलग्नमेवम् ।। १२ ।।
(Lalla, *ŚiDhVṛ.*, 4. 11-12)

Rising point of the ecliptic at any time

Multiply the remaining degrees etc. of the Sign of the zodiac in which the Sun is located by the time of rising (in *asus*) of that Sign and divide by 30. Subtract the quotient in *asus* from the given time, also in *asus*. Add the remaining degrees etc. of the sign to the longitude of the Sun. (Call the result i).

From the remaining *asus* subtract as many times of rising (*asus*) of the Signs as possible (beginning with the one next to the Sun), and add the same number of signs to i. (Call the sum ii).

Multiply the remaining time (in *asus*) by 30 and divide by the time of rising (in *asus*) of the Sign next to the one subtracted. Add the quotient in degrees etc. to ii.

The final result gives thus the longitude of the orient ecliptic point or rising ecliptic point. (11-12). (BC)

द्युगतकालः

15. 23. 6. चक्रार्धयुक्तमिदमस्तविलग्नमाहु-
र्लग्नोदयेन गुणितास्तनुभुक्तभागाः ।
स्वाग्न्युद्धृताः स्युरसवो रविभोग्यकाले
तन्मध्यगोदययुताः समयो विलग्नात् ।। १३ ।।

भानोरभुक्तसमयाल्पतरेष्टकालात्
त्रिंशद्गुणात् तदुदयाप्तलवान्वितोऽर्कः ।
लग्नं तदर्कविवरांशहृतोदयात् स्वात्
त्रिंशोद्धृतात् समय एकगृहेऽर्कतन्वोः ।। १४ ।।
(Lalla, *ŚiDhVṛ.*, 4. 13-14)

Time elapsed from Sunrise

When 180° is added to the longitude of the orient ecliptic point, the result is the longitude of the setting point of the ecliptic or *asta-lagna*. So they say.

Multiply the number of degrees etc. passed by the orient ecliptic point in the Sign of the zodiac by the local time of rising (in *asus*) of the Sign and divide by 30. The quotient is the time in *asus*. Then, find out (as before) the time of rising of the remaining portion of the Sign in which the Sun is. Add both these times, and to this sum add the local times of rising (in *asus*) of all the Signs intervening between the orient ecliptic point and the Sun. The sum is the required time (in *asus*, calculated) from the given orient ecliptic point. (13)

If the given time (when the longitude of the orient ecliptic point is required) is less than the time of rising of the remaining portion of the Sign of the zodiac in which the Sun is located as calculated above, multiply the given time by 30 and divide by the time of rising of this Sign. When the quotient in degrees etc. is added to the true longitude of the Sun, the result is the longitude of the orient ecliptic point.

If the Sun and the orient ecliptic point are in the same Sign of the zodiac, multiply the difference in their longitudes by the time of rising of this Sign and divide by 30. The quotient gives the required time (in *asus*).[1] (14). (BC)

चर:

15. 23. 7. व्यासक्रान्तिज्याघ्नी विषुवज्यालम्बकद्युदैर्घ्यंहृता ।
तच्चापकलात्र्यंशश्चरखण्डविनाडिकाः स्पष्टाः ।।२६।।

चरात् अक्षज्यादिकम्

चरखण्डखपक्षांशज्याघ्नमहर्व्यासमुद्धरेत् 'खजिनैः' ।
द्विः कृत्वा तद्वर्गात् क्रान्तिज्याकृतियुतान्मूलम् ।। २७ ।।
तेन विभजेत् स्थितज्यां व्यासार्धगुणामवाप्तमक्षज्या ।
नवतेरक्षोनायाः क्रमशो ज्या लम्बको भवति ।। २८ ।।
(Varāha, *PS*, 4. 26-28)

Cara

Multiply the sine of latitude by 240′ and by the sine of declination. Divide by the sine of colatitude and by the day-diameter. Find the arc of the sine obtained in minutes, (this arc is called half-*cara*), and divide by 3. The result are the accurate minutes of *cara*, (which might be called as ('day-difference'.) From the *cara* we can obtain the *cara* intervals (or *cara*-differences). (26)

Latitude from Cara

Divide the *vināḍīs* of *cara* by twenty and find the sine of the resulting degrees. Multiply the day-diameter by this, and divide by 240. Put the result in two places. In one place, square it and add the square of the sine of declination and find its root. Multiply the result kept in the other place by the radius, and divide by this root. The quotient is the sine of latitude. Its arc is the latitude. 90° minus latitude is the colatitude, and its sine, sine colatitude. (27-28). (TSK)

चरसंस्कार:

15. 23. 8. चरदलविनाडिकागति-
कलावधात् 'खखरसाग्नि'लब्धकलाः ।
ऋणमुदयेऽस्तमये
धनमुत्तरगोलेऽन्यथा याम्ये ।। ३९ ।।
(Deva, *KR*, 1.39)

Cara-correction

Multiply the Sun's ascensional difference in terms of *vināḍīs* by the (planet's) motion in terms of minutes and divide the product by 3600. The resulting minutes should be subtracted from the planet's longitude for sunrise or added to the planet's longitude for sunset when the Sun is in the northern hemisphere, and vice versa when the Sun is in the southern hemisphere. (39). (KSS)

लङ्कोदयासव:

15. 23. 9. एकस्य राशेर्बृहती ज्यका या
द्वयोस्त्रिभस्यापि कृतीकृतानाम् ।
स्वस्वापमज्याकृतिवर्जितानां
मूलानि तासां त्रिगुणाहतानि ।। ५४ ।।

स्वस्वद्युमौर्व्या विभजेत् फलानां
चापान्यधोऽधः परिशोधितानि ।
क्रमोत्क्रमस्थानि निरक्षदेशे
मेषादिकानामुदयासवः स्युः ।। ५५ ।।

तेऽ'भ्राद्रिभूपा' 'गुणगोऽद्रिचन्द्राः'
'सप्ताग्निनन्देन्दु'मिता अथैते ।
क्रमोत्क्रमस्थाश्चरखण्डकैः स्वैः
क्रमोत्क्रमस्थैश्च विहीनयुक्ताः ।। ५८ ।।

मेषादिषण्णामुदयाः स्वदेशे
तुलादितोऽमी च विलोमसंस्थाः ।
उदेति राशिः समयेन येन
तत्सप्तमोऽस्तं समुपैति तेन ।। ५९ ।।

क्षेत्राणां स्थूलत्वात्
स्थूला उदया भवन्ति राशीनाम् ।
सूक्ष्मार्थी होराणां
कुर्याद् दृक्काणकानां वा ।। ६० ।।
(Bhāskara II, *SiŚi.*, 1.2. 54-56, 58-60)

[1] For exposition and rationale, see *ŚiDhVṛ:BC*, II, pp. 66-72.

Rising at Laṅkā

The R sines of 30°, 60° and 90°, being squared and decreased by the squares of their respective declinations, the square root of the differences being taken, and the result being multiplied by the radius (3438) is to be divided by the respective R cosines of the declination. (54).

The first of the three results, the difference of the second and the first, and the difference of the third and the second, will give us the rising times of what are called the *sāyana-rāśis* of Meṣa, Vṛṣabha and Mithuna; their reverses will then give the rising times of the next three; then the original ones, those of the next three; and again their reverses, those of the last three. (55)

Magnitudes of the rising times

Those rising times are 1670, 1793, 1937; these in the same and reverse orders diminished or increased by their respective *cara* segments which are also in the same and reverse orders give the rising times of the *sāyana-rāśis* beginning from Meṣa for the locality. The *rāśis* from Tulā are in a reverse direction, i.e. as the Meṣa is projecting upwards above the horizon, Tulā will be projecting below the horizon, so that, the time taken by Meṣa to rise is exactly the time taken by Tulā to set. (58-59)

Computations of *lagna*, *udayāntara* and the like from the rising times of big arcs of the ecliptic like *rāśis* will be approximate, whereas one desirous of greater approximation has to determine the same from the rising times of smaller arcs like *horās* and *dṛkkāṇas*, so as to be more correct.[1] (60). (AS)

[1] For commentary and rationale, see *SiŚi*: *AS*, pp. 186-203.

16. ग्रहणम् – ECLIPSES

वेदनिर्दिष्टं रविग्रहणकारणम्

16. 1. 1. यत्त्वा सूर्यं स्वर्भानुस्तमसाविध्यदासुरः ।
अक्षेत्रविद् यथा मुग्धो भुवनान्यदीधयुः ॥ ५ ॥

स्वर्भानोरध यदिन्द्र माया अवो दिवो वर्तमाना अवाहन् ।
गूळ्हं सूर्यं तमसाऽपव्रतेन तुरीयेण ब्रह्मणाऽविन्दद् अत्रिः ॥

मा मामिमं तव सन्तं अत्र इरस्या द्रुग्धो भियसा नि गारीत् ।
त्वं मित्रो असि सत्यराधास्तौ मेहावतं वरुणश्च राजा ॥

ग्राव्णो ब्रह्मा युयुजानः सपर्यन्
कीरिणा देवान् नमसोपशिक्षन् ।
अत्रिः सूर्यस्य दिवि चक्षुराधात्
स्वर्भानोरप या अघुक्षत् ॥ ८ ॥

यं वै सूर्यं स्वर्भानुस्तमसाऽविध्यदासुरः ।
अत्रयस्तमन्वविन्दन् न ह्यन्ये अशक्नुवन् ॥ ९ ॥

(*ṚV*, 5.40. 5-9)

Cause stated in the Ṛgveda

O Sun, when the demon Svarbhānu overspread you with darkness, all the worlds stood as if not knowing where they were. (5)

O Indra, you destroy the illusions of Svarbhānu which exist under the sky. Sage Atri got back the Sun who was hidden by darkness, by means of the fourth *brahma* incantation. (6)

O sage Atri, may that malicious demon, desirous of food, not devour me with that dreadful darkness. You are a friend and truth is your wealth. May you and god Varuṇa protect me. (7)

The sage Atri set the press-stone, propitiated the gods with prayers and salutations and dispelled Svarbhānu and set his eye on Sun's light. (8)

Atri and his descendents alone could restore the Sun when the demon had overspread (the Sun) with darkness. None besides them had the power to do so. (9)

16. 1. 2. स्वर्भानुर्व्वा आसुर आदित्यं तमसाऽऽविध्यत् । तं देवा न व्यजानन् । ते अत्रिमुपाधावन् । तस्य अत्रिर्भासेन तमोऽपाहन्यत् । प्रथममपाहन् सा कृष्णाविरभवत् । द्वितीयं सा रजता, यत् तृतीयं सा लोहिती यथा वर्णमभ्यतृणत् सा शुक्लासीत् ॥

(*Tāṇḍya/Pañcaviṃśa Brāhmaṇa*, 6.6.8)

The Demoniac Svarbhānu struck the Sun with darkness; the gods did not discern it (the Sun, hidden as it was by darkness): they resorted to Atri; Atri repelled its darkness by the *bhāsa*. The part of the darkness he first repelled became a black sheep, what (he repelled) the second time (became) a silvery (sheep), what (he repelled) the third time (became) a reddish one, and with what (arrow) he set free its original appearance (colour), that was a white sheep.[1] (W. Caland).

चन्द्रग्रहणहेतुः

16. 2. 1. भूच्छायां स्वग्रहणे भास्करमर्कग्रहे प्रविशतीन्दुः ।
प्रग्रहणमतः पश्चान्नेन्दोर्भानोश्च पूर्वार्द्धात् ॥ ८ ॥

वृक्षस्य स्वच्छाया यथैकपार्श्वे भवति दीर्घचया ।
निशि निशि तद्वद् भूमेरावरणवशाद्दिनेशस्य ॥ ९ ॥

सूर्यात्सप्तमराशौ यदि चोदग्दक्षिणेन नातिगतः ।
चन्द्रः पूर्वाभिमुखश्छायामौर्वी तदा विशति ॥ १० ॥

चन्द्रोऽधस्थः स्थगयति रविमम्बुदवत्समागतः पश्चात् ।
प्रतिदेशमतश्चित्रं दृष्टिवशाद् भास्करग्रहणम् ॥ ११ ॥

आवरणं महदिन्दोः कुण्ठविषाणस्ततोऽर्द्धसञ्छन्नः ।
स्वल्पं रवेर्यतोऽतस्तीक्ष्णविषाणो रविर्भवति ॥ १२ ॥

एवमुपरागकारणमुक्तमिदं दिव्यदृग्भिराचार्यैः ।
राहुरकारणमस्मिन्नित्युक्तः शास्त्रसद्भावः ॥ १३ ॥

न कथञ्चिदपि निमित्तैर्ग्रहणं विज्ञायते निमित्तानि ।
अन्यस्मिन्नपि काले भवन्त्यथोत्पातरूपाणि ॥ १६ ॥

पञ्चग्रहसंयोगान्न किल ग्रहणस्य सम्भवो भवति ।
तैलं च जलेऽष्टम्यां न विचिन्त्यमिदं विपश्चिद्भिः ॥ १७ ॥

(Varāha, *Bṛ. Saṃ.*, 5.8-17)

Cause of the Lunar eclipse

At a lunar eclipse the Moon enters the shadow of the Earth, and at a solar eclipse, she enters the Sun's disc. That is the reason why the lunar eclipse does not begin at the western limb, nor the solar one at the eastern limb. (8)

Just as the shadow of a tree goes on increasing on one side as a result of the Sun's movement, even so is the case with the shadow of the Earth every night by its hiding the Sun during its revolution. (9)

In her course towards the east, if the Moon tenanting the seventh house from the Sun, does not swerve much either to the north or the south (when her declination is very little), she enters the Earth's shadow. (10)

[1] The interesting feature of the above passage is the detailed observation of the change of colour in the Sun's disc during the progress of an eclipse.

The Moon situated below and moving from the west, obstructs the solar disc like a cloud. The solar eclipse, therefore, is different in different countries according to the visibility of the eclipsed disc. (11)

When the lunar eclipse takes place, the obstructing agency is very large, whereas in the solar eclipse it is small. Hence in semi-lunar and semi-solar eclipses, the luminous horns become blunt and sharp, respectively. (12)

In this manner the ancient seers endowed with divine insight have explained the causes of eclipses. Hence the scientific fact is that *Rāhu* is not at all the cause of eclipses. (13)

An eclipse can by no means be ascertained through omens and other indications. For, the portents such as the fall of meteors and earth tremors occur at other times as well. (16)

Scholars should not believe the traditional statement to the effect that an eclipse cannot take place except when there is a combination of five planets in the same zodiacal Sign, and that a week before the eclipse, i.e. on the previous 8th lunar day, its characteristics can be inferred from the behaviour or appearance of a drop of oil poured on the surface of water. (17). (M.R. Bhat)

चन्द्रग्रहणगणनसिद्धान्तः

16. 3. 1. सार्धानि षट् सहस्राणि योजनानि विवस्वतः ।
विष्कम्भो मण्डलस्येन्दोः साशीतिस्तु चतुश्शती ।। १ ।।

स्फुटस्वभुक्तिगुणितौ मध्यभुक्त्या हतौ स्वकौ ।
रवेस्स्वभगणाभ्यस्तः शशाङ्कभगणोद्धृतः ।। २ ।।

शशाङ्ककक्ष्यागुणितो भाजितो वाऽर्ककक्ष्यया ।
विष्कम्भश्चन्द्रकक्ष्यायां तिथ्याप्तो मानलिप्तिकाः ।। ३ ।।

स्फुटेन्दुभुक्तिर्भूव्यासगुणिता मध्ययोद्धृता ।
लब्धं सूची महीव्यासस्फुटार्कश्रवणान्तरम् ।। ४ ।।

मध्येन्दुव्यासगुणितं मध्यार्कव्यासभाजितम् ।
विशोध्य लब्धं सूच्यास्तु तमोलिप्ताश्च पूर्ववत् ।। ५ ।।

भानोर्भार्धे महीच्छाया तत्तुल्येऽर्कसमेऽथवा ।
शशाङ्कपाते ग्रहणं कियद्भागाधिकोनके ।। ६ ।।

तुल्यौ राश्यादिभिः स्याताममावास्यान्तकालिकौ ।
सूर्येन्दू पौर्णमास्यन्ते भार्धे भागादिभिस्समौ ।। ७ ।।

गतैष्यपर्वनाडीनां स्वफलेनोनसंयुतौ ।
समलिप्तौ भवेतां तौ पातस्तात्कालिकोऽन्यथा ।। ८ ।।

(*SūSi.*, 4.1-11)

Lunar eclipse: Principle of computation

The diameter of the Sun's disc is six thousand five hundred yojanas; of the Moon's, four hundred and eighty. (1)

These diameters, each multiplied by the true motion, and divided by the mean motion of its own planet, give the corrected (*sphuṭa*) diameters. If that of the Sun be multiplied by the number of the Sun's revolutions in an Age, and divided by that of the Moon's, (2)

Or if it be multiplied by the Moon's orbit (*kakṣā*), and divided by the Sun's orbit, the result will be its diameter upon the Moon's orbit: all these, divided by fifteen, give the measures of the diameters in minutes. (3)

Multiply the Earth's diameter by the true daily motion of the Moon, and divide by her mean motion: the result is the Earth's corrected diameter (*sūci*). The difference between the Earth's diameter and the corrected diameter of the Sun is to be multiplied by the Moon's mean diameter, and divided by the Sun's mean diameter: subtract the result from the Earth's corrected diameter (*sūci*), and the remainder is the diameter of the shadow; which is reduced to minutes as before. (4-5)

The Earth's shadow is distant half the Signs from the Sun: when the longitude of the Moon's node is the same with that of the shadow, or with that of the Sun, or when it is a few degrees greater or less, there will be an eclipse. (6)

The longitudes of the Sun and the Moon, at the moment of the end of the day of new moon (*Amāvāsya*), are equal, in Signs, etc.: at the end of the day of full moon (*Paurṇamāsī*) they are equal in degrees, etc., at a distance of half the Signs. (7)

When diminished or increased by the proper equation of motion for the time, past or to some, or opposition or conjunction, they are made to agree, to minutes: the place of the node at the same time is treated in the contrary manner.[1] (8).

चन्द्रग्रहणगणनम्

—वासिष्ठ-पौलिशौ

समकलचन्द्रसूर्यौ

16. 4. 1. नैश्यास्तिथिनाड्योऽर्के देयाश्चान्द्रे समेन्दुरविविवरात् ।
दिवसोद्भवाश्च शोध्याः स भवति तत्कालशशिलिप्तः ।।

चन्द्रग्रहणसम्भवः

राहोः समषट्कृतिकलां हित्वांशं तच्छशाङ्कविवरांशैः ।
ग्रहणं त्रयोदशान्तः पञ्चदशान्तस्तमस्तस्य ।। २ ।।

ग्रहणस्थितिकालः

विक्षेपकलाकृतिवर्जितस्य पञ्चोनषष्टिवर्गस्य ।
मूलं द्विगुणं तिथिवद् विभज्य कालः स्थितेर्भवति ।। ३ ।।

[1] For notes and comments, see *Sū. Si.*: *Burgess*, pp. 143-45.

शशितिमिरविवरभागाः त्रयोदशोनाः शराहताः क्षेप्याः ।
स्थित्यां विनाडिकास्ताः राहावधिकेऽन्यथा हानिः ।। ४ ।।

विमर्दकालः

किन्त्वन्तरांशहीनैः पञ्चभिरूना हता दश 'कृत'घ्नाः ।
तत्पदमेकाश्विघ्नं पञ्चांशोऽस्माद्विमर्दकलाः ।। ५ ।।

स्पर्शमोक्षदिशः

स्थितिदलविमर्ददलयोर्विशेषके तमः सकलमत्तीन्दुम् ।
प्रग्रहणमोक्षशशिराहुविवरभागैश्च दिग् वाच्या ।। ६ ।।

विक्षेपविपर्यासान्तरीयभागे कृते त्रयोदशधा ।
परिधौ प्राक्प्रभृतीन्दोर्ग्रहणाशांशे वदेत् पर्व ।। ७ ।।

शशिपरिधिदलार्धघ्ने खेन्द्वन्तरभागसंगुणे चाक्षे ।
'खखरूपाष्ट'हृते प्राग्वलनं वामं च्युते सव्यम् ।। ८ ।।

ग्रहणकालं वर्णं च

तिथ्यन्ते ग्रहमध्यं प्राक्परतः स्थितिदलेन चात्यन्तौ ।
रक्तकपिलौ च वर्णावुच्चाधस्स्थे परे नितराम् ।। ९ ।।

राहुमुखोनं चक्रं 'धीद्वियम'गुणं शशाङ्कसंयुक्तम् ।
(जूकेत्थगेऽयमुच्चः) क्रियादिकन्यान्तगे नीचः ।। १० ।।

(Varāha, *PS*, 6.1-10)

Vāsiṣṭha-Pauliśa systems

Sun and Moon of equal longitude

Minutes of arc equal to the *nāḍīs* of the full moon *tithi*, to go, after sunset, are to be added to the Sun, (which has been computed for sunset). Minutes of arc, equal to the *nāḍīs* to go from the end of the full-moon or new-moon *tithi* in the day-time, upto sunset, are to be so added to the Sun. Thus corrected, the Sun becomes equal to the Moon in (degrees and) minutes at the end of the full or new moon *tithi*, (*i.e.*, at full or new moon). (1)

Possibility of a lunar eclipse

Deduct one degree and thirtysix minutes from Rāhu's Head or Tail, (whichever is near the Moon) and find the interval in degrees between that and the Moon (at full moon found above). If it is less than thirteen, a lunar eclipse will occur then. If it is less than fifteen, (and above thirteen) there will be a slight darkening, that is all. (2)

Duration of the eclipse

Square the Moon's latitude, subtract it from the square of 55, (*i.e.* from 3025), and find its square root. Double this, and multiplying by 60, divide by the difference of the daily motions of the Sun and Moon, in minutes. The approximate time of the duration of the eclipse is obtained in *nāḍīs*. (3).

If Moon∼Sun is less than 13°, multiply the degrees by 5. The result is in *vināḍīs*. Add these *vināḍīs* to the duration if the longitude of Rāhu is greater than that of the Moon, and subtract if the Moon is greater than Rāhu. Thus the time of duration becomes correct. (4)

Total obscuration

Deduct the difference of the longitudes between the Moon and Rāhu from five degrees. Deduct this from ten degrees, and multiply the remainder by this itself and by four. Find the square root of the result and multiply it by 21. The minutes of arc of total obscuration is obtained. This, divided by the daily motion, gives the time. (5)

Direction of the eclipse

During the interval from the time of first contact to the beginning of totality, Rāhu, (*i.e.* darkness), swallows the Moon completely. The directions of the points of first and last contacts are to be calculated from the Moon∼Rāhu of those times. (6)

Divide the semi-orbit of the Moon situated opposite to the direction of latitude into 13 parts by straight lines parallel to the east-west diameter, at equal distances from one another. At the part of the rim equal to the degrees of Moon∼Rāhu, on the eastern or western part of the orbit, are the points of first and last contacts, from which the direction can be read. (7)

Multiply a fourth of the Moon's rim, (in whatever unit taken, as for *e.g.* minutes or digits) by the latitude, and again by the degrees of the Moon east or west of the meridian. Divide this by 8100. By so many units is the east or west point of contact bent northward or away from the north, respectively, if the Moon is east of the meridian, and bent away from the north and northward, respectively, if the Moon is west of the meridian. (8)

Moment of the eclipse and colour

The middle of the eclipse is at the moment of new moon. The times of first and last contacts are earlier and later than the middle, by half the time of duration (9a-b).

When the eclipse is total, the colour of the Moon is red or brown as it is farthest or nearest to the earth respectively and mixed, more or less, in between. When the eclipse is near sunset or sunrise, the moon is smoky in colour. When the eclipse is partial, the Moon has the colour of rain-cloud. (9 c-d).

Subtract the Head of Rāhu from 12 *rāśis*, multiply it by 228, and add the Moon's longitude. If this is between 6 and 12 *rāśis*, the Moon is farther, and if between 0 and 6 *rāśis*, it is nearer.[1] (10). (TSK).

[1] For the elucidation of the several rationales involved, see *PS*: *TSK*: 6.1-10.

—सौरसिद्धान्तः

तमोबिम्बः

16. 5. 1. रविकक्ष्या नवतिगुणा 'षडष्टदस्रो'द्धृतेन्दुकक्षायाः ।
छेदः 'षट्ति'घ्नायाः लब्धेनोनश्च षड्वर्गः ।। १ ।।
'वियदर्क'गुणे शशिकक्ष्यया हृते कार्मुकं तमोव्यासः ।

स्थित्यर्धकालः

चन्द्रतमोव्यासयुतिं द्वाभ्यां हृत्वा ततो वर्गात् ।। २ ।।
विक्षेपवर्गहीनादासन्नपदे 'वियद्द्विचन्द्र'घ्ने ।
सूर्येन्दुभुक्तिविवरोद्धृते स्थितेर्नाडिका लब्धाः ।। ३ ।।
प्रग्रहणेन्दोः कृत्वा विक्षेपमतोऽनया स्थितैर्भवति ।
एवं भूयो भूयः स्थित्यविशेषः कृतो यावत् ।। ४ ।।

इष्टकालग्रासप्रमाणम्

अर्केन्दुभुक्तिविवरं वाञ्छितनाडीहतं तु षष्टिहृतम् ।
स्थितिलिप्तास्ताभ्यस्तत्तत्कालेन्दोश्च विक्षेपात् ।। ५ ।।
कृतियोगपदं शोध्यं शशिराहुकलाप्रमाणयोगदलात् ।
यच्छेषं तद् ग्रस्तं ज्ञेयं तत्कालमर्केन्द्वोः ।। ६ ।।

विमर्दकालः

अन्त्याद्ययोर्विशेषादवनतिविक्षेपवर्गविवरपदम् ।
द्विगुणं तिथिवत् कृत्वा विमर्दकालोऽर्कचन्द्रमसोः ।। ७ ।।
(Varāha, *PS*, 10.1-7)

—Saura Siddhānta

Diameter of the shadow

Multiply the Moon's true distance in its orbit by 36, and divide by the Sun's true distance multiplied by 90 and divided by 286. Subtract this result from 36, multiply by 120, divide by Moon's true distance and get the arc of the resulting sine. This is the angular diameter of the shadow. (1-2a)

Duration of the eclipse

Add the angular diameters of the Moon and the shadow, divide by two, and square it. Subtract the square of the Moon's latitude from this, and find the square root. Multiply this by 120 and divide by the difference of the motions per day of the Sun and the Moon, pertaining to the time of eclipse. The duration of the eclipse is obtained in *nāḍikās*. (2b-3).

Find the Moon's latitude at first contact, and using this find a more correct duration. Repeat this till there is no difference between the previous and the next durations. (4)

Obscuration at any desired moment

Take the *nāḍīs* before or after full or new moon up to the time for which the amount eclipsed is wanted. Multiply this by the difference of the Sun's and Moon's daily motions, (mentioned above), and divide by 60. The 'corresponding minutes of arc' are got. Square this, square the Moon's latitude for the moment, add them and extract the square root. Subtract this from the half-sum of the diameters of the eclipsing and the eclipsed bodies. The remainder is the minutes of arc eclipsed, at the moment taken, of the Moon in the case of the lunar eclipse, and of the Sun in the case of the solar eclipse. (5-6)

Time of obscuration

Take the difference of the angular semi-diameters, instead of the sum. Square it, subtract the square of the parallax-corrected latitude (in the case of the solar eclipse), of the latitude (in the case of the lunar), find the square root, double it, and treat it as *tithi, i.e.* multiply by 60 and divide by the difference of the parallax-corrected daily motions for the solar eclipse, or of the mere daily motions in the case of the lunar. The time of total obscuration is got.[1] (7) (TSK).

—आर्यभटीयम्

16. 6. 1. चन्द्रे जलमर्कोऽग्निः मृद्भूश्छायापि या तमस्तद्धि ।
छादयति शशी सूर्यं, शशिनं महती च भूच्छाया ।। ३७ ।।
स्फुटशशिमासान्तेऽर्कं पातासन्नो यदा प्रविशतीन्दुः ।
भूच्छायां पक्षान्ते तदाधिकोनं ग्रहणमध्यम् ।। ३८ ।।
भूरविविवरं विभजेद् भूगुणितं तु रविभूविशेषेण ।
भूच्छायादीर्घत्वं लब्धं भूगोलविष्कम्भात् ।। ३९ ।।
छायाग्रचन्द्रविवरं भूविष्कम्भेण तत् समभ्यस्तम् ।
भूच्छायया विभक्तं विद्यात् तमसः स्वविष्कम्भम् ।। ४० ।।
तच्छशिसम्पर्कार्धकृतेः शशिविक्षेपवर्गितं शोध्यम् ।
स्थित्यर्धमस्य मूलं ज्ञेयं चन्द्रार्कदिनभोगात् ।। ४१ ।।
चन्द्रव्यासार्धोनस्य वर्गितं यत्तमोमयार्धस्य ।
विक्षेपकृतिविहीनं तस्मान्मूलं विमर्दार्धम् ।। ४२ ।।
तमसो विष्कम्भार्धं शशिविष्कम्भार्धवर्जितमपोह्य ।
विक्षेपाद्यच्छेषं न गृह्यते तच्छशाङ्कस्य ।। ४३ ।।
विक्षेपवर्गसहितात् स्थितिमध्यादिष्टवर्जितान्मूलम् ।
सम्पर्कार्धाच्छोध्यं शेषस्तात्कालिको ग्रासः ।। ४४ ।।
(Āryabhaṭa I, *ABh.*, 4. 37-44)

—Āryabhaṭīya

Moon, Sun, Earth and Shadow

The Moon is water, the Sun is fire, the Earth is earth, and what is called Shadow is darkness (caused by the Earth's Shadow). The Moon eclipses the Sun and the great Shadow of the Earth eclipses the Moon. (37)

[1] For detailed elucidation, rationales and calculations, see *PS*: *TSK*, 10. 1-7.

Occurrence

When at the end of a lunar month, the Moon, lying near a node (of the Moon), enters the Sun, or, at the end of a lunar fortnight, enters the Earth's Shadow, it is more or less the middle of an eclipse, (solar eclipse in the former case and lunar eclipse in the latter case). (38)

Length of Shadow

Multiply the distance of the Sun from the Earth, by the diameter of the Earth and divide (the product) by the difference between the diameters of the Sun and of the Earth : the result is the length of the Shadow of the Earth (*i.e.* the distance of the vertex of the Earth's shadow) from the diameter of the Earth (*i.e.* from the centre of the Earth). (39)

Earth's shadow

Multiply the difference between the length of the Earth's shadow and the distance of the Moon by the Earth's diameter and divide (the product) by the length of the Earth's shadow: the result is the diameter of the *Tamas* (*i.e.*, the diameter of the Earth's shadow at the Moon's distance). (40)

Half-duration

From the square of half the sum of the diameters of that (*Tamas*) and the Moon, subtract the square of the Moon's latitude, and (then) take the square root of the difference; the result is known as half the duration of the eclipse (in terms of minutes of arc). The corresponding time (in *ghaṭis* etc.) is obtained with the help of the daily motions of the Sun and the Moon. (41)

Subtract the semi-diameter of the Moon from the semi-diameter of that *Tamas* and find the square of that difference. Diminish that by the square of the (Moon's) latitude and then take the square root of that: the square root (thus obtained) is half the duration of totality of the eclipse. (42)

Non-eclipsed portion

Subtract the Moon's semi-diameter from the semi-diameter of the *Tamas*; then subtract whatever is obtained from the Moon's latitude: the result is the part of the Moon not eclipsed (by the *Tamas*). (43)

Measure of the eclipse

Subtract the *iṣṭa* from the semi-duration of the eclipse; to (the square of) that (difference) add the square of the Moon's latitude (at the given time); and take the square root of this sum. Subtract that (square root) from the sum of the semi-diameters of the *Tamas* and the Moon; the remainder (thus obtained) is the measure of the eclipse at the given time. (44)

—आर्यभटीयार्धरात्रपक्षः

16. 7. 1a. रविचन्द्रौ समलिप्तौ तिथिगतगम्यघटिकाफलोनयुतौ ।
पातोनचन्द्रजीवा विक्षेपो नवगुणेषुहृता ।। १ ।।

'भवदश'गुणिते रविशशिगती 'नखैः' 'स्वरजिनै'र्हृते माने ।
षष्टया भक्तं 'तत्त्वाष्ट'गुणितयोरन्तरं तमसः ।। २ ।।

विक्षेपं संशोध्य प्रमाणयोगार्धतस्तमश्छन्नम् ।
सर्वग्रहणं ग्राह्यादधिके खण्डग्रहणमूने ।। ३ ।।
छाद्यार्धेन च्छादकदलस्य युक्तोनकस्य वर्गाभ्याम् ।
विक्षेपकृतिं प्रोह्य पदे तिथिवत् स्थितिविमर्दार्धे ।। ४ ।।

भुक्तिः षष्टिहृता स्थितिविमर्ददलनाडिकागुणार्केन्द्वोः ।
आदावृणमन्ते धनमसकृत् तेनान्यथा पाते ।। ५ ।।

वीष्टस्थितिदलविक्षेपलिप्तिकावर्गयुतिपदेनोनम् ।
मानैक्यार्धं छन्नं मध्ये विक्षेपलिप्तोनम् ।। ६ ।।

त्रिज्याप्तचापभागैर्नताक्षजीवावधादुदग्याम्यैः ।
पूर्वापरयोः पूर्वा त्रिभयुग्ग्राह्यायनांशैश्च ।। ७ ।।
(Brahmagupta, *KK*, 1.4.1-7)

—Āryabhaṭa's Midnight system

The degrees, minutes and seconds (omitting the Signs) in the longitudes of the Sun and Moon should be made equal by adding to or subtracting from them their respective motions during the *ghaṭikās*, which are to pass till the *pūrṇānta* (or time of opposition) or that have passed since then, respectively.

Subtract the longitude of the *pāta*[1] from that of the Moon. The *jyā* of the remainder, multiplied by 9 and divided by 5, gives the *vikṣepa* of the Moon in minutes. (1)

The true daily motions of the Sun and Moon multiplied, respectively, by 11 and 10, and divided by 20 and 247, give their angular diameters in minutes.

The difference between 8 times the true motion of the Moon and 25 times that of the Sun, when divided by 60, gives the angular diameter of the Earth's shadow in minutes. (2)

When the *vikṣepa* of the Moon is subtracted from half of the sum of the diameters of the obscuring and the obscured bodies, the remainder is the portion obscured by the shadow.

If the obscured portion is greater than the obscured body, there is total eclipse; if less, there is partial eclipse. (3).

Find the sum and difference of the semi-diameters of the obscuring and obscured bodies. Subtract the

[1] *Pāta* in this chapter is Moon's Node.

square of the *vikṣepa* of the Moon from the square of each of the results. Find the square roots of the remainders; and thus calculate, respectively, the half durations of the eclipse and of the total obscuration in the same way as in the case of *tithis*. (4)

Multiply the true daily motion of the Sun or the Moon by the number of *ghaṭikās*, etc., in the half duration of the eclipse or of total obscuration. Divide each product by 60. Add the result to or subtract from the respective longitude of the Sun or of the Moon. The first gives the longitude at the end of the eclipse, or the total obscuration, as the case may be; and the second gives the longitude at the beginning of the eclipse or the total obscuration. In the case of the *pāta*, the process must be reversed. The corrections should be applied repeatedly. (5)

From the half duration of the eclipse, whether at the beginning or at the end, subtract the given time, after which or before which, respectively, the obscured portion is wanted. From that time, calculate the arc in minutes gained by the Moon and also its *vikṣepa*. Find the square root of the sum of the squares of these two quantities. Subtract it from half the sum of the diameters of the obscuring and obscured bodies. The remainder is the obscured portion. At the time of the *madhyagrahaṇa*, the obscured portion is obtained by subtracting the number of minutes in the Moon's *vikṣepa* from half the sum of the diameters of the obscuring and obscured bodies. (6)

Multiply the *natajyā* (*samamaṇḍalīya-natāṃśa-jyā*) of the obscured body by the *akṣajyā* and divide by the *trijyā*. Considering the result as the *jyā*, find the number of degrees in the corresponding arc. This arc is to the north or south according as the obscured body is in the eastern or the western half of the celestial sphere. Take the sum of difference of the number of degrees in this arc and that in the *āyanavalana*, that is, the declination calculated from the longitude of the obscured body increased by 90°, according as they are of the same or of opposite denominations. The result is the variation of the eastward direction of the ecliptic from the eastward direction of the disc of the obscured body.[1] (7) (BC)

पर्वज्ञानम्

16. 7. 1b. पातोनरवेर्भार्धाच्चक्राच्चोनाधिकाः कला भक्ताः ।
तद्गतियुत्याप्तदिनैरासन्नेऽर्कस्य मासान्ते ॥ १६ ॥
पर्वेन्दोः पक्षान्ते प्रागधिकोना युतिर्भवति पश्चात् ।
तन्मध्ये न ग्रहणं यदि भानोः 'पञ्चजिनभरसाः' ॥२०॥
इन्दोर्विषया द्वियमा दिवाकरा'स्त्रिविषया'स्तदुच्चस्य ।
'व्योमातिधृतिद्वियुगानि' 'रसशराश्च' चन्द्रपातस्य ॥२१॥
खं नन्दा द्वियमा खाब्धयो गृहाद्यास्तथेष्टपर्वगुणाः ।
क्षेप्याः पर्वण्येष्यति शोध्याः पातेऽन्यथातीते ॥ २२ ॥
ग्रहणे यथा रवीन्द्वोः स्पष्टीकरणाद्यमुक्तवत् कृत्वा ।
एवं पर्वज्ञानं ग्रहणज्ञानं स्फुटं गणितात् ॥ २३ ॥

(Brahmagupta, *KK*, 2.4. 19-23)

Syzygy computation

Find the mean longitudes of the Sun and Moon's *pāta* on a given day. Subtract the longitude of the *pāta* from that of the Sun. Subtract the remainder completely from 6 and 12 Signs and express the difference in minutes. Divide this difference by the sum of the mean daily motions of the Sun and the *pāta*. The result is in days etc. If the difference in the longitudes of the Sun and the *pāta* is greater than 6 or 12 Signs the difference will be equal to it before the time in the result; if less, the difference will be equal to it after the time in the result. If the time is Amāvāsyā, there is the possibility of a solar eclipse; if near Pūrṇimā, there is the possibility of a lunar eclipse.

If there is no eclipse at present, solar or lunar, (it should then be examined whether there was one, 12, 18, 24, etc., months before or there will be one after the same intervals. For this purpose, the longitudes of the Sun, Moon its *ucca* and *pāta* before or after these intervals must be found). The half-yearly motions of the Sun, Moon, its *ucca* and *pāta* are respectively 5 signs 24° 27′ 6″, 5 signs 22° 12′ 53″, 19° 42′ 56″ and 9° 22′ 40″. Multiply these motions by the number of half years before or after which the possibility of an eclipse is to be determined. The products resulting from the motions of the Sun, Moon and its *ucca* should be added to the respective longitudes, if the time is after the given day; and deducted if the time is before the given day. The product resulting from the *pāta's* motion should be applied to its longitude in the reverse manner. (The final results are the mean longitudes at the time when the possibility of an eclipse is being determined. (19-23). (BC)

—आर्यभटोपरि ब्रह्मगुप्तकृतः शोधः

16. 8. 1. स्वफलमृणं चक्रार्धादूने केन्द्रेऽधिके धनं मध्ये ।
तत्तिथिनतकेन्द्रज्यावधो रवेः शशिनवेन्दुगुणः ॥ १ ॥
इन्दोर्नवनववेदैस्त्रिज्याकृतिलब्धविकलिकोनः प्राक् ।
पश्चादधिकोऽर्कोऽसकृदृणेऽन्यथेन्दुर्धने हीनः ॥ २ ॥
क्षयधनहानिधनानि प्राक् पश्चादन्यथा रवेरिन्दोः ।
प्राग्वत् पश्चात् स्वगतौ धनक्षयक्षयधनानि प्राक् ॥ ३ ॥
दिनगतशेषाल्पयुतं स्वपादयुक्तं दिनं दिनार्धहृतम् ।
अङ्गुललिप्ता वितुषैर्यवोदरैरङ्गुलं षड्भिः ॥ ४ ॥

[1] For the rationale and formulae involved, see *KK*: *BC*, I. 117-22.

ग्राह्यग्राहकदलतत्समासविक्षेपलिप्तिकाच्छेदः ।
अङ्गुललिप्ता त्रिज्यावलनज्यानां भवेदिष्टः ।। ५ ।।
(Brahmagupta, *KK*, 2.4.1-5)

—Emendation by Brahmagupta

If the *mandakendra* of the Sun or Moon is less than 6 Signs, the *mandaphala* of each should be subtracted from its mean longitude; if the *mandakendra* is greater than 6 Signs, the *mandaphala* should be added. The result in each case will be the corrected longitude.

From these longitudes calculate the time of opposition or conjunction, according as it is lunar or solar eclipse. Find the *natakāla* at that time. Multiply the *jyā* of the Sun's *mandakendra* by the *jyā* of the *natakāla*. Multiply the product again by 191 and divide by the square of the *trijyā*. Add or subtract the result in seconds, to or from the Sun's corrected longitude, according as it is in the western or eastern half of the sky. (The result gives the correct true longitude.)

Multiply the *jyā* of the Moon's *mandakendra* by the *jyā* of its *natakāla*. Multiply the product again by 499 and divide by the square of the *trijyā*. The result is in seconds. If the *mandaphala* of the Moon is subtractive, add or subtract the result to or from its corrected longitude, according as it is in the eastern or western half of the sky. If the *mandaphala* is additive subtract the result from the corrected longitude of the Moon, whether it is in the eastern or western half of the sky. (The result is its corrected true longitude.)

The process should be repeated till the longitudes are fixed. (1-2)

If the Sun is in the eastern half of the sky, the correction to its motion is subtractive, additive, subtractive and additive, according as its *mandakendra* is in the first, second, third and fourth quadrants, respectively. If the Sun is in the western half, the process is reversed.

In the case of the Moon, when it is in the western half, the correction to its motion is subtractive, additive, subtractive and additive, according as its *mandakendra* is in the first, second, third and fourth quadrants, respectively. If the Moon is in the eastern half, the correction is additive, subtractive, subtractive and additive, according as its *mandakendra* is in the first, second, third and fourth quadrants respectively. (3)

In the case of a solar eclipse, add the length of the day (on which the eclipse occurs) to its one-quarter. Add to the sum the *dinagata* or the *dinaśeṣa*, whichever is less, and divide by half the length of the day. The result is the number of minutes in an *aṅgula* on that day.

(In a lunar eclipse the same method must be followed using the length of the day of the Moon.)

The breadth of six grains of barley without the husk is equivalent to one *aṅgula*. (4)

The semi-diameters of the obscuring and the obscured bodies, their sum and the *vikṣepa* of the Moon are expressed in *aṅgulas*, when the number of minutes in these lengths is divided by the number of minutes in an *aṅgula*.

The *trijyā* and the *valanajyā* are expressed in *aṅgulas* by dividing each by any number. (It is 6 according to ancient astronomers.)[1] (5). (BC)

—भास्करः १

16. 9. 1a. पर्वनाड्यो रवौ देयास्ताः सलिप्ता निशाकरे ।
एवं प्रतिपदः शोध्याः समलिप्तादिदृक्षुणा ।। १ ।।
'पञ्चवस्विषुरन्ध्रेषुसागरा'स्तिग्मतेजसः ।
कर्णः 'पर्वतशैलाग्निवेदरामा' निशाकृतः ।। २ ।।
अविशेषकलाकर्णताडितौ त्रिज्यया हृतौ ।
स्फुटयोजनकर्णौ तौ तयोरेव यथाक्रमम् ।। ३ ।।
'पङ्क्तिसागरवेदाख्यो' रवे'स्तिथिशिखी'न्दुजः ।
व्यासो वसुन्धरायाश्च 'व्योमभूतदिशः' स्मृतः ।। ४ ।।
योजनव्याससंक्षुण्णं विष्कम्भार्धं विभाजयेत् ।
स्फुटयोजनकर्णाभ्यां लिप्ताव्यासौ स्फुटौ तयोः ।। ५ ।।
कर्णः क्षुण्णः सहस्रांशोर्मेदिनीव्यासयोजनैः ।
मेदिन्यर्कविशेषेण भूच्छायादैर्घ्यमाप्यते ।। ६ ।।
चन्द्रकर्णविहीनेऽस्मिन् भूमिव्यासेन ताडिते ।
छायादैर्घ्यहृते व्यासश्चन्द्रवत्तमसः कलाः ।। ७ ।।
(Bhāskara I, *LBh*. 4. 1-7)

—Bhāskara I

Longitudes of the Sun and the Moon

One who wants to obtain (the longitudes of the Sun and the Moon when there is) equality in minutes of arc[2] should add as many minutes of arc as there are *parvanāḍis*, to the Sun's longitude (at sunrise) and the same together with the minutes of arc (of the difference between the longitude of the Sun as increased by 6 signs, and the longitude of the Moon in the case of opposition, or of the difference between the longitudes of the Sun and the Moon in the case of conjunction) to the Moon's longitude (when opposition or conjunction of the Sun and Moon is to occur); similarly, (when opposition or conjunction of the Sun and Moon has occurred) one

[1] For the explanation of and proof, see *KK:BC*, I. pp. 149-51.

[2] When the Sun and the Moon are in opposition, their longitudes differ by six signs; when they are in conjunction, their longitudes are the same. The minutes, however, are the same. The equality in minutes of arc refers here to the time of opposition or conjunction.

should subtract the *pratipad-nāḍīs* (etc. from the longitude of the Sun and the Moon). (1)

Mean distances of the Sun and the Moon

4,59,585 is (in *yojanas*) the (mean) distance of the Sun and 34,377 that of the Moon. (2)

True distances of the Sun and the Moon

These (above-mentioned mean distances of the Sun and the Moon) multiplied by their true distances in minutes obtained by the method of successive approximations and divided by the radius (i.e., by 3438') give their true distances in *yojanas*. (3)

Diameters of the Sun, the Moon and the Earth

The diameter of the Sun is 4410 (*yojanas*); of the Moon, 315 (*yojanas*); and of the Earth, 1050 (*yojanas*). (4)

Angular diameters of Sun and Moon

Multiply the radius (i.e., 3438') (separately) by their diameters in *yojanas* and divide by their true distances in *yojanas*: then are obtained their true (i.e., angular) diameters in minutes of arc. (5)

Length of Earth's shadow

Multiply the Sun's (true) distance (in *yojanas*) by the diameter of the Earth in *yojanas* and divide by the differrence between (the diameters of) the Sun and the Earth. Then is obtained (in *yojanas*) the length of the Earth's shadow. (6)

Diameter of the Earth's shadow

This (length of the Earth's shadow) diminished by the (true) distance of the Moon and multiplied by the diameter of the Earth and (then) divided by the length of the Earth's shadow gives (in *yojanas*) the diameter of the Earth's shadow (at the point where the Moon crosses it). This should be reduced to minutes of arc like (the diameter of) the Moon. (7). (KSS)

चन्द्रविक्षेपः

16. 9. 1b. पातोनसमलिप्तेन्दोर्जीवा 'खत्रिघना'हता ।
कर्णेन ह्रियते लब्धो विक्षेपः सौम्यदक्षिणः ॥ ८ ॥

इन्दुहीनतमोव्यासदललिप्ताविवर्जिताः ।
विक्षेपस्य न गृह्यन्ते तमसा शशलक्ष्मणः ॥ ९ ॥

विक्षेपवर्गहीनायाः सम्पर्कार्धकृतेः पदम् ।
गत्यन्तरहृतं हत्वा षष्टया स्थित्यर्धनाडिकाः ॥ १० ॥

स्फुटभुक्तिहता नाडयः षष्टया नित्यं समुद्धृताः ।
लब्धलिप्ताः क्षयश्चन्द्रे क्षेपश्च स्पर्शमोक्षयोः ॥ ११ ॥

विक्षेपश्चन्द्रतस्तस्मान्नाडिका लिप्तिकाः शशी ।
आवृत्या कर्मणा तेन स्थित्यर्धमविशेषयेत् ॥ १२ ॥

स्थित्यर्धेनाविशिष्टेन हीनयुक्ता तिथिः स्फुटा ।
स्पर्शमोक्षौ तु तौ स्यातां पर्वमध्यं ग्रहस्य तत् ॥ १३ ॥

ग्राह्यग्राहकविश्लेषदलविक्षेपवर्गयोः ।
विश्लेषस्य पदं प्राग्वद् विमर्दार्धस्य नाडिकाः ॥ १४ ॥

तिथिमध्यान्तरालानामसूनामुत्क्रमज्यया ।
विषुवज्ज्या हता भाज्या त्रिमौर्व्या लब्धदिक्क्रमः ॥१५॥

प्राक्कपाले तु बिम्बस्य पूर्वपश्चिमभागयोः ।
उदग्दक्षिणतोऽक्षस्य वलनं पश्चिमेऽन्यथा ॥ १६ ॥

तत्कालेन्द्वर्कयोः कोटयोरुत्क्रमज्यापमो गुणः ।
अयनाद्बिम्बपूर्वार्धे पश्चार्धे व्यत्ययेन दिक् ॥ १७ ॥

योगस्तद्धनुषोः साम्ये दिशोर्भेदे विपर्ययः ।
सम्पर्कार्धहता तज्ज्या त्रिज्याप्तं वलनं हि तत् ॥ १८ ॥

एकदिक्कं क्षिपेत् क्षेपे विदिक्कं तद्विशोधयेत् ।
वलनं तत् स्फुटं ज्ञेयं सूर्याचन्द्रमसोर्ग्रहे ॥ १९ ॥

सम्पर्कार्धाधिकं तद्धि सङ्ख्यया यत्र लभ्यते ।
सम्पर्कात् सकलाद्धित्वा वलनं तत्र शिष्यते ॥ २० ॥

असंयुक्तमविश्लिष्टं स्पर्शवत् केवलं स्फुटम् ।
विक्षिप्या ग्रहमध्यस्य तस्य स्याद् व्यस्तदिक्क्रमः ॥ २१ ॥

भास्करेन्दुतमोव्यासविक्षेपवलनोद्भवाः ।
अङ्गुलान्यर्धिता लिप्तास्ता एव हरिजस्थिते ॥ २२ ॥

(Bhāskara I, *LBh.*, 4. 8-22)

Moon's latitude

Multiply the R sine of the difference between the longitudes of the Moon, when in opposition with the Sun, and its ascending node by 270 and divide (the product) by the true distance of the Moon, in minutes: the result is the Moon's (true) latitude north or south. (8)

Moon's diameter unobscured by the shadow

Diminishing the (minutes of arc of the) Moon's latitude (obtained above) by half of the minutes of arc resulting on diminishing the diameter of the shadow by that of the Moon are obtained those of (the diameter of) the Moon which remain unobscured by the shadow. (9)

Sparśa and mokṣa sthityardhas

Diminish the square of half the sum of the diameters of the Moon and the shadow (*samparkārdha*) by the square of the (Moon's) latitude (for the time of opposition of the Sun and Moon) and then take the square root (of that). That divided by the difference between the (true) daily motions (of the Sun and Moon) and multiplied by 60 gives, in *nāḍīs*, the (first approximation to the *sparśa* or *mokṣa*) *sthityardha*.

(Then) multiply those *nāḍīs* by the true daily motion (of the Moon) and always divide by 60. The resulting minutes should then be severally subtracted from, and added to the longitude of the Moon (calculated for the time of opposition) to get the longitudes of the Moon for the times of the first and last contacts.

From the Moon's longitude (for the first contact as also for the last contact) calculate the Moon's latitude; and from that successively determine the (corresponding (*sthityardha* in terms of) *nāḍīs*, the corresponding minutes of arc (of the Moon's motion), and the longitude of the Moon (for the first contact as also for the last contact). Repeating this process again and again, find the nearest approximations to the (*sparśa* and *mokṣa*) *sthityardhas*. (10-12).

First and the last contacts

Diminish and increase the true time of opposition by the (*sparśa* and *mokṣa*) *sthityardhas*, obtained by the method of successive approximations, (respectively): then are obtained the times of the first and the last contacts. The time of the middle of the eclipse is the same as that (of opposition of the Sun and the Moon).[1] (13)

Sparśa and mokṣa vimardārdhas

The square root of the difference between the squares of the Moon's latitude and half the difference between (the diameters of) the eclipsed and eclipsing bodies leads, as before, to the determination of the (nearest approximation in) *nāḍīs* of the (*sparśa vimardārdha* as also of the *mokṣa vimardārdha*. (14)

[1] This is how the exact times of the beginning and end of a lunar eclipse are determined. In practice, however, the exact beginning and end of an eclipse are not perceived with the unaided eye. A lunar eclipse is seen to begin after a portion of the Moon's disc is already obscured by the shadow. Śaṅkaranārāyaṇa tells us how to find the times when a lunar eclipse is actually seen to begin and end. He says:

"At the beginning, having diminished the sixteenth part of the Moon's diameter from half the sum of the diameters of the Moon and the shadow, (then) having squared it and subtracted from it the square of the Moon's latitude, one should obtain half the (apparent) duration of the lunar eclipse by the method of successive approximations. Or, one should multiply the sixteenth portion of that (semi-duration) in minutes by 60 and divide by the difference between the daily motions of the Sun and the Moon, and then reduce that to *vighaṭīs* etc. Having thus ascertained the corresponding time (in *vighaṭīs*) the apparent instant of the first contact should be declared by adding that to the instant of the first contact. After that, in order to determine the instant of the last contact, the *mokṣa-sthityardha* obtained by the method of successive approximations should be added to the instant of opposition and the result taken, as before, as the instant of the last contact. There also the (apparent) time should be announced after diminishing it by one-sixteenth (of the time corresponding to the *mokṣa-sthityardha*). Then adding the two *sthityardhas* (i.e., the *sparśa* and *mokṣa sthityardhas*), the sum should be declared, in *ghaṭīs* etc., to be the duration of the eclipse."

In support of his statement, Śaṅkaranārāyaṇa quotes the following verse of Ācārya Bhaṭṭa Govinda:

śaśidehāṣṭyaṁśonaṁ samparkadalaṁ yadā nater adhikam |
bhavati tadendugrahaṇaṁ na bhavatyalpe 'rdhasamparke ||

i.e., When half the sum of the diameters of the Moon and the shadow diminished by the sixteenth portion of the Moon's diameter is greater than the Moon's latitude (for the time of opposition), then does a lunar eclipse occur (i.e., is observed). When half the sum of the diameters of the Moon and the shadow (diminished by the sixteenth part of the Moon's diameter) is smaller, a lunar eclipse does not seem to occur (i.e., is not observed).

The statement that the time of the middle of the eclipse is the same as that of opposition of the Sun and Moon is only approximately true. An accutare expression for the difference between the two instants was first given by Gaṇeśa Daivajña (1520). (KSS)

Akṣa-valana and Ayana-valana

Multiply the R sine of the (local) latitude by the R versed-sine of the *asus* between the times of (the beginning, middle, or end of) the eclipse and the middle of the night or day,[1] and divide by the radius (i.e., 3438′): (the result is the R sine of the *akṣa-valana*). The direction of the result (i.e., *akṣa-valana*) is (determined) in the following manner:

(If the eclipsed body, at the time of the first or last contact, is) in the eastern half of the celestial sphere, the directions of the *akṣa-valana* for the eastern and western halves of the disc (of the eclipsed body) (i.e., of the *sparśa* and *mokṣa valanas* in the case of the Moon and *vice versa* in the case of the Sun) are north and south, (respectively); (if the eclipsed body is) in the western half of the celestial sphere, (they are to be taken) reversely. (15-16).

Magnitude and direction of the ayana-valana

The R sine of the declination calculated from the R versed sine of the *koṭi* of the tropical (*sāyana*) longitude of the Sun or Moon[2] for that time (i.e., for the beginning, middle, or end of the eclipse) (is the R sine of the *ayana-valana*). In the eastern half of the disc (of the Sun or the Moon), the direction (of the *ayana-valana*) is the same as that of the *ayana*[3] (of the Sun or the Moon). In the western half, the direction is contrary to that of the *ayana*. (17)

Resultant valana

Take the sum of their arcs (i.e., of the *akṣa-valana* and *ayana-valana* (when they are of like directions) and the difference when they are of unlike directions. Multiply the R sine of that (sum of difference) by the sum of the semi-diameters of the eclipsed and eclipsing bodies and divide by the radius: this result is the *valana*. (18)

Corrected valana: sphuṭa-valana

If the *valana* (obtained above) is of the same direction (as the Moon's latitude) add it to the Moon's latitude; if it is of the contrary direction, subtract it (from the

[1] Night when the eclipse is lunar, and day when the eclipse is solar.

[2] The Sun is taken when the eclipse is solar, and the Moon is taken when the eclipse is lunar.

[3] *Ayana* means "the northerly or southerly course (of a planet)". The course (*ayana*) is north or south according as the planet lies in the half orbit beginning with the tropical Sign Capricorn or in that beginning with the tropical sign Cancer.

Moon's latitude). The (sum or difference thus obtained is known as the corrected *valana* (*sphuṭa-valana*) in the case of solar and lunar eclipses.

In case that (corrected *valana*) is found to be greater than the sum of the semi-diameters of the eclipsed and eclipsing bodies, it should be subtracted from the entire sum of the semi-diameters of the eclipsed and eclipsing bodies and the remainder (thus obtained) should be taken as the (corrected) *valana*. (19-20)

Valana for the middle of the eclipse

The (resultant) *valana* for the middle of the eclipse obtained in the same way as for the first contact without any further addition or subtraction of the Moon's latitude tude is the corrected (*valana* for the middle of the eclipse). The direction of that (Moon's latitude) is to be taken reversely (in the projection of a lunar eclipse). (21)

Converting minutes of arc into aṅgulas

The minutes of arc of the diameters of the Sun, the Moon, and the shadow and those of the (Moon's) latitude and the (corrected) *valana* when divided by two are reduced to *aṅgulas*. (But when the Sun and Moon are) on the horizon, they (i.e., minutes of arc) are the same (as *aṅgulas*). (22). (KSS)

—लल्लः

ग्रहणकालः

16. 10. 1. दिनकरास्तसमये सविधुन्तुदौ
रविविधू विदधीत परिस्फुटौ ।
प्रथमपक्षजपञ्चदशे तिथौ
शशधरग्रहणावगमेच्छया ॥ १ ॥

आकाशकक्ष्या

दशगुणं गुणयेत् 'खखषड्घनं'
युगभवैर्भगणैः शिशिरद्युतेः ।
भवति योजनमानमहःपते-
र्द्युतियुजो नभसः परिधेरिदम् ॥ २ ॥

ग्रहभुक्तिलिप्ताः

'खखनखाद्रि'हृतं भयुजो भवेद्
ग्रहयुजो निजपर्ययहृत् पृथक् ।
'शरयमाङ्ग'हता 'भनवाग्नि'हृद्
ग्रहवृतेः श्रवणः फलमुच्यते ॥ ३ ॥

रविचन्द्रयोः भूमध्यान्तरम्

'विषयनागशराङ्कशराब्धयो'
दिनकृतः खलु योजनजा श्रुतिः ।
'तुरगशैलहुताशकृताग्नयः'
शशधरस्य कुमध्यतदन्तरम् ॥ ४ ॥

निजमृदुश्रवणेन हते श्रुती
त्रिभगुणेन हृते भवतः स्फुटे ।
श्रवणमध्यमभुक्तिहतोऽथवा
निजनिजस्फुटभोगविभाजितः ॥ ५ ॥

तिथिगुणं शशियोजनमण्डलं
दिनकृतो गगनेन्दुकृताब्धयः ।
दिनकृतः श्रवणः शरसङ्गुणो
नृपहृतो भवतीह महीप्रभा ॥ ६ ॥

स्फुटशशिश्रवणेन विवर्जिता
गगनपञ्चककुब्गुणिता कुभा ।
अपहृता च तया प्रभया भुवो
भवति योजनबिम्बमगोः फलम् ॥ ७ ॥

त्रिभगुणेन हतानि रविश्रुति-
स्मरसुहृच्छ्रवणोपहृतान्यतः ।
फलधनूंषि वपूंषि फलानि वा
हिमगु-तिग्ममयूख-शशिद्विषाम् ॥ ८ ॥

शिवहता द्विभहृच्छशिनो गति-
स्तनुकलाः स्युरिनस्य नखोद्धृताः ।
गुणितयोर्भुजगैः शरलोचनैः
'खरस'हृद् विवरं तमसोऽथवा ॥ ९ ॥

तिमिरमावरणं हिमदीधिते-
र्दिनकरस्य निशाकरमण्डलम् ।
भवति मण्डलखण्डयुतिस्तयो-
स्तदभिधावरणावरणीययोः ॥ १० ॥

रविसमानकलस्य कलावतो
वितमसो गुणितां भुजशिञ्जिनीम् ।
तिथिभिरिन्दुनवेन्दुभिर्हरेद्
व्यगुनिशाकरगोलवशाच्छरः ॥ ११ ॥

च्युतशरावरणावरणीययो-
र्दलयुतिः स्थगितस्य मितिर्भवेत् ।
समधिकावरणीयतनोर्यदा
सकलमेव तदावृतमादिशेत् ॥ १२ ॥

विवरमावरणावरणीययो-
रपनयेदथवा दलितं शरात् ।
तदवशेषमितिः प्रकटा भवे-
द्यदि न शेषमशेषतनुर्ग्रहः ॥ १३ ॥

दलितमावरणं युतमूनितं
पृथगथावरणीयदलेन च ।
स्वगुणितं शरवर्गविवर्जितं
कृतपदं गगनाङ्गहतं हरेत् ॥ १४ ॥

गतिवियोगकलाभिरतो भवेत्
स्थितिदलं घटिकादि समर्दलम् ।
रविशशाङ्कतमोगतिसङ्गुणं
'खरस'हृत् स्वफलानि पृथक् पृथक् ॥ १५ ॥

रविशशाङ्कफले समलिप्तयोः
क्षयधने भवतः प्रथमान्त्ययोः ।
धनमृणं तमसः स्वफलं कलाः
स्थितिविमर्ददलादसकृत् ततः ॥ १६ ॥

तिथ्यन्तं स्थितिदलवर्जितं युतं च
प्राग्ग्रासं क्रमश उशन्ति मोक्षकालम् ।
संयोगं स्थितिदलयोः स्थितेश्च कालं
मर्दार्धद्वययुतिमिन्द्वदर्शनस्य ॥ १७ ॥

मध्यग्रासः स्यात् तिथेश्चावसाने
मर्दार्धेनान्त्येन युक्तः स उक्तः ।
कालस्तज्ज्ञैर्नूनमुन्मीलनस्य
हीनश्चाद्येनेन्दुसम्मीलनस्य ॥ १८ ॥

इष्टोनितस्थितिदलेन वियोगलिप्ता-
भुक्त्योर्हता गगनषट्कहृता भुजः स्यात् ।
तात्कालिकोडुपशरं कथयन्ति कोटिं
दोःकोटिवर्गयुतिमूलमभीष्टकर्णः ॥ १९ ॥

एवं विमर्दार्धहृते च गत्योः
स्यादन्तरे दोः स्वशरश्च कोटिः ।
निमीलनोन्मीलनकर्णसिद्धयै
ग्रासस्तु मानैक्यदलं विकर्णम् ॥ २० ॥

वीष्टग्रासे मानयोरर्धयोगे
स्वघ्ने मूलं क्षेपवर्गोनितं यत् ।
तत् षष्टिघ्नं भुक्तिविश्लेषभक्तं
स्वस्थित्यर्धाच्छुद्धमिष्टस्तु कालः ॥ २१ ॥

स्पर्शाद्यातो ग्राह्यमानस्य खण्डे
दृष्टे शेषे मुच्यमानस्य शेषः ।
तत्कालेन्दोः क्षेपमानीय सम्यक्
कुर्यात् तावत् कर्म यावत् स्थिरं स्यात् ॥ २२ ॥

स्पर्शादिकालजनतोत्क्रमशिञ्जिनीभिः
क्षुण्णाक्षभा पलभवश्रवणेन भक्ता ।
चापानि पूर्वनतपश्चिमयोः फलानि
सौम्येतराणि समवेहि पृथक् क्रमेण ॥ ॥ २३ ॥

अहर्दलाद्रात्रिदलावसानं
यावत् कपालं कथयन्ति पूर्वम् ।
ततो दिनार्धान्तिमपूर्वमिन्दो-
र्भानोर्भवेतां ग्रहणेऽन्यथा ते ॥ २४ ॥

ग्राह्यात् सराशित्रितयाद् भुजज्या
व्यस्ता ततः प्राग्वदपक्रमज्या ।
तस्या धनुः सत्रिगृहेन्दुदिक् स्यात्
क्षेपो विपातस्य विधोर्दिशि स्यात् ॥ २५ ॥

अपक्रमक्षेपपलोद्भवानां
युतिः क्रमादेकदिशां कलानाम् ।
कार्या वियोगोऽन्यदिशां ततो ज्या
ग्राह्या भवत् सा वलनस्य जीवा ॥ २६ ॥

अङ्गुलानि वलनस्य च जीवा
'खेश्वरैर्वसुगुणाब्धिहुताशाः' ।
उन्नतो निजदिनार्धविभक्तः
सार्धयुग्मयुगथाङ्गुललिप्ताः ॥ २७ ॥

मानैक्यार्धं छादकच्छाद्ययोश्च
क्षेपश्छन्नं कर्णदोःकोटयश्च ।
भाज्याः सर्वेऽप्यङ्गुलानां कलाभि-
र्जायन्ते ते वाङ्गुलानि क्रमेण ॥ २८ ॥

(Lalla, *ŚiDhVṛ*., 5.1-28)

—Lalla

Time of the eclipse

If one wants to ascertain (the time of) a lunar eclipse, one must find the true longitudes of the Sun, the Moon, and its ascending node, on the fifteenth *tithi* (i.e., full moon day) in the light half of the lunar month, at sunset. (1)

Circumference of the sky

Multiply 21,600 by 10 and then by the revolutions of the Moon in a *yuga*. The result is the *yojanas* in the circumference of the sky up to which the rays of the Sun reach. (2)

Distance of the planet from the Earth

(The circumference of the sky in *yojanas*) divided by 72,000 gives the circumference of the orbit of the asterisms. Again, (the circumference of the sky in *yojanas*) divided by the number of revolutions of each planet in a *yuga*, gives the circumference of the planet's orbit (in *yojanas*). When this circumference is multiplied by 625 and divided by 3,927, the result is the distance of the planet from the earth (in *yojanas*). (3)

Distance of the Sun and the Moon from the Earth

The mean distance of the Sun from the Earth's centre is 4,59,585 *yojanas* and that of the Moon is 34,377 *yojanas*. (4)

When their mean distances are multiplied by their respective *mandasphuṭa* hypotenuse and divided by the radius, the results are their correct distances. Or, the mean distance multiplied by the mean motion and divided by the true motion, gives the correct distance (of the Sun or Moon) from the earth. (5)

The diameter of the Moon is 315 *yojanas* and that of the Sun is 4410. The Sun's distance from the earth multiplied by 5 and divided by 16 gives the height of the cone of the Earth's shadow. (6)

Diameter of the Earth's shadow in yojanas

The length of the earth's shadow (as found above) diminished by the correct distance of the Moon from the

centre of the earth, then multiplied by 1050 and divided by itself, gives, as result, the diameter of the earth's shadow in *yojanas* (in the Moon's orbit). (7)

Angular diameter of the Sun etc.

The diameters of the Sun, the Moon and the shadow, (each expressed in *yojanas*), and multiplied by the radius and divided, respectively, by the Sun's distance from the earth, the Moon's distance and the Moon's distance, (in *yojanas*), give the respective angular diameters in minutes, whence the arcs corresponding to the quotients as R sines are found. The results can also be treated approximately as diameters (without finding the arcs). (8)

Or, the true motion of the Moon multiplied by 11 and divided by 272 gives its angular diameter in minutes. That of the Sun is obtained by multiplying its true motion by 11 and dividing by 20. The difference between 8 times the true motion of the Moon and 25 times that of the Sun, divided by 60, gives the angular diameter of the shadow (in minutes). (9)

The eclipser and the eclipsed

The shadow is the Moon's obscuring body and the Moon is the Sun's obscuring body. There are total and partial eclipses of the obscured body caused by the obscuring body. (The eclipse) is named after the (eclipsed portion of the) obscured body.[1] (10) (BC)

Latitude of the Moon

When the Moon is equal to the Sun in respect of minutes etc., subtract from its longitude, the longitude of its ascending node. Find the R sine of the remaining arc, multiply it by 15 and divide by 191. The result is the latitude of the Moon, and its direction is according to the hemisphere in which the Moon happens to be diminished by its node. (11)

Obscured portion at mid-eclipse

When this latitude is subtracted from the sum of the semi-diameters of the obscuring and the obscured bodies, the remainder is the portion obscured (at the time of mid-eclipse). When the portion is greater than the obscured body, the latter is said to be completely obscured. (12)

Or, subtract half the difference of the diameters of the obscuring and the obscured bodies from the latitude of the Moon. The remainder is the portion not obscured. If there is no remainder, the obscured body is completely obscured. (13)

[1] For rationales and demonstrations, see *Śi.Dh.Vṛ: BC.* II.112-13.

Half-duration of the eclipse

Take the sum or difference in the semi-diameters of the obscuring and the obscured bodies. Square it and subtract from it the square of the Moon's latitude. Find the square root of the remainder. Multiply it by 60 and divide by the difference of the true motions in minutes of the two bodies. The results are, respectively, the approximate half durations of the eclipse and the total eclipse in *ghaṭikās*.

When these times are severally multiplied by the true motions of the Sun, Moon and its node and each product divided by 60, the results are, respectively, the motions of the Sun, Moon and its node during these times. (14-15)

The Sun's and Moon's motions in minutes should, respectively, be subtracted from their longitudes and the node's motion added to its longitude, if the first half of duration of the eclipse or of the total eclipse is required; the reverse process is to be followed if the second half is required.

From these longitudes, again calculate the half duration of the eclipse and of the total eclipse. Repeat the process till the times are fixed. (16)

When the first half of the duration of an eclipse is subtracted from the time of the full moon, the remainder gives the time when the eclipse began. When the second half of the duration of an eclipse is added to the time of the full moon, the sum gives the time when the eclipse ends. So the wise say.

The sum of the first and second half of the duration of the eclipse is its total duration. The sum of the first and second half of the duration of the total eclipse is the duration of the complete disappearance of the Moon. (17)

The end of the full moon is the time of mid-eclipse. That time, diminished by the first half of the duration of the total eclipse, gives the time when the Moon is completely obscured. Again, that time added to the second half of the duration of the total eclipse, gives the time when the Moon begins to reappear. So say those who know. (18)

Obscured portion at any time

When the given time (during an eclipse) is subtracted from the first or second half of the duration of an eclipse, and the remainder is multiplied by the difference in minutes of the motions (of the obscuring and the obscured bodies) and divided by 60, the result is called *bhuja* or base. The latitude of the Moon at that time is called *koṭi* or perpendicular. The square root of the sum of the squares of the base and the perpendicular is the hypotenuse at that time. (19)

In the same way, when the first or second half of a total eclipse is multiplied by the difference of the true motions (of the obscuring and the obscured bodies, and the product is divided by 60, the result is the base). The latitude of the Moon at these times is the perpendicular. (The square root of the sum of the squares of the base and the perpendicular is the hypotenuse at the beginning of the total eclipse (if the first half is taken), and is the hypotenuse for the end of the total eclipse (if the second half is taken).

(In both cases), when the hypotenuse is subtracted from the sum of the semi-diameters of the two bodies, the remainder is the obscured portion. (20)

Time from the obscured portion

Subtract the given obscured portion from the sum of the semi-diameters of the two bodies. Square the remainder. Subtract from it the square of the Moon's latitude (at the time of mid-eclipse). Find the square root of the remainder. Multiply it by 60 and divide by the difference of the true motions of the two bodies. Subtract the result from the first half of the duration of the eclipse, if the observed portion is between the beginning of the eclipse and the mid-eclipse. But if it is between the mid-eclipse and the end, subtract the quotient from the second half of the duration of the eclipse. The result is approximately the time when the given portion is obscured.

Find the Moon's latitude at this time and repeat the process till the time is fixed. (This then is the correct time.) (22)

Valana: Deflection due to latitude and declination

Multiply the equinoctial midday shadow by the R versed sine or the *utkramajyā* of the hour-angle at the beginning of the eclipse, etc. and divide by the hypotenuse of the equinoctial shadow. Remember that the arc corresponding to this quotient as the R sine, (which is called *ākṣavalana*), is to be taken as north or south according as the obscured body is in the eastern or western hemisphere. (23)

In a lunar eclipse, the Moon, (the obscured body), is said to be in the eastern hemisphere from midday till midnight. From midnight till midday it is said to be in the western hemisphere. In a solar eclipse, (for the Sun, which is the obscured body), the contrary is the case. (24)

Increase the number of degrees in the longitude of the obscured body by 3 Signs and then find its R versed sine or *utkramajyā*. Hence find, as before, the R sine of the declination. The corresponding arc (*āyanavalana*) has the same denomination as that of the Moon increased by 3 signs. The latitude of the Moon has the same denomination as that of the Moon diminished by its node. (25)

The *ākṣavalana*, the *āyanavalana* and the Moon's latitude, all in minutes, should be added together if they are of the same denomination but their difference must be taken when of different denominations. Then find the R sine of the sum or difference, as the case may be. The result is the R sine of the *valana* or variation of the eastward direction of the ecliptic from the eastward direction of the disc of the obscured body. (26)

Divide the R sine of the *valana* and 3438, the radius, by 110, to convert them into *aṅgulas*. When the altitude in time (*unnatakāla*) on any day at any given time, is divided by half the duration of the day and $2\frac{1}{2}$ added to the result, the sum is the number of minutes equivalent to one *aṅgula*. (27)

Conversion of aṅgular deflection to linear deflection

The sum of the radii of the obscuring and the obscured bodies, the latitude of the Moon, the obscured portion, the hypotenuse, the base and the perpendicular, (expressed in minutes) should be divided by the number of minutes in one *aṅgula*.[1] (28). (BC)

—भास्करः २

ग्रहणसंभवः

16. 11. 1a. कलेर्गताब्दा 'रवि'भिर्विनिघ्ना-
श्चैत्रादिमासैः सहिताः पृथक्स्थाः ।
द्विघ्नाः स्व'नागाङ्कगजां'शहीनाः
'पञ्चाङ्ग'भक्ताः प्रथमान्विताः स्युः ॥ १ ॥

मासाः पृथक् ते द्विगुणा'स्त्रिपूर्ण-
बाणा'धिका स्वा'ङ्कनृपां'शयुक्ताः ।
त्रिभिर्विभक्ताः फलमंशपूर्वं
मासौघतुल्यैश्च गृहैर्युतं स्यात् ॥ २ ॥

सपातसूर्योऽस्य भुजांशका यदा
'मनू'नकाः स्याद् ग्रहणस्य संभवः ।
गृहार्धयुक्तस्य सपातभास्वतो
भुजांशकान् गोलदिशोऽवगम्य च ॥ ३ ॥

ज्ञैयोऽर्को रविसंक्रमाद् गतदिनैर्दर्शान्तनाडीनता-
'द्वेदां'शेन गृहादिनोनसहितः प्राक्पश्चिमेऽस्यापमः ।
अक्षांशैः खलु संस्कृतो 'रस'लवेनास्याथ ते संस्कृताः
पाताढ्यार्कभुजांशका यदि 'नगो'नाः स्युस्तदार्कग्रहः॥

'रूपं' 'वियत्' 'पूर्णकृतान्' सपादान्
क्षिप्त्वा सपाते प्रतिमासमर्के ।

[1] For the rationale and exposition, see *ŚiDhVr*, : *BC*, II. 113-28.

तत्सम्भवं प्रागवलोक्य धीमान्
ग्रहान् ग्रहार्थं विदधीत तत्र ।। ५ ।।
(Bhāskara II, *SiŚi.*, 1. 4. 1-5)

—Bhāskara II

Possibility of an eclipse

Multiply the number of years that have elapsed from the beginning of the Kaliyuga by twelve and add the number of months elapsed from the beginning of the luni-solar year. Let the result be x. Then add $\frac{2x\left(1-\frac{1}{876}\right)}{65}$ to x. Let the result be y. Then the longitude of what is called *Sapāta-Sūrya* or the longitude of the Sun with respect to a node will be x *rāśis*$+$ $\frac{(2y+503)\left(1+\frac{1}{167}\right)}{3\times 30}$ *rāśis*. If this longitude be less than 14°, then a lunar eclipse is likely to occur. (1-3a)

Special note for the solar eclipse

Add half a *rāśi* to the longitude previously obtained; find out on which side the Sun lies, north or south; compute the longitude of the Sun from the number of days elapsed after the *Saṅkramaṇa* day (i.e. the day on which the Sun has left one *rāśi* and entered another. (3b)

Obtain the hour-angle in *nāḍīs* of the Sun at the ending moment of the Amāvāsyā, i.e. at the moment of new moon; add or subtract one-fourth thereof in *rāśis* from the position of the Sun according as the Sun is in the western or eastern hemisphere; then finding the declination of that point and from the sum or difference of the declination and latitude of the place, obtain the zenith-distance of the culminating point of the ecliptic; taking that point to be roughly the Vitribha, i.e. the point of the ecliptic which is 90° behind the Sun on the ecliptic, find one-sixth of the zenith-distance; taking the sum or difference of the result and the longitude of the Sun with respect to the node (obtained in the beginning by adding half a *rāśi* to its position at full moon) if the result happens to fall short of 7°, then we could expect a solar eclipse. (4)

If there be no eclipse at the current new moon, then go on adding one *rāśi*-0°-40′-15″ to the longitude of the Sun with respect of the Node (which will be its longitude for the moment of the next new moon) and repeating the procedure indicated, the occurrence of an eclipse or otherwise could be known. If such an occurrence be indicated, then compute the actual positions of the Sun, Moon and Rāhu and following the procedure to be indicated in the chapter on Solar Eclipses, the moment of the occurrence of the eclipse and other relevant details could be computed.[1] (5). (AS)

[1] For an exposition, see *SiŚi*: *AS*, pp. 331-45.

अर्केन्द्वोः कक्षाव्यासार्धः

16. 11. 1b. 'नगनगाग्निनवाष्टरसा' रवे
'रसरसेषुमहीषु'मिता विधोः ।
निगदितावनिमध्यत उच्छ्रितिः
श्रुतिरियं किल योजनसंख्यया ।। ३ ।।

मन्दश्रुतिर्द्राक्श्रुतिवत् प्रसाध्या
तया त्रिभज्या द्विगुणा विहीना ।
त्रिज्याकृतिः शेषहृता स्फुटा स्या-
ल्लिप्ताश्रुतिस्तिग्मरुचेर्विधोश्च ।। ४ ।।

लिप्ताश्रुतिघ्नस्त्रिगुणेन भक्तः
स्पष्टो भवेद्योजनकर्ण एवम् ।
बिम्बं 'रवेर्द्विशरर्तु'संख्या-
नीन्दोः 'खनागाम्बुधि'योजनानि ।। ५ ।।

भूव्यासहीनं रविबिम्बमिन्दु-
कर्णाहतं भास्करकर्णभक्तम् ।
भूविस्तृतिर्लब्धफलेन हीना
भवेत् कुभाविस्तृतिरिन्दुमार्गे ।। ६ ।।

सूर्येन्दुभूभातनुयोजनानि
त्रिज्याहतान्यर्कशशीन्दुकर्णैः ।
भक्तानि तत्कार्मुकलिप्तिकास्ता-
स्तेषां क्रमान्मानकला भवन्ति ।। ७ ।।

सपाततात्कालिकचन्द्रदोर्ज्या
'खभैः' हता व्यासदलेन भक्ता ।
सपातशीतद्युतिगोलदिक् स्या-
द्विक्षेप इन्दोः स च बाणसंज्ञः ।। १० ।।

ग्रासप्रमाणम्

यच्छाद्यसंछादकमण्डलैक्य-
खण्डं शरोनं स्थगितप्रमाणम् ।
तच्छाद्यबिम्बादधिकं यदा स्या-
ज्ज्ञेयं च सर्वग्रहणं तदानीम् ।। ११ ।।
(Bhāskara II, *SiŚi.*, 1.5.3-7,10-11)

Preliminaries

Orbital radius

The distances of the centres of the globes of the Sun and the Moon from the centre of the Earth in *yojanas* are respectively 689,377 and 51,566. (3)

Hypotenuse

The radius vector is to be computed even in the case of the Equation of centre as we did in the case of *śīghraphala*. If it be 'K', $\frac{R^2}{2R-K}$ will be what is *kalākarṇa* both in the case of the Sun, as well as the Moon. (4)

The above *kalākarṇa* multiplied by the *karṇa* given in *yojanas* and divided by the Radius gives the rectified *yōjanakarṇa*. (5)

Spherical radii of Sun and the Moon

The spherical diameters of the Sun and the Moon are respectively, 6522 and 480 *yojanas*. (5b)

$e-\frac{(S-E)\ Km}{Ks}=2a$ (where e=Earth's diameter, s= Sun's diameter; Km=Moon's distance from the Earth's centre; Ks=Sun's distance from the Earth's centre, and a=radius of the Earth's shadow cone at the lunar orbit). (6)

Angular measure

The diameters of the Sun, the Moon, and Rāhu in *yojanas* multiplied by R=3438′, and divided, respectively, by Ks, Km and Km, give their angular measures. (7)

Latitude of the Moon

Vikṣepa or *Śara*, as it is also called, i.e. the latitude β of the Moon, is obtained by the formula $\beta=\frac{R\ \sin\lambda\times 270}{R}$ and it will have the same direction as the Moon with respect to the ecliptic, where λ is the longitude of the Moon with respect to the nearer node and 270′ or $4\frac{1}{2}°$ is taken to be the inclination of the lunar orbit to the ecliptic or, what is the same, the maximum latitude of the Moon. (10)

Magnitude of a lunar eclipse

Sthagita or the magnitude of an eclipse is defined as $P+r+\beta$ (where P and r are, respectively, the radii of the eclipsing and eclipsed bodies and β is the latitude of the Moon). If the *Sthagita* is greater than 2r, then the eclipse is total.[1] (11). (AS)

स्थित्यर्धः

16. 11. 1c. मानार्धयोगान्तरयोः कृतिभ्यां
शरस्य वर्गेण विवर्जिताभ्याम् ।
मूले 'खषट्'संगुणिते विभक्ते
भुक्त्यन्तरेण स्थितिमर्दखण्डे ।। १२ ।।

स्थित्यर्धनाडीगुणिता स्वभुक्तिः
षष्ट्या हृता तद्रहितौ युतौ च ।
कृत्वेन्दुपातावसकृच्छराभ्यां
स्थित्यर्धमाद्यं स्फुटमन्तिमं च ।। १३ ।।

एवं विमर्दार्धफलोनयुक्त-
सपातचन्द्रोद्भवसायकाभ्याम् ।
पृथक् पृथक् पूर्ववदेव सिद्धे
स्फुटे स्त आद्यान्त्यविमर्दखण्डे ।। १४ ।।

(Bhāskara II, *SiŚi.*, 1.5. 12-14)

[1] For an exposition and the rationale of the several processes involved, see *SiŚi* : *AS*, pp. 347-63.

Duration of the eclipse

$$Sthiti\text{-}khaṇḍa=\frac{\sqrt{(P+r)^2-\beta^2}\times 60}{m_1-s_1}=\tfrac{1}{2}\text{ duration of the eclipse}$$

$$Marda\text{-}khaṇḍa=\frac{\sqrt{(P-r)^2-\beta^2}\times 60}{m_1-s_1}=\tfrac{1}{2}\text{ duration of totality}$$

where P is the radius of the shadow-cone, r the radius of the Moon's disc, β its latitude taken to be constant during the eclipse, m_1 and s_1 the daily motions of the Moon and Sun respectively. (12)

Rectification of the times

From the position of the Moon and that of the Node obtained for the moment of opposition, have to be computed their position for the moment of first contact and those for the moment of last contact. (13)

Proceeding on the same lines as above and obtaining β_2 and β_4 the rectified latitudes of the Moon for the moments of the commencement and end of totality of the eclipse, the *Sammīlana-marda-khaṇḍa* and *Unmīlana-marda-khaṇḍa*, T_3 and T_4, are to be rectified.[1] (14). (AS)

स्पर्शादिव्यवस्था

16. 11. 1d. मध्यग्रहः पर्वविरामकाले
प्राक् प्रग्रहोऽस्मात् परतश्च मुक्तिः ।
स्थित्यर्धनाडीष्वथ मर्दजासु
संमीलनोन्मीलनके तथैव ।। १९ ।।

'खाङ्का'हतं स्वद्युदलेन भक्तं
स्पर्शादिकालोत्थनतं लवाः स्युः ।
तेषां क्रमज्या पलशिञ्जिनीघ्नी
भक्ता द्युमौर्व्या यदवाप्तचापम् ।। २० ।।

प्रजायते प्रागपरे नते क्रमा-
दुदग्यमाशं वलनं पलोद्भवम् ।

(Bhāskara II, *SiŚi.*, 1.5.19-21b)

First contact etc.

The 'Middle of the eclipse' (or, strictly speaking) the moment when the portion eclipsed is a maximum) occurs at the moment of opposition. *Sparśa* or *pragraha* is at the moment of first contact and *mokṣa* is at the moment of last contact, separated from the moment of the middle of the eclipse by times equal to *sparśa-sthiti-khaṇḍa* and *mokṣa-sthiti-khaṇḍa* respectively before and after. Similarly, *sammīlana* and *unmīlana* or the moment of the commencement of totality and the end thereof occur before and after the moment of 'the middle of the eclipse' by times equal to *sammīlana-marda-khaṇḍa* and *unmīlana-marda-khaṇḍa*, respectively. (19)

[1] For the rationale involved, see *ŚiSi*: *AS*, pp. 363-70.

Valana or Deflection

The hour angle of the eclipsed body expressed in *nāḍīs*, multiplied by 90 and divided by half the duration of night (if it be lunar eclipse) or half the duration of day (if it be solar), as the case may be, will give the degrees of an angle, whose R sine being multiplied by the R sine of the latitude and divided by (R cos δ), (where δ is the declination of the eclipsed body), gives the R sine of what is called *Ākṣa-valana* which is north when the hour angle is east, and south otherwise. (20-21ab). (AS)

—करणरत्नम्

बिम्बव्यास:

16. 12. 1. 'दश' गुणितेन्दोर्भुक्ति:
'शशिशरयम'भाजिता निजं बिम्बम् ।
'नवनव'भि: स्वा राहो-
स्तया रवे: स्वा 'पुराणेन' ।। २ ।।

चन्द्रविक्षेप:

फणिरहितसमकलेन्दोर्ज्या स्वदशांशोनिताऽत्र विक्षेप: ।

स्पर्शमोक्षौ

तत्कृतिरहिते फणिशशिबिम्बसमासार्धवर्गे स्यात् ।। ३ ।।
मूलं स्थित्यर्धकला रविशशिभुक्त्यन्तरोद्धृता नाड्य: ।
पर्वणि तद्रहिते स्यात् प्रग्रहणं तत्र संयुते मोक्ष: ।। ४ ।।
प्रतिपदि विपरीतमिदं त्वविशिष्टं जायते ग्रहणमध्यम् ।
सूर्यग्रहणेऽप्येवं राहुस्थाने तु शशिबिम्बम् ।। ५ ।।

ग्रहणसम्भव:

सम्पर्कादल्पे विक्षेपे ग्रहणमस्ति नास्त्यधिके ।
राहोर्युतिश्च दृष्टिर्यदा रवे: स्यात् तदा भवति ।। ६ ।।

ग्रहणादेश्यता

द्वादशभागादूनं ग्रहणं तैक्ष्ण्याद्रवेरनादेश्यम् ।
षोडशभागादिन्दो: स्वच्छत्वादधिकमादेश्यम् ।। ७ ।।

विमर्दकाल:

विक्षेपकृतिं त्यक्त्वा फणिशशिविष्कम्भविवरदलवर्गात् ।
मूलं भुक्त्यन्तरहृतमथ घटिका: स्युर्विमर्दार्धे ।। ८ ।।
स्थितिदलघटिकासदृशी संख्या विषमे पदे स्वविक्षेपे ।
स्पर्शे शोध्या क्षेप्या समेऽन्यथा मोक्षकाले स्यात् ।। ९ ।।
कृत्वाऽविशेषमेवं यावद् द्वितुल्यरूपता भवति ।
स्थितिदलविक्षेपौ तौ पुन: पुन: तावदानीयात् ।। १० ।।
तेनानीतस्थितिदलघटिकाकालौ तु तौ स्फुटौ ज्ञेयौ ।

(Deva, *KR*, 2. 2-11b)

—Karaṇaratna

Angular diameters of Sun, Moon and Rāhu

Ten times the Moon's daily motion when divided by 251 gives the Moon's own diameter and when divided by 99, gives the diameter of *Rāhu* (i.e., Shadow); and 10 times the Sun's daily motion when divided by 18 gives the diameter of the Sun. (2)

Moon's latitude

From the longitude of the Moon at full moon subtract the longitude of the Moon's ascending node. The R sine of that diminished by one-tenth of itself is the Moon's latitude (at full moon). (3)

Times of first and last contacts

Subtract the square of that (Moon's latitude) from the square of half the sum of the diameters of the Moon and Shadow. The square root of that is half the duration of the eclipse in terms of minutes. This divided by the motion-difference of the Sun and Moon (in terms of degrees) gives the *nāḍīs* (of half the duration of the eclipse). (When the time is measured from sunrise) on the full moon *tithi*, these *nāḍīs* being subtracted from the time of opposition (of the Sun and the Moon), the result is the time of the first contact; and the same number of *nāḍīs* being added to the time of opposition, the result is the time of the last contact. (When the time is measured from sunrise) on the next *tithi* (called *Pratipad*), the process is just the reverse. (That is, the time of the last contact is obtained by subtracting the above *ghaṭīs* from the time of opposition and the time of the first contact is obtained by adding those *ghaṭīs* to the time of opposition). The time of the middle of the eclipse is obtained by iterating the above process. In the case of a solar eclipse, the process is the same, except for that in the place of *Rāhu* (Shadow) one has to use the Moon's disc. (3cd-5).

Possibility of a Lunar eclipse

When the Moon's latitude (for the time of opposition) is less than half the sum of diameters of the eclipsed and eclipsing bodies, an eclipse (of the Moon) is possible; when greater (or equal), it is not possible.

The conjunction of *Rāhu* (Shadow) with the Moon occurs when the Sun sees the Moon (i.e., when the Moon is diametrically opposite to the Sun). (6)

Prediction of an eclipse

A solar eclipse should not be predicted when it amounts to less than one-twelfth of the Sun's diameter (as it might not be visible to the naked eye) on account of the brilliancy of the Sun. But a lunar eclipse must be declared whenever it amounts to more than one-sixteenth of the Moon's diameter, as it will be visible (to the naked eye) on account of the transparency of the Moon. (7)

Duration of totality

Subtract the square of the Moon's latitude (for the time of opposition) from the square of half the difference

between the diameters of the eclipsed and eclipsing bodies and take the square root thereof, and then divide (that square root) by the motion-difference (of the Sun and Moon) (in terms of degrees): the quotient gives the *ghaṭikās* of half the duration of total eclipse. (8)

Moon's latitude for first or last contact

(In order to obtain the Moon's latitude) for the first contact, subtract as many minutes from the Moon's latitude (for the time of opposition) as there are *ghaṭīs* in half the duration of the eclipse if the eclipse occurs in an odd nodal quadrant, and add the same number of minutes if the eclipse occurs in an even nodal quadrant. (In order to find the Moon's latitude) for the last contact, proceed reversely. (9)

Semi-duration of eclipse by iteration

Having done this, apply the process of iteration in the following way: Calculate the semi-durations of the eclipse and the Moon's latitude (for the first and last contacts) again and again until the successive values are the same. The times, in *ghaṭīs*, of the semi-durations of the eclipse calculated from them (i.e., from the Moon's latitudes for the first and last contacts, obtained by iteration) arc the true values of the two (semi-durations).[1] (10-11 a). (KSS)

—श्रीपतिः

चान्द्रमासानयमम्

16. 13. 1a. 'चन्द्राङ्गनन्दो'नशको'ऽर्क' निघ्न-
श्चैत्रादिमासैर्युगधो 'द्वि'निघ्नः ।
'पञ्चो'नितः स्वीय'नृपाङ्क' भाग-
हीनः 'शराङ्गा'प्तफलेन युक्तः ॥ २ ॥
(Śrīpati, *Dhīkoṭi*, 1. 2)

—Śrīpati

Lunar months since Śaka 961, the epoch

Diminish the (current) year of the Śaka era by 961, (then) multiply (the remainder) by 12, (then) add (to the resulting product) the number of months elapsed since the beginning of Caitra; (then set down the resulting sum in two places one below the other.) Multiply the sum in the lower place by 2, (then) diminish (the product) by 5, and (then) diminish (the remainder obtained) by 961th of itself. Divide whatever is obtained (as the remainder) by 65 an add (the quotient to the sum standing in the upper place.[2] (2)

दर्शान्त-पूर्णिमान्त-भध्रुवाः ग्रहणसंभवश्च

16. 13. 1b. त्रिधा 'करा'भ्यां 'धृति'भि'र्भुवा' च
तं मासवृन्दं गुणयेत् ततोऽन्त्यः ।
निजा'भ्रनेत्रां'शयुगा'भ्रषड्भि'-
र्भक्तः फलाढ्यस्त्वथ मध्यराशिः ॥ ३ ॥

स 'षष्टि'भक्तः फलमूर्ध्वराशौ
दत्वा ततो 'भै'र्विभजेच्च शेषम् ।
दर्शान्तिको भध्रुवकस्तु सः स्यात्
'भुवा''ब्धिवेदै'श्च 'रसैः' समेतः ॥ ४ ॥

दर्शान्तकः स्या'च्छशिना'थनन्दे-
'रभ्रेण' युक्तः स च पूर्णिमान्तः ।
'चक्रार्ध'चक्रान्तरके ध्रुवस्य
'पूर्णं' यदोर्ध्वं ग्रहणं विचिन्त्यम् ॥ ५ ॥
(Śrīpati, *Dhīkoṭi*, 1. 3-5)

Darśānta-bhadhruva and Pūrṇimānta-bhadhruva

Set down the resulting "aggregate of months" in three places (one below the other) and multiply (the numbers in the upper-most, the middle and the lowest places) by 2, 18, and 1, respectively. Then increase the last result (which is in the lowest place) by one-twentieth of itself. Then divide that by 60, (retain the remainder) and add the quotient to the number in the middle place. (Then) divide that (i.e., the number now in the middle place) by 60, (retain the remainder) and add the quotient to the number in the uppermost place. Then divide the number now in the uppermost place) by 27 (discard the quotient and retain the remainder). (To the remainders standing in the uppermost, the middle and the lowest places denoting *nakṣatra*, *ghaṭī* and *pala* in order) add 1, 44 and 6, respectively. Thus is obtained the Darśānta-bhadhruva.[1] (3-5a)

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[1] For rationale, see: *KR*; *KSS*, pp. 42-47.

[2] This rule assumes, following Āryabhaṭa I, that one solar month is equivalent to

$$1+\frac{2}{65}\left(1-\frac{1}{916}\right) \text{ lunar months approx.}$$

The subtractive 5 in the text is meant to account for the intercalary excess at the beginning of the Śaka year 961.

[1] The Darśānta-bhadhruva being increased by 1, 9 and 0 (in the uppermost, the middle and the lowest places denoting *nakṣatra*, *ghaṭī* and *palā*, respectively) gives the *Pūrṇimānta-bhadhruva*.

The Darśānta-bhadruva denotes the longitudinal distance of the Mean Sun from the Moon's ascending node at the time of conjunction of the Sun and the Moon in terms of *nakṣatra*, *ghaṭī* and *pala*. The *nakṣatra*, *ghaṭī* and *pala* here are the divisions of the circle like the degrees, minutes and seconds. The whole circumference of the circle is divided into 27 equal parts called *nakṣatras*, each *nakṣatra* is subdivided into 60 equal parts called *ghaṭis*, and each *ghaṭī* is further subdivided into 60 equal parts called *palas*.

The Pūrṇimānta bhadhruva denotes the longitudinal distance of the mean Sun from the Moon's ascending node at the time of opposition of the Sun and the Moon, in terms of *nakṣatra*, *ghaṭī* and *pala*.

The above rule assumes, following the *Siddhāntaśekhara*, that the rate of separation of the Sun from the Moon's node is equivalent to 2 *nakṣatras* 18 *ghaṭis* and 1 1/20 *palas* per month. In a fortnight, likewise, this separation to 1 *nakṣatra* 9 *ghaṭis* 0 *pala*.

1 *nakṣatra* 44 *ghaṭis* and 6 *palas*, used as an additive in the above rule, is the *bhadhruva* for the beginning of Caitra Śaka 961, the starting point of our calculation.

Possibility of an eclipse

When the difference of the (Darśānta or Pūrṇimānta) bhadhruva from half a circle (i.e. 13 *nakṣatras* 30 *ghaṭīs*), or a full circle (i.e. 27 *nakṣatras*) yields zero in the uppermost place (denoting *nakṣatras*), an eclipse (of the Sun or the Moon) is to be considered (possible).[1] (5b). (KSS)

रविकनाड्य:

16. 13. 1c. 'वह्न्य''ष्ट' 'दिग्' 'रुद्र' 'गजा''ग्नि' संख्या:
कर्कादिकेऽर्के रविका ऋणाख्या: ।
'षट्' 'दिक्' 'भवा''ऽऽशा' 'रस' 'शून्य' संख्या
मृगादिकेऽर्के धनसंज्ञनाड्य: ।। ६ ।।
(Śrīpati, *Dhīkoṭi*, 1. 6)

Difference between true and mean Sun

When the Sun is in the six Signs beginning with Cancer (i.e. in Cancer, Leo, Virgo, Libra, Scorpio, and Sagittarius), the Ravikā Nāḍīs (i.e. *ghaṭīs* of the Sun's correction), which are then negative in sign, amount to 3, 8, 10, 11, 8 and 3 (respectively); when the Sun is in the six Signs beginning with Capricorn (i.e. in Capricorn, Aquarius, Pisces, Aries, Taurus and Gemini) the (corresponding Ravikā Nāḍīs, which are now positive in sign, amount to 6, 10, 11, 5, 6 and 0 (respectively).[2] (6)

ग्रहणगणनम्

16. 13. 1d. संस्कृत्य पूर्वं रविकाभिरस्य
चक्रार्धचक्रान्तरकेऽथ शेषम् ।
लिप्तादिकं मध्यशर: स याम्यो
भार्धाधिके, न्यूनतरेऽथ सौम्य: ।। ७ ।।

मध्येषुहीना 'रसबाण'लिप्ता-
श्छन्नं च तस्मिन् शशिमण्डलोने ।
स्यात्खण्डपर्वाभ्यधिकेऽथ पूर्णं
'कराग्नि'लिप्तामितमिन्दुमानम् ।। ८ ।।

'षडग्निचन्द्राग्नि'मितार्धयुक्त-
वर्गात्तु विक्षेपकृतिं विशोध्य ।
मूलेऽथ 'षष्ट्या' गुणिते'ऽभ्रराम-
नगै'श्च भक्ते स्थितिखण्डनाड्य: ।। ९ ।।

'रसाद्रिबाण'प्रमिताच्छरोनात्
प्राग्वत्कृते मध्यविमर्दनाड्य: ।
स्पर्शस्थिति: स्यात् स्थितिखण्डहीने
मोक्षस्थिति: स्यात् स्थितिखण्डयुक्ते ।। १० ।।

मर्दोनमध्ये तु निमीलनं स्या-
दुन्मीलनं मध्यविमर्दयुक्ते ।
मध्यग्रह: पर्वतिथे: समाप्तौ
शरादिके तु त्रिहतेऽङ्गुलादि ।। ११ ।।

पूर्वाश्रित: किंचिदिवेशदिक्स्थ:
स्पर्शश्च वायोर्दिशि मोक्ष उक्त: ।
स्यादुत्तरस्यां दिशि मध्यखण्डं
सौम्य: शशांकस्य यदा शर: स्यात् ।। १२ ।।

स्पर्शस्तथाग्नौ निर्ऋतौ च मोक्ष:
खण्डं च याम्ये यदि याम्यबाण: ।
निमीलनं पश्चिमदिग्विभागे
उन्मीलनं पूर्वदिशस्तथेन्दो: ।। १३ ।।
(Śrīpati, *Dhīkoṭī*, 1. 7-13)

Eclipse computation

Having first applied these Ravikā Nāḍīs (as a negative or positive correction) to the Bhadhruva, one should find the difference of the (corrected) Bhadhruva from half a circle or a full circle (as the case may be). The difference (in *ghaṭīs* etc. thus obtained) gives the Madhyaśara (i.e. the Moon's latitude for the middle of the eclipse) in terms of minutes etc.

When the (corrected) Bhadhruva is greater than half a circle, the Madhyaśara is south; when less it is north.[1] (7)

Moon's diameter

Subtract the Madhyaśara (i.e. the Moon's latitude for the middle of a lunar eclipse) from 56 minutes (denoting the sum of the semi-diameters of the Moon and the Shadow): then is obtained the measure of the lunar eclipse (in minutes). When it is less than the Moon's diameter, the eclipse is partial; when greater, the eclipse is total, the measure of the Moon's diameter is 32 minutes.[2] (8)

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[1] This means that an eclipse of the Sun or the Moon should be considered possibe if at the time of conjunction or opposition of the Sun and the Moon the distance of the Sun from the nearest node is less than one *nakṣatras*, i.e. 13° 20′.

[2] The *Ravikā-nāḍīs* above give the difference between the true and mean positions of the Sun, i.e. the Sun's correction. This correction can be easily identified with the Sun's equation of the centre. It is to be noted that the correction stated above is zero in Gemini, the sign occupied by the Sun's apogee, and is again zero somewhere in Sagittarius, the sign occupied by the Sun's perigee. In the other signs its variation behaves like that of the Sun's equation of the centre. Moreover, its maximum value is 11 *nāḍīs*, i.e. 2° 26′ 40″, which roughly corresponds to the Indian value of the Sun's equation of the centre. It may be added that 11 *nāḍīs* is the approximate maximum value of the correction in round figures.

[1] The (corrected) Bhadhruva denotes the longitudinal distance of the apparent (or true) Sun from the Moon's ascending node, in terms of *nakṣatra, ghaṭī and pala*.
The difference between the corrected Bhadhruva and half a circle or a full circle (as the case may be) gives the distance of the apparent Sun from the Moon's nearer node.
For demonstration see *Dhīkoṭi*: *KSS*, pp. 15-16.

[2] The following table gives the mean (angular) diameters of the Sun, the Moon, and the Shadow as used by Śrīpati in the present work and also the corresponding modern values.

	Śrīpati's value	*Modern value*
Sun's diameter	33′	32′ 4″
Moon's diameter	32′	31′ 7″
Diameter of shadow	80′	82′ approx.

Duration of the eclipse

Subtract the square of the Moon's latitude from 3136, which is the value of the square of the sum of the semi-diameters of the Moon and the Shadow (in minutes). Then multiply the square root of that (difference) by 60 and divide by 730: the quotient gives the duration of the (lunar) eclipse in *nāḍīs*. (9)

Duration of totality for a total lunar eclipse

Subtracting (the square of) the Moon's latitude from 576 (i.e. the square of the difference of the semi-diameters of the Moon and the Shadow in minutes)[1] and proceeding as before, are obtained the *nāḍīs* of half the totality (for a total lunar eclipse). (10a)

First and last contacts and immersion and emersion

(The time of the middle of the lunar eclipse) being diminished by half the duration of the lunar eclipse gives the time of first contact: the same being increased by half the duration of the lunar eclipse gives the time of separation of the Moon from the Shadow (i.e. the time of the last contact.)

(The time of the middle of the lunar eclipse) being diminished by half of the duration of totality gives the time of immersion; and the same increased by half the duration of totality gives the time of emersion.

The middle of the eclipse happens to be (approximately) at the end of the Parva-tithi (i.e. at the time of opposition of the Sun and the Moon).

(The minutes of) the Moon's latitude etc. being divided by 3 are reduced to *aṅgulas*. (10b-11)

When the Moon's latitude is north, the first contact occurs in the east slightly deviated towards the north-east; the separation occurs in the north-west; and the middle of the eclipse in the north. (12)

When the (Moon's) latitude is south, the first contact occurs in the south-east; the separation in the south-west; and the middle of the eclipse, in the south. The immersion of the Moon takes place in the west and emersion, in the east. (13). (KSS)

—वाक्यकरणम्

ग्रहणोपकरणम्

16. 14. 1a. प्रतिपत्पर्वघटिकाः कला ऋणधनं रवौ ।। ५ ।।
भोग'नीचा'न्तरघ्नास्ताः विकलाः पूर्ववद् रवौ ।
'नीचा'न्न्यूने तु भोगे स्युर्व्यत्ययात् स समो रविः ।।६।।
चन्द्रेण पौर्णमास्यन्ते चक्रार्धेन समन्वितः ।
बिम्बं रवेर्गतिकला 'मान'घ्ना 'धन'भाजिताः ।। ७ ।।
इन्दोः 'शस्त्र'हृता भुक्तिः बिम्बं, त'न्मान'ताडितम् ।
अर्धीकृतं तमोबिम्बं स'यज्ञं' छादकं विधोः ।। ८ ।।
पातोनचन्द्रबाहुज्या 'स्तन'घ्ना क्षेपचापकम् ।
स्वस्व'पुत्रां'शसहितं धनर्णं जूकमेषतः ।। ९ ।।
(*VK*, 4. 5b-9)

—Vākyakaraṇa

The Ecliptic Elements

Deduct from the Sun's longitude at sunrise as many minutes as the *nāḍīs* gone in *Prathamā*, or add as many minutes as the *nāḍīs* to go for the end of the *Parva*. Deduct or add seconds equal to the product of the *nāḍīs* and the excess in minutes of the daily motion over 60′. If the daily motion is less than 60′ use the defect, and add or deduct, respectively. The longitude of the Sun at the end of the *Parva* is got. This will be equal to the longitude of the Moon, if the *Parva* is Amāvaśyā. If it is Pūrṇimā, this will be equal to the Moon *plus* 6 *rāśis*. (5b-7a)

Multiply the Sun's daily motion in minutes by 5 and divide by 9. The angular diameter of the Sun is in minutes is got. Divide the daily motion of the Moon in minutes by 25. The Moon's angular diameter is obtained. The Moon's angular diameter multiplied by 5 divided by 2 and increased by 1, is the diameter in minutes of the shadow which hides the Moon (in the Lunar eclipse. (7b-8)

Deduct Rāhu from the Moon find its *bhuja* and the sine of the *bhuja*. Multiply this by 6 and add a 21st part of the result. This is the latitude of the Moon in minutes. This is positive (i.e. South) if Moon *minus* Rāhu is from 6 to 12 *rāśis* and negative (i.e., North) if from 0 to 6 *rāśis*. (9).[1] (TSK-KVS)

ग्रहणकर्म

16. 14. 1b. ग्राह्यग्राहकसंयोगदलं विक्षेपवर्जितम् ।
शिष्टं ग्रासाङ्गुलं, ग्राह्यादधिके सकलग्रहः ।। १० ।।
ग्राह्यग्राहकसंयोगदलवर्गाद् विवर्जितात् ।
विक्षेपवर्गेण पदं हृतं गत्यन्तरांशकैः ।। ११ ।।
स्थित्यर्धनाडिकास्तद्वद् विमर्दार्धं तदन्तरात् ।
स्थित्यर्धेनोनितं पर्व स्पर्शकालः प्रकीर्तितः ।। १२ ।।
संयुक्तो मोक्षकालः स्यान्मध्यकालः स्वयं भवेत् ।
मीलनाख्यो भवेत्कालो विमर्दार्धेन वर्जितः ।। १३ ।।
उन्मीलनाख्यः संयुक्तः सर्वग्रासे तु ते उभे ।
विक्षेपार्धविनाडीभिर्वितनांशाभिरूनितौ ।। १४ ।।
स्पर्शमोक्षौ, युतौ युग्मे मीलनोन्मीलने अपि ।
नत्यार्कग्रहणे, क्षेपे नतिशुद्धे पदान्तरम् ।। १५ ।।
(*VK*, 4. 10-15)

[1] The semi-diameter of the shadow has been taken to be 40 minutes.

[1] For worked out examples see *VK*: *TSK-KVS*, pp. 278-279.

The circumstances of the Eclipse: General

Add the angular diameters of the eclipsing and the eclipsed bodies, and divide by 2. Deduct the latitude of the Moon. The remainder is the Magnitude in minutes or *aṅgulas*. (If the latitude is greater, there is no eclipse). If the remainder is greater than the eclipsed body, the eclipse is total. (10)

Add the eclipsed and the eclipsing and divide by 2. Square it. Deduct the square of the latitude from this, and find the square root. Divide this by the difference of the daily motions of the Sun and the Moon, in degrees. The result are *nāḍikās* of half-duration of the eclipse. If, instead of adding the eclipsed and the eclipsing, we subtract one from the other and do the calculation, we get the half-duration of the total phase. (11-12a)

The end of the *parva* is the Middle of the eclipse. Deducting the half-duration of the eclipse from this, the aproximate time of the First contact or the beginning of the eclipse is got. Adding, the approximate time of the Last Contact or the ending of the eclipse is got. (12b-13a)

Deducting and adding the half-duration of the total phase from the end of the *parva*, the approximate times of Immersion and Emergence are got. (13b-14a)

Take *vināḍīs* equal to half the latitude of the Moon in minutes, and deduct from it 1/6th of itself. Deduct these *vināḍīs* from the times of the First contact and Last contact and from the times of Immersion and Emergence, if any, if (Moon *minus* Rāhu) is in the odd quadrants; add if in even quadrants. The respective correct times are got. In the case of the solar eclipse, use of the Moon's latitude corrected for parallax (PCL) here. If it happens, in this case, that the latitude has changed sign by the parallax-correction, add for odd quadrants and subtract for even quadrants.[1] (14b-15)

—ग्रहलाघवम्

16. 15. 1. गतगम्यदिनाहतद्युभुक्ते:
'खरसा'प्तांशवियुग्युती ग्रह: स्यात् ।
तत्कालभवस्तथा घटिघ्न्या:
'खरसै'र्लब्धकलोनसंयुत: स्यात् ।। १ ।।

एवं पर्वान्ते विराह्वर्कबाहो-
'रिन्द्रा'ल्पांशा: सम्भवश्चेद् ग्रहस्य ।
तेंऽशा निघ्ना: 'शंकरै:' 'शैल'भक्ता
व्यग्वर्काश: स्यात् पृषत्कोंऽगुलादि: ।। २ ।।

'व्यसुशर'गती'ष्वं'शो 'दिग्'युग्भवेद्वपुरुष्णगो-
रथ सितरुचो बिम्बं भुक्ति'युगाचल'भाजिता ।
तदपि हिमगोर्बिम्बं त्रिघ्नं नि'जेश'लवान्वितं
वि'वसु' भवति क्ष्माभाबिम्बं किलांगुलपूर्वकम् ।। ३ ।।

छादयत्यर्कमिन्दुर्विधुं भूमिभा
छादकच्छाद्यमानैक्यखण्डं कुरु ।
तच्छरोनं भवेच्छन्नमेतद्यदा
ग्राह्यहीनावशिष्टं तु खच्छन्नकम् ।। ४ ।।

मानैक्यखण्ड'मिषु'णा सहितं दशघ्नं
छन्नाहतं पदमत: स्व'रसां'शहीनम् ।
ग्लौबिम्बहृत् स्थितिरियं घटिकादिका स्या-
न्मर्दं तथा तनुदलान्तरखग्रहाभ्याम् ।। ५ ।।

युग्माहतैर्व्यगुभुजांशसमै: पलै: सा
द्विष्ठा स्थितिर्विरहिता सहिताऽर्कषड्भात् ।
ऊने व्यगावितरथाऽभ्यधिके स्थिती स्त:
स्पर्शान्तिमे क्रमगते च तथैव मर्दे ।। ६ ।।

तिथिविरतिरयं ग्रहस्य मध्य:
स च रहित: सहितो निजस्थितिभ्याम् ।
ग्रहणमुखविरामयोस्तु काला-
विति पिहितापिहिते स्वमर्दकाभ्याम् ।। ७ ।।

पिहितहतेष्टं स्थितिविहृतं तत् ।
सचरण'भू'युग् ग्रसनमभीष्टम् ।। ८ ।।

त्रिभयुतोनरवि: स्वविधुग्रहे-
ऽयनलवाढ्य इतश्चरवह्लै: ।
'नगशरेन्दु'मितैर्वलनं भवेत्
स्वरविदिक् त्वक्ष मध्यनताच्च यत् ।। ९ ।।

विषयलब्धगृहादित उक्तवद्
वलन'मक्ष'हृतं पलभाहृतम् ।
उदगपागिह पूर्वपरे क्रमाद्
'रस'हृतोभयसंस्कृतिरङ्घ्रय: ।। १० ।।

मानैक्यार्धहृतात् 'खपड्'घ्नपिहितान्मूलं तदाशांघ्रय:
खच्छन्नं सदलैकयुक् च गदिता: खच्छन्नजाशांघ्रय: ।
सव्यासव्यमपागुदग्वलनजाशांघ्रीन् प्रदद्याच्छरा-
शाया: स्याद् ग्रहमध्यमन्यदिशि खग्रासोऽथवाशेषकम् ।।

मध्याच्छन्नाशांघ्रिभि: प्राक् च पश्चा-
दिन्दोर्व्यस्तं तूष्णगो: स्पर्शमोक्षौ ।
खग्रस्तात् खच्छन्नपादै: परे प्राग्-
दत्तैरिन्दोर्मीलनोन्मीलने स्त: ।। १२ ।।

(Gaṇeśa, *GL*, 5. 1-12)

—Grahalāghava

Multiply the interval that has elapsed or yet to elapse (from sunrise or any standard time) by the daily motion of the planet. Divide by 60. The result in degrees is to be added to the position of the planet, if the previous interval is yet to elapse, it should be subtracted otherwise. The result gives the position of the planet.

[1] For worked out examples, see *VK*: *TSK-KVS*, pp. 280-81.

If the interval (gone or to elapse) is in *ghaṭis*, multiply the same by the daily motion of the planet and divide by 60. The quotient is to be added to subtracted from the position of the planet in minutes or seconds. After this correction, the true position of the planet is obtained. (1)

At the end point of full or new moon, subtract the position of Rāhu from that of the Sun. Find the value of the *bhuja* of the Sun. If it is less than 14 degrees, an eclipse will occur. In the case of the occurrence the eclipse, multiply the *bhuja* thus got by 11 and divide by 7. The quotient in *aṅgulas* give the *śara*. Its direction is the same as that of the Sun *minus* Rāhu. (2)

Subtract 55 minutes from the true daily motion of the Sun (x). Take ($x/5+10$). This in *aṅgulas* gives the diameter of the Sun. 1/74 of the daily motion of Moon gives diameter of Moon in *aṅgulas*, (y). Find ($3y+3y/11-8$). This gives the diameter of Earth's shadow. (3)

The Moon eclipses the Sun, and the Earth's shadow eclipses the Moon. Find the sum of the radii of the Sun and the Moon (during the solar eclipse) and that of the Moon and the Earth's shadow (during the lunar eclipse). This is termed *mānaikya-ardha* (half-sum of the diameters). Subtract from it the *śara* obtained earlier. The result gives in *aṅgulas* the position of the eclipsed body (x).

Subtract the angular diameter of the eclipsed body from x. The result is called (*kha-cchannaka* or *sarva-grāsa* or (occurring at a total eclipse). (4)

Find the sum of the *śara* and the sum of the radii of the Sun and the Moon. Multiply the result by 10. Let it be x. Multiply x by the *grāsa*, the position that is eclipsed. And take its square root, (y). Subtract $y/6$ from it. Divide ($y-y/6$) by the angular diameter of Moon. The quotient gives the duration of eclipse in *ghaṭikās*.

Add the *śara* to the difference of the radii of the Sun and Moon. Multiply by 10. Find the square of its product by *khagrāsa* (y). Divide ($y-y/6$) by the angular diameter of the Moon. The result gives *marda* in *ghaṭis*. (This is for total eclipse). (5)

The first and last points of contact

When the position of the Sun with Rāhu (Node) is less than 12 *rāśis* or 6 *rāśis*, multiply the above *bhuja* by 2, (x). Take the number of *palas* equivalent to the number of degrees in x. Subtract this value from the time, *ghaṭis* of the middle of the eclipse to get the time of first contact; by adding the time of last contact is to be had. The process should be reversed if Rāhu *plus* Sun is more than 12 *rāśis*.

The process is the same for finding the commencement and end of the eclipse. (6)

The end of a *tithi* is the middle of the eclipse. The time of first point of contact is obtained by subtracting the *sparśa-tithi* from this; the time of the last point is got by adding its *sthiti*.

The instants of the commencement and the end are obtained in the same manner by taking the *nimīlana* and *unmīlana marda* instead of the *sthiti*. (7)

Eclipsed body at any time

Multiply the desired time in *ghaṭikās* by the value of *grāsa*, and divide it by the duration (*stithi*). Add 1° 25′ to the quotient. The result in *aṅgulas* gives the position of the eclipse at the desired time. (8)

Valana: deflection

Ayana-valana. In the case of the solar eclipse add 3 *rāśis* to that of the Sun; in respect of the lunar eclipse subtract 3 *rāśis*. Add the *ayanāṃśa*. Following the process to find *cara*, find the result by using 7, 5 and 1 as *cara-khaṇḍas*. That equals *ayana-valana*. Its direction is the same as that of the Sun *plus* 3 *rāśis* (solar eclipse) or the Sun *minus* 3 *rāśis* (lunar eclipse). (9)

Ākṣa-valana

Divide by 5 the hour angle of the middle of the eclipse (*madhya-kāla-nata*) and the result is in *rāśis* etc. As in verse 9, take 7, 5, 1 as *carakhaṇḍas* and repeat the process to find cara (x). Multiply x by the equinoctical shadow and divide by 5. The result gives *ākṣa-valana*. In the case of eastern hour angle it is northern and for western hour angle it is southern. The *sphuṭa-valana* is the algebraic sum of these two. One-sixth of *sphuṭa-valana* is called *sphuṭa-valanāṅghri*. (10)

The *grāsa* multiplied by 60 is to be divided by the sum of the radii. Take the square root of the quotient. That equals *grāsāṅghri*. In the case of total eclipse, replace the sum of the radii by their difference . Then *kha-grāsāṅghri* (*kha-channāṅghri*) is obtained.

Draw a circle with any centre, and radius equal to (the disc) of the eclipsed body. If the *valana* is southwards, mark a point equal to *valanāṅghri* from the southern tip and to the right of the *śara;* reverse the process for northern valanas, i.e. mark the point to the left of the northern tip of the *śara*. The middle point indicates the middle of the eclipse.

Total eclipse is in a direction opposite to that of the middle. In case it is not a total eclipse, the remainder after the middle is in the same direction. (11)

The direction of first and last points of contacts

In the case of lunar eclipse: On the eastern and western sides of the point denoting the middle eclipse, mark the points equal to the *āśāṅghri*. They denote respectively the first and last points. In the case of the solar eclipse reverse the process. In the case of a total lunar eclipse: On the western and eastern sides the point denoting *khagrāsa*, mark the points equal to the *khagrāsāṅghri*. Then denote respectively the immersion and emersion of the total eclipse. (12) (VSN)

चन्द्रग्रहणलेखनम्

—वासिष्ठ-पौलिशौ

16. 16. 1a. सप्तदशाष्टत्रिंशत्तद्द्वयलिप्तायुतोनसूत्रेण ।
शशिनवराहुस्थितिवृत्तान्येकस्थानानि चालिख्य ॥११॥

प्रोक्ताशांशकलङ्कापूर्वापरायाश्च पार्श्वयोश्चापि ।
आयामिन्यो रेखास्त्रयोदश समान्तराः कार्याः ॥ १२ ॥

चन्द्रच्छेदकमेतद् व्याख्यागम्यं समासतोऽभिहितम् ।
ग्रासविमर्दस्थितयः संस्थानेनात्र दृश्यन्ते ॥ १३ ॥

(Varāha, *PS*, 6. 11-13)

Lunar eclipse diagram

Draw three concentric circles with radii 17, 38+17 (= 55), and 38 − 17 (= 21), minutes of arc. These circles relate to the Moon, the duration and obscuration, respectively.

(Drawing the parts of the Moon's orbit forming the path of the Moon), mark the points of first and last contacts, and also those of immersion and emergence if any. (11)

Draw the diameter making an angle equal to the *valana* given in verses 7-8 with the ecliptic to which, (according to this siddhānta) is east-west with reference to the equator. This diameter shows the east-west of the place. Draw thirteen equally spaced lines parallel to the east-west diameter. (12)

Here the graphical representation of the lunar eclipse has been described briefly, and can be understood properly only by explanation (followed by demonstration.) From this, the total duration the total obscuration, the magnitude etc., can be found by inspection.[1] (13). (TSK)

रविचन्द्रग्रहणयोर्विशेषः

16. 16. 1b. स्वे भूच्छायामिन्दुः स्पृशत्यतः स्पृश्यते न पश्चार्धे ।
भानुग्रहेऽर्कमिन्दुः प्राक् प्रग्रहणं रवेर्नातः ॥ १४ ॥

(Varāha, *PS*, 6. 14)

[1] For the rationale, see *PS*: *TSK*: 6. 11-13.

Difference between solar and lunar eclipses

In the lunar eclipse, the Moon, (moving eastward), contacts the Earth's shadow. Therefore the 'first contact' (occurs at the eastern limb of the Moon and so) does not occur at the Moon's western limb. In the solar eclipse, the moon meets the Sun, and therefore, (the Sun being contacted at its western limb), the first contact does not occur at the eastern limb of the Sun. (14). (TSK)

—आर्यभटार्धरात्रपक्षः

16. 17. 1. ग्राह्यसमासव्यासार्धाङ्गुलतुल्येन कर्कटेन भुवि ।
वृत्तत्रितयं कृत्वा तस्मिन् दिक्साधनं कुर्यात् ॥ ६ ॥

प्राक्प्रभृतीन्दोः पश्चादर्कस्य दिशः स्ववलनजीवाभिः ।
विक्षेपा विपरीताश्चन्द्रस्य यथादिशं सवितुः ॥ ७ ॥

सूर्यस्य प्रग्रहणे मोक्षे शशिनो विपर्ययाद्वलना ।
देया शशिनो ग्रहणे मोक्षे सूर्यस्यानुलोमात् ॥ ८ ॥

व्यासार्धे वलनज्यां दत्वा ज्यावत् समाप्यते यत्र ।
तस्मान्मध्यं यावद्रेखां नीत्वा तया च यत्र ॥ ९ ॥

संयोगस्तस्मादपि मानैक्यार्धे पृथक् स्वविक्षेपः ।
ग्राह्यो विक्षेपाग्रात् परिलेख्यो ग्राहकार्धेन ॥ १० ॥

तीक्ष्णकिरणस्य मध्ये दिशं दक्षिणोत्तरां ज्ञात्वा ।
विक्षेपवशात्तस्यां वलना देया विपर्ययाच्छशिनः ॥११॥

प्रग्रहमोक्षानुगतां तदग्रतो वृत्तमध्यगां रेखाम् ।
दत्वा विक्षेपमितं सूत्रं निःसारयेत् प्रतीपं तत् ॥ १२ ॥

मध्ये कृत्वा ग्राह्यं परिलिख्य ग्राहकप्रमाणेन ।
प्रग्रहमोक्षा सा दिग्भूपरिलेखे भवत्येवम् ॥ १३ ॥

पश्चात् प्रग्रहणे प्राङ्मोक्षे रविबिम्बमध्यतो बाहुः ।
स्ववलनसिद्धायां दिशि विपरीतः शीतकरमध्यात् ॥१४॥

भानुमतो बाह्वग्राद्यथादिशं कोटिरन्यथा शशिनः ।
रविशशिमध्यात् कर्णस्तिर्यक्कर्णाग्रकोटियुतेः ॥ १५ ॥

परिलेखं ग्राह्यस्य ग्राहकमानेन पूर्ववत् कृत्वा ।
तात्कालिकसंस्थानं निमीलनोन्मीलने चैवम् ॥ १६ ॥

(*Brahmagupta*, *KK*, 2. 4. 6-16)

—ABh. Midnight System

Draw by means of *karkaṭa* (a kind of compass), on the ground, three concentric circles, whose radii are, respectively, equal to the radius of the obscured body, the sum of the radii of the obscuring and the obscured bodies and the *trijyā*. (These circles are, respectively, called *grāhayavṛtta*, *samāsavṛtta* and *trijyāvṛtta*). Then mark the directions north, south etc., in these circles. (6)

In a lunar eclipse, for contact, the *valanajyā* should be marked along the *trijyāvṛtta*, from the east point in its own direction; and for separation, from the west point in the opposite direction. In a solar eclipse, for contact,

the *valanajyā* should be marked along the *trijyāvṛtta* from the west point in a direction opposite to its own; and for separation, from the east point in its own direction.

In a lunar eclipse, the *vikṣepa* is marked along the *samāsavṛtta* in a direction opposite to its own, both for contact and separation. In a solar eclipse, the *vikṣepa* is marked along the *samāsavṛtta* in its own direction both for contact and separation.

Following the above rules, mark off on the circumference of the *trijyāvṛtta* a length equal to the *valanajyā* beginning from the east or west point, as the case may be. Join the point thus marked and the centre of the concentic circles by a straight line. (This line is called *valanasūtra*). Mark the point where this line cuts the *samāsavṛtta*. From this point along the circumference of the *samāsavṛtta* cut off a length equal to the *vikṣepa* (according to the rules given above). (The point thus marked is the centre of the obscuring body. With this as centre and the radius of the obscuring body as radius, describe a circle, representing the obscuring body. (The respective diagrams give the positions during contact and separation.) (7-10)

For a diagram at the *madhyagrahaṇakāla*, that is, at *pūrṇānta* or *darśānta*, first mark the north and the south points in the concentric circles already drawn. In a solar eclipse, the *valanajyā* should be marked along the *trijyāvṛtta* from the north point, if the Moon's *vikṣepa* is south. The *valanajyā* should be marked eastward, if its direction is opposite to that of the *vikṣepa*, and westward if same. In a lunar eclipse the reverse process must be followed.

Join the point thus marked and the centre of the concentric circles by a straight line. From the centre along this line cut off the length of the *vikṣepa*, in its own direction in the case of a solar eclipse, and in an opposite direction in the case of a lunar eclipse. Mark this point which is the centre of the obscuring body. With this point as centre, and the radius of the obscuring body as radius describe a circle. This represents the obscuring body. Thus one should draw the diagrams on the ground to represent contact, separation and the middle of an eclipse. (11-13)

In a solar eclipse, from the centre of the Sun or *grāhyavṛtta* mark along the *valanasūtra* a length equal to a *bhuja*. If the given time is between the *madhyagrahaṇakāla* and the beginning of the eclipse, the length must be marked to the west; and to the east, if the time is between the *madhyagrahaṇakāla* and the end of the eclipse. At the point thus marked, draw a line perpendicular to the *bhuja* and equal to the length of the *koṭi*, in the same direction as that of the Sun's *koṭi*. The straight line joining the centre of the Sun to the end of the *koṭi* is called *karṇa*. With that point of intersection of the *koṭi* and the *karṇa* as centre, and with the radius of the Moon or obscuring body as radius, draw a circle. Thus is found the obscured portion of the obscured body at a given time. In the same manner the diagrams for immersion and emergence may be drawn.

In a lunar eclipse, from the centre of the Moon or *grāhyavṛtta* mark along the *valanasūtra* a length equal to the *bhuja*. If the given time is between the beginning of the eclipse and the *madhyagrahaṇakāla*, the length must be marked to the east; and to the west, if the time is between the *madhyagrahaṇakāla* and the end of the eclipse. At the point thus marked, draw a line perpendicular to the *bhuja*, and equal to the length of the Moon's *koṭi*, in a direction opposite to its own. The straight line joining the centre of the moon to the end of the *koṭi* is called *karṇa*. (The remaining construction is the same as that in a solar eclipse). 14-16. (BC)

—भास्करः १

16. 18. 1. ग्राह्याङ्गुलार्धविस्तृत्या वृत्तं सूत्रेण लिख्यते ।
ग्राह्यग्राहकसम्पर्कदलसङ्ख्येन चापरम् ।। २३ ।।

पूर्वापरायतं सूत्रं तन्मत्स्यात् सौम्यदक्षिणम् ।
कृत्वा यथादिशं केन्द्राद्वलनं नीयते स्फुटम् ।। २४ ।।

विन्यस्तमत्स्यमध्येन सूत्रं पूर्वापरे दिशौ ।
नीत्वा तु बाह्यवृत्तान्तं ततः केन्द्रं समानयेत् ।। २५ ।।

ग्राह्यमण्डलतद्योगो व्यक्तं यत्रोपलक्ष्यते ।
प्रग्रासग्रहमोक्षौ स्तस्तत्र देशे निशाकृतः ।। २६ ।।

तुल्यदिग्वलनक्षिप्त्योर्वलनं वारुणीं नयेत् ।
अन्यथैन्द्रीं रवेर्व्यस्तं सूत्रं तन्मत्स्यतो बहिः ।। २७ ।।

विक्षेपस्य वशात् केन्द्रमानयेत् तत् यथादिशम् ।
विक्षेपं केन्द्रतो नीत्वा बिन्दुं तत्र प्रकल्पयेत् ।। २८ ।।

ग्राहकाङ्गुलविष्कम्भदलसङ्ख्येन खण्डयेत् ।
ग्राह्यबिम्बं तथा मध्ये ग्राहकस्यावतिष्ठते ।। २९ ।।

प्रग्रासमध्यमोक्षाणां बिन्दूनां मस्तकानुगम् ।
मत्स्यद्वयोत्थवृत्तं यद् वर्त्म स्यात् ग्राहकस्य तत् ।। ३० ।।

स्थित्यर्धेनेष्टहीनेन हत्वा गत्यन्तरं हरेत् ।
षष्ट्या लब्धकृतिं युक्त्वा विक्षेपस्य कृतेः पदम् ।। ३१ ।।

तन्नयेत् केन्द्रतो वर्त्म यत्र सम्यक् तयोर्युतिः ।
तत्रेष्टकालजो ग्रासो ग्राहकार्धेन लिख्यते ।। ३२ ।।

(Bhāskara I, *LBh.*, 4. 23-32)

—Bhāskara I

Draw a circle with a thread equal in length to half the *aṅgulas* of the diameter of the eclipsed body (as radius) and another (concentric circle) with a thread equal in length to half the sum of the diameters of the eclipsed and eclipsing bodies.

(Then) having drawn (through the common centre) the east-west line and with the help of a fish-figure the north-south line, lay off from the centre (of the circle) the corrected *valana* (for the first or last contact) according to its directions.

About that point draw a fish-figure (in the east-west direction). (Then) passs a thread through the middle of that fish-figure and produce it towards the east or west (as the case may be) to meet the outer and from there carry it to the centre.

The point where the junction of the circle of the eclipsed body and that (thread) is clearly seen (in the figure) is the place where the Moon is eclipsed or is separated (from the shadow).

When the *valana* and the Moon's latitude (for the middle of the eclipse) are alike in direction, the *valana* should be laid off towards the west (from the centre); otherwise, towards the east. In the case (of the eclipse) of the Sun, it should be done reversely. (Then) through the fish-figure drawn (along the north-south direction) about that point, pass a thread and extend it beyond the fish-figure (towards the north or south), according to (the direction of) the Moon's latitude to meet the outer circle, and from there carry the thread to the centre. Then from the centre along that thread lay off the Moon's latitude in the proper direction and put there a point.

(With that point as centre and) with the *angulas* of the semi-diameter of the eclipsing body (as radius), draw a circle cutting the disc of the eclipsed body. The portion of the eclipsed body thus cut off lies submerged in the eclipsing body.

The circle which is drawn through the points (*i.e.*, the centres of the eclipsing body) corresponding to the beginning, middle, and end of the eclipse, with the help of two fish-figures, is the path of the eclipsing body. (23-30)

Phase of the eclipse for given time

Multiply the difference between the (true) daily motions (of the Sun and Moon) by the *sthiyardha* minus the given time and divide that (product) by 60. Then adding the square of that to the square of the Moon's latitude (for the given time), take the square root (of that sum). (The square root thus obtained is the distance between the centres of the eclipsed and eclipsing bodies at the given time.)

Lay that off from the centre so as to meet the path of (the centre of) the eclipsing body. With the meetimg point as centre and half the diameter of the eclipsing body and radius, draw the eclipsed portion for the given time. (31-32). (KSS)

—लल्ल:

16. 19. 1. पूर्वाशायां प्रग्रह: शीतरश्मे:
पश्चान्मोक्षस्तिग्मगोरन्यथा तौ ।
क्षेपा: सर्वे व्यत्ययेन स्युरिन्दो-
र्यद्वद् भानोरागतास्तद्वदेव ।। २९ ।।

ग्राह्यं वृत्तं मानयोगार्धवृत्तं
त्रिज्यावृत्तं चालिखेत् साधिताशम् ।
त्रिज्यावृत्ते शीतगो: पूर्वभागे
ज्यावद् दद्याद् वालनान्यङ्गुलानि ।। ३० ।।

पश्चाद्भागे तिग्मगोश्चन्द्रभान्वो:
पश्चाद् व्यस्तान्यन्तजान्यादिजानि ।
याम्यात् सौम्यान् मध्यमानि प्रदेशा-
दन्यैकाशान्यानयेत् प्राक्प्रतीच्यो: ।। ३१ ।।

क्षेपं ज्ञात्वा तिग्मगोरन्यथेन्दो-
स्तेभ्य: सूत्राण्यानयेत् केन्द्रभाञ्जि ।
आद्यादन्त्यात् सूत्रमानैक्ययोगा-
ज्ज्यावत् क्षेपौ सारयेदाद्यमोक्षौ ।। ३२ ।।

मध्यक्षेपं मध्यतो मध्यसूत्रे
क्षेपाग्रेभ्यो ग्राहकार्धेन तेभ्य: ।
विज्ञायन्ते खण्डिते तु क्रमेण
ग्राह्ये स्पर्शो मध्यमग्रासमोक्षौ ।। ३३ ।।

क्षेपाग्रत्रयमण्डलैस्तिमियुगस्यास्यस्थितासक्तयो
रज्ज्वोर्योगभुव: शरत्रयशिर:प्राप्यालिखेन्मण्डलम् ।
तत् स्यात् ग्राहकवर्त्म केन्द्रविसृतां युक्तां श्रुतिं तत्स्पृशां
कृत्वा ग्राहकमालिखेदभिमतग्रासादिसंसिद्धये ।। ३४ ।।

मुच्यमानमुडुपे पराङ्मुखीं
छाद्यमानमिह शक्रदिङ्मुखीम् ।
संप्रसार्य विधिवच्छ्रुतिं ततो
विद्धयभीष्टदितमन्यथा रवौ ।। ३५ ।।

(Lalla, *ŚiDhVṛ.*, 5. 29-35)

—Lalla

In a lunar eclipse, the contact takes place in the eastern portion of the disc of the Moon and separation in the western portion. The contrary is the case in a solar eclipse.

The latitudes of the Moon should always be drawn in a direction contrary to their own (in the projection) of a lunar eclipse. (But in the projection) of a solar eclipse, the latitudes should be drawn in their own direction. (29)

Draw three (concentric) circles, with radii respectively equal to the radius of the obscured body, the sum of the radii of the obscuring and the obscured bodies, and

the radius (*trijyā*). Mark the directions (north, etc.) in these circles. (In the projection) of a lunar eclipse, for contact, the *valana* or deflection should be marked along the third circle, from the east point, in its own direction. For separation, it should be marked from the west point in a direction opposite to its own. In both the cases it should be expressed as an R sine.

In a solar eclipse, for contact, the *valana* should be marked from the west point in a direction opposite to its own; and for separation, it should be marked from the east point in the same direction as its own. Here again, it should be expressed as an R sine.

In the projection of a solar eclipse, at the time of mid-eclipse, if the Moon's latitude is north, the *valana* or deflection should be marked from the north point eastward, if its own direction is opposite to that of the latitude, and westward, if its own direction is the same. Again, if the Moon's latitude is south, the *valana* or deflection should be marked from the south point eastward, if its own direction is opposite to that of the latitude, and westwards, if its own direction is the same. In the projection of a lunar eclipse, the contrary is the case.

(In each case, from the end of the *valana* or deflection thus marked), draw a straight line passing through the centre of the concentric circle. (This is called *valana-sūtra*). Mark the point where it cuts the circle with radius equal to the sum of the radii of the obscuring and the obscured bodies. From this point mark along the same circle the latitude for contact and separation, each expressed as an R sine. In a solar eclipse the latitudes should be drawn in their own direction and in a lunar eclipse in the opposite direction.

At mid-eclipse, the latitude should be marked along the *valanasūtra* from the centre of the (concentric) circles, (towards the *valana*).

In each case, with the extremity of the latitude as the centre and the radius of the obscuring body as radius, describe a circle (cutting the obscured body). Thus are known the points of contact and separation and also the obscured part at the mid-eclipse. (30-33)

(Mark) the three extremities of the latitudes, (at the beginnning, middle and end of the eclipse). (Draw) two fish-figures, (one passing through the first two points and the other through the last two points). Draw two lines passing through the mouth and tail of each fish-figure. (With the point of intersection of these two lines as centre) draw a circle passing through the three extremities of the latitudes. This is the path of the obscuring body. Then place the hypotenuse from the centre (of the concentric circles) just touching the path.

(In the projection of a lunar eclipse, when the obscured portion is increasing, that is between the beginning and middle of the eclipse), the hypotenuse must be drawn eastward. But when the obscured portion is decreasing, (that is, between the middle and end of the eclipse), the hypotenuse must be drawn westward. In the projection of a solar eclipse, the contrary is the case.

With this point of intersection as the centre, draw the obscuring body. Thus is found the obscured portion at any time. (34-35). (BC)

—सूर्यसिद्धान्तः

16. 20. 1. न छेद्यकमृते यस्मात्क्षेपा ग्रहणयोः स्फुटाः ।
ज्ञायन्ते तत्प्रवक्ष्यामि छेद्यकज्ञानमुत्तमम् ।। १ ।।

सुसाधितायामवनौ बिन्दुं दत्वा ततो लिखेत् ।
सप्तवर्गाङ्गुलेनादौ मण्डलं वलनाश्रितम् ।। २ ।।

ग्राह्यग्राहकयोगार्धसम्मितेन द्वितीयकम् ।
मण्डलं तत्समासाख्यं ग्राह्यार्धेन तृतीयकम् ।। ३ ।।

याम्योत्तरा प्राच्यपरा साधनं पूर्ववद् दिशाम् ।
प्रागिन्दोर्ग्रहणं पश्चान्मोक्षोऽर्कस्य विपर्ययात् ।। ४ ।।

यथादिशं प्राग्ग्रहणं वलनं हिमदीधितेः ।
मौक्षिकं तु विपर्यस्तं विपरीतमिदं रवेः ।। ५ ।।

वलनाग्रान्नयेन्मध्यं सूत्रं तद्यत्र संस्पृशेत् ।
तत्समासे ततो देयौ विक्षेपौ ग्रासमौक्षिकौ ।। ६ ।।

विक्षेपाग्रात् पुनस्सूत्रं मध्यबिन्दुं प्रवेशयेत् ।
तद्ग्राह्यवृत्तसंस्पर्शे ग्रासमोक्षौ विनिर्दिशेत् ।। ७ ।।

नित्यशोऽर्कस्य विक्षेपाः परिलेखे यथादिशम् ।
विपरीतं शशाङ्कस्य तद्वशादथ मध्यमम् ।। ८ ।।

वलनं प्राङ्मुखं नेयं तद्विक्षेपैकता यदि ।
भेदे पश्चान्मुखं नेयम् इन्दोर्भानोर्विपर्ययात् ।। ९ ।।

वलनाग्रात् पुनः सूत्रं मध्यबिन्दुं प्रवेशयेत् ।
मध्यात् सूत्रेण विक्षेपं वलनाभिमुखं नयेत् ।। १० ।।

विक्षेपाग्राल्लिखेद्वृत्तं ग्राहकार्धेन तेन यत् ।
ग्राह्यवृत्तं समाक्रान्तं तद्ग्रस्तं तमसा भवेत् ।। ११ ।।

—Sūryasiddhānta

Since, without a projection (*chedyaka*), the precise (*sphuṭa*) differences of the two eclipses are not understood, I shall proceed to explain the exalted doctrine of the projection. (1)

Having fixed, upon a well prepared surface, a point, describe from it, in the first place, with a radius of forty-nine digits (*aṅgula*), a circle for the deflection (*valana*). (2)

Then a second circle, with a radius equal to half the sum of the eclipsed and eclipsing bodies; this is called

the aggregate-circle (*samāsa*); then a third, with a radius equal to half the eclipsed body. (3)

The determination of the directions, north, south, east, and west, is as formerly. In a lunar eclipse, contact (*grahaṇa*) takes place on the east, and separation (*mokṣa*) on the west; in a solar eclipse, the contrary. (4)

In a lunar eclipse, the deflection (*valana*) for the contact is to be laid off in its own proper direction, but that for separation in reverse; in an eclipse of the Sun, the contrary is the case. (5)

From the extremity of either deflection draw a line to the centre: from the point where that cuts the aggregate-circle (*samāsa*) are to be laid off the latitudes of contact and of separation. (6)

From the extremity of the latitude, again, draw a line to the central point: in either case, where that touches the eclipsed body, there point out the contact and separation. (7)

Always, in a solar eclipse, the latitudes are to be drawn in the figure (*parilekha*) in their proper direction; in a lunar eclipse, in the opposite direction. In accordance with this, then, for the middle of the eclipse, the deflection is to be laid off—eastward, when it and the latitude are of the same direction; when they are of different directions, it is to be laid off westward: this is for a lunar eclipse; in a solar, the contrary is the case. (8-9)

From the end of the deflection, again, draw a line to the central point, and upon this line of the middle lay off the latitude, in the direction of the deflection. (10)

From the extremity of the latitude describe a circle with a radius equal to half the measure of the eclipsing body: whatever of the disc of the eclipsed body is enclosed within that circle, so much is swallowed up by the darkness (*tamas*).[1] (11). (Burgess)

—भास्करः २

16. 21. 1. ग्राह्यासूर्धंत्रेण विधाय वृत्तं
मानैक्यखण्डेन च साधिताशम् ।
बाह्येऽत्र वृत्ते वलनं ज्यकावत्
प्राक्चिह्नतः स्पर्शभवं हिमांशोः ॥ २६ ॥

सव्यापसव्यं खलु याम्यसौम्यं
मौक्षं तदा पश्चिमतश्च देयम् ।
रविग्रहे पश्चिमपूर्वतस्ते
विक्षेपदिक्चिह्नत एव साध्यम् ॥ २७ ॥

सूत्राणि केन्द्राद्वलनाग्रसक्ता-
न्यङ्क्यान्यतः स्पर्शविमुक्तिबाणौ ।
ज्यावन्निजाभ्यां वलनाग्रकाभ्यां
देयौ यथाशावथ मध्यबाणः ॥ २८ ॥

केन्द्रात् प्रदेयो वलनस्य सूत्रे
तेभ्यः पृथग्ग्राहकखण्डकेन ।
वृत्तैः कृतैः स्पर्शविमुक्तिमध्य-
ग्रासाः क्रमेणैवमिहावगम्याः ॥ २९ ॥

केन्द्राद् भुजं स्वे वलनस्य सूत्रे
शरं भुजाग्राच्छ्रवणं च केन्द्रात् ।
प्रसार्य कोटिश्रुतियोगचिह्नाद्
वृत्ते कृते ग्राहकखण्डकेन ॥ ३० ॥

संमीलनोन्मीलनकेष्टकाल-
ग्रासाश्च वेद्या यदि वान्यथामी ।
ये स्पर्शमुक्त्योर्विशिखाग्रचिह्ने
ताभ्यां पृथङ्मध्यशराग्रयाते ॥ ३१ ॥

रेखे किल प्रग्रहमोक्षमार्गौ
तयोश्च माने विगणय्य वेद्ये ।
बिम्बान्तरार्धेन विधाय वृत्तं
केन्द्रेऽथ तन्मार्गयुतिद्वयेऽपि ॥ ३२ ॥

भूभार्धसूत्रेण विधाय वृत्ते
सम्मीलनोन्मीलनके च वेद्ये ।

इष्टग्रासः

मार्गाङ्गुलघ्नं स्थितिखण्डभक्त-
मिष्टं स्युरिष्टाङ्गुलसंज्ञकानि ॥ ३३ ॥

इष्टाङ्गुलानीष्टवशात् स्वमार्गे
दत्त्वात्र च ग्राहकखण्डवृत्तम् ।
कृत्वेष्टखण्डं यदि वावगम्यं
स्थूलः सुखार्थं परिलेख एवम् ॥ ३४ ॥

(Bhāskara II, *SiŚi.*, 1.5. 226-34)

—Bhāskara II

Draw a circle with radius equal to that of the radius of the disc of the eclipsed body and also a circle of radius equal to r+p, the sum of the radii of the eclipsed and eclipsing bodies; let directions (east etc.) be marked in the figure. In the outer circle, draw the *valanajyā* or the R sine of the *sphuṭavalana*. In the case of the Moon, the *valanajyā* pertaining to the moment of first contact should be marked from the east point and that pertaining to the moment of last contact should be marked from the west point. In the case of the Sun the reverse is to be done. If the *valana* is south, it should be marked in the clockwise direction, otherwise anticlockwise. (26-27)

Having marked the *valanajyā* in the form of an R sine, draw the line joining the centre to the top of the *valanajyā*, i.e. to the point of intersection of the R sine with the outer

[1] For elucidation, see *Su.Si*: *Burgess*, pp. 178-83.

circle. The celestial latitude of the Moon is to be drawn from this top of the *valanajyā* in the form of an R sine again. If the latitude pertains to the moment of first contact, it should be drawn from the top of the *valanajyā* pertaining to that moment, and if it pertains to the moment of last contact, it should be laid off from the top of the *valanajyā* pertaining to the moment of last contact. (28)

The celestial latitude pertaining to the middle of the eclipse should be drawn from the centre along the line of *valanasūtra* or the line joining the centre to the top of the *valanājyā*. Taking the extremities of these latitudes, circles are to be drawn with the radius of the eclipsing body to depict the eclipse at the respective moments. (29)

First contact etc.

The *bhuja* is to be laid from the centre of the Moon along its *valanasūtra* or the line indicating the direction of the ecliptic; the latitude is to be drawn from the end of the *bhuja* and perpendicular to the *bhuja*. The hypotenuse is to be drawn from the centre of the Moon. Taking the point of intersection of the latitude (*koṭi*) and the hypotenuse, as centre, and radius p equal to that of the eclipsing body, if circles be drawn, from these circles could be known the points where totality begins and ends as well as the magnitude of the eclipse at any given moment. Or, these could be found in another way as follows. (30-31a)

Joining the upper end of the latitude of the middle moment of the eclipse to those of the first and last contacts, we have what are called the *pragrahamārga* and *mokṣamārga*, i.e. the path of the centre of the eclipsing body from the first contact to the middle moment to the last contact. The lengths of these paths could be computed and they could be drawn beforehand. Then, with the centre of the Moon as centre and radius equal to $(p—r)$ if a circle be drawn, it cuts the paths described above each in one point. With these points as centre and radii equal to p, if circles be drawn, they will touch the Moon's disc each in one point which are respectively the points of *sammīlana* and *unmīlana*. (31b-33a)

Eclipse at any moment

Let the product of the time elapsed from the moment of first contact and the length of the path of the eclipsing body traced from the moment of the first contact to the middle of the eclipse divided by the time between the moment of first contact and the middle of the eclipse, be x. Similarly, let the product of the time before the end of last contact and the path of the eclipsing body traced between the middle moment of the eclipse and the moment of last contact divided by the time between the middle moment and the moment of last contact be y. Lay off x and y units of length from the first and last points of the path of the eclipsing body along the path, respectively. Then we get the points of the centre of the eclipsing body at the required moments. With these points as centre and radius p, if circles be drawn, they represent the eclipsing body. The length of the diameter of the eclipsed body shaded gives the magnitude of the eclipse called *grāsa*.[1] (33b-34). (AS)

—करणरत्नम्

16. 22. 1. शशिरविवृत्तं लेख्यं स्फुटबिम्बदलेन दिक्चतुष्टयवत् ॥
प्रग्रहणमुक्तिवलने प्रागपरां तन्नयेत् परिधौ ।
अन्यदिशीन्दोरर्कग्रहणे परपूर्वयोः समानदिशि ॥ १२ ॥
तत्र रवीन्द्वोर्वाच्यौ स्पर्शविमोक्षप्रदेशौ तौ ।
तद्द्विप्रदेशमध्यादारभ्येन्द्वर्कपरिधिमध्यगता ॥ १३ ॥
रेखा चोत्तरदक्षिणसमदिक्स्था संविधेयेन्दोः ।
तत्रान्यद्दिशि न्यसेन्मध्याद्विक्षेपमात्मदिशि भानोः ।।१४
कृत्वाऽत्राङ्कं भ्रमयेद् ग्राहकबिम्बार्धसदृशसूत्रेण ।
छन्नपतिते रवीन्द्वोर्यथा यथा च्छेदिते प्रदृश्येते ॥ १५ ॥
नभसि च तथा तथा ते भवतः संलक्षिते ग्रहणे ।
बिम्बद्वययुतिदलसमसूत्रं प्रागपरतो नयेन्मध्यात् ॥ १६ ॥
प्रग्रहणमोक्षबिन्दू, स्पर्शत्वाङ्कौ तदग्रस्थौ ।
अङ्कत्रयद्विमत्स्यान्मुखपुच्छस्पृक्सूत्रसङ्गमे न्यस्य ॥१७॥
सूत्रात् त्रितयाङ्कस्पृग्रेखा या ग्राहको मार्गः ।
सहितबिम्बार्धसम्मितसूत्राग्रं मध्यबिन्दुतः प्राग्वत् ॥१८॥
ग्राहकमार्गं यत्र स्पृशति तमस्तत्र परिलेख्यम् ।
परिलेख्यमानमेतच्छशिपरिधिं यत्र संस्पृशेत्तत्र ॥ १९ ॥
संच्छादितौ प्रदेशौ पश्चादेवं प्रदृश्येते ।

इष्टग्रासः

इष्टघटिकाविहीनं स्थितिदलमर्केन्दुभुक्तिविवरेण ॥२०॥
संगुण्य 'खरस'लब्धं तत्कृतिविक्षेपवर्गयुतम् ।
यत्तत्र भवति मूलं तेनोनं बिम्बमानयोगदलम् ॥ २१ ॥
यच्छेषं तद्ग्रासं विक्षेपकलाविवर्जितं मध्ये ।
तन्मूलसदृशसूत्रं मध्यात् प्राक्पश्चिमं नयेदिन्दोः ।
व्यस्तं रवेरिह, पथं स्पृशति यथास्थं तमो विलिखेत् ॥
इष्टेन स्थित्यर्धे मध्यग्रासाङ्गुलानि सङ्गुण्य ।
हृत्वा स्थितिदलकालैरिष्टग्रासाङ्गुलं भवति ॥ २३ ॥

(Deva, *KR*, 2. 11b-23)

—Karaṇaratna

With half the diameter of the Moon (in the case of a lunar eclipse) or with half the diameter of the Sun (in the case of a solar eclipse), draw a circle, and furnish it with the four cardinal points. (11b)

In the case of a lunar eclipse, lay off the resultant *valanas* for the first and last contacts towards the east and

[1] For explanation, see *ŚiŚi* : *AS*, pp. 396-402.

west respectively along the circumference in the opposite direction[1] (i.e., towards the north or south according as the *valana* is of south or north direction); and in the case of a solar eclipse, towards the west and east respectively, in its own direction (north or south). (And set down points there). (12)

These points should be declared as the points of the first and last contacts of the Moon (in the case of a lunar eclipse) or of the Sun (in the case of a solar eclipse). Then draw lines proceeding from these two points and reaching the centre of the circle representing the Moon or Sun. (13)

Also draw another line joining the north and south cardinal points. Starting from the centre, lay off along this line the Moon's latitude (for the middle of the eclipse) in the contrary direction in the case of the Moon, and in its own direction in the case of the Sun. (14)

Put down a point there. Taking it as centre and the semi-diameter of the eclipsing body as radius draw a circle by revolving the compass. As is a portion of the Sun or Moon seen intercepted by the eclipsing body in the diagram, just so is the (actual) Sun or Moon seen eclipsed in the sky during the eclipse. (15-16a)

Path of the eclipsing body

From the centre (of the circle) draw two lines, each equal to half the sum of the diameters of the eclipsed and eclipsing bodies, towards the east and west, one towards the point of the first contact and the other towards the point of the last contact. The extermities of these lines are the points (denoting the positions of the centre of the eclipsing body at the times) of the first and last contacts. (The point at the extremity of the Moon's latitude for the middle of the eclipse is the third point). (16b-17a).

Now, with the help of these three points construct two fish-figures, and keeping one end of a thread at the intersection of the head and tail lines of the two fish-figures, draw a circular arc (*lit.* line) through the above three points: this is the path of the eclipsing body. (17b-18a)

Now take a thread of length equal to half the sum of the diameters of the eclipsed and eclipsing bodies, and stretch it from the centre (of the circle towards the east and west), as before. Where the other extremity of this thread meets the path of the eclipsing body (towards the east or west), taking that as centre draw a circle with radius equal to that of the Shadow. The point where this circle touches the circumference of the Moon, there lies the point of the first or last contact. This is how the points of the first and last contacts are seen afterwards (in the sky). (16b-20a)

Obscuration at the given time (Iṣṭa-grāsa)

Diminish (the *ghaṭīs* of) half the duration of the eclipse by the given *ghaṭīs*, then multiply by the motion-difference of the Sun and Moon, and then divide (the product) by 60. Add the square of that to the square of the Moon's latitude, and take the square root (of that sum). By that (square root) diminish half the sum of the diameters of the eclipsed and eclipsing bodies. The remainder is the measure of eclipse at the given time. (The minutes of half the sum of the diameters of the eclipsed and eclipsing bodies) diminished by the minutes of the Moon's latitude give the measure of eclipse at the time of the middle of the eclipse. (20b-22a)

Graphical representation of Iṣṭa-grāsa

Stretch a thread of length equal to the square root (obtained in the previous rule) from the centre towards the east or west of the Moon, or towards the west or east of the Sun (according as the given time relates to the first or second half of the eclipse), so as to meet the path of the eclipsing body as it stands. At that point draw the Shadow. (22)

Method for Iṣṭa-grāsa

Multiply the *aṅgulas* of the measure of eclipse at the time of the middle of the eclipse by the given time (elapsed since the first contact or to elapse before the last contact) in *ghaṭīs* and divide (the product) by (the *ghaṭīs* of) half the duration of the eclipse: the quotient gives the measure of eclipse at the given time, in terms of *aṅgulas*. (23). (KSS)

सूर्यग्रहणगणनम्

पौलिशसिद्धान्तः

लम्बननाड्यः

16. 23. 1. दिनमध्यमसंप्राप्त्या यावत्या नाडिका व्यतीता वा ।
ताभ्यः षड्गुणिताभ्यो ज्यात्रिंशांशस्तिथेर्नाम ॥ १ ॥

नतिसंस्कारः

पञ्चघ्नात् त्रिघनाप्तादक्षान्मुखपुच्छयोर्धनर्णे तत् ।
सशशिचरणापमगुणा धनर्णनाड्यो धृतिभक्ताः ॥ २ ॥

उदगयने पूर्वार्धे धनमृणं दक्षिणे प्राच्याम् ।
पश्चादूनं तु याम्ये दिगुदगृणं वामतः पुच्छे ॥ ३ ॥

दिनयातशेषनाड्यश्चन्द्रापमसंगुणास्त्वशीतिहृताः ।
मेषतुलादि ऋणधनं विपरीतं वामतः पुच्छे ॥ ४ ॥

ग्रहणकर्म

राहोः सषट्कृतिकलां हित्वांशं तच्छशाङ्कविवरांशैः ।
ग्रहणं त्रयोदशान्तः शशिनो भानोस्तथाष्टान्तः ॥ ५ ॥

[1] In fact, only the *vikṣepa-valana* should be of the opposite direction; the other two *valanas* should be of their own direction.

तद्वर्गमपास्येन्दोर्नवतुरूपाच्छुतिरसाच्च ।
तन्मूलं पादोनं स्थितिकालश्चन्द्रभान्वोश्च ॥ ६ ॥

(Varāha, *PS*. 7.1-6)

Solar eclipse computation: —Pauliśa Siddhānta

Parallax of longitude

Find the interval between midday and the sine of new moon, in *nāḍīs*. Multiply this by 6. Degrees are got. Find its sine. Divide it by 30. The result is the parallax in *nāḍīs*, to be deducted from the time of new moon is before midday, and to be added to the time of new moon, if after midday. The new moon corrected for parallax in longitude is obtained. (1)

Parallax in latitude

(i) Multiply the degrees of latitude by 5 and divide by 27. Add or subtract the resulting degrees, respectively, to Rāhu's head or from Rāhu's tail, where the moon is situated. (ii) Add three *rāśis* to the Moon, and find its declination in degrees. This multiplied by the *nāḍīs* of parallax (given by verse 1) and divided by 18, are to be added to the Head if it is forenoon, and *Uttarāyaṇa* (*i.e.*, the Sun is in his northward course), or afternoon and *Dakṣiṇāyana*. The degrees are to be subtracted from the Head, if it is forenoon and *Dakṣiṇāyana* or afternoon and *Uttarāyaṇa*. For the Tail, the addition and subtraction should interchanged. (2-3)

(iii) Take the *nāḍīs* from sunrise to new moon, if forenoon, the *nāḍīs* from new moon to sunset if afternoon. Multiply these by the degrees of the moon's declination, and divide by 80. The resulting degrees are to be added to the Head if the moon's longitude is between 6 and 12 *rāśis*, and subtracted if between 0 and 6 *rāśis*. For the Tail, interchange the addition and subtraction. (4)

Computation

Deduct 1° 36′ from Rāhu, and find the Moon$\sim$Rahu, in the case of the lunar eclipse. Deduct 1° 36′ from Rāhu corrected (by verses 2-4), and find Moon$\sim$Rāhu, in the case of the solar eclipse. If the difference is less than 13° there is a lunar eclipse. If the difference is less than 8°, there is a solar eclipse; (otherwise not). (5)

For the lunar eclipse, deduct the square of the difference from 169, find its square root, and take three fourths of it. This is the total duration in *nāḍīs*. For the solar eclipse, deduct the square of the difference from 64, find its square root, and take three fourths of it. This is the total duration in *nāḍīs*.[1] (6). (TSK)

[1] i.e., *nāḍīs* of total duration $= \frac{3}{4}\sqrt{169-(\text{moon}\sim\text{Rāhu})^2}$ or $\frac{3}{4}\sqrt{64-(\text{moon}\sim\text{Rāhu})^2}$ respectively. Half this subtracted or added to the full moon, or parallax corrected new moon, gives the times of first and last contacts. For details see *PS*: *TSK*, 7. 1-6.

—रोमकसिद्धान्तः

लम्बिततिथिः

16. 24. 1. दिनमध्यमसम्प्राप्ता यावत्यो नाडिका व्यतीता वा ।
ताभ्यः षड्गुणिताभ्यो ज्यात्रिशांशस्तिथेर्नाम ॥ ९ ॥

दृक्क्षेपज्याचापम्

उदयात् प्रभृति च नाड्यो याः स्युः प्राग्लग्नमानयेत्ताभिः ।
तस्मात्तु नवसमेतादपक्रमांशान् विनिश्चित्य ॥ १० ॥

लग्नल्यगुविवरज्या द्विगुणा स्व'रसां'शसंयुतामपमात् ।
जह्याद् दिग्व्यत्यासे विक्षेपैक्ये तयोर्योगः ॥ ११ ॥

उत्तरमक्षाच्छुद्धं याम्यं साक्षं च दक्षिणं विद्यात् ।
उत्तरमक्षाद्यदधिकमुत्तरमेवं विजानीयात् ॥ १२ ॥

अवनतिः, बिम्बमानं च

तज्ज्याघ्नीं शशिभुक्तिं हृत्वा 'धृतिभिश्शतैः' स्मृतावनतिः ।
मध्यममानं त्रिशद् भानोः शशिनश्चतुस्त्रिशत् ॥ १३ ॥

अवनतिसंस्कृतविक्षेपः

समलिप्तराहुविवरज्याभ्यस्ता 'मूर्च्छना' नवहृतश्च ।
अवनत्यायुतविश्लेषिताश्च दिक्साम्यवैलोम्ये ॥ १४ ॥

स्फुटबिम्बमानम्

मध्यममानाभ्यस्ता स्फुटभुक्तिर्मध्यभुक्तिभक्ता च ।
भवति कलापरिमाणं तत्कालीनं रविहिमांश्वोः ॥ १५ ॥

ग्रहणकालः

अवनतिवर्गं जह्याद् रवीन्दुपरिमाणभोगदलवर्गात् ।
तन्मूलात्तु द्विगुणात् तिथिभुक्तवदादिशेत् कालम् ॥१६॥

(Varāha, *PS*, 8. 9-16)

—Romakasiddhānta

Parallax in longitude

Find the interval between mid-day and the time of new moon, in *nāḍīs*. Mulitply this by 6. Degrees are got. Find its sine. Divide it by 30. The result is the parallax in *nāḍīs* to be deducted from the time of new moon if new moon is before mid-day, and to be added to the time of new moon if after mid-day. The new moon corrected for parallax in longitude is obtained. (9)

Declination of the Nonagesimal

At any time (for which the zenith distance of the nonagesimal, ZDN, is derived), find the orient ecliptic point (OEP). Add nine Signs to it. (This point is called the nonagesimal). Find its declination. (10)

Subtract the Head of Rāhu from the nonagesimal, find its sine, double it, and add a sixth of the quantity got by doubling, (*i.e.*, find the latitude of the Moon supposing it to be situated at the nonagesimal). Add this to the declination found above if both are of the same direction, and subtract it from the declination if they are of different directions. (Thus the declination of the nonagesimal is corrected.) (11)

The north declination, being less, and therefore deducted from the latitude of the place, the remainder (which is the ZDN) is south. The south declination must be added to the latitude, and the sum (forming the ZDN) is north. The part of the north declination greater than the latitude, (*i.e.*, the remainder after deducting the latitude from the north declination, which forms the ZDN), is north. (12)

Correction to the parallax and diameter of the orbit

Multiply the true daily motion of the Moon by the sine of the ZDN thus found, and divide by 1800. This is the parallax correction for latitude. The mean angular diameter of the Sun is 30 minutes, and that of the Moon, 34 minutes, (according to the Romaka). (13)

Twentyone, multiplied by the sine of (Sun or Moon at new moon$\sim$Rāhu) and divided by nine is the latitude. This, with the parallax correction added is the parallax-corrected latitude, when both are of the same direction. When of different directions, their difference is the corrected latitude. (14)

True diameter of the orbits

The mean angular diameters of the Sun and the Moon, respectively, multiplied by their true daily motions and divided by their mean daily motions, the true angular diameters at the time of eclipse.[1] (15)

Moment of the eclipse

Subtract the square of the parallax-corrected latitude from the square of the sum of the semi-diameters. The square root of the remainder, multiplied by two, is the number of minutes of arc giving the duration. These minutes multiplied by 60 and divided by the minutes of the relative true daily motion gives the time of duration in *nāḍikās*.[2] (16). (TSK)

—सौरसिद्धान्तः

रविचन्द्रकक्षे

16. 25. 1. 'मुनिकृतगुणेन्द्रिय'घ्नः स्फुटकर्णः 'खकृत'भाजितोऽर्कस्य ।
कक्षेति चन्द्रकर्णो 'दिग्'घ्नः कक्षा शशाङ्कस्य ।। १५ ।।

बिम्बमानम्

'स्वरवसुमुनीन्द्रविषया' भानोः 'खकृततर्तुवसुगुणाः' शशिनः।
तात्कालिकमानार्थं स्फुटकक्षाभ्यां पृथग् विभजेत् ।।१६।।

मध्यज्या

मध्यार्कलम्बिततिथेरनक्षराश्युद्गमैः प्रतीपांशाः ।
प्राक् समलिप्ता हानिः क्रमेण पश्चाद्धनं कार्यम् ।। १७ ।।

[1] *I.e.*, (i) The angular diameter of the Sun=30′×Sun's true daily motion÷59.
(ii) The angular diameter of the Moon=34′×Moon's true daily motion÷791.

[2] For the rationales and the working, see *PS*: *TSK*, 8. 9-16.

तन्मध्यविलग्नाख्यं तस्माच्चापक्रमांशकाः क्रमशः ।
तैरक्षवियुतयुक्तैर्या ज्या मध्याभिधाना सा ।। १८ ।।

रवेर्दृक्क्षेपः

तिथ्यन्तविलग्नज्या काष्ठान्तज्याहता स्वलम्बहृता ।
मध्यज्याघ्नी व्यासार्धभाजिता वर्गिता सा च ।। १९ ।।

मध्यज्याकृतिविश्लेषितां पृथक् स्थाप्य मूलमेकस्याः ।
सवितुर्दृक्क्षेपाख्यं संस्कृत्यर्थं पृथक् स्थाप्यम् ।। २० ।।

शङ्कुः

दृक्क्षोपकृतिं जह्यात् त्रिज्यावर्गात् ततोऽस्य यन्मूलम् ।
लग्नार्कविवरमौर्व्या गुणितं त्रिज्योद्धृतं शङ्कुः ।। २१ ।।

लम्बितपर्वान्तः

शङ्क्वङ्गुलाख्यविंशतिशतकृत्योरन्तरेण विश्लेषात् ।
स्थितवर्गान्मूलं द्विनवकाहतं तद्विभज्य कक्ष्याभ्याम् ।। २२

भागाविशेषात्तिथिवत्तिथ्यन्तनाम पुनः पुनस्तत् स्यात् ।
एवं मृग्यः कालस्तूत्पन्नो यावदविशेषः ।। २३ ।।

नतिः

अविशेषाद् दृक्क्षेपं 'वस्वेक'घ्नं विभज्य कक्षाभ्याम् ।
लब्धान्तरचापांशा मध्यज्यादिग्वशेन नतिः ।। २४ ।।

ज्याविधिना विक्षेपं तत्कालं प्राप्य तेन सहितोना ।
स्पष्टा नतिः प्रमाणैः स्वैः स्वैर्ग्रासं स्थितं च वदेत् ।।२५।।

अवनतिवर्गं जह्याद् रवीन्दुपरिमाणभोगदलवर्गात् ।
तन्मूलात्तु द्विगुणात् तिथिभुक्तवदादिशेत् कालम् ।।२६।।

तिथ्यवनामो ग्रहणादिनामविश्लेषितो युतस्थित्याम् ।
गोलान्यत्वे देयस्त्ववनामौक्षिकस्यैवम् ।। २७ ।।

(Varāha, *PS*, 9. 15-27)

—Saurasiddhānta

Kakṣā of the Sun and the Moon

The Sun's radius vector multiplied by 5347 and divided by 40 is called its *kakṣā*. The Moon's radius vector multiplied by 10 is its *kakṣā*. (15)

Measure of the orbit

Divide 5,14,787 by the Sun's *kakṣā*, and 38,640 by the Moon's, to get the respective angular diameters in minutes at the time. (16)

Find the interval between midday and the moment of new moon. If the Sun is east of the meridian, (*i.e.*, if new moon falls in the forenoon), find the degrees of right ascension corresponding to this time using the ascensional differences of zero latitude, (*Laṅkodaya-māna*), backwards from the Sun. Subtract these degrees from the Sun (=Moon) of the moment of new moon. If the Sun is west of the meridian, (*i.e.*, if new moon is in the afternoon), find the degrees corresponding to the interval counting forward from the Sun and add to the Sun (=Moon). (17).

The merdian point of the ecliptic (*madhya-lagna*) is obtained. Find its declination, north or south. If north, find the difference between the declination and the latitude of the place. If south, add them. The sine of the result is called *madhyajyā*, (*i.e.*, sine zenith distance) of the point. (18)

Dṛkkṣepa of the Sun

Find the sine of the longitude of the Orient Ecliptic point (OEP) at new moon, multiply by the sine of maxium declination, 48′ 48″, and divide by the sine of the colatitude. (This is sine amplitude of OEP, called *udayajyā*.) Multiply this by the sine of the zenith distance, (ZD) of MEP, already found, and divide by 120′. Square the result and subtract from the square of the sine ZD, of the MEP. (19)

Set the remainder in two places. In one place, find its square root. This is the sine of the zenith-distance of the nonagesimal (ZD of N) called the Sun's *dṛk-kṣepa*. Keep this aside for future work. (20)

Gnomon

Subtract from 14,400 the square of sin ZD of N, (kept unused in the other place in the previous work,) and find its square root. Multiply this by the sine of the distance between the Sun and the OEP, and divide by 120′. The result, which is the sine of the Sun's altitude, is called *śaṅku*, i.e., the Sun's *śaṅku*. (21)

Subtract the square of the Sun's *śaṅku* obtained above from 14,400. From the remainder subtract the square of the Sun's *dṛk-kṣepa* kept apart in the previous work and find its square root, technically called *dṛggati*. Multiply this by 18 and divide by each of the *kakṣās* of the Sun and the Moon. (22)

Find the respective arcs (in minutes) and get their difference. Treat this as the minutes of *tithi* and find the *tithi-nāḍikās* for this. Subtract the *nāḍikas* from the time of new moon if forenoon, and add, if afternoon. The parallax-corrected new moon (PCN) is determined. Repeat the operation of finding the PCN, till there is no difference (in time) in two successive operations. This is the PCN (to be used in the subsequent work.). (23)

Parallax in latitude

Take the sine ZD of N last obtained in the successive approximation, multiply by 18, and divide by the respective *kakṣās*. The respective sine parallax in latitude is got. The arc of their difference is the relative parallax in latitude and its direction is that of sine ZD of MEP (*i.e.*, of M from Z). (24)

The Moon's latitude at the time taken is to be got by using the sine (of Moon⌐Rāhu), and this is to be added to or subtracted from the parallax correction in latitude, (according to their direction). This is the parallax-corrected latitude. This is to be determined separately for each of the times separately, and from them the times of total obscuration and total duration are to be got. (25)

Subtract the square of the parallax-corrected latitude from the square of the sum of the semi-diameters of the Sun and the Moon and find the square root. Double this, and find the time for it, treating it as the motion of *tithi*. (The duration of the eclipse is got.) (26)

Find the *nāḍīs* of parallax for the time of the beginning. If the time of beginning and the new moon are both in the forenoon or both in the afternoon, find the difference of the *nāḍīs* of parallax and add it to the half duration to get the correct half duration, (to be subtracted from the time of the corrected new moon). If one is before noon and the other afternoon, add the *nāḍīs* of parallax, and add it to the half-duration, to get the correct half duration (to be subtracted from the time of parallax-corrected new moon). Do the same for the sine of the end of the eclipse (to find the correct half duration to be added to the parallax-corrected new moon, to get the correct last contact).[1] (27). (TSK)

—आर्यभटीयार्धरात्रपक्षः

16. 26. 1. वित्रिभलग्नापक्रमविक्षेपाक्षांशयुतिविशेषोनात् ।
भनितयाज्ज्या छेदस्त्रिज्यार्धकृतेः फलेन हृता ॥ १ ॥
वित्रिभलग्नार्कान्तरजीवा घटिकादि लम्बनं सूर्ये ।
ऋणमधिके धनमूने वित्रिभलग्नात्तिथावसकृत् ॥ २ ॥
ये युतिविशेषभागास्तज्ज्यावनतिर्गुणा त्रयोदशभिः ।
'खाब्धि'हृता विक्षेपं कृत्वा तात्कालिकशशाङ्कात् ॥
संयोगान्तरमवनतिशशाङ्कविक्षेपयोः समान्यदिशोः ।
स्फुटविक्षेपः शशिवत् स्थित्यर्धविमर्ददलनाड्यः ॥ ४ ॥
प्राग्वल्लम्बनमसकृत् तिथ्यन्तात् स्थितिदलेन हीनयुतात् ।
तन्मध्यान्तरयुक्तं स्थितिदलमेवं विमर्दार्धम् ॥ ५ ॥
अधिकेऽधिकान्तरज्यालम्बनमेवं तदृणधनैकत्वे ।
हीने हीनं भेदे तदैक्ययुतमुक्तवत्ते च ॥ ६ ॥
(Brahmagupta, *KK*, 1. 5. 1-6)

—Āryabhaṭa's Midnight system

Find the sum or difference of the *krānti* and *vikṣepa* of the *vitribha-lagna* and the latitude of the place. Subtract the result from 90° and find the *jyā* of the remainder. Divide the square of half the *trijyā* by this *jyā*. Divide the *jyā* of the difference of the longitudes of the Sun and *vitribha-lagna* by this result. Thus is obtained the

[1] For detailed elucidation and rationale involved, see *PS:TSK*, 9. 15-27.

lambana in terms of *ghaṭikās*, etc. This time should be added to or subtracted from the instant of conjunction, according as the Sun is less or greater than the *vitribha-lagna*. This process should be repeated (till the time is fixed). (1-2)

The *jyā* of the degrees, etc., in respect of the Sun or difference of the *krānti* and the *vikṣepa* of the *vitribha-lagna* and the latitude of the place, multiplied by 13 and divided by 40, gives the *avanati*.

Find the *vikṣepa* of the Moon from its longitude at the instant of conjunction. The sum or difference of the *avanati* and the *vikṣepa*, according as they are in the same or different directions, gives the *sphuṭavikṣepa* of the Moon. This should be used to calculate, as in the case of the lunar eclipse, the half duration in *ghaṭikās* of the solar eclipse or of the total obscuration. (3-4)

As before, the *lambana* should be calculated by repeated process from the instant of apparent conjunction of the Sun and Moon, decreased or increased by the duration of the first or second half of the eclipse, respectively, till it is fixed. When the *lambanas* for the beginning and middle of the eclipse, that is the *sparśā-lambana* and the *madhyalambana*, are both subtractive, and the former is greater than the latter, and when both are additive, and the former is less than the latter, then their difference, when added to the duration of the first half of the eclipse, gives its corrected duration. When the *sparśalambana* and the *madhyalambana* are both subtractive, and the former is less than the latter, and when both are additive, and the former is greater than the latter, then their difference, when subtracted from the duration of the first half of the eclipse, gives its corrected duration. Again, when the *sparśalambana* and the *madhyalambana* are of different denominations, then their sum, when added to the duration of the first half of the eclipse, gives its corrected duration. In the same manner, the correct duration of the first half of the total eclipse is calculated. Similarly, one can find the correct duration of the second half of the eclipse or of the total obscurations. (5-6).[1] (BC)

—भास्करः १

16. 27. 1a. लम्बकाभिहता त्रिज्या परमक्रान्तिसंहृता ।
लब्धं स्वदेशसम्भूतो व्यवच्छेदः प्रकीर्तितः ॥ १ ॥

लङ्कोदयानुपाताप्तानवगम्य रवेरसून् ।
तिथिमध्यान्तरासुभ्यो हित्वा शोध्यं गतं ततः ॥ २ ॥

शेषेऽपि यावतां सन्ति व्युत्क्रमात् तावतस्त्यजेत् ।
भागा लिप्ताश्च पूर्वाह्णे मध्यलग्नमुदाहृतम् ॥ ३ ॥

अपराह्णे चयः कार्यो गन्तव्यादेर्विवस्वतः ।
पातहीनात्ततः कल्प्यो विक्षेपः सौम्यदक्षिणः ॥ ४ ॥

(Bhāskara I, *LBh.*, 5. 1-4)

—Bhāskara I

Specialities

Multiply the radius by the R sine of the colatitude and divide by the R sine of the (Sun's) greatest declination: the result is called the local divisor. (1)

Having calculated the *asus* (of the right ascension) of the traversed portion of the Sun's Sign, by proportion with the right ascensions of the Sun's Sign, and (then) having subtracted them from the *asus* between the times of geocentric conjunction of the Sun and the Moon and midday, subtract the traversed portion of the Sun's Sign from the Sun's longitude. From the remainder, also subtract in the reverse order, as many Signs as have their right ascensions included (in the remaining *asus*) (as also)) the degrees and minutes (of the fraction) of a Sign, if any. The result (thus obtained) is known as the (tropical) longitude of the meridian ecliptic point in the forenoon.

(When the geocentric conjunction of the Sun and the Moon occurs) in the afternoon, addition should be made of the untraversed portion of the Sun's Sign, etc. (2-4a)

From that (tropical longitude of the meridian ecliptic point) diminished by the longitude of the Moon's ascending node, calculate the celestial latitude north or south, (as the case of the Moon). (4b). (KSS)

दृक्क्षेपः

16. 27. 1b. मध्यलग्नापमक्षेपपलज्याधनुषां युतिः ।
तुल्यदिक्त्वे विदिक्कानां विश्लेषश्शेषदिग्वशात् ॥ ५ ॥

मध्यजीवा तया क्षुण्णां प्राग्विलग्नभुजां हरेत् ।
व्यवच्छेदेन यल्लब्धं वर्गीकृत्य विशोधयेत् ॥ ६ ॥

मध्यज्यावर्गतः शेषो वर्गो दृक्क्षेपसंभवः ।

दृग्गतिज्या

तत्कालशङ्कुवर्गेण युक्त्वा तं प्रविशोधयेत् ॥ ७ ॥

लम्बनम्

विष्कम्भार्धकृतेर्मूलं 'रूपरन्ध्रनिशाकरैः' ।
हृत्वा लब्धस्य भूयोंऽशो विज्ञेयो योऽर्धपञ्चमैः ॥ ८ ॥

लम्बनाख्यो भवेत्कालो नाडिकाद्यो रवेर्ग्रहे ।
पर्वणः शोध्यते प्राह्णे दीयते मध्यतोऽपरे ॥ ९ ॥

एवं कृतेन भूयोऽपि पर्वणा कर्म कल्प्यते ।
कालस्य लम्बनाख्यस्य निश्चलत्वं दिदृक्षुणा ॥ १० ॥

(Bhāskara I, *LBh.*, 5. 5-10)

[1] For formulae involved and the rationale, see *KK:BC*, I.123-28.

Dṛkkṣepa

Take the sum of the declination of the meridian ecliptic point and the celestial latitude (calculated from the tropical longitude of the merdian ecliptic point), and of the (local) latitude when they are of like direction and the difference when they are of unlike directions, the directions of the remainder (in the latter case) being that of the minuend. (The R sine of the sum or difference is) the *madhyajyā*. By that multiply the R sine of the *bhujā* of the tropical longitude of the rising point of the ecliptic and divide (the product) by the (local) divisor (defined in stanza 1). Square whatever is thus obtained and subtract that from the square of the *madhyajyā*. The remainder is the square of the R sine of the *dṛkkṣepa*. (5-7a)

Dṛggatijyā

Having added that (square of the *dṛkkṣepajyā*) to the square of the R sine of the instantaneous altitude (of the Sun), subtract that from the square of the radius: (the result is the square of the *dṛggatijyā*). (7b-8a)

Lambana

Having divided the square root thereof by 191, further divide the quotient by 4 and a half; the result in *nāḍīs* is the time known as *lambana* in the case of a solar eclipse. It is subtracted from the time of (geocentric) conjunction if the latter occurs in the forenoon and is added to that if that occurs in the afternoon. To get the nearest approximation for the *lambana*, (i.e., the *lambana* for the time of apparent conjunction of the Sun and Moon), one should similarly perform the above operation again and again with the help of the time of (geocentric) conjunction. (8-10). (KSS)

नतिः

16. 27. 1c. दृक्क्षेपज्यामविश्लिष्टां गत्यन्तरहतां हरेत् ।
'खस्वरेष्वेकभूताख्यै'र्लब्धास्ता लिप्तिकादयः ।। ११ ।।
तत्कालशशिविक्षेपसंयुक्तास्तुल्यदिग्गताः ।
भिन्नदिक्का विशेष्यन्ते रवेरवनतिः स्फुटा ।। १२ ।।

(Bhāskara I, *LBh.*, 5. 11-12)

Nati

Multiply the R sine of the *dṛkkṣepa* obtained by the method of successive approximations,[1] (i.e., multiply the R sine of the *dṛkkṣepa* for the time of apparent conjunction) by the difference between the daily motions (of the Sun and Moon) and divide by 51,570: the result is (the *nati*) in minutes of arc, etc. (11)

[1] While finding the nearest approximation to the *lambana* for the time of apparent conjunction by the method of successive approximations, the R sines of the *dṛkkeṣepa* and the *dṛggati* were calculated at every stage. By the R sine of the *dṛkkeṣepa* obtained by the method of successive approximation is here meant the value of the R sine of the *dṛkkṣepa* calculated at the last stage, which corresponds to the time of apparent conjunction.

(The *nati*) and the Moon's latitude for that instant should be added if they are of like directions and subtracted if they are of unlike directions: thus is obtained the true latitude (of the Moon) in the case of a solar eclipse. (12). (KSS)

—लल्लः

रविग्रहणसाधनानि

16. 28. 1. अथ रविग्रहणावगमोद्यमी तदुदये विदधीत परिस्फुटान् ।
दिनकरेन्दुनिशाकरविद्विषश्चरमपक्षजपञ्चदशे तिथौ ।।१।।

मध्यलग्नम्

तिथ्यन्तं रजनीदलेन सहितं कृत्वा सभार्धं रविं
पूर्वाह्णे क्रियते यदुक्तविधिना लग्नं निरक्षोदयैः ।
मध्याह्नात् परतो दिनार्धरहितं कृत्वावसानं तिथेः
सूर्यं चाविकृतं विधाय सुधियस्तन्मध्यलग्नं जगुः ।। २ ।।

मध्यलग्नशङ्कुः

तत्क्रान्तिकाष्ठसहिताः स्वदिशोऽक्षभागा
मध्याह्वयाः स्युरथ भिन्नदिशोर्वियुक्ताः ।
तज्ज्या भवेत्तदभिधा त्रिभमध्यभाग-
विश्लेषमौर्व्यपि च मध्यविलग्नशङ्कुः ।। ३ ।।

उदयज्या

इनोदयात्तिथ्यवसानलग्नं
यत् स्वोदयैस्तस्य भुजांशमौर्वी ।
जिनांशमौर्व्या गुणिता विभक्ता
लम्बज्यया स्यादुदयाभिधा ज्या ।। ४ ।।

दृक्क्षेपः

तां मध्यजीवागुणितां विभक्तां
त्रिभज्यया बाहुमुदाहरन्ति ।
तन्मध्यजीवाभववर्गयोश्च
दृक्क्षेपमाहुर्विवरस्य मूलम् ।। ५ ।।

लम्बनशुद्धपर्व

तद्वर्गतिथ्यन्तजदृष्टिजीवा-
वर्गान्तरं दृग्गतिवर्गमुक्तम् ।
तन्मूलनिघ्नान् शरदृष्टिबाणान्
द्विष्ठान् भजेत् सूर्यशशिश्रुतिभ्याम् ।। ६ ।।

फलान्तरे षष्टिगुणे विभक्ते
भुक्त्यन्तरेणेन्दुसहस्ररश्म्योः ।
विलम्बनं स्याद् घटिकादि कृत्वा
नते तिथौ स्वं विदधीत भूयः ।। ७ ।।

तिथेर्नतस्य क्रमशिञ्जिनी हता
स्वमध्यलग्नप्रभवेन शङ्कुना ।
'क्षमाषडङ्काब्धिशराङ्कनेत्र'हृद्
बिलम्बने स्याद् घटिकादि वा फलम् ।। ८ ।।

हृताथवा दृष्टिगतिः 'खषड्गजै'-
र्विलम्बनं तत् स्वमृणं क्रमाद् भवेत् ।

तिथेर्विरामे परपूर्वभागयो-
 र्मुहुस्तदुत्थांशकलाः शशीनयोः ।। ९ ।।

खमध्यगे तिग्मकरे यदा भवेद्
 विलम्बनं स्वं विदधीत तत्तदा ।
तिथौ रवेर्दक्षिणगे निशाकरे
 निशाकराद् दक्षिणगे रवावृणम् ।। १० ।।

दृक्षेपे शरयुग्मबाणगुणिते द्विःस्थे शशाङ्केनयोः
 कर्णाभ्यां विहृते फलान्तरकला मध्यांशदिक् सा नतिः ।
दृक्क्षेपः स्फुटभुक्तिजान्तरहतः 'खाद्रीषुरूपेषु'हृद्
 वा नत्येन्दुशरः समान्यककुभोर्युक्तो वियुक्तः स्फुटः ।।

चन्द्रग्रहोक्तविधिना स्थितिमर्दखण्डे
 संसाध्य तत्स्थितिदलोनयुतात् पृथक्स्थात् ।
स्पष्टात् तिथेरसकृदेव विलम्बनादी-
 नानीय तत्स्थितिदले सकृदेव साध्ये ।। १२ ।।

प्राग्लम्बनं समधिकं यदि मध्यमात् स्या-
 दूनं च मौक्षमृणसंज्ञितयोस्तयोश्च ।
प्राग्ग्रासमूनमधिकं यदि वापि मौक्षं
 स्यान्मध्यमाद्धनगयोश्च तदन्तरेण ।। १३ ।।

युक्तं निजं स्थितिदलं स्फुटमन्यथोनं
 योगेन वार्णधनलम्बनयोर्युतं स्यात् ।
दृक्क्षेपदृग्गुणसमत्वविलम्बनं स्यात्
 संयोजयेत् स्थितिदलेऽस्ति विलम्बनं यत् ।। १४ ।।

एवं विमर्ददलयोरपि संस्थितिः स्यात्
 तेनोनितादथ युतात् स्फुटपर्वणोऽन्तात् ।
साध्यौ च मध्यशरवत् प्रथमान्त्यबाणौ
 ग्रासं तथेष्टमवगन्तुमभीष्टकोटिः ।। १५ ।।

बाहुः स्पष्टशरोद्भवः स्थितिदलक्षुण्णः स्फुटो जायते
 स्थित्यर्धेन हृतः स्फुटेन शशिवच्छेषस्य कार्यो विधिः ।
ग्रासात् पूर्ववदागतश्च समयः क्षुण्णः स्फुटेनासकृत्
 स्थित्यर्धेन हृतः स्फुटेषुजनितेनोनः स्थितेः स्वाद् दलात्।।

(Lalla, *ŚiDhVṛ.*, 6. 1-16)

—Lalla

Data for computing the solar eclipse

One who intends to ascertain a solar eclipse, must first find the true longitudes of the Sun, the Moon and its Ascending Node, on the fifteenth day of the dark half of the lunar month at sunrise. (1)

Meridian ecliptic point

(To find the meridian ecliptic point or *madhyalagna*), the time when the Amāvāsyā or new moon ends, if before midday, should be added to half the duration of the night and the true longitude of the Sun should be increased by 6 Signs. The *lagna* calculated from these by means of the times of rising of the Signs of the zodiac at Laṅkā, according to the methods given above, is called meridian ecliptic point by the wise. If the time when the Amāvāsyā ends is after midday, it should be diminished by half the duration of the day and the true longitude of the Sun should be considered without any change and the *lagna* calculated. (This, again, would be the meridian ecliptic point). (2)

R sine altitude at the Meridian ecliptic point

The latitude of the observer's station, expressed in degrees, increased or diminished by the declination corresponding to the longitude of the meridian ecliptic point, according as they are in the same or opposite direction, is called *madhya*. Its R sine is called *madhyajyā*. The R sine of 90° minus the *madhya* is called the *śaṅku* or R sine of the altitude of the *madhya*. (3)

R sine amplitude of the rising point of the ecliptic

Calculate the *lagna* for the time between sunrise and the end of the *amāvāsyā*, using the local times of rising of the Signs of the zodiac. Multiply the R sine of the longitude of this *lagna* by the R sine of 24° and divide by the R sine of the colatitude. The result is called *udayajyā* (or R sine of the amplitude of the rising point of the ecliptic). (4)

Ecliptic zenith distance

When the *udayajyā* is multiplied by the *madhyajyā* and divided by the radius, the result is called *bāhu* or base. The square root of the difference of the squares of the *madhyajyā* and the *bāhu* is called *dṛkkṣepajyā* (or R sine of the ecliptic zenith distance). (5)

Syzygy corrected for parallax in longitude

The difference between the squares of the *dṛkkṣepajyā* and that of the R sine of the Sun's zenith distance at the end of the Amāvāsyā, is called the square of the *dṛggatijyā*. Find its square root and multiply it by 525. Divide the product severally by the distances of the Sun and the Moon from the Earth.

Find the difference of the two quotients. Multiply it by 60 and divide by the difference of the motions of the Sun and the Moon. The result is the parallax in longitude in *ghaṭikās* or *lambana* (at the mid-eclipse). It should be applied positively, (or negatively as the case may be), to the calculated time when the Amāvāsyā ends. The parallax should be repeatedly calculated (and applied till the time is fixed). (6-7)

Or, the R sine of the hour-angle at the Amāvāsyā multiplied by the R sine altitude of the meridian ecliptic point and divided by 29,54,961, gives the parallax in *ghaṭikās* at the mid-eclipse. (8)

Or, the *dṛggatijyā* divided by 860 gives the parallax in *ghaṭikās*. It should be added to the calculated time

when the Amāvāsyā ends, if the Sun is in the western hemisphere, and subtracted, if in the eastern hemisphere. (The result is the time once corrected.) The longitudes of the Sun and the Moon must be found for this corrected time by adding or subtracting the minutes resulting (from the motions according as the parallax is additive or subtractive) and hence again the parallax. This process must be repeated (till the parallax and the time are fixed). (9)

When the Sun is on the meridian, the parallax, if any, should be added to the (calculated time) when the Amāvāsyā ends, provided that the Moon is to the south of the Sun. If the Sun is to the south of the Moon, the parallax should be subtracted. (10)

Parallax in latitude

Multiply the *dṛkkṣepajyā* by 525 and divide severally by the distances of the Sun and Moon from the Earth. The difference of the results in minutes is called *nati* or parallax in latitude. Its direction is the same as that of the *madhyajyā*.

Or, the *dṛkkṣepajyā* multiplied by the differences in the true motions of the Sun and Moon and divided by 51,570 gives the parallax in latitude.

The latitude of the Moon increased or diminished by this parallax according as they are in the same or opposite directions, is the corrected latitude. (11)

Application of the parallaxes

Calculate the first and second half of the duration of the eclipse and of the total eclipse, following the method given for the lunar eclipse. From the corrected time when the Amāvāsyā ends subtract the first and add to it the second half of the duration of the eclipse. (The results are approximately the times when the eclipse begins and ends, respectively.) Then, from these times calculate the parallax at the beginning and end of the eclipse, and apply them (in the manner given in the next two verses) to the approximately calculated half durations.

Repeat the process till these times are fixed, (which are then the apparent or *sphuṭa* durations of the first and second halves of the eclipse). (12)

If the parallax for the beginning of the eclipse is greater than that for the middle of the eclipse and both are subtractive, and if the parallax for the end of the eclipse is less than that for the middle of the eclipse and both are subtractive, add their differences to the approximately calculated first and second half of the duration of the eclipse.

If the parallax for the beginning of the eclipse is less than that for the middle of the eclipse and both are additive, and if the parallax for the end of the eclipse is greater than that for the middle of the eclipse and both are additive, then also add their differences respectively to the approximately calculated first and second half of the duration of the eclipse.

If the parallax for the beginning of the eclipse is less than that for the middle of the eclipse and both are subtractive, and if the parallax for the end of the eclipse is greater than for the middle of the eclipse and both are subtractive, then, subtract their differences respectively from the first and second half of the duration of the eclipse.

If the parallax for the beginning of the eclipse is greater than that for the middle of the eclipse and both are additive, and if the parallax for the end of the eclipse is less than that of the middle of the eclipse and both are additive, then also subtract their differences respectively from the first and second half of the duration of the eclipse.

If the parallax for the beginning of the eclipse is different in denomination from that for the middle of the eclipse or if the parallax for the end of the eclipse is different in denomination from that for the middle of the eclipse, then always add their sums to the first or second half of the duration of the eclipse (as the case may be). The result in each case is the apparent (*sphuṭa*) half duration of the eclipse.

If there is parallax when *dṛkkṣepa* and *dṛggati* are equal it should be added to the half duration of the eclipse. (13-14)

Computation of the eclipse

The same rule is applicable for calculating the first and second half of the duration of the total eclipse. When the apparent duration for the first half and the second half of the eclipse are respectively subtracted or added to the apparent time when the Amāvāsyā ends, (the results are the apparent times for the beginning and end of the eclipse, respectively).

Then calculate the Moon's correct latitude at these times in the same manner as the latitude at the mid-eclipse is calculated.

When the obscured portion at any given time is required, calculate the Moon's correct latitude for that time, and this is the *koṭi* or perpendicular. (15)

Then calculate the *bāhu* or base (as above). Multiply it by the (approximately calculated first or second) half of the duration of the eclipse, as the case may be, using the corrected latitude, and divide by the apparent duration of the first or second half. The result is the

corrected base. The remaining process (to find the obscured portion at any given time) is the same as that in the case of the Moon.

Again when the obscured portion is given, and the corresponding time is required, follow the process as given above. Then multiply this time by the apparent duration of the first or second half of the eclipse, as the case may be; and, using the corrected latitude, divide by the approximately calculated duration. The result is the more correct time. This should be subtracted from the half duration. Repeat the process till the time is fixed.[1] (16). (BC)

—महासिद्धान्तः

आयनदृक्कर्म

16. 29. 1a. प्राक् शृङ्गोन्नतिमुख्ये कर्मणि सूर्यग्रहाविनोदयजौ ।
कृत्वा चन्द्रादीनां बाणः साध्योऽस्तजौ पश्चात् ॥ १ ॥

दत्तायनजव्यस्तज्योनां गज्यां शरेण संगुणयेत् ।
क्लधजै च हरेद् गज्यावर्गेणायनमिदं कलादिफलम् ॥

कोटिज्येषुवधो वा जढममभक्तोऽयनेषु दिक्साम्ये ।
शोध्यं खगे त्वसाम्ये योज्यं स्यादायनः खेटः ॥ ३ ॥

(ABh. II, *Mahā.*, 7. 1-3)

—Mahāsiddhānta

Parallax of longitude

In the operation, (in which the projection of) the elevation of the lunar cusps is the main (part), determine the latitude of the Moon etc., after finding out (the longitudes of) the Sun and the planet, in east at sunrise and in west at sunset. (1)

Diminish the radius by the versed sine (*vyastajyā*) (of the planet in a) given progress. Multiply (the remainder) by the latitude (*śaro*) and by 1398, and divide by the square of the radius: the result in minutes etc. is the correction for ecliptic deviation (*āyana*).[2] (2). (SRS)

Multiply the perpendicular-sine by the latitude (*iṣu*) and divide by 8455. (The quotient) is to be subtracted from (the longitude of) the planet, when the correction and the latitude are of the same direction, and added when of opposite (direction; the result) is the planet's longitude, as corrected for ecliptic deviation (*āyana-kheṭa*).[3] (3). (SRS)

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[1] For elucidation and demonstration, see, *ŚiDhVṛ: BC.*, II. 131-42

[2] For the rationale, see *Mahā: SRS*, II. 130-33.

[3] From the previous verse, the correction for ecliptic deviation in

$$\text{minutes} = \frac{\text{sin gr. decl.} \times (\text{R—versed sine}) \times \text{latitude}}{R^2}$$

$$= \frac{1398 \times \text{perpendicular-sine} \times \text{latitude}}{1{,}18{,}19{,}844}$$

$$= \frac{\text{perpendicular-sine} \times \text{latitude}}{8455} \text{ approximately. (3)}$$

For the rationale, see *Mahā:SRS*, II. 130-36.

आक्षदृक्कर्म

16. 29. 1b. विषुवद्भाशरघातं प्रहृतं खेटे क्षिपेच्चरे सौम्ये ।
पश्चाद् याम्ये जह्याद् व्यस्तं प्रागक्षकर्मैतत् ॥ ४ ॥

(ABh. II, *Mahā*, 7.4)

Parallax of latitude

Multiply the equinoctial shadow (*viṣuvadbhā*) by the latitude, and divide by 12; (the result, when the latitude is north, is to be added to the (longitude) of) the planet; (when the latitude is) south, is to be subtracted in the western (hemisphere. The operation is) reverse in the eastern (hemisphere). This is the operation for latitude (*akṣakarman*).[1] (4). (SRS)

दृक्संस्कारः

16. 29. 1c. दिक्साम्ये विश्लेषोऽर्केन्दुक्रान्त्योरसाम्य ऐक्यं तत् ।
व्यर्केन्दुज्याक्षज्याहतेः गमौर्व्याप्तयाम्यांशैः ॥ ५ ॥

संस्कृत्य भजेद् व्यर्केन्दुज्यातांशेन चन्द्रबिम्बघ्नम् ।
पौरै भक्तं वलनं संस्कारवशेन दिग् ज्ञेया ॥ ६ ॥

(ABh. II., *Mahā.*, 7. 5-6)

Application of parallax

Of (the sines of) the declination of the Sun and of the Moon (take) the difference, (when both are) of the same (direction)and the sum, (when they are) of opposite (direction) (A). Multiply the sine of the difference (in longitudes) of the Moon and the Sun by the sine of latitude and divide by the radius. By the quotient in degrees, which are south, correct (the above difference or sum, A) and divide by one sixth of the sine of the difference (in longitude of) the Moon and the Sun. (The quotient), multiplied by the diameter of the Moon and divided by 12, is the deflection (*valana*), (whose) direction is to be known as the same as the correction above. (5-6). (SRS)

—भास्करः २

लम्बननती

16. 30. 1a. दर्शान्तकालेऽपि समौ रवीन्दू
द्रष्टा नतौ येन विभिन्नकक्षौ ।
क्वर्धोच्छ्रितः पश्यति नैकसूत्रे
तल्लम्बनं तेन नतिं च वच्मि ॥ १ ॥

दर्शान्तलग्नं प्रथमं विधाय
न लम्बनं वित्रिभलग्नतुल्ये ।
रवौ तदूनेऽभ्यधिके च तत् स्या-
देवं धनर्णं क्रमतश्च वेद्यम् ॥ २ ॥

त्रिभोनलग्नं तरणिं प्रकल्प्य
तल्लग्नयोर्यः समयोऽन्तरेऽसौ ।
त्रिभोनलग्नस्य भवेद् द्युयातः
शङ्क्वाद्यतस्तस्य चरान्त्यकाद्यैः ॥ ३ ॥

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[1] For the rationale, see *Mahā: SRS*, II. 137-46.

त्रिभोनलग्नार्कविशेषशिञ्जिनी
'कृता'हता व्यासदलेन भाजिता ।
हतात् फलाद्वित्रिभलग्नशङ्कुना
त्रिजीवयाप्तं घटिकादि लम्बनम् ॥ ४ ॥

तत्संस्कृतः पर्वविराम एवं
स्फुटोऽसकृत् स ग्रहमध्यकालः ॥ ७c-d ॥

नतिः

दृग्ज्यैव या वित्रिभलग्नशङ्कोः
स एव दृक्क्षेप इनस्य तावत् ।
सौम्येऽपमे वित्रिभजेऽधिकेऽक्षात्
सौम्योऽन्यथा दक्षिण एव वेद्यः ॥ १० ॥

चापीकृतस्यास्य तु संस्कृतस्य
त्रिभोनलग्नोत्थशरेण जीवा ।
दृक्क्षेप इन्दोर्निजमध्यभुक्ति-
तिथ्यंशनिघ्नौ त्रिगुणोद्धृतौ तौ ॥ ११ ॥

नती रवीन्द्वोः समभिन्नदिक्त्वे
तदन्तरैक्यं तु नतिः स्फुटात्र ॥ १२a-b ॥

स्पष्टोऽत्र बाणो नतिसंस्कृतोऽस्मात्
प्राग्वत् प्रसाध्ये स्थितिमर्दखण्डे ॥ १४c-d ॥

(Bhāskara II, *ŚiSi*, 1.6. 1-4, 7c-d, 10-12b, 14c-d)

—Bhāskara II

Parallax in longitude and latitude

Inasmuch as the observer who is situated on the surface of the Earth and as such elevated by the radius of the Earth from the centre thereof, perceives not the Sun and the Moon having the same longitude at the moment of conjunction, to be in the same line of sight, their height being depressed unequally having different orbits, so I proceed to elucidate what are called *lambana* and *nati*, i.e. parallax in longitude and latitude, on which account they are not in the same line of sight. (1)

Compute the *lagna* at the moment of conjunction of the Sun and the Moon. There will be no parallax in longitude when the Sun is situated at the point called *vitribha* or the point whose longitude is $=L-90^\circ$, (L being the longitude of the *lagna*, i.e. the ascendant which is the point of intersection of the ecliptic with the horizon). If the Sun's longitude falls short of the longitude of the *vitribha* or exceeds it, there will be parallax in longitude which will be positive in the former case and negative in the latter. (2)

Compute the R cosine of ZV, by calculating the rising time of AV, the *kujyā*, *dyujyā*, and *antyā* pertaining to V, (as was formulated in the *Tripraśnādhikāra*), then R sin V., multiplied by 4 and divided by R, and again multiplied by R cos ZV and divided by R again gives the parallax in longitude. (3-4)

The time of the ending moment of new moon, i.e. the moment of geocentric conjunction, is to be rectified by this parallax in longitude, to get the moment of apparent conjunction by the method of successive approximation. (7c-d)

Parallax of latitude

R sine ZV is called the *Dṛkkṣepa* of the Sun, which is considered to be north in case the northern declination of the *vitribha* is greater then ϕ, the latitude, otherwise south. (10)

Then the sum of ZV and the latitude of V, assuming V to be the Moon or the difference of the above two, as the case may be, according as both of them are north or of opposite directions, gives the arc whose R sine is the *dṛk-kṣepa* of the Moon. The *dṛk-kṣepas* of the Sun and the Moon multiplied, respectively, by 1/15 part of their daily motions and divided by the radius R (equal to 3438') are the parallaxes of the Sun and the Moon in latitude. The sum or difference of these parallaxes according as they are of opposite or the same direction, is the true parallex in latitude in the context of a solar eclipse. (11b-12b)

The apparent latitude of the Moon is equal to the algebraic sum of its geocentric latitude and the parallax in latitude. From this apparent latitude are to be ealculated the *sthiti-khaṇḍa* and *marda-khaṇḍa* of the solar eclipse (by the method described in the chapter on lunar eclipse, taking the eclipsing body or *grāhaka* to be the Moon and the eclipsed or *grāhya* to be the Sun).[1] (14c-d). (AS)

स्पर्शमुक्तिसंमीलनोन्मीलनकालाः

16. 30. 1b. तिथ्यन्ताद् गणितागतात् स्थितिदलेनोनाधिकाल्लम्बनं
तत्कालोत्थनतीपुसंस्कृतिभवस्थित्यर्धहीनाधिके ।
दर्शान्ते गणितागते धनमृणं वा तद्विधायासकृज्-
ज्ञेयौ प्रग्रहमोक्षसंज्ञसमयावेवं क्रमात् प्रस्फुटौ ॥१५॥

तन्मध्यकालान्तरयोः समाने
स्पष्टे भवेतां स्थितिखण्डके च ।
दर्शान्ततो मर्ददलोनयुक्तात्
संमीलनोन्मीलनकाल एवम् ॥ १६ ॥

(Bhāskara II, *SiŚi.*, 1.6. 15-16)

Sparśakāla, Mokṣakāla, Sammīlanakāla and Unmīlanakāla

First compute the time called *sthiti-khaṇḍa* (as mentioned in the chapter on lunar eclipses). The ending moment of local Amāvāsyā or what is called the moment of local conjunction is known as the *madhya-grahakāla* or the moment of the middle of the eclipse.

[1] For a detailed exposition and rationale of processess, see *SiŚi*: *AS*, pp. 408-36. See also fig. 92 there.

Subtract the *sthiti-khaṇḍa* from the computed time of geocentric conjunction; the result will be the approximate *sparśakāla*. This has to be rectified for parallax in longitude as well as the approximate *madhyagrahakāla* of geocentric conjunction to obtain the local *sparśakala* and the local *madhyagrahakāla*. Similarly, the *mokaṣkāla*, the *sammīlana* and the *unmīlanakālas* are to be rectified for parallax in longitude. (15)

But while effecting this correction for the parallax in longitude, the Moon's latitude also differs for the corrected time which, in turn, affects the durations of *sthiti-khaṇḍa*, *mokṣa-khaṇḍa* etc. Correcting the first computed *sthiti-khanda*, *mokṣa-khaṇḍa* etc. for this variation in the latitude, and subtracting the *sthiti-khaṇḍa* from the time of *madhya-graha*, we have better approximation for the *sparśa-kāla*. Inasmuch as parallax in longitude, that in latitude, and the Moon's latitude vary from time to time, and the times of *sparśa*, *madhyagraha* etc. are affected by them, the process of computation proceeds by the method of successive approximation. Subtracting the rectified *marda-khaṇḍa* from the rectified *madhyagrahakāla*, we have the true *sammīlanakāla*; similarly adding the former to the latter we have the true *unmīlanakāla*. (16). (AS)

ग्रहणकर्म

16. 30. 1c. शेषं शशाङ्काङ्कग्रहणोक्तमत्र
स्फुटेषुजेन स्थितिखण्डकेन ।
हतोऽथ तेनैव हृतः स्फुटेन
बाहुः स्फुटः स्याद् ग्रहणेऽत्र भानोः ।। १८ ।।

ग्रासाच्च कालानयने फलं यत्
स्फुटेन निघ्नं स्थितिखण्डकेन ।
स्फुटेषुजेनासकृदुद्धृतं तत्
स्थित्यर्धशुद्धं भवतीष्टकालः ।। १९ ।।
(Bhāskara II, *SiŚi.*, 1.6. 18-19)

Computation of the eclipse

The remaining work proceeds on the lines indicated in the chapter on 'Lunar eclipses' (i.e. the computation of the *bimba-valana*, *bhuja*, *koṭi* and the like is to be done as indicated there). The *bhuja* will be rectified by multiplying it by the *sthiti-khaṇḍa* obtained by adopting the latitude of the Moon effected by parallax in latitude and divided by the *sthiti-khaṇḍa* rectified for parallax in longitude. (18)

Similarly, given the *grāsa*, i.e. the magnitude of the eclipse, the result found before by verse 15 in the chapter on 'Lunar eclipses', is to be multiplied by the *sthiti-khaṇḍa* rectified for parallax in longitude and divided by that obtained adopting the latitude of the Moon effected by parallax in latitude, and the result so obtained being subtracted from the *sthiti-khaṇḍa*, we get the *iṣṭa-kala*.[1] (19). (AS)

वलनम्

16· 30. 1d. युतायनांशोडुपकोटिशिञ्जिनी
'जिनां'शमौर्व्या गुणिता विभाजिता ।। २१ ।।

द्युजीवया लब्धफलस्य कार्मुकं
भवेच्छशाङ्कायनदिक्कमायनम् ।
तयोः पलोत्थायनयोः समाशयो-
र्युतेर्वियुक्तेस्तु विभिन्नकाष्ठयोः ।। २२ ।।

या शिञ्जिनी मानदलैक्यनिघ्नी
त्रिज्योद्धृता तद्वलनं स्फुटं स्यात् ।
यैरुत्क्रमज्याविधिनैतदुक्तं
सम्यङ न ते गोलगतिं विदन्ति ।। २३ ।।
(Bhāskara II, *SiŚi.*, 1.5. 21c-23)

$$\frac{R \cos \lambda \times R \sin \omega}{R \cos \delta} = R \sin \theta$$, where θ is the *ayana-valana*. The direction of this *valana* is that of the hemisphere north or south in which the Moon lies.[2] (21-c 22a)

The R sine of the sum of difference of the two *valanas* according as they are of the same or opposite directions, multiplied by the sum of the angular radii of the Moon and Rāhu, and divided by the radius gives the R sine of the *Sphuṭa-valana*. Those who said that the *valana* is proportional to the R versine, do not know spherical geometry properly.[3] (22b-23). (AS)

—करणरत्नम्

16. 31. 1. पर्वाह्दलविवरजनाड्यस्त्वविशेषलम्बनपदानि ।
त्रिशच्छिष्टानि तदा पञ्चदशभ्योऽधिकास्तु यदा ।। १ ।।

मनु-रष्टाश्वि-खवेदाः-खशर-नवपञ्च-रसषडे-कमुनिः ।
शरगिरि-वसुमुनि-नवमुन्य-शीतिरथ पञ्चसु त्रिगुणः ।।२।।

पूर्वकपाले हीनं युतमपरे लम्बनेन पर्व स्यात् ।
तद्विषयांशेन तथा चन्द्रस्तस्मात्तु विक्षेपः ।। ३ ।।

विषुवच्छायाव्यङ्गुलपिण्डं स्वत्र्यंशभागसंयुतया ।
सप्तत्या हृतलब्धं विषुवन्नाड्यः सदा याम्या ।। ४ ।।

मध्यलग्नम्

दिनदलपर्वविशेषे षड्गुणितेंऽशा रवौ विशोध्यास्ते ।
पूर्वकपाले पश्चाद् देयाः तन्मध्यलग्नं स्यात् ।। ५ ।।

मध्यविलग्ना जीवा स्वशरांशोना विनाडिकापूर्वैः ।
समदिशि युता विशोध्या भिन्नायामधिकदिग् ग्राह्या ।।६।।

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[1] For the rationale, see *SiŚi*:*AS*, pp. 438-45.

[2] For a detailed demonstration and rationale, see *SiŚi*: *AS*, pp. 375-91.

[3] For an exposition, see *SiŚi*:*AS*, pp. 391-92.

अवनतिः

दशभक्ता तज्जीवा रविशशिनोर्भुक्तिविवरसंगुणिता ।
हृत्वा 'विद्येषुकृतै'र्लब्धाऽवनतिः सुसूक्ष्मतरा ॥ ७ ॥

विक्षेपस्यावनतेः प्रयुतिर्वियुतिः समान्यदिशोः ।
एवं स्फुटविक्षेपो दृक्क्षेपज्यां विनाऽपि धिया ॥ ८ ॥

सम्पर्कार्धकलायास्तुल्यायां वाऽथवाऽधिकायां वा ।
स्फुटविक्षेपावनतौ शशिमण्डलं रवेर्न रुणद्धि ॥ ९ ॥

स्थित्यर्धस्य शरांशं स्पर्शे लम्बनविशुद्धचन्द्रमसि ।
हित्वा दत्वा मोक्षे शशिविक्षेपस्ततः कार्यः ॥ १० ॥

समदिशि वलनत्रितयं संयोज्यं भिन्नदिशि तु विश्लेष्यम् ।
ग्राहक इन्दुग्रहणे राहुः, सूर्यग्रहे चन्द्रः ॥ ११ ॥

प्रग्रहणमोक्षकालिकविक्षेपादानयेद्वलनान् ।
युतबिम्बार्ध-प्रग्रहमोक्षस्थित्यर्धलिप्तिकाविवरम् ॥ १२ ॥

वर्गीकृतं च साग्रं निजविक्षेपस्य कृतिसहितम् ।
मूलं ग्राह्यतनुघ्नं ग्राह्यग्राहकसमेतबिम्बहृतम् ॥ १३ ॥

विक्षेपवलनमेतद्विक्षेपसमा दिगस्य स्यात् ।
त्रिभवनरहिताच्चन्द्रात् प्रग्रहणे तैः समन्वितान्मोक्षे ॥१४॥

बाहुज्यां कृतवेदैर्हृत्वाऽऽगतमयनवलनं स्यात् ।
कालविनाड्यो द्युदलत्रिभागभक्तास्तु राशयो ज्ञेयाः ॥१५

राशित्रितयं हित्वा स्पर्शे मोक्षे च दत्वा दिक् ।
तद्बाहुज्यां विषुवच्छायोत्थविनाडिकागुणां कृत्वा ॥१६॥

'रसकृतमुनिगगनेन्दु'भिराप्तं स्यादक्षवलनं तत् ।
लम्बनान्तरसंयुक्ते स्थित्यर्धे विनिर्दिशेत् स्फुटे ॥ १७ ॥

युतबिम्बार्धविक्षेपविश्लेषो ग्रास उच्यते ।
'पक्षाग्नि'गुणितो ग्राह्यबिम्बभक्तः स्फुटः स्मृतः ॥१८॥

ग्रासैः सप्तभिरष्टमं 'द्विकु'लवैर्भागं चतुर्थं वदेद्
'वेदैकै'स्तु तृतीय'मङ्गशशिभि'श्चार्धं गृहीतं रवेः ।
सर्वं 'लोकयमै'स्त्रिभागरहितं पादोन'मङ्गाश्विभि'-
र्हीनं स्वाष्टमभागतो 'नवयमै'रिन्दोः तथैकान्वितम् ॥

(Deva: *KR*. 3.1-19)

Karaṇaratna

The iterated lambana

The *nāḍīs* lying between midday and the end of the new moon *tithi* (*parva*) are the *padas* of the iterated *lambana*. When they exceed 15, they are subtracted from 30. (1)

(The *vināḍīs* of the iterated *lambana* for 1, 2, . . . , 15 *padas* are): 14, 28, 40, 50, 59, 66, 71, 75, 78, 79, 80, 80, 80, 80, and 80—each multiplied by 3. (2)

Application of Iterated lambana

In the eastern hemisphere (i.e., in the forenoon), the *parva* (i.e., the time of geocentric conjunction of the Sun and Moon) should be diminished by the (*vināḍīs* of the iterated) *lambana* and in the western hemisphere (i.e., in the afternoon), the *parva* should be increased by the (*vināḍīs* of the iterated) *lambana*. (Thus is obtained the time of apparent conjunction of the Sun and the Moon).

The longitude of the Moon (for the time of conjunction should be diminished or increased in the same way by 1/5 of that (*lambana* in *ghaṭīs*). From the resulting longitude (of the Moon) should be calculated the Moon's latitude. (3)

Local latitude

Divide the equinoctial midday shadow (of the gnomon), in terms of *vyaṅgulas*, by 70 (1+1/3); the quotient gives the (local) latitude in terms of *nāḍīs*. It is always south. (4)

Meridian ecliptic point

The time (in *ghaṭīs*) between midday and the *parva* (i.e., the time of conjunction of the Sun and the Moon) should be converted into degrees by multiplying it by 6. These degrees should be subtracted from the longitude of the Sun for the time of conjunction (if the Sun is) in the eastern hemisphere and added to that (if the Sun is) in the western hemisphere. The result is (the longitude of) the meridian ecliptic point. (5)

Zenith distance of meridian ecliptic point

Diminish the R sine of the longitude of the meridian ecliptic point by one-fifth of itself: (the result is the declination of the meridian ecliptic point, in terms of *vināḍīs*.) Take the sum or difference of this (declination) and the (local latitude in) *vināḍīs* (already obtained in vs. 4), according as they are of like or unlike directions: (the result is the zenith distance of the meridian ecliptic point in terms of *vināḍīs*). Its direction is that of the greater of the two. (6)

Parallax in latitude

Divide the result (obtained last) by 10, and then find the R sine of the quotient. Multiply the R sine obtained by the motion-difference of the Sun and Moon and divide by 4518; the quotient is the more accurate value of the *avanti*. (7)

Moon's true latitude

Take the sum or difference of the Moon's latitude and *avanati*, according as they are of like or unlike directions. This is how the Moon's true latitude is obtained without using the *dṛkkṣepajyā*, by the application of intellect. (8)

Impossibility of a solar eclipse

When the Moon's true latitude equals or exceeds half the sum of (the diameters of) the eclipsed and eclipsing bodies, the Moon's disc does not hide the Sun's disc. (9)

Moon's latitude for first and last contacts

(To obtain the Moon's latitude for the first or last contact:) First subtract one-fifth of the semi-duration of the eclipse (in terms of *vināḍīs*) from the Moon's longitude (for the time of conjunction) corrected for *lambana*, in the former case, and add the same in the latter case, and then find the Moon's latitude. (10)

The three valanas

One should take the sum or difference of the three *valanas* (taking two at a time) according as they are of like or unlike directions.

Rāhu (i.e., Shadow) is the eclipser in the lunar eclipse and Moon in the solar eclipse. (11)

Calculate the *valanas* (for the first and last contacts) with the help of the Moon's latitude for those times, as follows: (12ab)

Vikṣepa-valana

Find the difference of (i) half the sum of the eclipsed and eclipsing bodies, and (ii) half the duration of eclipse towards the first or last contact, in terms of minutes. Square it and then increase it by the square of the Moon's own latitude (for that time). Multiply the square root of that (sum) by the diameter of the eclipsed body and divide by the sum of the diameters of the eclipsed and eclipsing bodies. This is the *vikṣepa-valana* and its direction is the same as that of the Moon's latitude. (12cd-14ab)

Āyana-valana

In the case of the first contact, subtract three Signs from the Moon's longitude, and in the case of the last contact, add three Signs to the Moon's longitude. Find the R sine of the *bāhu* thereof and divide that by 44. What is thus obtained is the *āyana-valana*. (14cd-15ab)

Ākṣa-valana

Divide the *vināḍīs* of the hour angle by (the *vināḍīs* of) one-third of half the duration of the day: the result is (the hour angle) in terms of Signs. In the case of the first contact, subtract 3 Signs from that and in the case of the last contact, add three Signs to that. Multiply the R sine of the *bāhu* of that by the *vināḍīs* (of the local latitude) arising from the equinoctial midday shadow and divide by 10,746: the result is the *ākṣa-valana*. (15cd-17ab)

True semi-durations in a solar eclipse

Half the semi-duration of the eclipse (towards the first contact) should be increased by the difference between the *lambanas* for the first contact and the middle of the eclipse; and half the semi-duration of the eclipse (towards the last contact) should be increased by the difference between the *lambanas* for the middle of the eclipse and the last contact. The results thus obtained should be declared as the true values of the two semi-durations of the eclipse. (17cd)

Measure of eclipse (Grāsa)

The difference of (i) half the sum of the diameters of the eclipsed and eclipsing bodies and (ii) the Moon's latitude (both for the time of conjunction and in terms of minutes of arc) is called the measure of eclipse. That multiplied by 32 and divided by the diameter of the eclipsed body is called the true value thereof. (18)

The eight phases of a solar eclipse

When the eclipsed portion (of the Sun's diameter) amounts to 7′, 1/8 (of the Sun's diameter) should be declared as eclipsed; when 12 , 1/4; when 14′, 1/3; when 16′, 1/2; when 23′, 1—1/3 (=2/3); when 26′, 1—1/4 (=3/4); when 29′, 1—1/8 (=7/8); when 32′, 1.[1] (19). (KSS)

—श्रीपतिः

16. 32. 1. रवेस्तु पर्वण्यथ पर्वकालः स्पष्टो भवेल्लम्बनसंस्कृतश्च ।
साधं घटीनां त्रितयं त्रयं च द्वयं तथैका घटिका क्रमेण ॥१॥

आद्ये द्वितीये च तृतीयतुर्ये यामार्धके लम्बनकं ऋणं स्यात् ।
धनं तथैका युगलं त्रयं च सार्धत्रयं पञ्चमकात् क्रमेण ॥२॥

मीनेऽथ मेषे नतिर'ङ्करामा'
मिता'ब्धिरामा' गवि चाथ कुम्भे ।
युग्मे मृगे 'सप्तकरा' 'धृति'श्च
कर्केऽथ चापे नतिरत्र याम्या ॥ ३ ॥

सिंहे तथाऽलौ दशकं कलानां
कलाश्चतस्त्रस्तुलकन्ययोश्च ।
शरावनत्योर्युतिरेकदिक्के-
ऽन्यत्वेऽन्तरं स्पष्टतरः शरः स्यात् ॥ ४ ॥

'त्रिराम'लिप्ताप्रमिता शरोना
छन्नं रवेर्मण्डलमिन्दुमानम् ।
'नवाष्टखेन्दु'प्रमितोऽत्र युक्त-
वर्गः शशांकग्रहवत् स्थितिश्च ॥ ५ ॥

सूर्यग्रहेऽपि द्युमणेर्विमर्दः
स्यात्सम्भवश्चन्द्रमितेर्बहुत्वात् ।
तिथ्यन्ततो लम्बनसंस्कृताच्च
स्थितेर्वशात् तत्समयो विचिन्त्यः ॥ ६ ॥

स्पर्शस्थितौ चन्द्रमसोऽर्कमुक्तिः
स्पर्शोब्जमुक्ताविति सूर्यपर्व ।

[1]For the rationale of the several processes, see *KR:KSS*, pp. 52-62.

धीकोटिदं सत्करणं प्रसिद्धं
श्रीश्रीपतिः सारतरं चकार ॥ ७ ॥

(Śrīpati, *Dhīkoṭi*, 7.1-7)

—Śrīpati

Time of apparent conjunction

In the case of an eclipse of the Sun, the time of conjunction of the Sun and the Moon becomes apparent when *lambana* (i.e. correction for parallax in longitude) is applied to it. (1)

In the first, second, third and fourth *yāmārdhas*, the *lambana* is negative and its values are 3½ *ghaṭīs*, 3 *ghaṭīs*, 2 *ghaṭīs*, and 1 *ghaṭī* respectively. In the four subsequent *yāmārdhas*, beginning with the fifth, the Lambana is positive and its values are 1 *ghaṭi*, 2 *ghaṭis*, 3 *ghaṭis* and 3½ *ghaṭīs* respectively.[1] (2)

Moon's true latitude

When the *lagna*, i.e. the point of ecliptic lying on the eastern horizon, is) in Pisces or Aries, the value of *nati* (i.e. correction for parallax in latitude) is 39′; in Taurus or Aquarius it is 34′; in Gemini or Capricorn, it is 27′; in Cancer or Sagittarius, it is 18′; in Leo or Scorpio, it is 10′; and in Libra or Virgo it is 4′. The direction of nati here (i.e. in this work) is always south.

When the Moon's latitude and *nati* are of like directions, they are to be added together; when of unlike directions, their difference is to be taken. Whatever is thus obtained is the Moon's true latitude. (4)

First and last contacts etc.

33 minutes *minus* the Moon's (true) latitude is the measure of the eclipse of the Sun. The Sun's diameter is (roughly) equivalent to the Moon's diameter (already stated).

1089 is here the value of the square of the sum of the diameters of the eclipsed and eclipsing bodies. The duration of the eclipse is obtained as in the case of a lunar eclipse. (5)

The measure (of the diameter) of the Moon being (at times) greater (than that of the Sun), in the case of a solar eclipse too, total obscuration of the Sun is possible.

The times (of first contact and separation, or of immersion and emersion) should be obtained with the help of the time of conjunction as corrected for *lambana* and the duration (of the eclipse or totality). (6)

The position of the Sun's separation (from the Moon is the same as that of the Moon's (first) contact (with the Shadow); and the position of (the Sun's first) contact (with the Moon) is the same as that of the Moon's separation. (from the Shadow).

This is the situation in a solar eclipse.

This excellent Karaṇa (i.e. calendaric work), which is entitled *Dhīkoṭi* and is highly condensed is composed by Śrī Śrīpati. (7). (KSS)

[1] *Yāmārdha* is a unit of time which is equivalent to one-eighth of the day measured from sunrise to sunset. The first *yāmārdha* begins at sunrise and the eighth *yāmārdha* ends at sunset.

—ग्रहलाघवम्

लम्बनसंस्कारः

16. 33. 1. लग्नं दर्शान्ते त्रिभोनं पृथक्स्थं
तत् क्रान्त्यंशैः संस्कृतोऽक्षो नतांशाः ।
तद्द्विघ्नचंशो वर्गितश्चेद् द्विकोर्ध्वो-
ऽधोऽसौ द्व्यूनः खण्डितस्तद्युतः सः ॥ १ ॥

सार्को हारः स्यात् त्रिभोनोदयार्क-
विश्लेषां'शा'शांशहीनघ्नशक्राः ।
हाराप्ताः स्याल्लम्बनं नाडिकाद्यं
तिथ्यां स्वर्णं वित्रिभेऽर्काधिकोने ॥ २ ॥

'त्रिकु'निघ्नविलम्बनं कलास्तत्
सहितोनस्तिथिवद् व्यगुः शरोऽतः ।
अथ षड्गुणलम्बनं लवास्तै-
र्युगवित्रिभतः पुनर्नतांशाः ॥ ३ ॥

नतिः, शरश्च

दशहृतनतभागोनाहता'ष्टेन्दव'स्तद्-
रहितसधृतिलिप्तैः षड्भिराप्तास्त एव ।
स्वदिगिति नतिरेतत्संस्कृतः सोऽङ्गुलादिः
स्फुट इषुरमुतोऽत्र स्यात् स्थितिच्छन्नपूर्वम् ॥ ४ ॥

ग्रहणकालः

स्थिति'रस'हतिरंशा वित्रिभं तैः पृथक्स्थं
रहितसहितमाभ्यां लम्बने ये तु ताभ्याम् ।
स्थितिविरहितयुक्तः संस्कृतो मध्यदर्शः
क्रमश इति भवेतां स्पर्शमुक्त्योस्तु कालौ ॥ ५ ॥

मर्दादेवं मीलनोन्मीलने स्तो
ग्रासो नादेश्योऽङ्गुलाल्पो रवीन्द्वोः ।

वर्णः

धूम्रः कृष्णः पिङ्गलोऽल्पार्धसर्व-
ग्रस्तश्चन्द्रोऽर्कस्तु कृष्णः सदैव ॥ ६ ॥

इष्टग्रासः

इष्टं द्विघ्नं छन्नक्षुण्णं स्पर्शान्त्यान्तर्नाडीभक्तम् ।
रूपार्धेनोपेतं विद्यादिष्टे कालेऽर्कस्य ग्रासम् ॥ ७ ॥[1]

(Gaṇeśa, *GL*, 6.1-7)

—Grahalāghava

Parallax in Longitude and Latitude

Find the *lagna* at the end of new moon and subtract from it 90°. Keep it separate. Find the declination

[1] For elacidation and rationale see *GL:RCP*, II, pp. 1-15.

and correct it for latitude; that gives *nati* in degrees (say x). (If *krānti* and *akṣa* are in the same direction, take their sum; if of different signs take their difference). (1)

Find the square of $x/22$. If this is greater than 2, subtract 2 from it. Add half of it to the square. Add 12. This becomes the 'divisor'.

If $(x/22)^2$ is less than 2, add 12 to it. That is the 'divisor'.

Find the difference in degrees between the *lagna* of the Nonagesimal and the Sun. Find 1/10th of this (x). Subtract x from 14. Multiply the remainder by x, i.e. find $(14—x)\ x$. Divide this by the 'divisor'. The result in *nāḍikās* gives the *lambana*. Add the *lambana* to the end of *tithi* in *ghaṭīs*. If the *lagna* of the nonagesimal is less than that of the Sun, the *lambana* is to be subtracted from the time in *ghaṭīs* of the end of the *tithi*. (2)

Correction for the lambana for Sun minus Node

Multiply the *lambana* by 13. The result is in minutes etc. Carry out the correction of this product just as in the case of the *tithi*, (i.e. if the *lambana* is positive add; if negative subtract), to the position of the Sun *minus* node. Convert this corrected value into *śara*.

Multiply the *lambana* by 6, and add this to the *lagna* of the nonagesimal if the *lambana* is positive, (subtract otherwise). The declination (*krānti*) is to be calculated from this value. From the values of the *krānti* and *akṣa* (declination and latitude) the corrected value of *natāṁśa* can be had. (3)

Nati (parallax) and śara from natāṁśa

Divide *natāṁśa* by 10. Let it be x. Find $(18—x).\ x$. Subtract the same from 6° 18′, (y). Divide y by $(18—x).\ x$. The result in *aṅgulas* gives the parallax, its direction being the same as that of *natāṁśa*.

For the *śara* already found out, carry out the parallax correction; i.e. take the sum if the directions are the same and take the difference otherwise. Only from the *śara* thus corrected, is the portion of the body eclipsed and the duration of eclipse are to be calculated. (4)

Times of first and last contacts

Multiply the duration of the eclipse (already found) in *ghaṭīs* by 6. It is thus converted into degrees. Take the *lagna* of the nonagesimal at the end of the *tithi*. Let it be x. The *lagna* of the nonagesimal at the first and last contacts are to be had by respectively substracting and adding the duration from and to this value x.

The parallax is to be calculated separately from the two found above.

The time of first contact is found by correcting the middle of the eclipse minus the duration with the *lambana* of first contact, i.e. if *lambana* is positive, add them; if it is negative, take the difference.

For the last point of contact take the middle of the eclipse plus the duration. Carry out the correction for *lambana* as mentioned above. (5)

Immersion and Emergence

When *marda* is multiplied by six, degrees are obtained. Repeat the same process for determining the time of immersion or emergence. If the portion eclipsed is less than one *aṅgula* it is not necessary to find *grāsamāna* for both lunar and solar eclipses. (5b)

Colour of the eclipse

If Moon is eclipsed partially it is smoky in colour; if half is eclipsed it is black and a full eclipse is reddish brown. The solar eclipse is always balck in colour. (6b)

Portion eclipsed at any time

Double the desired time in *ghaṭīs*. Multiply this by the *grāsamāna*. (x) Find the difference between the times of last and first contacts (y). Find x/y. Add to this half-*aṅgula* (or 30 *vyaṅgulas*), to get the portion of the body eclipsed at any desired time. (7). (VSN)

सूर्यग्रहणलेखनम्

—रोमकसिद्धान्तः

16. 34. 1. रविशशिमानदलादवनतिहीनाद् भवन्ति या लिप्ताः ।
तान्यङ्गुलानि विद्याद् भानोश्छन्नानि चन्द्रमसा ।। १७ ।।
अर्धेनालिख्य रविं दत्वावनतिं यथादिशं मध्यात् ।
अवनत्यन्ताच्चन्द्रं विलिखेद् ग्रासार्थमर्धेन ।। १८ ।।
(Varāha, *PS*, 8. 17-18)

—Romaka Siddhānta

Subtract the parallax corrected-latitude for the time of parallax-corrected new moon, from the sum of the semi-diameters. The remainder in minutes are the digits of obscuration of the sun by the moon. (17)

To represent the amount of obscuration graphically, draw a circle of radius equal to the semi-diameter of the Sun, measure the parallax-corrected latitude, north or south according as where the Moon is situated, and with the point marking its end as centre, draw a circle of radius equal to the Moon's semi-diameter, to represent the Moon. (The part common to both the circles is the part obscured, and its measure in digits is its width in minutes of arc).[1] (18). (TSK)

[1] For elucidation and worked out examples, see *PS: TSK*, 8.17-18

—सौरसिद्धान्तः

अपमण्डलाद्यङ्कनम्

16. 35. 1. यष्टचा विद्धाङ्गुलया वृत्तं परिलिख्य संप्रसार्य दिशम् ।
अन्त्याद्यदलैक्येनाथ यदपरमर्धेन चाद्यस्य ॥ १ ॥

चन्द्राम्बरान्तरांशोत्क्रमज्यया ज्यां निहत्य वैषुवतीम् ।
'खार्क'शांशानुदयास्तमयोदग्याम्यतो दद्यात् ॥ २ ॥

सत्रिगृहस्य हिमांशोरपक्रमांशान् यथादिशं कुर्यात् ।
प्रागपरसिद्धिरेवं चक्राद् याम्योत्तरे ज्ञेये ॥ ३ ॥

स्पर्शमोक्षबिन्द्वङ्कनम्

दिग्व्यत्ययेन शशिनो विक्षेपान्ताद् दिगन्तकं सूत्रम् ।
स्पर्शो द्वितीयवृत्ते तस्मादन्यत् स्पृशेन्मध्यम् ॥ ४ ॥

तत्सम्पाते स्पर्शो मोक्षोऽप्येवं विपर्ययात् साध्यः ।
तात्कालिकात् स्वबुद्ध्या मोक्षो दिक् संविधातव्या ॥५॥

कलानामङ्गुलीकरणम्

लिप्ताद्वयेन हरिजे त्रयेण मेषूरणेऽङ्गुलं भवति ।
अनुपातोऽन्तरसंस्थे कर्तव्यो दृष्टियुक्तार्थम् ॥ ६ ॥

(VM, *PS*, 11.1-6)

—Saurasiddhānta

Diagram for ecliptic etc.

Using the stick-instrument with notch-marks of digits, draw the circle called the 'sum-circle', having for its radius the half-sum of the diameters converted into digits. Mark the east-west and north-south lines. Similarly, using the semi-diameter of the eclipsed body converted into digits as radius, draw the 'eclipsed body circle' concentric with the sum-circle. (1)

Find the versine of the hour-angle (of the Moon at mid-eclipse), and multiply this by the tabular sine of the latitude of the observer and divide by 120. Find the arc in degrees of the resulting sine. If the hour-angle is east, lay the degrees north of the east-point, if west, south of the east-point. The east-point with reference to the equator is thus obtained. (2)

Add three *rāśis* to the Moon's longitude and find the degrees of declination of this point. If the declination is north, lay the degrees north of E′, if south, south of E′. This is the east-point with respect to the ecliptic. Draw the straight line through the centre, E″OW″. E″-W″ is the ecliptic east-west. By means of circles, (*i.e.*, by drawing the perpendicular bisector) get the ecliptic north-south, *viz.*, N″-S″. (3)

Marking points of contact release etc.

In the case of the lunar eclipses, mark on the 'eclipsed body circle' the direction points in reverse of the points on the sum-circle. (4)

On the N S line, mark the Moon's latitude at first contact (converted into digits) according to its direction, and take it (westward) to the sum-circle. Join this point on the sum-circle and the centre with a straight line. (5)

Conversion of minutes into angulas

In order that the graphical representation may appear as the eclipse is seen actually, the minutes of arc are to be converted into digits, at 2′ per digit when the Moon is near the horizon, and at 3′ per digit when it is on the tenth sign, *i.e.* meridian, and proportionately in between.[1] (6). (TSK)

ग्रहणवर्णः

16. 36. 1. प्रग्रहणान्ते धूम्रः खण्डग्रहणे शशी भवति कृष्णः ।
सर्वग्रासे कपिलः सकृष्णताम्रस्तमोमध्ये ॥ ४६ ॥

(Āryabhaṭa I, *ABh.*, 4. 46)

Colour of the eclipse

At the beginning and end of its eclipse, the Moon (*i.e.*, the obscured part of the Moon) is smoky; when half obscured, it is black; when (just) totally obscured, (*i.e.* at immersion), it is tawny; when far inside the Shadow, it is copper-coloured with blackish tinge. (46). (KSS)

16. 36. 2. आद्यन्तयोः स धूम्रः कृष्णः खण्डग्रहेऽर्धतोऽभ्यधिके ।
ग्रासे स कृष्णताम्रः सर्वग्रहणे कपिलनर्णः ॥ १७ ॥

(Brahmagupta, *KK*, 2.4.17)

Both at the beginning and end of the eclipse, the Moon is dusky; it is dark, when the obscured portion is less than half and is of dark copper colour, when the obscured portion is greater then half; it is tawny, when it is completely obscured. (17). (BC)

16. 36. 3. आद्यन्तयोर्बहुलधूम्रलवानुकारी
खण्डग्रहे नियतमञ्जनपुञ्जवर्णः ।
ग्रासे दलात् समधिकेऽरुणकृष्णवर्णः
सर्वग्रहे भवति शीतकरः पिशङ्गः ॥ ३६ ॥

(Lalla, *ŚiDh Vṛ.*, 5.36)

At the beginning and end of an eclipse, the Moon is of dense smoky colour. In a partial eclipse, it is always dark as a mass of collyrium. When the obscured portion is greater than half, it is dark red. When it is completely obscured, it is tawny. (36). (BC)

16. 36. 4. स्वल्पे छन्ने धूम्रवर्णः सुधांशो-
रर्धे कृष्णः कृष्णरक्तोऽधिकेऽर्धात् ।
सर्वच्छन्ने वर्ण उक्तः पिशङ्गो
भानोश्छन्ने सर्वदा कृष्ण एव ॥ ३६ ॥

(Bhāskara II, *SiŚi.*, 1. 5.36)

[1] For the rationale and the diagram, see *PS: TSK*, 11. 1-6.

Colour

When less than half the disc of the Moon is eclipsed, the colour will be what is called *dhūmra*, i.e. of the colour of smoke; when the disc is half eclipsed, the colour is black; when more than half is eclipsed, the colour would be a blend of black and red, and, when the entire disc is eclipsed, the colour will be what is called *piśaṅga* or reddish-brown. (36). (AS)

अदर्शनतया अनादेश्यं ग्रहणम्

16. 37. 1. सूर्येन्दुपरिधियोगेऽर्काष्टमभागो भवत्यनादेश्यः ।
भानोर्भासुरभावात् स्वच्छतनुत्वाच्च शशिपरिधेः ।।
(Āryabhaṭa I, *ABh.*, 4.47)

Eclipses: Conditions when not to be predicted

When the discs of the Sun and the Moon come into contact, a solar eclipse should not be predicted when it amounts to one-eighth of the Sun's diameter (or less) (as it may not be visible to the naked eye) on account of the brilliancy of the Sun and the transparency of the Moon. (47). (KSS)

16. 37. 2. द्वादशभागादूनं ग्रहणं तैक्ष्ण्याद्रवेरनादेश्यम् ।
षोडशभागादिन्दोः स्वच्छत्वादधिकमादेश्यम् ।। १८ ।।
(Brahmagupta, *KK*, 2.4. 18)

If the obscured portion of the Sun is less than its twelfth part, the eclipse is ignored, because the obscured portion is so small that it cannot be seen owing to the brightness of the Sun. If the obscured portion of the Moon is greater than its sixteenth part, the eclipse is considered because though the portion is small, it is visible owing to the clearness of the Moon. (18). (BC)

ताम्रलेखादिषु ग्रहणनिर्देशः—चन्द्रग्रहणम्

—कालचूरिसंवत् ८८०

16. 38. 1. तेनाशीत्यधिकाष्टवत्सरे शके जाते दिने गीःपतेः
कार्तिक्यामथ रोहिणीभसमये रात्रेश्च यामत्रये ।
श्रीमद्-रत्ननरेश्वरस्य सदसि ज्योतिर्विदामग्रतः
सर्वग्रासमनुष्णगोः प्रवदता तीर्णा प्रतिज्ञानदी ।।
(Sarkho plates of Ratnadeva II, Kalachuri year 880: A.D. 1128, lines 23-24, *Corpus Ins. Indicarum*, IV, p. 427.)

Inscriptional references—Lunar eclipse

—Kalachuri 880 : A.D 1128

He (Padmanābha) declaring in the assembly of the illustrious Ratnadeva, in the presence of all astronomers, that when the year eight hundred increased by eighty had passed, on the day of the Lord of Speech (i.e. Thursday), on the full moon day of Kārttika, during the third quarter of the night when (the Moon would be in) the constellation Rohiṇī, there would be a complete eclipse of the Moon, crossed the river of assertion (i.e. vindicated his prediction).

—शकसंवत् १११९

16. 38. 2. मार्गशीर्षपौर्णमास्यां शनैश्चरवारे सोमग्रहणे . . . . ।
(Gadag stone inscription of Vīra-Ballāla II, Śaka 1119: A.D. 1197, lines 43-44, *Indian Antiquary*, 2 (1873) 298ff.)

—Śaka 1119 : A.D. *1197*

On the occasion of a lunar eclipse on Saturday, the day of the full moon in the month of Mārgaśīrṣa . . .

—शकसंवत् ११५१

16. 38. 3. मार्गशीर्ष-पौर्णमास्यां गुरुवारे चन्द्रोपरागे . . . . ।
(Nagari copper plate inscription of Anaṅgabhīma III, Śaka 1151 and 1152: A.D. 1230-31, line 135, *Epigraphica Indica*, XXVIII, pp. 235ff.)

—Śaka 1151 : A.D. 1230-31

On the occasion of a lunar eclipse on Thursday, the bright half of the month of Mārgaśīrṣa . . .

ताम्रलेखादिषु ग्रहणनिर्देशः—रविग्रहणम्

—कालचूरिसंवत् ३२२

16. 39. 1. एतेषां ब्राह्मणानां . . . . उत्सर्पणार्थं आषाढसंवत्सरे चैत्रामावास्यायां जाह्नवीमध्ये चट्कवटसंस्थितेन ग्रहोपरागे . . . ।
(Nagardhan plates of Svāmirāja, Kalachuri year 322: A.D. 580, line 14, *Ep. Ind.* XXVIII, pp. 1ff).

Inscriptional references: Solar eclipse

—Kalachuri year 322: A.D. 580

And, to these (same) Brāhmaṇas, while staying at the Caṭuka-vaṭa village on the Gaṅgā, on the occasion of the eclipse on the new moon day of Caitra in the year Āṣāḍha (donated, with a libation of water, according to the maximum of uncultivated land, the village Amkollikā).

—कालचूरिसंवत् ४०४

16. 39. 2. महाबलाधिकृत-श्रीवासवसमादेशात् लिखितमिदं देवदिन्नेनेति । सं ४०४ दे आषाढ व अमावास्या सूर्यग्रहोपरागे . . . . ।
(Kasare plates of Allasakti, Kalachuri year 404: A.D. 662, line 30, *Corpus Ins. Indicarum*, IV. i, p. 110).

—Kalachuri year 404 : A.D. 662

This charter is written by Devadinna by the order of the *mahābalādhikṛta* Vāsava in the year four hundred and four, on the new moon day in the dark (fortnight) of Āṣāḍha, on the occasion of a solar eclipse.

—शकसंवत् ११११३

16. 39. 3. शकनृपकालातीतसंवत्सरशतेषु त्रयोदशाधिकेष्वेकादशसु वर्तमान-विरोधिकृत्-संवत्सरान्तर्गत-ज्येष्ठमासामावास्यायाम् आदित्य-वारे सूर्यग्रहणे . . . . ।

(Gadag stone inscription of Bhillana V, Śaka 1113: A.D. 1192, *Ep. Ind.* III, pp. 219ff.)

—Saka 1113 : A.D. 1192

On the occasion of the solar eclipse on Sunday, the new moon *tithi* of the month of Jyeṣṭha of the year Virodhikṛt in Śaka 1113 . . .

—शकसंवत् ११५१

16. 39. 4. कर्कटकामावास्यायां सूर्योपरागे . . . . ।

(Nagari copper plate inscription of Anaṅgabhīka III, Śaka 1151 and 1152, A.D. 1230-31), line 142, *Ep. Ind.*, XXVIII, pp. 235ff).

—Saka 1151: A.D. 1230-31

On the occasion of a solar eclipse on the Karkaṭaka amāvāsyā (i.e. new moon day when the moon's in the zodiac Karkataka).

17. चन्द्रशृङ्गोन्नतिः – PHASES OF THE MOON

चन्द्रदर्शनम्

17. 1. 1. चन्द्रमा अमावास्यायां आदित्यमनुप्रविशति, आदित्याद् वै चन्द्रमा जायते ।

(*Taitt. Brāhmaṇa*, 40. 28)

Periodical appearance of the Moon

The Moon enters the Sun on the new moon day; and (later) the Moon comes out of the Sun.

17. 1. 2. तद्वा एष एवेन्द्रः य एष तपति, यथैष एव वृत्रो यच्चन्द्रमाः सोऽस्यैष भ्रातृव्यजन्मैव । तस्माद् यद्यपि पुरा विदूरमिवोदितोऽथैनमेतां रात्रिमुपैव न्याप्लवते, सोऽस्य व्यात्तमापद्यते । तं ग्रसित्वोदेति । स न पुरस्तान्न पश्चाद्ददृशे . . . . . तं निर्धीय निरस्यति । स एष धीतः पश्चाद्ददृशे । स पुनराप्यायते । स एतस्यैवान्नाद्याय पुनराप्यायते ।

(*Śatapatha Brāhmaṇa*, 1.6.4. 18-20)

Now, (the Sun) which burns there, is, indeed, none other than Indra, and the Moon is none other than Vṛtra. But the former is hostile to the latter, and for this reason, though (the Moon, Vṛtra) had previously risen at a great distance from him (the Sun, Indra), he now swims towards the latter and enters into his open mouth.

Having swallowed him, he (the Sun) rises; and the other one (Moon) is not to be seen either in the east or in the west.

Having sucked him empty, he (the Sun) throws him (the Moon) out; and the latter, thus sucked out, is seen in the western sky, and again increases; he again increases to serve that (Sun) as food.

चन्द्रशौक्ल्यम्

17. 2. 1. नित्यमधस्थस्येन्दोः भाभिर्भानोः सितं भवत्यर्धम् ।
स्वच्छाययान्यदसितं कुम्भस्येवातपस्थस्य ॥ ३५ ॥

सलिलमये शशिनि रवेर्दीधितयो मूर्च्छितास्तमो नैशम् ।
क्षपयन्ति दर्पणोदरनिहिता इव मन्दिरस्यान्तः ॥ ३६ ॥

प्रतिदिवसमेवमर्वाक्स्थानविशेषेण शौक्ल्यपरिवृद्धिः ।
भवति शशिनोऽपराह्ने पश्चाद्भागे घटस्येव ॥ ३७ ॥

असितात् सिताच्च पक्षात् असितं पक्षार्धमर्कमीक्षन्ते ।
राशित्रयादुभयतो नभो यतः शीतकरसंस्थाः ॥ ३८ ॥

(Varāha, *PS*, 13.35-38)

Moon's luminosity

The Sun always lights up one half of the Moon situated below it, (at any position round the earth), and the other half is dark by its own shadow, (i.e. the Moon obstructing the sunlight by its own body), just like a pot placed in sunlight. (35)

The Sun's rays, reflected in the watery Moon dispels the darkness on the Earth, as the rays of the Sun falling on a mirror in the interior of a house, does. (36)

According to the position of the Moon underneath the Sun every day, the lighted up part increases (from the time of new moon, as seen from the Earth), as the lighted portion increases on the pot, on the western side, in the afternoon. (37)

Anywhere on the Moon, its denizens (the *pitṛs*, in this case,) see the Sun for half the time during each fortnight, (on the whole not seeing the Sun for a fortnight's time, and seeing it for a fortnight's time) because the visible part of the sky extends only upto 90° from the zenith. (38). (TSK)

चन्द्रदर्शनसम्भवासम्भवः

17. 3. 1. चन्द्रोंऽशैर्द्वादशभिरविक्षिप्तोऽर्कान्तरस्थितो दृश्यः ।
नवभिर्भृगुर्भृगोस्तैर्द्व्यधिकैर्द्व्यधिकैर्यथाश्लक्ष्णाः ॥ ४ ॥

(Āryabhaṭa I, *ABh.*, 4. 4)

Conditions of Moon's visibility

When the Moon has no latitude it is visible when situated at a distance of 12 degrees (of time)[1] from the Sun. Venus is visible when 9 degrees (of time) distant from the Sun. The other planets, taken in the order of decreasing sizes, (*viz.*, Jupiter, Mercury, Saturn, and Mars), are visible when they are 9 degrees (of time) increased by two-s (*i.e.*, when they are 11, 13, 15 and 17 degrees of time) distant from the Sun. (4)

17. 3. 2. भूग्रहभानां गोलार्धानि स्वच्छायया विवर्णानि ।
अर्धानि यथासारं सूर्याभिमुखानि दीप्यन्ते ॥ ५ ॥

(Āryabhaṭa, *ABh.*, 4. 5)

Halves of the globes of the Earth, the planets and the stars are dark due to their own shadows; the other halves facing the Sun are bright in proportion to their sizes. (5). (KSS)

17. 3. 3. सार्कांशाविह कुरु पक्षतिक्षयेऽर्क-
व्यगवर्कौ चरमथ केवलाद् व्यगोर्यत् ।

[1] One degree of time is equivalent to 4 minutes.

'षड्बाणै'र्विहृतमिदं क्रमाल्लवाद्यं
स्वर्णं स्याद् व्यगुरविगोलयोः पृथक् तत् ।। १ ।।

त्रिभायनलवान्वितारुणचराहतं द्व्यक्षभा-
हतेः कृतिहृतं धनर्णमसमैकगोले व्यगोः ।
'खखानल'विशेषितः स'रस'भायनार्कोदयः
'शरद्विक'हृतो धनाधनमनल्पकाल्पोदये ।। २ ।।

द्युमितिप्रतिपद्गमान्तरं य-
'च्छर'भक्तं स्वमृणं दिनेऽधिकोने ।
धनमत्र चतुष्कसंस्कृतिश्चेत्
तपनास्ते विधुरीक्ष्यतेऽन्यथा न ।। ३ ।।[1]

(Gaṇeśa, *GL*, 9. 1-3)

At the end of the first day in the bright fortnight, add 12° to Sun and Sun *minus* Rāhu (node). Without adding *ayanāṁśa*, find the *cara*. Divide by 56. The result is positive if the Sun *minus* node is in the northern and negative if southern hemisphere. This is the first (*phala*) (say $x1$). Keep it separately. (1)

Take the Sun with *ayanāṁśa*. Add 3 *rāśis* to it. Find the *cara-phala* for this and multiply the *cara* by $x1$. Divide this by the square of twice the equinoctial shadow. The result gives the second *phala* ($x2$). This is positive if the Sun and Sun *minus* Rāhu are in different hemispheres and negative if in the same hemisphere.

Add 6 *rāśis* to the Sun with *ayanāṁśa*. Find the value of the rising time in that particular *rāśi*. Take the difference between this and 300. Divide by 25. If the *udaya-māna* is more, this *phala* is positive; if otherwise negative ($x3$). (2)

Find the difference between the measures of the day and the *tithi*. Divide it by 5. It is positive if the day-measure is greater than the *tithi*-measure; negative otherwise. This gives the fourth *phala* ($x4$)

If correction is done with the four *phalas* (results) mentioned above (i.e. taking their algebraic sum), and the final result is positive then Moon is visible on the first *tithi* of the bright fortnight after the setting of the Sun. Otherwise it is not visible. (3). (VNS)

17. 3. 4. अपमान्तरसंयुक्तात् तदूनगुणिताच्छशाङ्करविविवरात् ।
मूलेनापमविवरे छिन्ने विक्षेपसंगुणिते ।। १ ।।
फलमिन्द्वर्कविशेषाच्छोध्यं त्वयनानुकूलविक्षिप्ते ।
तद्व्यत्यासे देयं विपरीतं पूर्वसन्ध्यायाम् ।। २ ।।
दिनकृत्सप्तमभवनात् तेनोदयनाडिकाद्वयं यदि वा ।
वियति विमले तदेन्दोर्लोकस्यालोकमायाति ।। ३ ।।

(Varāha, *PS*, 5. 1-3)

[1] For elucidation and rationale, see *GL: RCP*, II, pp. 56-61.

Find the difference in longitude of the Sun and the Moon, as also the difference of their declinations, (the mean declination of the Moon being used for this purpose). Multiply the sum of these two differences by their difference and find the square root of the product. By this 'square root' divide the product of the Moon's latitude and the difference of declination already found. (1)

The 'result' is to be subtracted from the difference in longitude, if visibility in the west is in question, and the latitude and *ayana* (*i.e.*, course northward or southward) of the Moon are of the same direction, or added to the difference in longitude if of opposite directions. If visibility in the east is in question, reverse the subtraction and addition. (2)

In the case of the visibility pertaining to the west, if a segment equal to the corrected difference in longitude takes at least two *nāḍīs* to rise in the east as reckoned by using the ascensional difference (for the place) of the seventh *rāśi* from the Sun, then the Moon will be visible provided the sky is clear. In the case of visibility in the east, use the ascensional difference of the Sun's *rāśi* itselt.[1] (3). (TSK)

सामान्यक्रिया:

—शङ्क्वग्राद्य:

17. 4. 1. कृत्वाऽत्र पौर्णमासीविधानकथितं स्फुटत्रयं शशिनि ।
अस्य दिवसप्रमाणं बिम्बं च प्राग्वदानीयात् ।। २ ।।
रविचन्द्रान्तरकाले राशीनामुदयास्तमयपिण्डम् ।
प्रागपरयोर्यदि स्याद् दृश्यो नाडीद्वयं चन्द्रः ।। ३ ।।
ता विघटिका नवत्या हताः शशिदिनदलविघटिकालब्धाः ।
भागास्तेषां जीवा शङ्कुरयं चन्द्रबिम्बघ्नः ।। ४ ।।
षड्भिः शतैर्विभक्तं सितमानं बिम्बशुद्धमसितं स्यात् ।
विषुवद्भवगुणशङ्कुस्त्रिशतविभक्तस्तु शङ्क्वग्रम् ।। ५ ।।
अर्केन्दुबाहुजीवा निजविषयांशोनिता तयोरग्रे ।
चन्द्राग्रे शङ्क्वग्रं युतवियुतं स्यात् समान्यदिशोः ।। ६ ।।
षड्भागो विक्षेपस्यापि तदा तद्विशुद्धमिन्द्वग्रम् ।
तस्य च सूर्याग्रस्य च युतिवियुतिर्भिन्नसदृशदिशोः ।। ७ ।।
तद्बिम्बगुणं कृतहतशङ्कुविभक्तं तु भवति कोट्याख्यम् ।
सूर्याग्रस्याधिक्ये दिग् विपरीताऽत्र विज्ञेया ।। ८ ।।

(Deva, *KR*, 6.2-8)

Preliminaries for computation

Śaṅkvagra etc.

Having applied the three (visibility) corrections, stated in connection with the instructions pertaining

[1] For elucidation and worked examples, see *PS: TSK*, 5. 1-3.

to the full moon day, to the Moon's longitude (or sunrise or sunset), find the measure of its (i.e., Moon's) day as also the diameter of its disc in the manner described heretofore. (2)

If the sum of the times of rising or setting (at the local place of the Signs intervening between the Sun and the Moon (as corrected for the visibility corrections) amounts to two *nāḍīs*, the Moon shall be visible towards the east (before sunrise) or towards the west (after sunset). (3)

Moon's illuminated part and Śaṅkvagra

Multiply those *vighaṭikās* (i.e., the *vighaṭikās* intervening between the Sun and the Moon) by 90 and divide by the *vighaṭikās* of half the Moon's day. Thus are obtained the degrees (intervening between the Sun and the Moon). Their R sine is to be designated as *śaṅku*. (4)

Multiply that (*śaṅku*) by the diameter of the Moon and divide by 600: the result is the illuminated part of the Moon. That subtracted from the diameter of the Moon is the unilluminated part.

The *śaṅku* multiplied by the R sine of latitude and divided by 300 is the (Moon's) *śaṅkvagra*. (5)

Agrās of Sun and Moon and Koṭi of elevation triangle

Find the R sines of the *bāhus* of the longitudes of the Sun and the Moon, and diminish them by one fifths of themselves: (the results are the declinations of the Sun and the Moon, in terms of *vināḍīs*). From these (declinations) calculate the *agrās* of the Sun and the Moon. Take the sum or difference of the Moon's *agrā* and Moon's *śaṅkvagra*, according as they are of like or unlike directions: (the result is the so called Moon's *bhuja*). If one-sixth of the Moon's latitude (in terms of minutes of arc) is also applied to the Moon's declination (in terms of *vināḍīs*), then the value of Moon's *agrā* becomes accurate. Now take the sum or difference of that (Moon's *bhuja*) and the Sun's *agrā*, according as they are of unlike or like directions: (the result is the *koṭi* of the elevation triangle in terms of *vināḍīs*). (6-7)

The true koṭi

That (*koṭi*) multiplied by the Moon's diameter and divided by 4 times the *śaṅku* gives the so called (true) *koṭi*. When the Sun's *agrā* exceeds the Moon's *agrā*, the direction of the *koṭi* is to be reversed. (The R sine of the Moon's altitude similarly reduced is called the true *bāhu*).[1] (8). (KSS)

[1] For the rationale of the several processes involved see *KR:KSS*, pp. 76-79.

क्रान्तिज्यादय:

17. 4. 2. शुभ्रभानुभुजशिञ्जिनी हृता 'कृष्णवर्त्मकुमुदाकरप्रियै:' ।
भाजिता 'नयनपावकै'र्भवेत् क्रान्तिमौर्व्यशिशिरद्युतेरपि ।।

चन्द्रस्य दिनमानम्

शश्यपक्रमधनु: समान्यदिक्क्षेपयुक्तवियुतं स्फुटं भवेत् ।
शिञ्जिनी च तत इन्द्वपक्रमज्या स्फुटान्यदत उक्तकर्मणा ।।

चन्द्रस्य चरार्धम्

आयनेतरजदृक्फलोनयुग् भिन्नतुल्यशरचन्द्रगोलयो: ।
वक्ष्यमाणचरखण्डसाधितं स्याच्चरार्धमथवैन्दवं स्फुटम् ।। ३

चन्द्रशङ्कु:

अर्कवत् स्वदिनयातयेययो: शङ्कुरस्य धनुषोऽर्कपूर्ववत् ।
सिद्धदृग्गतिकलोनितस्य या ज्या स्फुट: स खलु शङ्कुरैन्दव:।।
(Lalla, *ŚiDhVṛ.*, 9. 1-4)

R sine declination of the Moon

The R sine of the longitude of the Moon multiplied by 13 and divided by 32 gives the R sine of the declination of the Moon's (position on the ecliptic) or its *madhyamakrāntijyā*. (1)

Measure of the Moon's day

The sum or difference of the Moon's *madhyamakrānti* and its latitude, according as they are in the same or opposite directions, gives the Moon's *sphuṭakrānti* or the declination of the Moon's centre. Find its R sine. The result is the correct R sine declination. Hence find (the half variation of the Moon's day) following the methods given before. (2)

Half variation of the Moon's day

Or, calculate the Moon's half variation of the day (from the *madhyamakrānti* or the declination of its position on the ecliptic) using the tabular differences to be given later on. Then add to it or subtract from it the Moon's second visibility correction, according as the Moon and its latitude are in the same or different spheres. The result is the correct half variation of the Moon's day. (3)

R sine altitude of the Moon

As in the case of the Sun, calculate the R sine altitude of the Moon for the time passed or to pass in the day of the Moon. Then find the arc corresponding to this as the R sine. Subtract from it the longitudinal parallax in minutes of the Moon as calculated before. Find the R sine of the remainder. The result is the correct R sine altitude of the Moon. (4). (BC)

दृक्कर्म

—१. आक्षदृक्कर्म

17. 5. 1. विक्षेपगुणाक्षज्या लम्बकभक्ता भवेद् ऋणमुदक्स्थे ।
उदये धनमस्तमये दक्षिणगे धनमृणं चन्द्रे ।। ३५ ।।
(Āryabhaṭa I, *ABh.*, 4. 35)

Visibility corrections

—1. Ākṣadṛkkarma

Multiply the R sine of the latitude of the local place by the Moon's latitude and divide (the resulting product) by the R sine of the colatitude: (the result is the *ākṣadṛkkarma* for the Moon). When the Moon is to the north (of the ecliptic), it should be substracted from the Moon's longitude in the case of the rising of the Moon and added to the Moon's longitude in the case of the setting of the Moon; when the Moon is to the south (of the ecliptic), it should be added to the Moon's longitude (in the case of the rising of the Moon) and subtracted from the Moon's longitude (in the case of the setting of the Moon). (35)

17. 5. 2. विक्षेपज्यां क्षपाभर्तुरक्षज्याक्षुण्णविग्रहाम् ।
लम्बकेन हरेल्लब्धं विशोध्यं तत्स्फुटेन्दुत: ।। १ ।।

उदये सौम्यविक्षेपे देयमस्तमये सदा ।
व्यस्तं तद्याम्यविक्षेपे कार्यं स्यादुदयास्तयो: ।। २ ।।

(Bhāskara I, *LBh.*, 6. 1-2)

Multiply the R sine of the Moon's latitude by the R sine of the (local) latitude and divide (the product) by the R sine of the colatitude. Whatever is thus obtained should be subtracted from the Moon's longitude in the case of the rising of the Moon (i.e., in the eastern hemisphere) and added to that in the case of the setting of the Moon (i.e., in the western hemisphere), provided that the Moon's latitude is north. When the Moon's latitude is south, the above correction is applied reversely in the cases of rising and setting (both). (1-2). (KSS)

17. 5. 3. विषुवच्छायागुणितो विक्षेपो रविहृत: शशिन्युदये ।
उत्तरतो विश्लेषो याम्ये योज्योऽस्तगे व्यस्तम् ।। २ ।।

(Deva, *KR*, 5.2)

Multiply the Moon's latitude by the equinoctial midday shadow and divide (the product obtained) by 12; (the result is the *ākṣa-dṛkkarma* for the Moon). In the case of moonrise, subtract it from or add it to the Moon's longitude according as the Moon's latitude is north or south. In the case of moonset, the law of addition and subtraction is just the reverse.[1] (2). (KSS)

आयनदृक्कर्म

17. 5. 4. विक्षेपापक्रमगुणमुत्क्रमणं विस्तरार्धकृतिभक्तम् ।
उदगृणधनमुदगयने दक्षिणगे धनमृणं याम्ये ।। ३६ ।।

(Āryabhaṭa I, *ABh.*, 4. 36)

ii. Āyanadṛkkarma

Multiply the R versed sine of the Moon's (tropical) longitude (as increased by three signs) by the Moon's latitude and also by the (R sine of the Sun's) greatest declination and divide (the resulting product) the by square of the radius. When the Moon's latitude is north, it should be subtracted from or added to the Moon's longitude, according as the Moon's *ayana* is north or south (*i.e.* according as the Moon is in the six signs beginning with the tropical sign Capricorn or in those beginning with the tropical sign Cancer); when the Moon's latitude is south, it should be added or subtracted, (respectively). (36). (KSS)

17. 5. 5. त्रिराश्यूनोत्क्रमक्षुण्णां तत्कालक्षिप्तिमाहताम् ।
क्रान्त्या परमया भूयो हरेद् व्यासदलस्य ताम् ।। ३ ।।

कृत्या लब्धकला: शोध्या विक्षेपायनयोर्दिशो: ।
तुल्ययोर्व्यत्यये क्षेप्यं शीतांशोस्तत्फलं सदा ।। ४ ।।

(Bhāskara I, *LBh.*, 6. 3-4)

Multiply the (Moon's) instantaneous latitude by the R versed sine (of the Moon's longitude) as diminished by three Signs and then by the R sine of the (Sun's) greatest declination and divide that (product) by the square of the radius. The resulting minutes of arc should be subtracted from the longitude of the Moon when the latitude and *ayana* (of the Moon)[1] are of like directions. In the contrary case, they should always be added to the longitude of the Moon. (3 4). (KSS)

17. 5. 6. एवं कर्मक्रमात् सिद्धो दृश्यतेऽन्तरित: शशी ।
भागैर्द्वादशभि: सूर्याद् व्यभ्रे नभसि निर्मले ।। ५ ।।

(Bhāskara I, *LBh.*, 6. 3-5)

When the Moon obtained by applying these (two visibility) corrections is found to be twelve degrees (of time) distant from the Sun, she shall be (just) visible in clear cloudless sky. (5). (KSS)

17. 5. 7. विक्षेपो राशित्रययुतशशिजीवाहतोऽ'द्रिगुणशैलै:' ।
लब्धो विक्षेपायनसमदिशि शोध्योऽन्यत्र विक्षेप: ।। ३ ।।

(Deva, *KR*, 5.3)

Multiply the Moon's latitude by the R sine of the Moon's longitude as increased by 3 signs and divide by 737: the result (which is known as the *āyana-dṛkkarma*) should be subtracted from the longitude of the Moon, if the Moon's latitude and *ayana* are of like directions. In the contrary case, it should be added.[2] (3). (KSS)

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[1] That is:

$$\text{Moon's } \bar{a}k\d{s}a\text{-}d\d{r}kkarma = \frac{\text{Moon's latitude} \times \text{equinoctial midday shadow}}{12}$$

[1] The Moon's *āyana* is north or south according as the Moon is in the half-orbit beginning with the tropical (*sāyana*) sign Capricorn or in that beginning with the tropical (*sāyana*) sign Cancer.

[2] For the rationale, see *KR: KSS*, p. 73.

—३. तृतीयं कर्म

17. 5. 8. 'मङ्गलवेदा'श्च कला प्राक्पश्चाद्भागयोः ऋणधनं स्यात् ।
एभिस्त्रिभिर्विधानैः दर्शनयोग्यो भवति चन्द्रः ।। ४ ।।
(Deva, *KR*, 5.4)

iii. The Third correction

48 minutes too should be subtracted from the Moon's longitude or added to that according as the Moon is in the eastern or western hemisphere. By applying these three corrections, the Moon becomes fit for observation (at sunset).[1] (4). (KSS)

चन्द्रोदयास्तमयकालः

17. 6. 1. सूर्यश्चक्रार्धयुतः तदा विलग्नं यदीन्दुरूनोऽस्मात् ।
पूर्वमुदेत्यधिकश्चेत् पश्चात् दिवसाधिपास्तमयात् ।। ५ ।।

राश्युदयैरानीयात् तत्कालं चन्द्रलग्नतः प्राग्वत् ।
रात्रौ तु नाडिकाः स्युः क्षेप्यास्त्याज्या दिवाऽभ्युदितैः ।।६।।
(Deva, *KR*, 5.5-6)

Rising and setting of the Moon

Increase the longitude of the Sun for sunset by 6 Signs: the result is the longitude of the horizon-ecliptic point in the east. If the longitude of the Moon (for sunset) is less than that, the Moon rises before sunset; if greater, after sunset. (5). (KSS)

The time of moonrise should be obtained from the Moon's longitude, the longitude of the rising point of the ecliptic, and the oblique ascensions of the Signs, as before. When moonrise occurs in the night (i.e., after sunset) the *nāḍīs* (intervening between the Moon and the rising point of the ecliptic) should be added to the measure of the day; when moonrise occurs in the day (i.e., before sunset) the *nāḍīs* (intervening between the Moon and the setting point of the ecliptic) should be subtracted from the measure of the day: (the result in either case is the time of moonrise reckoned since sunrise). (6). (KSS)

17. 6. 2. याम्योदग्विक्षेपाद् विषुवद्भाघ्नाद् रविभिरवाप्तांशः ।
उदये शशिनो वृद्धिः क्षयो विपर्यस्तमस्तमये ।। ८ ।।

एवं व्यर्कच्चिन्द्राद् यद्यूना राशयः षडधिका वा ।
तदुदयकालेन दिवा निशि च शशाङ्कोदयो वाच्यः ।। ९ ।।

कृत्वैवं क्षयवृद्धी व्यार्काच्चन्द्राद् विशोध्य चक्रार्धम् ।
शेषोदयकालसमे शशिदिवसान्ते शशी मध्ये ।। १० ।।
(Varāha, *PS*, 5.8-10)

Multiply the Moon's latitude in degrees by the equinoctial shadow and divide by twelve. Add the resulting degrees to the longitude of the Moon, or subtract from it, according as the Moon's latitude is south or north, if the times of daily moonrise is to be computed. If the times of daily moonset is to be found, reverse the addition and subtraction, (*i.e.* subtract and add respectively). (8)

Subtract the longitude of the Sun from that of the Moon corrected thus. Find the time for this segment of the ecliptic to rise, after sunrise. By so much time after sunrise, the Moon will rise. If this segment is less than six *rāśis*, then the moonrise will fall in the daytime, if greater, the Moon will rise at night. (9)

In the manner given in verse 8, correct the Moon for moonset, deduct the Sun from this corrected Moon, and deduct 6 *rāśis* from the remainder. Find the time by which the remaining segment will rise, after sunrise. This is the time from sunrise when the Moon will set. At the time exactly midway between moonrise and moonset, the Moon will reach the meridian, *i.e.*, will be at upper culmination).[1] (10). (TSK)

चन्द्रोदयास्तमयस्फुटकालः

17. 6. 3. भवनषट्कयुतेन विवस्वता
भवति चेत् सदृशो हिमदीधितिः ।
सममुदेति तथास्तमुपेयुषा
निशि महानमहानपि वासरे ।। ९ ।।

शशधरो गृहषट्कसमन्वितो
यदि समो रविणोदयमेष्यता ।
युगपदस्तमुपैति महान् दिवा
निशि लघुः समयोऽत्र च पूर्ववत् ।। १० ।।

अह्नो निशश्च घटिकाभिरितागताभिः
सङ्गुण्य भुक्तिमुडुपस्य 'खषड्'विभक्ताम् ।
कृत्वा क्रमाद्धनमृणं द्युनिशान्तजस्य
दृक्कर्म चापि समयोऽसकृदेव साध्यः ।। ११ ।।

ऊनो विलग्नादधिकोऽस्तलग्नाद्
दृश्यः शशी स्यात् कृतदृष्टिकर्मा ।
यथान्तरेऽर्कग्रहयोस्तथानयोः
प्रसाध्यते यः समयः समुन्नतः ।। १२ ।।
(Lalla, *ŚiDhVṛ.*, 8. 9-12)

Corrected time for the rising and setting of the Moon

If the true longitude of the Moon, (corrected by the two visibility corrections), is the same as the true longitude of the Sun at sunset, increased by 6 Signs, the Moon rises at the same time as the Sun sets; if greater, it rises after, and if less, it rises before sunset. (9)

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[1] This correction is probably meant to account for the difference between the horizontal parallaxes of the Moon and the Sun. 48′ is evidently the difference between the horizontal parallaxes of the Moon and the Sun. For, according to Āryabhaṭa I:
Moon's horizontal parallax = 52′ 30″
Sun's horizontal parallax = 3′ 56″

[1] For the working, see *PS: TSK*, 5. 8-10.

If the true longitude of the Moon (corrected by the two visibility corrections) and increased by 6 Signs is the same as the true longitude of the Sun while rising, the Moon sets at that time; if greater, it sets after and if less, before sunrise. The times (for these phenomena) should be calculated as before. (10)

Multiply the daily motion of the Moon by the number of hours elapsed or to elapse, as (calculated above), either in the days or in the night, and divide the produt by 60. The quotient should be added to or subtracted from the true longitude of the Moon, either at the end of the day or at the end of the night (as the case may be). Then again, the longitude of the Moon corrected by the two visibility corrections should be calculated and hence the time. The process must be repeated till the time is fixed. (11)

If the true longitude of the Moon, corrected by the two visibility corrections, is less than the *udayalagna* or the longitude of the rising point of the ecliptic and greater than the *astalagna* or the longitude of the setting point of the ecliptic, the Moon is visible.

The time passed since the Moon was on the eastern horizon and to pass when it will be on the western horizon, can be found from the longitudes of these points and of the Moon; and so also in the case of a planet.[1] (12). (BC)

चन्द्रच्छाया

17. 7. 1. खद्युखण्डसदृशो यदोन्नतः स्यात् तदा गगनमध्यगः शशी ।
तत्र चोक्तवदहर्दलप्रभा चन्द्रवच्च भनभःसदामपि ।। ५ ।।
(Lalla, *SiDhVṛ.*, 9.5)

Moon's shadow

When the time equal to half the day of the Moon has passed since its rising, then it is midday. Find the shadow of the gnomon caused by the Moon for that time as explained before (in the case of the Sun). In the same manner, the shadows caused by the stars and planets must be found. (5).[2] (BC)

17. 7. 2. ताभिः शशिनश्च्छाया 'षड्गुणिते'तिक्रमेण बोद्धव्याः ।
पञ्चप्रश्नविधानं सूर्यवदत्रापि सञ्चिन्त्यम् ।। ७ ।।
(Deva, *KR*, 5.7)

From those *nāḍīs* (intervening between the Moon and the rising or setting point of the ecliptic), one may obtain the gnomonic shadow due to moonlight by applying the rule: "Multiply the given *vināḍīs* by 6, etc." (*vide supra*, ch. iv. vss. 8-9). Methods for solving the five problems (*vide supra*, ch. iv. of vss. 16-17) should also be contemplated here (in the case of the Moon) too as in the case of the Sun. (7). (KSS)

[1] For elucidation and demonstration see, *ŚiDhVṛ:BC*, II. 159-61.

[2] For detailed elucidation see *ŚiDhVṛ:BC*, II. 162-66.

17. 7. 3. उदयेन्द्वन्तरप्राणैरस्तचन्द्रान्तरैरपि ।
स्वाहोरात्रादिभिश्चान्द्रैः शङ्कुदृग्ज्ये ततः प्रभा ।। २२ ।।
(Bhāskara I, *LBh.*, 6. 22)

From the *asus* (of the oblique ascension of the portion, the ecliptic) lying between the rising point of the ecliptic and the Moon (corrected for the visibility corrections) or from those (taken in setting at the local place by the portion of the ecliptic) lying between the setting point of the ecliptic and the Moon (corrected for the visibility corrections) (according as the Moon is above the eastern or western horizon), and from the Moon's day radius, etc., determine (the R sine of) the (Moon's) altitude and zenith distance and therefrom the shadow of the gnomon (due to the Moon). (22). (KSS)

ग्रहस्य दृश्यादृश्यत्वम्

17. 7. 4. निशीष्टलग्नादुदयास्तलग्ने न्यूनाधिके यस्य खगः स दृश्यः ।
दिनेऽपि चन्द्रे रविसन्निधानान्नास्तं गतश्चेत् सति दर्शने भा ।।

ग्रहस्य द्युगतम्

ज्ञातुं यदा भाभिमता ग्रहस्य तत्कालखेटोदयलग्नलग्ने ।
साध्ये तयोरन्तरनाडिका यास्ताः सावनाः स्युर्द्युगता ग्रहस्य।।

ता एव खेटद्युतिसाधनार्थं
क्षेत्रात्मकत्वात् सुधिया नियोज्याः ।
ऊनस्य भोग्योऽधिकभुक्तयुक्तो
मध्योदयाढ्योऽन्तरकाल एवम् ।। १२ ।।

स्पष्टा क्रान्तिः स्फुटशरयुतोनैकभिन्नाशभावे
तज्ज्या स्पष्टोऽपमगुण इतो द्युज्यकाद्यं ग्रहस्य ।
कृत्वा साध्या तदुदितघटीभिः प्रभा भानुभाव-
च्चन्द्रादीनां नलकसुषिरे दर्शनायापि भानाम् ।। १३ ।।

स्वभुक्तितिथ्यंशविवर्जितो ना
महांल्लघुः 'खाग्निकृतां'शहीनः ।

स्फुटक्रान्तिः

स्पष्टो भवेदस्फुटजातदृग्ज्या
सन्ताडितार्कैः स्फुटशङ्कुभक्ता ।। १४ ।।

प्रभा भवेन्ना तिथिभागतोऽल्पो
यावद्विधुस्तावदसावदृश्यः ।
एवं किल स्यादितरग्रहाणां
स्वल्पान्तरत्वान्न कृतं तदाद्यैः ।। १५ ।।
(Bhāskara II, *SiŚi.*, 1. 7.10-15)

Visibility of the planet

During the night, if, at a given moment, the computed *udayalagna* of the planet is less than the particular *lagna* of the moment, (i.e. if the longitude of the *udayalagna* is less than that of the current *lagna*), and also if the *astalagna* of the planet computed is greater than that of the then current *lagna* (i.e. if the longitude of the *astalagna* is greater than that of the current *lagna*,) the planet is visible (i.e. above the horizon).

In the case of the Moon, however, if it is not eclipsed by the rays of the Sun, it may be visible even shortly after sunrise or a little before sunset in contradistinction to a planet which could not be seen at all during day time (except perhaps Venus). When a planet is visible, its gnomonic shadow could be computed.[1] (10)

Time after rise

If it be required to find the gnomonic shadow of the planet, then the current *lagna* and the *udayalagna* of the planet *at the moment* are to be computed. The time in between the two *lagnas*, which will be in *sāvana* measure pertaining to the planet gives the time that has elapsed after the rise of the planet. (11)

The *sāvana* measure alone is to be employed while finding the gnomonic shadow (i.e. while the zenith distance of the planet is to be computed), because the arc of the diurnal circle of the planet indicates only *sāvana* measure. Suppose the *udayalagna* falls short of the current *lagna*; then the *bhōgyakāla*, i.e. the remaining rising time of the *rāśi* in which the planet is situated added to the elapsed time of the *rāśi* of the current *lagna* together with the sum of the rising times of the *rāśis* in between, gives the difference of the *udayalagna* of the planet and the current *lagna*. (12)

Declination

The *krānti* of the planet or the declination of the foot of the latitude of the planet added to the latitude rectified called *sphuṭaśara*, gives what is called *spaṣṭa-krānti* of the planet. Its R sine is called *spaṣṭa-krāntijyā*. From this, R sin δ, R cos δ etc. are to be computed (as mentioned in *Trīpraśnadhikāra* in the context of finding the zenith-distance of the Sun).

From the time that has elapsed after the rise of the planet called the *unnata*, the shadow is to be computed as in the case of the Sun's shadow. Having thus computed the shadow or what is the same, the zenith distance of the Moon or that of the stars, the instrument called Nalaka could be pointed to the spot where that celestial body is situated. (13)

The Mahāśaṅku of the Moon or planets (R cos Z) reduced by one fifteenth of the respective daily motion gives the visible *śaṅku* when the radius is taken to be 3438. If the radius is taken as 120, 1/430th of the daily motion is to be subtracted. If the Mahāśaṅku is less then 1/15th (or 1/430th) of the daily motion, then the Moon is not visible. This applies to the other planets as well, but as this is a neglible matter, the earlier teachers did not instruct on its determination.[1] (14-15). (AS)

[1] This does not mean that the gnomon casts a shadow of the planet; it only means that it zenith distance could be computed according to the methods described in *Trīpraśnādhikāra of SiŚi* etc.

चन्द्रशृङ्गोन्नतिगणनम्
—आर्यभटीयार्धरात्रिकपक्षः

17. 8. 1. 'गुणखाद्रि'-'शराग्निशराः'
'पक्षनखाः' शोधिताः शशिक्रान्त्याः ।
चरखण्डकानि तद्वत् क्रमतो-
त्क्रमात् पिण्डिताश्चान्द्रम् ।। १ ।।
अर्कशशिक्रान्तिकलान्तरैक्यजीवा समान्यगोलकयोः ।
चन्द्रच्छायाकर्णेन सङ्गुणा लम्बकविभक्ता ।। २ ।।
सदृशैर्युतान्यथोना विषुवच्छायाङ्गुलैर्भुजो याम्ये ।
यत्रेन्दुर्द्वादशका कोटिस्तद्वर्गयुतिपदं कर्णः ।। ३ ।।
अर्कचन्द्रयोरन्तरांशकास्तिथिविभाजिताः शुक्लम् ।
कर्णेन चन्द्रबिम्बे द्वादशके सूर्यवच्छाया ।। ४ ।।
(Brahmagupta, *KK*, 1.7.1-4)

Computation of Moon's cusps
—Āryabhaṭa's Midnight system

Subtract 703′, 535′ and 202′ (as many times as possible) from the *sphuṭakrānti* of the Moon, first in the order as the are stated and then in the reverse order. (These minutes are the tabular differences of the *krāntis* of the last points of Meṣa, Vṛṣa and Mithuna). Calculate from the remainder the *caradala* of the Moon using the *caradalas* of the *rāśis* at the observer's station and the above numbers. Adding the result to the *caradalas* passed over, the Moon's *caradala* is obtained. (1)

Multiply the *jyā* of the difference or sum of the *krāntis* of the Sun and the Moon, according as they are in the same or opposite directions, by the hypotenuse (of the right-angled triangle, whose other sides are the gnomon and its shadow caused by the Moon), and divide by the *jyā* of the colatitude of the place. Add to or subtract from the result the number of *aṅgulas* in the *palabhā*, in the same or different directions respectively. Thus is obtained the *bhuja*, which is the base, and is to the south of the Moon's place. The perpendicular is 12 *aṅgulas*. The hypotenuse is the square root of the sum of the squares of the base and the perpendicular. (2-3)

The differences in degrees between the longitudes of the Sun and Moon, divided by 15, gives in terms of *aṅgulas*, etc., the illuminated portion of the Moon along the hypotenuse calculated above.

The obscured part in the disc of the Moon, which is of 12 digits, may be found as in the case of the Sun.[2] (4). (BC)

[1] For comments and rationale, see *SiŚi:AS*, pp. 466-70.

[2] For a demonstration of the formulae and proofs, see *KK:BC*, I. 132-34.

—लल्ल:

17. 8. 2. शुक्लपक्षदिवसे द्वितीयके
भास्वदस्तसमये प्रसाधयेत् ।
तिग्मशीतकिरणौ परिस्फुटौ
श्रृङ्गमानमवगन्तुमैन्दवम् ॥ ६ ॥

तिग्मशीतमहसोरपक्रम-
ज्ये हते त्रिभगुणेन भाजिते ।
लम्बकेन भवतोऽग्रकागुणौ
चन्द्रशङ्कुतलमुक्तवद् धनुः ॥ ७ ॥

अग्रकानृतलयोः समाशयोः
संयुतिर्वियुतिरन्यदिक्स्थयोः ।
चन्द्रयोर्भवति बाहुरैन्दवो-
ऽग्रैव भास्करभुजस्ततोऽन्यदा ॥ ८ ॥

प्राक्कपालयुजि शीतदीधितौ
लग्नमिष्टसमयादथानयेत् ।
योजयेच्चरदलेन पश्चिमे
साधयेद्रविवदग्रकां ततः ॥ ९ ॥

तां भुजां दिनकरस्य कल्पयेत्
तद्युते शशिभुजेऽन्यदिग्गते ।
चन्द्रसूर्यभुजयोः समाशयो-
र्जायते यदवशेषमन्तरे ॥ १० ॥

स्यात् स्फुटस्तु स भुजोऽत्र योगजे
चन्द्रबाहुदिगथो वियोगजे ।
शीतदीधितिभुजेऽधिकेऽपि वा
व्यत्ययेन सवितुर्भुजेऽधिके ॥ ११ ॥

कोटिमाहुरथ शङ्कुमैन्दवं
तद्भुजाकृतियुतेः पदं श्रुतिः ।
अङ्गुलानि 'खनखै'र्विभाजिताः
सन्ति ते श्रवणकोटिबाहवः ॥ १२ ॥

व्यर्कशीतकरभागसञ्चयः
शुक्लमिन्दुदलसङ्गुणो भवेत् ।
'व्योमनन्द'विहृतः सितासितं
तद्वदेव भगणार्धवर्जितम् ॥ १३ ॥

रविशीतिकरान्तरांशजीवा
विपरीता शशिखण्डताडिता वा ।
विहृता त्रिभजीवया सितं स्या-
च्छशलक्ष्माङ्कवदङ्गुलानि तस्मिन् ॥ १४ ॥

(Lalla, *ŚiDhVṛ.*, 9. 6-14)

—Lalla

To know the cusps of the Moon, one must first find the true longitudes of the Sun and the Moon on the second day of the light half of the lunar month, at sunset. (6)

Multiply the R sine of the Sun's declination and that of the Moon separately by the radius. Divide each product by the R sine of the local colatitude. The results are respecitvely the R sines of their amplitudes. (Hence find the amplitudes). Then find the base of the altitude of the Moon as exlpained in the case of the Sun. (7)

In the case of the Moon, the sum or difference of the R sine of the amplitude and the base of the altitude, according as they are in the same or opposite directions, is called *bāhu* (*bhuja*). In the case of the Sun, the R sine of the amplitude itself is the *bāhu* (*bhuja*). (8)

(If at any other time besides sunrise and sunset, the cusps of the Moon are to be known), calculate the *lagna* for that time if the Moon is in the eastern hemisphere. But if the Moon is in the western hemisphere, increase the *lagna* by 6 Signs. (Each result should be taken as the longitude of the Sun.) Then calculate the R sines of the amplitudes as explained in the case of the Sun. (9)

The Sun's R sine amplitude is its *bāhu* (*bhuja*). If the *bāhu* (*bhuja*) of the Moon is in a direction different from the Sun, find their sum. If their directions are the same, then take their difference. The result in each case is called the 'corrected *bāhu*' (*bhuja*) or *sphuṭabāhu*.

In the first case, its direction is the same as that of the Moon's *bāhu* (*bhuja*). In the second case, its direction is the same as that of the Moon's *bāhu* (*bhuja*) if the latter is greater than the Sun's is *bāhu* (*bhuja*); if the Sun's is greater, the direction is opposite to that of the Moon's *bāhu* (*bhuja*).

(Subtract the square of the *bāhu* (*bhuja*) of the Sun and that of the Moon, respectively, from the squares of the R sines of their zenith distance. Find the square root of each difference. Find the sum or difference of these results according as the Sun and the Moon are in the same or different hemisphere. The result is called *prathama* or 'first'.

Again, if it is daytime, find the difference of the R sine altitudes of the Sun and Moon. But if it is night, then find their sum. The result is called *anya* of 'the other'. The square root of the sum of the squares of the *prathama* and the *anya* is called *koṭi* or perpendicular (and is parallel to the east-west line). (10-11)

Now, they say that the Moon's R sine altitude is the perpendicular. The square root of the sum of the squares of the base (*bāhu*) and perpendicular (*koṭi*) is the hypotenuse.

When the lengths of the hypotenuse, perpendicular and base are each divided by 200, the results are respectively the lengths in *aṅgulas*. (12)

The difference in the true longitudes of the Sun and Moon in degrees, multiplied by the radius of the Moon

and divided by 90, gives the illuminated portion (in the light half). When the above difference is diminished by 180° and the remainder is multiplied by the radius of the Moon and divided by 90, the result is the dark portion in the dark half. (13)

Or, multiply the R versed sine of the difference (in degrees) in the longitudes of the Sun and the Moon by the Moon's radius and divide by the radius. The result is the breadth of the illuminated portion of the Moon. This may be converted into *aṅgulas* as before.[1] (14). (BC)

—भास्करः १

17. 8. 3. अन्तरांशोत्क्रमां जीवां स्फुटेन्दुव्यासताडिताम् ।
षण्णगाष्टरसैर्हृत्वा सितमानं पदाधिके ।। ६ ।।

क्रमज्यामधिकोत्पन्नां त्रिज्यया योज्य तत् सितम् ।
आनयेदसितेऽप्येवमुत्क्रमक्रमतोऽसितम् ।। ७ ।।

(Bhāskara I, *LBh.*, 6. 6-7)

—Bhāskara I

(In the light half of the month) the R versed-sine of the difference (between the longitudes of the Moon and the Sun) multiplied by the true diameter of the Moon and divided by 6876 gives the measure of the illuminated part (of the Moon). When the difference exceeds a quadrant, one should add the radius to the R sine of the excess and from that (find) the measure of the illuminated part. In the dark half of the month, one should obtain, in the same way, the unilluminated part (of the Moon) with the help of the R versed sine (of the difference between the longitudes of the Moon and the Sun diminished by 6 Signs) and from the R sine (of the excess of that difference over a quadrant). (6-7) (KSS)

—भास्करः २

17. 8. 4. मासान्तपादे प्रथमेऽथवेन्दोः
शृङ्गोन्नतिर्यद्दिवसेऽवगम्या ।
तदोदयेऽस्ते निशि वा प्रसाध्यः
शङ्कुर्विधोः स्वोदितनाडिकाद्यैः ।। १ ।।

निशावशेषैरसुभिर्गतैर्वा
यथाक्रमं गोलविपर्ययेण ।
रवेरधः शङ्कुरथाक्षभाघ्नो
नरोऽर्क'हृच्छङ्कुतलं यथाशम् ।। २ ।।

सौम्यं त्वधोमुखनरस्य तलं प्रदिष्टं
स्वाग्रास्वशङ्कुतलयोः समभिन्नदिक्त्वे ।
योगोऽन्तरं भवति दोरिनचन्द्रदोष्णो-
स्तुल्याशयोर्विवरमन्यदिशोस्तु योगः ।। ३ ।।

स्पष्टो भुजो भवति चन्द्रभुजाश इन्दोः
शुद्धे तु रविभुजाद्विपरीतदिक्कः ।

योऽधो नरो दिनकृतः स विधोरुदग्र-
शङ्क्वन्वितो मम मता खलु सैव कोटिः ।। ४ ।।

दोःकोटिवर्गैक्यपदं श्रुतिः स्याद्
भुजो 'रसघ्नः' श्रवणेन भक्तः ।
प्रजायते दिग्वलनं हिमांशोः
शृङ्गोन्नतौ तत् स्फुटबाहुदिक्कम् ।। ५ ।।

चन्द्रस्य योजनमयश्रवणेन निघ्नो
व्यर्केन्दुदोर्गुण इनश्रवणेन भक्तः ।
तत्कार्मुकेण सहितः खलु शुक्लपक्षे
कृष्णेऽमुना विरहितः शशभृद्विधेयः ।। ६ ।।

(Bhāskara II, *SiSi*, 1.9. 1-6)

—Bhāskara II

Either in the last quarter of a lunation, or in the first quarter, on the day when the elevation of the cusps of the crescent Moon is to be determined, then either at the moment of moonrise or moonset or (for the matter of that, during any part of the night) the R sine of the altitude of the Moon is to be computed by noting the time from the moment of moonrise. (1)

The R sine of the altitude of the Sun is to be computed, assuming the rising Sun to be in the opposite hemisphere, south or north (i.e. if it be originally in the northern, assume it to be in the south) (and using the formula given in verse 54 of *Trīpraśnādhyāya*, given the time measured in *asus* that has elapsed after sunset). (2)

The *śaṅkutala* of the inverse altitude or the latitude below the horizon, is north (in contradistinction to what it is above the horizon). The sum or difference of the *agrā* and *śaṅkutala* according as they are of the same direction or of opposite directions, is the *bhuja*. The sum or difference of the *bhujas* of the Sun and Moon, according as they are of opposite or the same directions, is what is called the *spaṣṭa-bhuja* whose direction is to be construed as that of the Moon. If the *bhuja* of the Moon falls short of that of the Sun, then the direction of the *spaṣṭa-bhuja* is that opposite to that of the Moon. (3-4a)

I deem that the *koṭi* should be taken as the sum of the *śaṅkus* of the Sun and the Moon, the one being below the horizon and the other above, respectively. (4b)

The hypotenuse or *karṇa* is the square root of the sum of the squares of the *bhuja* and *koṭi*. The *bhuja* multiplied by 6 and divided by the *karṇa* gives what is known as the *dik-valana* of the Moon. The direction of the *valana* has the same direction as the *spaṣṭa-bhuja* defined before. (5)

The R sine of the elongation of the Moon is to be multiplied by the radius vector of the Moon measured

[1] For elucidation, see, *SiDhVr:BC*, II. 166-73.

in *yojanas*, and divided by the radius vector of the Sun, also measured in *yojanas*; the arc of the R sine so obtained is to be added to the longitude of the Moon in the bright half of the lunation and is to be subtracted from the same in the dark half.[1] (6). (AS)

—ग्रहलाधवम्

17. 8. 5. मासस्य प्रथमेऽन्तिमेऽथ वाङ्घ्रौ
विधुश्शृङ्गोन्नतिरीक्ष्यते यदह्नि ।
तपनास्तमयोदयेऽवगम्या-
स्तिथयः सावयवाः क्रमाद् गतैष्याः ।। १ ।।

'रवि'हततिथयोंऽशास्तद्वियुग्युक् क्रमेण
द्युमणिरपरपूर्वे मासपादे विधुः स्यात् ।
'नृप'गुणतिथिरूना स्वघ्नतिथ्यक्षभाघ्नी
'शरकु'हृदुदगाशा संस्कृतार्कापमांशैः ।। २ ।।

चन्द्रस्य च व्यस्तशरापमांशै-
र्द्विनिघ्नतिथ्या विहृताऽङ्गुलाद्यम् ।
संस्कारदिक्कं वलनं स्फुटं स्यात्
स्वेष्वंशहीनास्तिथयः सितं स्यात् ।। ३ ।।

उन्नतं वलनाशायामन्यस्यां स्यान्नतं विधोः ।
वलनस्याङ्गुलैः शृङ्गैः किमत्र परिलेखतः ।। ४ ।।[2]

(Gaṇeśa, *GL*, 12. 1-4)

—Grahalāghava

On the first or last quarter of the (lunar) month when the horns of the Moon are visible on any particular day during the bright and dark fortnights, the *tithis*, with their fractions, elapsed or yet to elapse, are to be calculated from sunset and sunrise, respectively. (1)

Multiply by 12 the number of *tithis* gone. To the product, add the true position of the Sun in the first quarter and subtract it in the fourth quarter. The result gives the position of the Moon.

Multiply the *tithi* by 16. From the product subtract the square of the *tithi*. Multiply it by the equinoctial shadow. Divide by 15. The result in degrees is in the northern direction. This value is to be corrected with the Sun's declination—the sum of the two if uni-directional, otherwise their difference. (Let it be x). (2)

For the Moon the process is otherwise—difference in the case of the same direction, sum otherwise. (Let it be x). Find $2x/15$. This gives the true deflection (*dik-valana*) in *aṅgulas*. From the *tithi* subtract 1/3 thereof (i.e. find $t—t/5$). That would give the phase in *aṅgulas*. (3)

The elevation of the cusp is in the direction as that of the deflection. In the oppositive direction it is depressed. The elevation of the cusp is to be had only by the *valana* in *aṅgulas*. Hence what is the need of drawing a graph in this connection? (4). (VSN)

चन्द्रशृङ्गोन्नतिलेखनम्

—पौलिशः

17. 9. 1. द्विगुणेऽक्षे 'तिथ्यंशः' शृङ्गमुदक्तुङ्गमुडुगणाधिपतेः ।
देयं च भुजादेतच्छौक्ल्यं कर्णाद् द्विषट्कांशम् ।। ४ ।।

अपमान्तरविक्षेपावेकान्यत्वे युतोनितौ कोटिः ।
कर्णो रवीन्दुविवरं तत्कृतिविवरात् पदं बाहुः ।। ५ ।।

सविता यतः शशाङ्कात् कोटचा परिकल्पितस्ततः कोटिः ।
देयांशकाङ्गुलसमा भुजकर्णौ चाङ्गुलैरेव ।। ६ ।।

शशिमध्यात् प्राक्कर्णः कोटिरतोऽतो भुजः शशाङ्कगतः ।
परिधावक्षोन्नामः शौक्ल्यं मध्याद्धनुस्तत्र ।। ७ ।।

(Varāha, *PS*, 5. 4-7)

Moon's cusps—Diagram

—Pauliśa

Multiply the latitude of the place in degrees by two and divide by fifteen. By the resulting number of *aṅgulas* or digits (measured along the rim), the northern tip of the horn of the Moon should be raised upwards (as caused by the latitude at the time of first visibility). This raising should be directed upwards like the '*Bhuja*' which we are going to mention. The number of digits of illumination of the Moon's orbit, (usually called merely 'digits'), is the twelfth part of the difference in longitude in degrees, last found, and should be directed like the 'hypotenuse', which we are going to mention. (4)

The difference in declination last found should be added to the Moon's latitude or subtracted from it, as the directions of the Moon's *ayana* and its latitude by the same or different. (This refers to the visibility in the west in the evening. With reference to the visibility in the east in the morning, the addition and subtraction is done *vice versa*). The result is called '*koṭi*'. The difference in longitude is called the 'hypotenuse'. The '*Bhuja*' is the square root of the difference of the squares of the hypotenuse and the '*Koṭi*'. (5)

The '*Koṭi*' to be drawn on that side of the Moon towards the sun, north or south, which is got in computing it, using the scale, one *aṅgula*—one degree of *koṭi*. The *Bhuja* and the hypotenuse also should be drawn to the same scale. (6)

Thus, first there is the hypotenuse from the centre of the Moon to that of the Sun. From the centre of the Sun the '*Koṭi*' is laid in the direction computed for it. Then from its termination the '*Bhuja*' is laid towards the

[1] For detailed exposition and rationale, see *SiŚi:AS*, pp. 482-95.

[2] For the rationales, see *GL:RCP*, II. 108-12.

Moon's centre. On the rim of the Moon represented by a circle of fifteen *aṅgulas*, the raising of the horn in *aṅgulas* due to the latitude of the place is to be done. At the centre of the two ends of the horns the illumination in digits is to be represented on the diameter. There the arc (forming the upper boundary of the illumination) is to be drawn (by making the arc pass through the two ends of the horn and the point in the middle to which the illumination extends.[1] (7). (TSK)

—लल्लः

17. 9. 2. यच्चिह्नं समभुवि भानुमान् स तस्माद्
दातव्यः स्वदिशि भुजस्ततोऽपि कोटिः ।
प्रागिन्दावपरककुङमुखी प्रतीच्यां
प्रागग्राद् दिनकरचिह्नतश्च कर्णः ॥ १५ ॥

श्रवणकोटियुतैः शशिमण्डलं
श्रवणसूत्रमिहापरपूर्वकम् ।
झषवशेन च शेषदिशौ ततः
खटिकया सुपरिस्फुटमालिखेत् ॥ १६ ॥

अपरतः श्रवणेन सितं नये-
दसितमप्यसिते सितदीधितौ ।
धनददिग्भवदक्षिणदिग्भवैः
परिधिभिर्जनयेच्च झषद्वयम् ॥ १७ ॥

तिमियुगमुखपुच्छसक्तरज्ज्वो-
र्भवति च यत्र समागमः प्रदेशे ।
तत उडुपतिलग्नशुक्लचिह्नं
समभिलिखेत् सितसिद्धये सुवृत्तम् ॥ १८ ॥

बाह्वङगुलानि यत एव निवेशितानि
शृङ्गं तु तन्नमति शेषमिहोन्नतं स्यात् ।
शुक्लेऽर्धबिम्बसदृशे दलितेऽर्धमौर्व्या
लाटीललाटतटरूपधरः शशाङ्कः ॥ १९ ॥

प्रथमतरमुदेति शृङ्गमुच्चं
हरिणभृतोऽस्तमुपैति पृष्ठतस्तत् ॥
अनुपचिततनोस्तु केतकाग्र-
श्रियमविसंश्रयसम्भवां दधानम् ॥ २० ॥

(Lalla, *ŚiDhVṛ.*, 9. 15-20)

—Lalla

Moon diagram

Mark a point on a level surface and assume it to be the Sun. From this point draw a straight line equal to the correct base (*bhuja*) and in its own direction. At the end of this line draw another straight line at right angles to it and equal to the perpendicular. If the Moon is in the eastern hemisphere, the perpendicular should be drawn to the west. If the Moon is in the western hemisphere, the perpendicular should be drawn to the east. Then draw the hypotenuse from this point to the Sun. (15)

The Moon's centre is at the point of intersection of the perpendicular and the hypotenuse. The direction of the hypotenuse is the east-west line (in the Moon's disc). Draw the north-south line at right angles to it (and passing through the centre of the Moon's disc) by means of fish-figures. All this should be drawn with chalk and very clearly. (16)

If the breadth of the illuminated portion as calculated above is for the bright half of the lunar month, it should be marked along the hypotenuse from the west point. From the same point should be marked the breadth of the dark portion, if it is for the dark half of the lunar month. Then draw two fish-figures using this point and the north and south points. (17)

With the point of intersection of the two lines joining the mouths and tails of the two fish-figures as centre, draw a circle to pass through the point marked above. This is the circle which shows the illuminated portion in the Moon's disc. (18)

The lower cusp of the Moon is on that side on which lies the base (*bhuja*) measured in *aṅgulas*, and the other is the higher cusp.

When the illuminated portion is equal to half of the Moon's disc which is bisected by the diameter, the Moon has the beauty of the forehead of a woman belonging to the *Lāṭa country*. (19)

The Moon's higher cusp rises first and sets last. The cusp of the Moon, when its crescent is very narrow, has the rich beauty of the middle portion (centre) of *ketakī* flowers. (20). (BC)

—भास्करः १

17. 9. 3. कोटिसूत्रं तदग्रोत्थमत्स्यपुच्छास्यनिःसृतम् ॥ १२ ॥
चन्द्रशङ्कुमिता कोटिः पूर्वतो नीयते स्फुटम् ।
तद्भुजामस्तकासक्तं कर्णसूत्रं विनिर्गतम् ॥ १३ ॥
कर्णकोटयग्रसम्पातकेन्द्रेणालिख्यते शशी ।
कर्णानुसारतस्तस्य सितमन्तः प्रवेश्यते ॥ १४ ॥
कर्णः पूर्वापरे काष्ठे तन्मत्स्याद्दक्षिणोत्तरे ।
दक्षिणोत्तरयोर्बिन्दू तृतीयः सितमानजः ॥ १५ ॥
त्रिशर्कराविधानोत्थमत्स्यद्वयविनिःसृतम् ।
बिन्दुत्रयशिरोग्राहिवर्त्मवृत्तं समालिखेत् ॥ १६ ॥
वृत्तान्तरसितोद्भासिशृङ्गोन्नत्या प्रदृश्यते ।
ज्योत्स्नाप्रसरनिर्धूतध्वान्तराशिर्निशाकरः ॥ १७ ॥

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[1] For elucidation, see *PS:TSK*, 5. 4-7.

प्राक्कपाले शशाङ्कस्य लग्नेन्द्वग्रादिभिः स्फुटः ।
साध्यो बाहुरनादिष्टमपराभिमुखं स्मृतम् ।। १८ ।।
(Bhāskara I, *LBh.*, 6. 12-18)

—Bhāskara I

Lay that (base) off from the Sun in its own direction. (Then) draw a perpendicular line passing through the head and tail of the fish-figure constructed at the end (of the base). (This) perpendicular should be taken equal to the R sine of the Moon's altitude and should be laid off towards the east. The hypotenuse-line should (then) be drawn by joining the ends of that (perpendicular) and the base. (12b-13)

The Moon is (then) constructed with the meeting point of the hypotenuse and the perpendicular as centre; and along the hypotenuse (from the point where it intersects the Moon's circle) is laid off the measure of illumination towards the interior of the Moon. (14)

The hypotenuse (indicates) the east and west directions: the north and south directions should be determined by means of a fish-figure. (Thus are obtained the three points, viz.) the north point, the south point, and a third point obtained by laying off the measure of illumination. (15)

(Then) with the help of two fish-figures constructed by the method known as *triśarkarāvidhāna* draw the circle passing through the (above) three points. Thus is shown, by the elevation of the lunar horns which are illumined by the light between two circles, the Moon which destroys the mound of darkness by her bundle of light. (16-17)

(When the Moon is) in the eastern half of the celestial sphere, the true base should be found out with the help of the rising point of the ecliptic and the Moon's *agrā*, etc.; and the unmentioned element (i.e., the upright) should be laid off towards the west. (18). (KSS)

—करणरत्नम्

17. 9. 4. पूर्वापरेन्दोरपरत्र दिक्स्थां
कोटिं यथादिक् परिधौ निधाय ।
तन्मत्स्यसूत्रोपरि बाहुमानं
दत्वार्ऽधबिम्बेन लिखेच्छशाङ्कम् ।। ९ ।।
फलमिह कथितं यत् तत्तु विक्षेपजातं
विविधमतिविधेयं चक्रिणं चाम्बुदिग्धम् ।। १०a-b ।।
(Deva, *KR*, 6.9-10a)

—Karaṇaratna

Of the Moon lying in the eastern or western hemisphere, the true *koṭi*, which has already been (calculated and) set down at another place, should be laid off in its own direction. Then, along the head and tail line of the fish-figure drawn at the extremity of that *koṭi*, should be laid off the (true) *bāhu*. Thereafter, taking (the extremity of the true *bāhu* as centre and) the Moon's semi-diameter as radius, one should draw the Moon, which is circular, smeared with water, deflected from the ecliptic, and pertaining to which a variety of results have been set out here. (9-10ab). (KSS)

—आर्यभटः २

17. 9. 5. समभुवि बिन्दुं दत्त्वा तस्माद् वृत्तं धनाङ्गुलैः कार्यम् ।
दिक्सिद्धि तद्वृत्ते वलनं प्राच्यां यथाशमर्केन्द्वोः ।। १ ।।

दद्याद् वरुणाशायां व्यस्ताशं सर्वदा वलनम् ।
स्पर्शविमोक्षाविन्दोः प्राक् पश्चादन्यथा भानोः ।। २ ।।

मानैक्यार्धेन लिखेद् वृत्तं च ग्राह्यखण्डसूत्रेण ।
वलनाग्रबिन्दुसूत्रस्य युतिर्मानैक्यखण्डवृत्तेन ।। ३ ।।

या तस्यास्तद्वृत्ते लेख्यौ व्यस्ताशकौ शरौ शशिनः ।
भानोर्यथागताशौ बाणाग्राद् बिन्दुगं सूत्रम् ।। ४ ।।

धार्यं तद्ग्राह्यार्धजवृत्तयुतौ स्पर्शमोक्षकौ स्याताम् ।
वलनाग्राभ्यां मत्स्यं विलिख्य तत्पुच्छमुखसूत्रे ।। ५ ।।

दद्याद् बिन्दोर्मध्यं व्यस्ताशेषु विधौ रवौ स्वाशम् ।
तद्बाणाग्राद् विलिखेद् ग्राहकखण्डेन वृत्तं च ।। ६ ।।

तद्ग्राह्यवृत्तयुतिवच्छन्नं स्यात्परममर्केन्द्वोः ।
बिन्दोर्बाहुं दद्याद् वलनसूत्रेऽथ तस्याग्रात् ।। ७ ।।

दद्यात् कोटिशलाकां यथाशकां सौम्ययाम्यायाम् ।
श्रवणशलाकां बिन्दोर्दद्यात् कोटयग्रगां तयोर्योगात् ।। ८ ।।

लेख्यं ग्राहकमण्डलदलेन वृत्तं भवेदसौ ग्रासः ।
इष्टोऽथ निमीलनकं ह्युन्मीलनकं च मर्दभवैः ।। ९ ।।

शृङ्गोन्नतौ हिमांशोर्मण्डलखण्डेन मण्डलं कुर्यात् ।
सितपक्षे प्राग्वलनं दद्याद् असिते दिगङ्कितं पश्चात् ।।१०।।

बिन्दोर्वलनसूत्रे कोटिं दद्यात् तदग्रतो वृत्तम् ।
कर्णजसूत्रे विलिखेदिन्दोः स्याच्छृङ्गयोः संस्था ।। ११ ।।

शशिशुक्लत्वेऽर्धोने साध्या शृङ्गोन्नतिर्गणकैः ।
बिम्बादौ परिलेखेऽङ्गुलानि लिप्तासमान्यत्र ।। १२ ।।
(ABh. II, *Mahā.*, 8. 1-12)

—Āryabhaṭa II

Upon an even surface, having fixed a point (*bindu*), describe from it a circle with (a radius of) 40 digits (*aṅgulas*). Fix the cardinal points thereon. In that circle, in the east, the deflection (*valana*) is to be laid off in the same direction as the Sun and Moon; and in the west always in the opposite direction. In a lunar (eclipse) contact (*sparśa*) and separation (*vimokṣa*) take place in the east and west (respectively); in a solar (eclipse) the contrary (is the case). (1-2)

Draw (with the same centre, a second) circle with (a radius equal to) half the sum of the diameters (of the eclipsing and eclipsed bodies) (*mānaikyārdha*), and (a third) with a line (equal to) half (the diameter of) the eclipsed body (*grāhyakhaṇḍa*) (as the radius). Where the line (joining) the extremities of the deflection and the centre (*valanāgrabindusūtra*) meets the circle (with the radius of) half the sum of the diameters (of the eclipsing and eclipsed bodies) (*mānaikyakhaṇḍavṛtta*), there draw the latitudes of the Moon onto that circle, with opposite directions (as calculated); and (those) of the Sun in the direction as obtained. From (either) extremity of the latitude a line is to be drawn to the centre.

(The two points, at which) these (lines) cut the circle (with the radius equal to) half (the diameter of) the eclipsed body (*grāhyārdhajavṛtta*) are (the points of) contact and separation. (3-5ab)

From the extremities of the deflection, draw a fish figure. On the line joining the mouth and tail of it, lay off from the centre, the latitude for the middle (of the eclipse), in the opposite direction for the Moon, and in the same direction for the Sun. From the extremity of this latitude draw a circle with (a radius equal to) half (the diameter of) the eclipsing body (*grāhakakhaṇḍa*). (5cd-6)

(The portion of) the circle of the eclipsed body (*grāhyavṛtta*), cut by this (circle), is the maximum obscuration of the Sun or Moon.

From the centre lay off the base (*bāhu*) on the line (joining the extremity) of the deflection (and the centre). From its end lay off a stick (equal in length to) the perpendicular (*koṭiśalākā*) in the proper direction, either north or south. Place a stick (equal in length to) the hypotenuse (*śravaṇaśalākā*) from the centre touching the end of the perpendicular. From (the point) where they touch draw a circle (with a radius equal to) half the diameter of the eclipsing body. (The portion it cuts from the circle of the eclipsed body) is the (amount of) obscuration at a given time. (Similarly), the points of immersion (*nimīlanaka*) and emergence (*unmīlanaka*) (can be projected with the deflection and the latitudes) of a total obscuration. (7-9)

(For the projection of) the elevation of the lunar cusps (*śṛṅgonnati*) draw a circle (with a radius equal to) half the diameter of the Moon. In the bright half of the lunar month, the deflection is to be laid off in the east; and in the dark (half), the direction be marked in the west. (10)

On the line (joining the centre and the extremity) of the deflection lay off the perpendicular from the centre. From its end draw a circle with a line (equal in length to) the hypotenuse, (as radius. The points, at which it cuts the main circle, indicate) the position of the lunar cusps. (11)

The elevation of the lunar cusps is to be drawn by astronomers only when the illuminated part is less than half (the Moon's diameter). In drawing the diameter etc. digits (*aṅgula*) are (to be considered) equal to minutes.[1] (12). (SRS)

—भास्करः २

17. 9. 6. व्यर्केन्दुकोटचंश'शरेन्दु'भागो
हारोऽमुना 'षट्कृति'तो यदाप्तम् ।
द्विष्ठं च हारोनयुतं तदर्धे
स्यातां क्रमादत्र विभास्वभाख्ये ।। ७ ।।

सूत्रेण बिम्बमुडुपस्य षडङ्गुलेन
कृत्वा दिगङ्कमिह तद्वलनं ज्यकावत् ।
मासस्य तुर्यचरणे वरुणेशदेशात्
प्राग्भागतः प्रथमके सुधिया प्रदेयम् ।। ८ ।।

केन्द्राद्विभां तद्वलनाग्रसूत्रे
कृत्वा विभाग्रे स्वभया च वृत्तम् ।
ज्ञेयेन्दुखण्डाकृतिरेवमत्र
स्यात् तुङ्गशृङ्गं वलनान्यदिक्स्थम् ।। ९ ।।

(Bhāskara II, *SiŚi.*, 1. 9. 7-9)

—Bhāskara II

Let the compliment of the elongation (corrected as directed in verse 6) divided by 15 be the denominator; let the numerator be 36; take the result after division and put it in two places; with this and the complement of the elongation divided by 15, using *saṅkramagaṇita*, we have successively *Vibhā* and *Svabhā*. (7)

Draw a circle with radius 6 *aṅgulas* to represent the disc of the Moon. Mark the disc with the cardinal points signifying the east, west, north and south. Compute the *valana* as directed in verse (5) (which represents E′ G′ in fig. 119) and mark it off as a R sine from the west point W in the last quarter and from the east point E in the first quarter. (8)

From the centre M of the Moon's disc, mark off the *Vibhā* MC (computed as directed) along the join of MG′. With centre C and radius *Svabhā* (already computed as directed) draw a circle. The spherical sector of the Moon's globe thus demarcated by the arc of the circle ADB, namely AGBD, is found to have the elevated cusp in the opposite direction in which the *valana* has been marked.[2] (9). (AS)

[1] For diagrams and rationale, see *Mahā:SRS*, II. 138-43.

[2] For details, see, *SiŚi:AS*, pp. 494-98. See also the fig. therein, no. 119.

18. ग्रहास्तोदयः – HELIACAL RISING AND SETTING OF PLANETS

ग्रहास्तोदयः

18. 1. 1. चरनाडीक्रमविधिना द्युव्यासाद्यथामति च विक्षेपात् ।
अस्तमयोऽप्यध्वविधिः शेषाणां युक्तितश्चिन्त्यम् ।।५१।।

(Varāha, *PS*, 4.51)

Heliacal rising and setting

For the luminaries other than the Sun and the Moon, (*i.e.* for the star-planets), while carrying out the several operations, and using their respective latitude and day-diameter, and getting the *caranāḍīs* etc. in terms of their respective *sāvana* days, not only the work of finding the shadow for the given time and time for the given shadow as above, but also their daily risings and settings and reduction to different localities, should be thought out and done. (51). (TSK)

ग्रहाणां उदयास्तदिक्

18. 1. 2. लघुगोऽल्प इनादुदेति पूर्वे
भूयान् भूरिगतिर्ग्रहः प्रतीच्याम् ।
भूयांल्लघुगः परत्र चास्तं
प्राच्यां भूरिजवो लघुः प्रयाति ।। ११ ।।

(Gaṇeśa, *GL*, 9. 11)

Direction of rising and setting

The planet which moves slower than the Sun, rises heliacally in the east if its *rāśimāna* is also lesser than that of the Sun. The planet having a quicker motion than the Sun and a greater *rāśimāna* riser in the west.

The planet with slower motion and greater *rāśimāna* sets in the west; that with quicker motion and lesser degree sets in the east.[1] (11). (VSN)

ग्रहाणामस्तोदयदिनानि

18. 2. 1. 'नखेन्दवो'-'ष्टिः' 'खगुणा' 'द्विजिह्वाः'
'षट्पुष्करा' व्यस्तदिनानि पश्चात् ।
प्राच्यां च चन्द्रात्मजदैत्यगुर्वो-
'र्दन्ताः' 'शरव्योम्निचराः' प्रदिष्टाः ।। १४ ।।
'खाङ्गर्तुभिः' 'कृतगुणैः' 'नवाङ्गरामैः' 'महीविशिखपक्षैः'।
'द्विकृतगुणैः'-भौमाद्याः पश्चात् पुनरस्तमायान्ति ।। १५ ।।

(Vaṭeśvara, *Vsi.*, 2-4. 14-15)

Days of setting and rising

120, 16, 30, 8 and 36 are the days during which the planets, Mars etc., remain in heliacal setting in the west. Mercury and Venus are said to remain in heliacal setting in the east for 32 and 75 days, respectively. (14)

After 660, 34, 369, 251, and 342 days (since rising in the east in the case of Mars, Jupiter and Saturn and in the west in the case of Mercury and Venus) the planets, Mars etc., again set in the west. (15). (KSS)

18. 2. 2. उदयास्तविलोमयानसीम्ना-
मधिकोनाः कलिका हृता दिनानि ।
मृदुसंस्कृतया ग्रहस्य गत्या
रहिताशूच्चजभुक्तिलिप्तकाभिः ।। २६ ।।

(Lalla, *ŚiDhVṛ.*, 3. 26)

The difference in minutes between the (last) *śīghra-kendra* of a planet (obtained while calculating its true longitude), and that for its heliacal rising, setting or retrograde motion, when divided by the difference in minutes between the motion of the *śīghrocca* and the planet's motion after correction or *mandasphuṭagati*, gives the days passed or to be passed in respect of any of the above phenomena. (26). (BC)

ग्रहास्तोदयकालभागाः

—भास्करः १

18. 3. 1. कृतदर्शनसंस्कारो भार्गवोऽर्कान्तरस्थितैः ।
अंशकैर्नवभिस्तेभ्यो द्व्यधिकैर्द्व्यधिकैः क्रमात् ।। १ ।।
दृश्यन्ते सूरिवित्सौरिमाहेया निर्मलेऽम्बरे ।
कालभागा दिगभ्यस्ता विज्ञेयास्ता विनाडिकाः ।। २ ।।
राशेस्तस्यैव पूर्वस्यां सप्तमस्यापरोदये ।
स्वदेशभोदयैः कालं ज्ञात्वा दर्शनमादिशेत् ।। ३ ।।

(Bhāskara I, *LBh.*, 7. 1-3)

Planetary elongation for setting and rising
—Bhāskara I

If Venus corrected for the visibility corrections is 9 degrees (of time) distant from the Sun, it is visible. Jupiter, Mercury, Saturn, and Mars are visible in the clear sky when their distance (from the Sun) are nine degrees increased successively by twos (i.e., when they are respectively at the distances of 11, 13, 15 and 17 degrees of time from the Sun). The degrees of time multiplied by 10 are known as *vināḍikās*. (1-2)

(When the planet is to be seen) in the east, (its) visibility should be announced by calculating the time

[1] For elucidation and rationale, see, *GL:RCP*, II, p.70.

(of rising of the part of the ecliptic between the Sun and the planet) by using the oblique ascension of that very Sign (in which the Sun and the planet are situated); (when the planet is to be seen) in the west, (its) visibility should be announced by calculating the time (of setting of the part of the ecliptic between the Sun and the planet) by using the oblique ascension of the seventh Sign. (3). (KSS)

—सौरः

18. 3. 2. स्फुटदिनकरान्तरांशाः
चन्द्रादीनां च दर्शने ज्ञेयाः ।
विंशतिरूना 'वसु'-'शिखि'-
'मुनि'-'नव'-'रुद्रे'-'न्द्रियैः' क्रमशः ।। १२ ।।

(Varāha, *PS*, 17-12)

—Saura

The heliacal rising and setting of the Moon, Mars, Mercury, Jupiter, Venus and Saturn are, when their elongation (from the true Sun) are 20 less, respectively, by 8, 3, 7, 9, 11 and 5 (i.e. when they are 12°, 17°, 13°, 11°, 9° and 15°). (12). (TSK)

—करणरत्नम्

18. 3. 3. 'भ'-'मनु'-'नखां'-शैः कुजगुरु-
शनयः केन्द्रैः पुरन्दराशायाम् ।
उदयं कुर्युश्च तदा
तद्वच्चक्रे विहीनेऽस्तम् ।। १ ।।

'नन्दशरै'-'रपरदिशो' बुध उदयं 'नगशरेन्दु'-भिश्चास्तम् ।
'रुद्रभुजैः' प्रागुदयं 'शून्याकाशाग्नि'-भिश्चास्तम् ।। २ ।।

भृगुजः पश्चादुदयं 'जिनै'-'र्नगागेन्दु'भिस्तथा चास्तम् ।
प्रागुदयं 'त्रिपुराणै'-'र्नगहुतभुक्पावकै'रस्तम् ।। ३ ।।

'स्वरशशि'-'शिखिशिखि'-'शशिशर'-
'शरशशि'-'शररस'-विहीनचक्रार्धे ।
केन्द्राख्ये भौमाद्या
वक्रिण एतैर्युतैर्मुक्ताः ।। ४ ।।

(Deva, *KR*, 8. 1-4)

—Karaṇaratna

At 27, 14 and 20 degrees, (respectively), of (*śīghra*) anomaly, Mars, Jupiter and Saturn rise in the east: and at 360° diminished (respectively) by the same (degrees), they set (in the west). (1)

At 59°, Mercury rises in the west; at 157°, it sets (in the west); at 211°, it rises in the east; and at 300°, it sets (in the east). (2)

Venus rises in the west at 24°, and sets (in the west) at 177°. It rises in the east at 183° and sets (in the east) at 337°. (3)

At 180° diminished (respectively) by 17, 33, 51, 15 and 65 (degrees) of (*śīghra*) anomaly, Mars etc. take up retrograde motion; and at 180° increased, (respectively), by the same (degrees), they abandon it.[1] (4). (KSS)

—भास्करः २

18. 3. 4. प्राच्यामुदेति क्षितिजोऽ'ष्टदस्रैः'
'शक्रै'र्गुरुः 'सप्तकुभि'श्च मन्दः ।
स्वस्वोदयांशोनितचक्रभागै-
स्त्रयो व्रजन्त्यस्तमयं प्रतीच्याम् ।। ४२ ।।

'खाक्षैः' 'जिनैः' ज्ञसितयोरुदयः प्रतीच्या-
मस्तश्च 'पञ्चतिथिभि': 'मुनिसप्तभूभिः' ।
प्रागुद्गमः 'शरनखैः' 'त्रिधृति'प्रमाणै-
रस्तश्च तत्र 'दशवह्निभि'-'रङ्गदेवैः' ।। ४३ ।।

अवक्रवक्रास्तमयोदयोक्त-
भागाधिकोनाः कलिका विभक्ताः ।
द्राक्केन्द्रभुक्त्याप्तदिनैर्गतैष्यै-
रवक्रवक्रास्तमयोदयाः स्युः ।। ४४ ।।

(Bhāskara II, *SiŚi.*, 1.2. 42-44)

—Bhāskara II

Mars rises heliacally in the East by 28°, Jupiter by 14°, Saturn by 17° of *śīghra* anomaly and set heliacally in the west by the degrees which are the differences of the above and 360°, respectively. (42)

Mercury and Venus rise in the West by 50° and 24° of *śīghra* anomaly, respectively, and set in the West by 155° and 177°, respectively. They rise in the east by 205° and 183° of *śīghra* anomaly and set there by 310° and 336°, respectively. (43)

When the *śīghra* anomalies have particular values, to decide when the planets rise or set heliacally, we have to take the difference of those particular values and the numbers given above for the respective *śīghra* anomalies, convert them into minutes of arc and divide the results by the daily motion in the *śīghra* anomalies in minutes of arc. Then we have the number of days in which the rising or setting takes place thereafter.[2] (44). (AS)

दृक्कर्म

18. 4. 1. प्राक् त्रिभेण वर्जितात् संयुतात् तु पश्चिमे ।
खेटतोऽपमाक्षयोः संस्कृतिर्नता लवाः ।। १७ ।।

'षट्शैलाष्टनवार्कधृत्यदितिजाः' खण्डानि कार्यं नतां-
'शा'शांशप्रमखण्डकैक्यमगतोच्छिष्टांशघाताद् युतम् ।
'आशा'प्त्या रविहृच्छराङ्गुलहृतं लिप्ता ग्रहे ता नतां-
शेष्वोः स्वर्णमभिन्नभिन्नदिशि स व्यस्तं परे दृग्ग्रहः ।।[3]

(Gaṇeśa, *GL*, 9. 17-18)

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[1] For tables of planetary motion, see *KR:KSS* p. 94-95.

[2] For an exposition, see *SiŚi:AS*, pp. 167-70.

[3] For elucidation and rationale see *GL:RCP*, II, pp. 80-83.

Visibility corrections

Subtract 3 *rāśis* from the planet's eastern rising point and add the same to its western rising point. From these find the *krānti*. Apply the correction with the latitude in degrees, being the sum of the two in case both are uni-directional, difference otherwise. *Natāṁśa* is thus got. (17)

6, 7, 8, 9, 12, 17, 33 are the *khaṇḍas*. Divide the *natāṁśas* by 10. Add to the quotient the value of the *khaṇḍa* specified for it. Keep it separate (and call it, say x).

Multiply the remainder by the succeeding value of the *khaṇḍa*. Again find 1/10th of this product. Add this to the *khaṇḍa* obtained earlier. Multiply by the *śara* in *aṅgulas*. Divide the product by 12. The result gives the visibility correction, *Dṛkkarma*, in minutes. These minutes are to be added to the true position of the planet, if the *śara and natāṁśa* are in the same direction; otherwise *dṛkkarma* is to be subtracted.[1] (18). (VSN)

ग्रहोदयास्तमयगणनम्

—लल्लः

18. 5. 1. ऊनः प्रागुदयं प्रयाति सवितुः प्रागेव चास्तं ग्रहः
पश्चादभ्यधिकोऽर्कसन्निधिवशान्नित्यं प्रवाहेण च ।
दृश्याः प्राचि कुजार्यसूर्यतनयाः पश्चाददृश्याः सदा
वक्रस्थौ ज्ञसितौ च तौ क्रमगतौ चन्द्रश्च तद्व्यत्ययात् ।।

अयनसंस्कृतग्रहः

'शैलाङ्कत्रिकु'भिर्यथागतभुजज्योनां त्रिभज्यां हतां
विक्षेपेण च 'वेदवेदवसुगोरूपाष्टरुद्रै'र्भजेत् ।
विक्षेपापमयोर्विभिन्नककुभोर्लिप्तादि लब्धं धनं
कर्तव्यं द्युचरे त्यजेत् समदिशोः स्यादायनः स ग्रहः ।। २ ।।

दृक्कर्मशुद्धः ग्रहः

व्यस्तज्यामथवा गृहत्रययुतात् क्षेपाहतामुद्धरेद्
'रूपाङ्काब्धिगजै'र्ग्रहे धनमृणं कुर्यात् कलाद्युक्तवत् ।
अक्षज्यागुणितेऽथ लम्बकहृते क्षेपे फलं सौरिका
जायन्ते 'रवि'भिर्हृतेऽथ विषुवच्छायाङ्गुलैर्वा हते ।। ३ ।।

प्रागृणं स्वमयनग्रहे फलं
सौम्ययाम्यशरसम्भवं सदा ।
अन्यथापरकपालसंस्थिते
कारयेदिति भवेत् स दृग्ग्रहः ।। ४ ।।

कालांशै 'रवि'भिः शशी 'महि'सुतो-
ऽत्य'ष्टचा'थ 'नन्दैः' सितः
शाशाङ्कि'र्गुणभूमि'भि'रक्षशशि'भिः
सौरिः 'शिवै'रङ्गिराः ।

वक्रस्थौ बुधभार्गवौ 'रवि'-'गजैः'
सूर्यान्तरैर्दृश्यतां
यान्ति प्रत्यगिनो भषट्कसहितः
कार्यस्तथा दृग्ग्रहः ।। ५ ।।

गम्योऽल्पादसुनिवहोऽधिकाद् गतो यो
मध्यस्थैरसुभिरसौ युतो विधेयः ।
कालांशा 'गगनरसै'र्भजेदमीभि-
श्चोक्ताल्पैर्द्युचरमवेहि तैरदृश्यम् ।। ६ ।।

स्फुटग्रहोदयास्तमयौ

उक्तेष्टकालांशविशेषलिप्ता
गत्योर्वियोगेन हृता दिनानि ।
विलोमगेऽर्कग्रहभुक्तियुत्या
गतानि गम्यानि च विद्धि युक्त्या ।। ७ ।।

अस्तेऽधिकाः स्युर्ध्रुवकालभागा
गतानि विद्धीह दिनानि तानि ।
अल्पास्तथा गम्यदिनानि तानि
तथोदये गम्यगतानि तानि ।। ८ ।।

(Lalla, *ŚiDhVṛ*., 8. 1-8)

Computation of rising and setting

—Lalla

The planet, which has a motion slower than that of the Sun, always rises (heliacally) in the east and sets in the west. The planet which has a faster motion rises in the west and sets in the east. So, Mars, Jupiter, and Saturn, (which have a motion slower than that of the Sun), always rise heliacally in the east and set in the west. Mercury and Venus, when retrograde, do the same. But when they have direct motion, (which is faster than that of the Sun), they rise heliacally in the west and set in the east. And so does the Moon.

(The daily rising and setting) is due to the wind called *Pravaha*. Then the planet with a lesser longitude than that of the Sun rises before sunrise and sets before sunset; and that with a greater longitude rises after sunrise and sets after sunset. (1)

Planet corrected for ecliptic deviation

Multiply the radius diminished by the R sine of the true longitude of a planet, as calculated, by 1397 and by the latitude of the planet. Divide the product by 1,18,19,844. The quotient in minutes etc. should be added to the true longitude of the planet if its latitude and declination are in opposite directions, and should be subtracted if they are in the same direction. The result is the longitude of the planet corrected by the visibility correction due to the ecliptic deviation or *Āyanadṛkkarmakalā*. (2)

[1] This procedure is for knowing the heliacal rising and setting in the east. The process is to be reversed for the west.

Planet corrected for visibility

Or, multiply the R versed sine of the true longitude of a planet increased by 3 Signs by the latitude and divide by 8461. Add the quotient in minutes to or subtract it from the true longitude of the planet following the rule given above. Multiply the latitude (of the planet) by the R sine of the local latitude and divide by the R sine of the colatitude.

Or, multiply it, (the latitude), by the equinoctial midday shadow expressed in *aṅgulas* and divide by 12. (The result in each case is the visibility correction due to the local latitude) or *Ākṣadṛkkarmaka*. It is called *Saurikā* or 'due to the Sun'. (3)

If the second visibility correction is calculated from a northern latitude (of a planet), it should always be subtracted from the corrected longitude of the planet on the eastern horizon. If, however, it is calculated from a southern latitude, it should be added to the same. The reverse is the process for the western horizon. The planet thus corrected is called *Dṛggraha*. (4)

Corrected by the two visibility corrections, the Moon, Mars, Jupiter and Saturn are heliacally visible when separated from the Sun by a *kālāṁśa* (of 'degrees of time') of 12°, 17°, 11° and 15°, respectively. Mercury and Venus (are heliacally visible) when in direct motion (when separated from the Sun) by a *kālāṁśa* of 13° and 90°, respectively, and, when retrograde, by a *kālāṁśa* of 12° and 8°, respectively.

(If the heliacal rising and setting of a planet) on the western horizon is considered, the true longitude of the Sun and that of the *dṛggraha* should each be increased by 6 Signs. (5)

(Find the Sun's true longitude at sunrise or sunset and the *dṛggraha* whether on the eastern or the western horizon, as the case may be). From the lesser of the two, find the time to be passed in *asus*, and, from the greater, the time passed. To the sum of the two times must be added the times of rising of the intervening Signs of the zodiac. The result divided by 60 gives the required *kālāṁśa* or degrees of time of the planet.

Remember that when a planet is separated from the Sun by a *kālāṁśa* less than its *kālāṁśa* for visibility, as mentioned above, the planet remains invisible. (6)

Rising and setting corrected by degrees of time

Find the difference expressed in minutes between the *kālāṃśa* of a planet at any time and the *kālāṁśa* for its visibility. Divide it by the difference of the true motions of the Sun and the planet, if they are moving in the same direction, and by the sum, if they move in opposite directions. The quotient in days gives the time elapsed or to come (as explained in the next verse). This is to be understood logically. (7)

When the setting of a planet is considered, if the given *kālāṁśa* for visibility is greater than its calculated *kālāṁśa*, know then that the planet has set heliacally before the number of days as found above; if the former is less, the planet will set after these days. When the rising is considered, in the former case, the planet will rise after the days calculated, and, in the latter case, the planet has risen before the days calculated.[1] (8). (BC)

—आर्यभटार्धरात्रिकपक्षः

18. 5. 2. शुक्रगुरुज्ञार्किकुजाः कालांशैर्द्व्युत्तरैर्नवभिः ।
दृश्यादृश्या दृक्कर्मणा रवेर्द्वादशभिरिन्दुः ॥ १ ॥
विक्षेपसत्रिराशिज्याघाता'दिन्द्रगाग्नि'लब्धकलाः ।
विक्षेपायनसाम्ये शोध्या भेदे ग्रहे क्षेप्याः ॥ २ ॥
विषुवच्छायागुणिताद्विक्षेपाद् द्वादशोद्धृतात् सौम्यात् ।
फलमृणधनं धनर्णं याम्यादुदयास्तमयलग्ने ॥ ३ ॥
प्रागूनमाद्यमधिकं पश्चाल्लग्नाद् ग्रहोदयोऽस्तमयः ।
षड्भयुतमन्यदुदयैर्घटिकाः कृत्वोनमधिकसमम् ॥ ४ ॥
(Brahmagupta, *KK*, 1.6. 1-4)

—Āryabhaṭa's Midnight system

Venus, Jupiter, Mercury, Saturn and Mars, corrected by the two *dṛkkarmakalās*, become heliacally visible, when separated from the Sun by *kālāṁśas* of 9°, 11°, 13°, 15°, and 17°, respectively. They become invisible, when separated by less. The Moon becomes visible, when separated from the Sun by a *kālāṁśa* of 12°. (1)

The product of the *vikṣepa* of a planet and the *jyā* of its longitude, increased by 90°, should be divided by 371. The minutes thus obtained are called *āyana-dṛkkarmakalā*. These minutes should be subtracted from the longitude of the planet if the *vikṣepa* and the *ayana* are of the same denomination (that is, if the *vikṣepa* is to the north, and the longitude of the planet increased by 90° is between Meṣa and Tulā, or, if the *vikṣepa* is to the south and the planet increased by 90° is between Tulā and Meṣa. These minutes should, however, be added to the longitude of the planet, if the *vikṣepa* and the *ayana* are of different denominations. (2)

Multiply the *vikṣepa* of a planet by the *palabhā* at the observer's station and divide by 12. If the *vikṣepa* is to the north, subtract the result from the longitude of the planet on the eastern horizon corrected by the *āyana-dṛkkarmakalā*. Thus is obtained the *udayalagna* of the planet. The previous result should be added to the longitude of the planet on the western horizon corrected

[1] For detailed exposition and demonstration see *ŚiDhVṛ:BC*, II, 151-58.

by the *āyana-dṛkkarmakalā*. Thus is obtained the *asta-lagna* of the planet. If the *vikṣepa* is to the south, the previous result should be added to obtain the *udayalagna* and subtracted for the *astalagna*. (3)

If the *udayalagna* of a planet is less than the *lagna*, the planet has already risen; if greater, the planet will rise later. If its *astalagna* increased by 6 Signs or 180° is less than the *lagna*, the planet has already set; if greater, the planet will set later. These times in *ghaṭikās* can be ascertained by means of the difference between the planet's *udayalagna* or *astalagna* and the *lagna*, using the times of the risings of the *rāśis*.[1] (4). (BC)

—ब्रह्मगुप्तकृतः शोधः

18. 5. 3. वेदा 'द्वयष्टर्तुदिशः' पातांशा 'दि'ग्गुणाः कुजादीनाम् ।
'नवतिथिरसार्कमासा' विक्षेपकला 'दिग'भ्यस्ताः ।। १ ।।

ग्रहवद् बुधसितपातौ तृतीयलब्ध्याधिकोनकौ स्पष्टौ ।
कुजजीवसौरपातास्तद्विपरीतं चतुर्थाप्त्या ।। २ ।।

मानाल्पत्वात् पश्चा-
दुदयोऽस्तमयः सितस्य दशभिः प्राक् ।
पश्चान्मानमहत्वा-
दस्तमयोऽष्टाभिरुदयः प्राक् ।। ३ ।।

ज्ञस्यैवं 'मनुसूर्यैः' पठितैः कुजजीवसूर्यपुत्राणाम् ।
उदयः प्रागस्तमयो मानसमत्वाद् भवति पश्चात् ।। ४ ।।

मिथुनस्य सप्तविंशे भागेऽगस्त्यस्य याम्यविक्षेपः ।
'मुनिनग'संङ्ख्या भागा रविसङ्ख्याश्चास्य कालांशाः ।।

षड्विंशे मिथुनांशेऽशंकचत्वारिंशता मृगव्याधः ।
दक्षिणतो विक्षेपस्त्रयोदशास्यैव कालांशाः ।। ६ ।।

ऋक्षाणां 'मनु'सङ्ख्या कालांशाः प्रथमकर्म सर्वेषाम् ।
दृक्संज्ञकं न कार्यं द्वितीयमेषां सदा कार्यम् ।। ७ ।।

उदयाख्यमगस्त्यस्य च लग्नं वृद्धि नयेत्तथास्ताख्यम् ।
हीनं कार्यं नित्यं घटिकाद्वितयेन राश्युदयैः ।। ८ ।।

उदयास्तसूर्यसंज्ञौ ज्ञेयावेतौ भषट्कयुतमन्यत् ।
घटिकाद्वितयेनैवं षड्भागयुतेन मृगहर्तुः ।। ९ ।।

एवं नक्षत्राणां घटिकाद्वितयेन सत्रिभागेन ।
दर्शनमदर्शनं स्यादन्योन्यं समे रविलग्ने ।। १० ।।

सर्वेषां नतभागान्नवतेः प्रोह्योन्नतास्तु शेषाः स्युः ।
उदयार्कोऽस्तमयार्काद्यस्योनस्तत् सदा द्वश्यम् ।। ११ ।।

उदयास्तसूर्ययोरन्तरे रवौ दृश्यतेऽन्यथास्तमितिः ।
उनाधिका रविकला रविभुक्त्या भाजिता दिवसाः ।।१२।।

(Brahmagupta, *KK*, 2. 5. 1-12)

—Emendation by Brahmagupta

The longitudes of the *pātas* of Mars etc., are 40°, 20°, 80°, 60° and 100°, respectively. The mean *vikṣepas* of Mars etc. are 90′, 150′, 60′, 120′ and 120′, respectively. (1)

The second *mandaphalas* calculated during the process of finding the true longitudes of Mercury and Venus should be added to or subtracted from the longitudes of their respective *pātas* (as given above), according as the *mandaphalas* are additive or subtractive. The results are their correct longitudes.

The second *śīghraphala* calculated during the process of finding the true longitudes of Mars, Jupiter and Saturn should be added to or subtracted from the longitudes of their respective *pātas* (as given above), according as the *śīghraphalas* are subtractive or additive. The results are their correct longitudes. (2)

When Venus has direct motion, its diameter appears smaller, and it rises in the west and sets in the east by a *kālāṁśa* of 10°. When its motion is retrograde, its diameter appears bigger, and it rises in the east and sets in the west by a *kālāṁśa* of 8°. Mercury rises and sets in a similar manner, with the difference that when it is direct, its *kālāṁśa* is 14°, and 12° when retrograde. The diameters of Mars, Jupiter and Saturn always appear the same. (There is, therefore, no change in their *kālāṁśas*). They rise in the east and set in the west by the *kālāṃśas* already given in *KK* 1.6.1. (3-4)

The *dhruvaka* of Agastya is 2 signs 27° and its distance from the ecliptic measured on the declination circle is 77° to the south. Its *kālāṁśa* is 12°. (5)

The *dhruvaka* of Mṛgavyādha is 2 Signs 26° and tis distance from the ecliptic measured on the declination circle is 40° to the south. Its *kālāṁśa* is 13°. (6)

The *kālāṁśa* of each *nakṣatra* is 14°. No *āyana-dṛkkarma* correction need be applied to their *dhruvakas*, as they are already corrected. The *ākṣa-dṛkkarma* correction only should be applied. (7)

From the *udayalagna* of Agastya calculate the *lagna* at 2 *ghaṭikās* after sunrise by means of the times of the rising of the *rāśis* (according to the method explained before). The result is the *udayasūrya* of Agastya. Again, from the *astalagna* calculate the *lagna* at 2 *ghaṭikās* before sunrise. Add 6 Signs to it. The result is the *astasūrya* of Agastya.

In the same manner the *udayasūrya* and *astasūrya* of Mṛgavyādha may be found. In this case 2 *ghaṭikās* and 10 *vināḍīs* should be used.

Similarly, the *udayasūrya* and the *astasūrya* of other *nakṣatras* should be calculated. In this case 2 *ghaṭikās* 20 *vināḍīs* should be used.

[1] For the formulae and proofs, see, *KK:BC*, I. 129-31.

Agastya, Mṛgavyādha or any of the *nakṣatras* rises or sets according as its *udayasūrya* or *astasūrya* is the same as the true longitude of the Sun. (8-10)

Subtract the number of degrees in the *natāṁśa* of a *nakṣatra* from 90°. The remainder is its *unnatāṁśa*.

The *nakṣatra* whose *udayasūrya* is less than its *astasūrya* is always heliacally visible. (11)

A *nakṣatra* is visible, as long as the true longitude of the Sun lies between its *udayasūrya* and *astasūrya*. Otherwise it is invisible.

Find the difference between the *udayasūrya* of a *nakṣatra* and the Sun, or the *astasūrya* and the Sun. Express the difference in minutes. Divide each difference by the motion of the Sun. The results give, respectively, the number of days passed since the visibility of the *nakṣatra* and that which will pass before it is invisible.[1] (12). (BC)

—आर्यभटः २

18. 5. 4. कुजजीवार्कजमुनयः शुक्रज्ञौ वक्रिणौ च सूर्याल्पाः ।
यान्ति प्राच्यामुदयं पश्चादस्तं व्रजन्त्यधिकाः ॥ १ ॥

ऋजुगौ ज्ञसितौ चेन्दुः प्राच्यामूना रवेर्व्रजन्त्यस्तम् ।
अधिकाः पश्चादुदयं सान्निध्ये लक्षणं चिन्त्यम् ॥ २ ॥

कोठा क्सा क्ला प्रा दा पोमा कालांशकाः शशाङ्काद्याः ।
लूनकलोना वक्रगबुधसितयोः सम्भवन्त्युक्ताः ॥ ३ ॥

रविदृक्खेटौ पश्चात्कार्यौ भगणार्धसंयुक्तौ ।
तद्विश्लेषांशहतं स्वदृकाणं चीननै विभजेत् ॥ ४ ॥

फलमिष्टांशा एतैरुक्तांशेभ्योऽधिकैरेष्यः ।
अस्तो न्यूनैर्यातो व्यस्तोऽस्माल्लग्नादुदयः ॥ ५ ॥

इष्टोक्तांशवियोगः कार्योऽथ प्रागिनादधिकः ।
पश्चादूनो वा चेद् दृक्खेटः स्यात्तदा योगः ॥ ६ ॥

तल्लिप्तौघं विभजेद् गत्योः स्वदृकाणसङ्गुणयोः ।
तननै हृतयोर्युत्या वक्रिणि खेटेऽन्यथा वियोगेन ॥ ७ ॥

लब्धैर्दिवसैः कथितवद् एष्यगतत्वं विचिन्त्यमिह ।
घटजध्रुवको म्रांशाः शरोऽन्तकस्थाः ससोऽपमजात् ॥८॥

(ABh. II, *Mahā*., 9. 1-8)

—Āryabhaṭa II

Mars, Jupiter, Saturn, Canopus (Muni), and Venus and Mercury in retrograde motion (*vakrin*), when (of) lesser (longitude) than the Sun, rise in the east, and when (of) greater, set in the west. (1)

Mercury and Venus in direct motion (*ṛjuga*), and the Moon, when (of) lesser (longitude) than the Sun, set in the east; when (of) greater, rise in the west. When they are nearer (to the Sun), the rule has to be duly reconsidered. (2)

The degrees of time (*kālāṁśaka*) of the Moon etc. are 12, 17, 13, 12, 8 and 15. The said (degrees of time), reduced by 30 minutes, (are those) of Mercury and Venus in retrograde motion: Moon 12°; Mars 17°; Mercury 13°; retrograde 12° 30′; Jupiter 12°; Venus 8°; retrograde 7° 30′; Saturn 15°. (3)

Find out (the longitude of) the Sun and (that of) a planet (corrected for ecliptic deviation and for apparent latitude) (*dṛkkheṭa*) (in the east at sunrise). These, increased by half a revolution, (are the respective longitudes) in the west (at sunset). Multiply (the ascensional equivalent of) the *dṛkāṇa* (in which they are situated) by the difference in degrees (of the longitude of the Sun and the corrected longitude of the planet) and divide by 600. (4)

The quotient gives the "required degrees" (*iṣṭāṃśa*). When these are greater than the degrees listed above, the setting is to come; and when less, it is past. The rule is reverse for the rising. (5)

If (the longitude of) the planet, (corrected for ecliptic deviation and for apparent latitude) (*dṛkkheṭa*), is greater than (that of) the Sun in the east, or less in the west, take the difference of the required degrees (*iṣṭāṃśa*) and those listed above. (6)

(Find out) the minutes (of the distance in ascension of two planets). Multiply the daily motions (*gati*) (of the two planets, each by the ascensional equivalent of) the *dṛkāṇa* (in which it is situated) and divide by 600. By the sum (of the two quotients), if one planet (of the two) is retrograde, otherwise, by their difference, (divide the minutes obtained above). By the quotient in days, it can be determined, as shown before, whether (the conjunction) is to come or is past. (7-8ab)

The polar longitude (*dhruvaka*) of Canopus (*ghaṭaja*) is 85°, and the (polar) latitude is 77 (degrees) south of the declination. (8cd). (SRS)

—वाक्यकरणम्

18. 5. 5. 'खाङ्गा''ष्टि' 'तिथि' 'शून्याग्नि' 'तिथिभि'र्युतशेषितैः ।
उदयास्ते कुजादीनां, वक्रे चन्द्रजशुक्रयोः ॥ १ ॥

'खेन्द्रियै'-'रद्रचष्टदृग्भि'र्दिनैरस्तोदयौ तथा ।
सूर्यादभ्यधिकाः पश्चादस्तं जीवकुजार्कजाः ॥ २ ॥

ऊनाः प्रागुदयं यान्ति, शुक्रज्ञौ वक्रिणौ तथा ।
अवक्रगौ तु तावूनौ रवेः प्राच्यामदर्शनम् ॥ ३ ॥

व्रजतोऽभ्यधिकौ पश्चादुदयं शीघ्रयायिनौ ।
कुजादीनां 'सटा' 'लोकः' 'पुण्यं दानं' 'शुकों'ऽशकाः ॥

[1] For worked out examples see, *KK:BC*, I. 152-54.

भृगोः 'षड्' वक्रिणो 'राज्यं' विदः कालांशका मताः ।
'मान'घ्ना यदि पूर्वस्यां ग्रहाक्रान्तघटीहृताः ॥ ५ ॥

यदि पश्चाद् ग्रहाक्रान्तसप्तमस्य घटीहृताः ।
क्षेत्रभागा रवेः शोध्याः प्राग्योज्याः पश्चिमे रवौ ॥ ६ ॥

संस्कृतार्कग्रहौ तुल्यौ उदयास्तमयौ तदा ।
समागमवदत्रापि तत्कालज्ञानमिष्यते ॥ ७ ॥

(*VK*, 5. 1-7)

—Vākyakaraṇa

For Mars, direct Mercury, Jupiter, direct Venus, and Saturn, the approximate dates of heliacal setting and rising are 60, 16, 15, 30 and 15 days respectively, before and after the days of the ends of cycles. For retrograde Mercury and Venus they are 50 and 287, respectively. (1-2a)

Mars, Jupiter and Saturn, and retrograde Mercury and Venus, when having longitudes greater than the Sun set in the west, and when having longitudes less than the Sun rise in the east. Mercury and Venus, in direct motion, set in the east when less than the Sun, and rise in the west when greater. (2b-4a)

The *kālāṁśa-s* are: For Mars 17°, for Mercury 13°, for Jupiter 11°, for Venus 8°, for Saturn 15°, for retrograde Mercury 12°, and for retrograde Venus 6°. (4b-5a)

If the rising or setting pertains to the east, multiply the *kālāṁśa-s* by 5 and divide by the *rāśimāna*, in *nāḍīs*, of the *rāśi* in which the body is situated. If the rising or setting pertains to the west, divide by the *rāśimāna* in *nāḍis* of the 7th *rāśi* from that in which the body is situated. *Kṣetrāṁśas* are obtained. (4b-5a)

In the case of the east, the *kṣetrāṁśa-s* are to be deducted from the Sun, and in the case of the west, added. When this corrected Sun and the body are equal, i.e. in conjunction, then is the time of the heliacal setting or rising. Here too, the time is to be found as that of conjunction.[1] (6b-7). (TSK-KVS)

—भास्करः २

18. 5. 6. प्राग्दृग्ग्रहः स्यादुदयाख्यलग्न-
मस्ताख्यकं पश्चिमदृग्ग्रहः सः ।
प्राग्दृग्ग्रहोऽल्पोऽत्र यदीष्टलग्नाद्
गतो गमिष्यत्युदयं बहुश्चेत् ॥ १ ॥

ऊनोऽधिकः पश्चिमदृग्ग्रहश्चे-
दस्तं गतो यास्यति चेति वेद्यम् ।
तदन्तरोत्था घटिका गतैष्या-
स्तच्चालितः स्यात् स निजोदयेऽस्ते ॥ २ ॥

तल्लग्नयोरन्तरतोऽसकृद्याः
कालात्मिकास्ता घटिकाः स्युराक्ष्यः ।
अभीष्टकालद्युचरोदयान्त-
र्यद्वेष्टकालद्युचरास्तमध्ये ॥ ३ ॥

निरुक्तौ ग्रहस्येति नित्योदयास्ता-
विनासन्नभावेन यौ तौ च वक्ष्ये ।
रवेरूनभुक्तिर्ग्रहः प्रागुदेति
प्रतीच्यामसावस्तमेत्यन्यथान्यः ॥ ४ ॥

ज्ञशुक्रावृजू प्रत्यगुद्गम्य वक्रां
गतिं प्राप्य तत्रैव यातः प्रतिष्ठाम् ।
ततः प्राक् समुद्गम्य वक्रावृजुत्वं
समासाद्य तत्रैव चास्तं व्रजेताम् ॥ ५ ॥

'दस्रेन्दवः' 'शैलभुवश्च' 'शक्रा'
'रुद्राः' 'खचन्द्राः' 'तिथयः' क्रमेण ।
चन्द्रादितः काललवा निरुक्ता
ज्ञशुक्रयोर्वक्रगयोर्द्विहीनाः ॥ ६ ॥

यत्रोदयो वास्तमयोऽवगम्य-
स्तद्दिग्भवो दृक्खचरो रविश्च ।
अस्तोदयासन्नदिने कदाचित्
साध्यस्तु पश्चात् तरणिः सषड्भः ॥ ७ ॥

(Bhāskara II, *SiŚi.*, 1.8. 1-7)

—Bhāskara II

The *udayalagna* of a planet is termed *Prāk-dṛk-graha* and the *astalagna* is termed the *Paścima-dṛk-graha*. If the *Prāk-dṛk-graha* happens to be less than the current *lagna* the planet had already risen. If it be greater, the planet is still to rise. Similarly, if the *Paścima-dṛk-graha* is less than the curernt *lagna*, the planet had already set; otherwise is yet to set. (1-2a)

Find the time that has elapsed after the planet's rise. From this time, obtain the arc that would have been traversed in between the moments. Subtracting this arc from the planet's position at the moment, the planet's position at rising would be obtained approximately. From this approximate time again, compute the arc that would have been traversed by the planet during that time. Subtracting this arc, which is nearer the truth from the planet's present position, we obtain a more approximate position of the rising planet. Again computing the *Prāk-dṛk-graha* from this position, calculate the time from this *Prāk-dṛk-graha* and the *lagna* of the moment. Repeating the process we will obtain the actual sidereal time in between the given time and the actual rising time of the planet. (2b-3)

Heliacal rising and setting of planets

The times of rising and setting of a planet have been dealt with. Now I shall tell the procedure to be adopted in computing the times of heliacal rising and setting of

[1] For worked out examples see, *VK:TSK-KVS*, p. 285.

a planet. If a planet has a daily motion less than that of the Sun, it rises heliacally in the east, and sets in the west; otherwise the reverse. (4)

Speciality of Mercury and Venus

Mercury and Venus rise heliacally in the west in their direct motion, (attain maximum elongation before they) become retrograde, set heliacally there itself, then rise heliacally in the east continuing to be retrograde, (attain maximum elongation there before they) next become direct and gradually set there (to rise again in the west) as before. (5)

Distance in degrees

The *kālāṁśas* with respect to the Moon, Mars, Mercury, Jupiter, Venus and Saturn, or the degrees of distance from the Sun within which they rise or set heliacally are 12, 17, 14, 11, 10, 15, respectively. In the case of Mercury and Venus when they are retrograde the *kālāṃśas* are 12, and 8, respectively. (6)

Moment of rising or setting

If it is to be known when a planet rises or sets heliacally, the position of the *Prāk-dṛk-graha* or the *Paścima-dṛk-graha*, as the case may be, (*Prāk-dṛk-graha* in case the rising or setting takes place in the east or the other in the other case), and that of the Sun are also to be computed on a day a little before the day of rising or setting as prognosticated by the *śighra* anomaly. In case the planet rises or sets in the west, to obtain the *lagna* the position of the Sun isto be increased by 180.[1] (7). (AS)

गुरोरुदयास्तमयः

18. 6. 1. चक्राढ्यो मधुवक्त्रमासनिचयो 'विश्वा'प्तचक्रोनितो
द्विघ्नो युक् दशमास'धूर्जटि'दिनै'र्भैः' शेषितो 'भ'च्युतः ।
द्व्याप्तः स्याद्भमुखः पृथक् 'तिथि'लवैरूनोऽस्य बाह्वंशका-
'र्कप्तांशोनयुतो धटाजरसभे मासादिकः स्यान्मधोः ।।
'तिथि'दिनरहिताद्योऽसौ द्विधा तैश्च मासैः ।
क्रमश इह भवेतां मन्त्रिणोऽस्तोदयौ च ।। ५a ।।
(Gaṇeśa, *GL*, 9.4-5a)

Jupiter's rising and setting

From Caitra month onwards calculate the *māsagaṇa* and add the *cakra* for that year to it (say, x). Divide by 13 and find (x—x/13). Multiply this by 2. Add 10 months and 11 days. Divide by 27 and subtract the remainder from 27. Dividing this by 2, *rāśis* etc. are obtained (say, y).

Subtract 15 degrees from y. Take the *bhujāṃśa* of the remainder. Divide the same by 12. (Let z be the result). In the case of the six months beginning from Meṣa and the other *rāśis*, add z to y. In the case of Tulā etc. find (y—z). (4)

Find ($y \pm z$) *minus* fifteen days. This gives the setting of Jupiter. ($y \pm z$) + 15 days gives its rise.[1] (5a). (VSN)

शुक्रोदयास्तौ

18. 7. 1. अथ मधुमुखमासाः 'सप्तभू'निघ्नचक्रैः
'स्वशर'युगलवाढ्यैः संयुता 'मार्गण'घ्नाः ।। ५ ।।
'उदधिरस'समेता'श्छिद्र'खेगाभितष्टा
नव नव परिशुद्धाः पञ्चभक्ताः पृथक्स्थाः ।
'रसगुण'दिनहीनाढ्या द्विधा चैत्रतस्तै-
र्भृगुजहरिदिगस्ताम्बूदयौ स्तः क्रमेण ।। ६ ।।
नवमास'भ'घस्रतोऽल्पपुष्टाः
पृथक्स्थाः क्रमशस्तु तैर्युतोनाः ।
द्वेधा युगवासरोनयुक्ता-
स्तोयास्तैन्द्रयुदयौ क्रमाद् भृगोः स्तः ।। ७ ।।
(Gaṇeśa, *GL*, 9. 5b-7)

Venus: Rising and setting

Find the *māsagaṇa* from Citrā onwards and the *cakra* also. Multiply the *cakra* by 17. Add 1/45th part of the product to the product itself. Again multiply 5. Add 64 to the result. Divide by 99. Let the remainder be subtracted from 99 and divide it by 5. Keep the result, (say x), separate. (5b-6a)

Find x—36 days. This gives the month during which Venus sets in the East. Similarly x+36 days denotes the months during which Venus rises in the west. (6b)

If x is less than 9 months and 27 days, then add them both. If x is more, take their difference. Keep it separate, (say y). Find (y+14) days and (y—14) days. These give, respectively, heliacal setting in the west and heliacal rising in the east, of Venus.[2] (7). (VSN)

भानामुदयास्तमयौ

18. 8. 1. उदयसवितुर्यस्यास्तार्को ध्रुवस्य महान् भवेद्
दिनकरवशादस्तं गच्छत्यसौ न कदाचन ।
यदपमधनुः क्षेपस्पष्टं पलेन च संस्कृतं
त्रिभवनमहद् यस्मिन् देशे स तत्र न दृश्यते ।। २० ।।
(Lalla, *ŚiDhVṛ*., 11.20)

Rising and setting of stars

The *nakṣatra*, whose *astārka* is greater than its *udayārka*, never sets heliacally. When the declination of a *nakṣatra*, corrected by its latitude and the local latitude, is greater than 90°, the *nakṣatra* is not visible at the place.[3] (20). (BC)

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[1] For comments, see *SiŚi:AS*, pp. 471-76.

[1] For elucidation and rationale, see *GL:RCP*, II. 61-64.

[2] For elucidation and rationale, see *GL:RCP*, II. pp. 65-66.

[3] For exposition, see *ŚiDhVṛ: BC*, II. p. 197-98.

18. 8. 2. उदयास्तमयार्कमध्यगेऽर्के
दृश्यः स्याद् ध्रुवकः स्फुटोपमश्चेत् ।
फलहीनयुतोऽन्यतुल्यदिक्स्थो
लघुतामेति गृहत्रयांशकेभ्यः ।। १८ ।।

अस्तोदयसूर्यमध्यगेऽर्के
ध्रुवकस्तिग्मकरे न दृश्यते ।
उभयोरपि मध्यगाः कला
गतिभक्ता दिवसाः सहस्रगोः ।। १९ ।।

(Lalla, *ŚiDhVṛ.*, 11. 18-19)

As long as the true longitude of the Sun is between *udayārka* (i.e. that longitude of the Sun with which the *nakṣatra* rises) and *astārka* (i.e. that longitude of the Sun with which the *nakṣatra* sets), so long the *nakṣatra* is heliacally visible.

The sum or difference of its correct declination and the local latitude, according as they are in the same or opposite directions, must then be less than 90°. (18)

As long as the true longitude of the Sun is between the *nakṣatra's astārka* and *udayārka*, so long is the latter heliacally invisible.

The difference between the *udayārka* and *astārka* of a *nakṣatra*, expressed in minutes, when divided by the true daily motion of the Sun, gives the days (for which the *nakṣatra* is visible or invisible). (19). (BC)

18. 8. 3. दृक्कर्मणा पलभवेन तु केवलेन
भानां मुनेर्मृगरिपोरुदयास्तलग्ने ।
कृत्वा तयोरुदयलग्नमिनं प्रकल्प्य
लग्नं ततो निजनिजे पठितेष्टकाले ।। १२ ।।

यत् स्यादसावुदयभानुरथास्तलग्नाद्
व्यस्तं विभार्धमपि लग्नकमस्तसूर्यः ।
इष्टोनषष्टिघटिकास्वथ वास्तलग्ना-
ल्लग्नं क्रमेण भदलोनितमस्तसूर्यः ।। १३ ।।

स्यादुद्गमो निजनिजोदयभानुतुल्ये
सूर्येऽस्तभास्करसमेऽस्तमयश्च भानाम् ।
अत्राधिकोनकलिका रविभुक्तिभक्ता
यातैष्यवासरमितिश्च तदन्तरे स्यात् ।। १४ ।।

यस्योदयार्कादधिकोऽस्तभानुः
प्रजायते सौम्यशरातिदैर्घ्यात् ।
तिग्मांशुसान्निध्यवशेन नास्ति
धिष्ण्यस्य तस्यास्तमयः कथंचित् ।। १५ ।।

यस्य स्फुटा क्रान्तिरुदक् च यत्र
लम्बाधिका तत्र सदोदितं तत् ।
न दृश्यते तत् खलु यस्य याम्या
भं लुब्धकः कुम्भभवो ग्रहो वा ।। १६ ।।

(Bhāskara II, *SiŚi*, 1.11.12-16)

Stars in general

Compute the *udayalagna* and the *astalagna* of Agastya and Lubdhaka doing *ākṣa-dṛk-karma* alone. Assuming the *udayalagna* to be the Sun, compute the *lagna* for the *iṣṭa-kāla-nāḍīs* (i.e. after a lapse of *iṣṭa-nāḍīs* after the moment) given before (namely 2 *nāḍīs* for Agastya and 2 1/6 for Lubdhaka) which will be the longitude of the Sun, when the star (Agastya or Lubdhaka or whatever it be) rises heliacally. Thus the *udayārka* is a point of the ecliptic which rises when the star rises heliacally. (12)

Having computed the *asta-lagna* of the star, taking it to be the Sun, compute the *lagna* in the reverse direction, i.e. the *lagna* which precedes it by the *iṣṭa-kāla* given. If this *lagna* be decreased by 180°, it will give the longitude of the setting Sun at the time of the heliacal setting of the star.

Or, again, find the longitude of the point of the ecliptic which is ahead of the *astalagna* which takes (60—*iṣṭanāḍīs*) to rise after the *astalagna*; this longitude decreased by 180° gives the longitude of the setting Sun at the time of the heliacal setting of the star. (13)

The heliacal rising or setting takes place when the longitude of the Sun equals the longitude of the point of the ecliptic which is technically called the *udayārka* or the *astārka*, respectively. The difference between the longitude of the Sun and that of the *udayārka* or the *astārka* divided by the daily motion of the Sun gives approximately the number of days that have elapsed or are to elapse for the heliacal rising or setting as the case may be. (14)

If in the case of a star, the *astārka* happens to have a longitude greater than the *udayārka* on account of a very big northern latitude, that star does not set heliacally (and so the question of heliacal rising does not arise.) (15)

Circumpolar stars or Sadōdita stars

If a star has a northern declination greater than $90-\phi$ (i.e. $\phi > 90-\delta$), it will be always above the horizon; also if the southern declination is greater than $90-\phi$, such a star will never be seen in a northern latitude, be it Lubdhaka or Agastya or even a planet for the matter or that.[1] (16)

अगस्त्योदयास्तमयौ

18. 9. 1. संख्याविधानात्प्रतिदेशमस्य
विज्ञाय सन्दर्शनमादिशेज्ज्ञः ।
तच्चोज्जयिन्यामगतस्य कन्यां
भागैः स्वराख्यैः स्फुटभास्करस्य ।। १५ ।।

(Varāha, *Bṛ. Saṃ.*, 12.15)

[1] For exposition and rationale, see *SiŚi:AS*, pp. 513-18.

Canopus: Rising

The time of rising of Agastya-Canopus for each region should be determined by calculation and announced by an astronomer. Now, for Ujjain, it takes place when the Sun's true position is 7° short of Sign Virgo (*Kanyā*). (15). (M.R. Bhatt)

18. 9. 2. पलभाऽष्टवधोनसंयुता
'गजशैला' 'वसुखेचरा' लवाः ।
इह तावति भास्करे क्रमाद्
घटजोऽस्तं ह्युदयं च गच्छति ।। २२ ।।
(Gaṇeśa, *GL*, 9.22)

Canopus : Setting

Multiply the equinoctial shadow by 8 and subtract from 78. When the Sun has this much degree, then Canopus sets. Similarly, 98 is added to eight times the equinoctial shadow. When the Sun equals this value, then Canopus rises. (22).[1] (VSN)

[1] For elucidation and rationale, see *GL:RCP*, II. pp. 87-88.

19. ग्रहयुतिः – CONJUNCTION OF PLANETS

ग्रहयुतिभेदाः

19. 1. 1. वियति चरतां ग्रहाणामुपर्युपर्यात्ममार्गसंस्थानाम् ।
अतिदूराद् दृग्विषये समतामिव सम्प्रयातानाम् ।। २ ।।
आसन्नक्रमयोगाद् भेदोल्लेखांशुमर्दनासव्यैः ।
युद्धं चतुष्प्रकारं पराशराद्यैर्मुनिभिरुक्तम् ।। ३ ।।
(Varāha, *Bṛ. Saṃ.*, 17. 2-3)

Types of Planetary conjunctions

When the planets move in the sky along their orbits lying over one another, they appear to our eyes to move on an even surface or plane, as a result of their great distance (from the earth). According to the degree of their seeming approachment, there are four kinds of wars as stated by Parāśara and other sages, viz. *Bheda* (occultation, cleaving), *Ullekha* (grazing), *Aṁśumardana* (clashing of the rays) and *Apasavya* (passing southward).[1] (2-3). (M.R. Bhat)

ग्रहयुतिसाधनतत्त्वम्

19. 2. 1. शीघ्रग्रहाधिके मन्दे योग एष्यन्, गतोऽन्यथा ।
द्वयोर्वक्रेऽन्यथैऽकस्य वक्रे वक्रग्रहेऽधिके ।। १ ।।
अन्यस्माद् योग एष्यन् स्याद् भूतोऽस्मिन्नल्पकेऽन्यतः ।
ग्रहान्तरकला भाज्या भुक्तियोगेन भिन्नयोः ।। २ ।।
भुक्त्यन्तरेण समयोः, दिनाद्यं लभ्यते फलम् ।
एष्यद्योगे गतिस्तेषां योज्या, शोध्या गते सति ।। ३ ।।
विपर्ययाद् वक्रगयोरेकस्मिन्नथ वक्रगे ।
धनमेष्यत्यृणं भूते प्राग्यायिनि फलं ग्रहे ।। ४ ।।
विपर्ययाद् वक्रगते स्यातां समकलावुभौ ।
(*VK*, 4. 1-5a)

Conjunction : Principle of computation

When the longitude of a body having a slower direct motion is greater, the conjunction is to come; otherwise the conjunction is gone. But if the bodies are in retrograde motion, then the conjunction is gone, and to come, respectively. If one body alone is retrograde and its longitude is greater, then the conjunction is to come and if less, it is gone. (1-2a)

Find the difference in longitude between the two bodies in minutes. Divide this by the difference of their daily motion in minutes if both are either direct or retrograde; divide by the sum of their motion if one alone is retrograde and the other direct. Days etc. are obtained. (Thus the time of conjunction is got). (2b-3a)

In the case of direct motion add it to the longitude of the body if the conjunction is to be and subtract if the conjunction has gone. In the case of retrograde motion, subtract and add, respectively. The longitude of the two bodies at conjunction is got, which, or course, will be equal.[1] (3b-5a). (TSK-KVS)

ग्रहयुतिगणनम्
—करणरत्नम्

19. 3. 1. गत्यधिकहीनभावाच्छीघ्रो मन्दो भवेद् ग्रहश्च सदा ।
शीघ्रेऽधिके गतः स्यादेष्यो मन्देऽधिके योगः ।। १३ ।।
ज्ञात्वाऽनतिः समागमकालमथासन्नयोगिनोरुभयोः ।
अधिकादल्पं हित्वा लिप्तीकृत्वाऽन्तरं विभजेत् ।। १४ ।।
भुक्त्यन्तरेण लब्धं फलमन्तरजं दिनानि योगेन ।
वक्रगतावथ भुक्त्या पृथक् पृथक् ताडयेदनयोः ।। १५ ।।
अथ तं विभजेत् षष्ट्या लब्धफलं लिप्तिकास्तु विज्ञेयाः ।
यदि चरतः समगत्या हेया देया गतेऽगते तत्र ।। १६ ।।
हेयधनं विपरीतं वक्रगतौ चेत्स्थितस्तयोरेकः ।
इति समलिप्ताभावः समासतः सूरिभिः ख्यातः ।।१७।।
प्राग्वत् सवर्णयित्वा स्फुटभुक्त्या तद्दिनानि सङ्गुण्य ।
षष्ट्या हृत्वा तत्फलमानीयादृणं धनं तद्वत् ।। १८ ।।
एवं क्रियया विधिवत् संस्कृत्य पृथक् पृथक् क्षिपेच्छोध्यम् ।
तत्कालस्फुटमध्यात् पातोनाच्छनिकुजाङ्गिरसाम् ।।१९।।
सितबुधयोः शीघ्रोच्चाद् बाहुज्यां गुणितपरमविक्षेपः ।
विक्षेपः श्रुतिलब्धः, स्फुटक्रियार्थं समान्यदिशोः ।। २० ।।
रहितः सहितः कार्यस्त्वेकैकस्मिन् क्रमेण विक्षेपः ।
अन्तरम'ब्धि'-प्रहृतम् अङ्गुलयः स्युः ग्रहान्तरजाः ।।२१।।
उभयोरेकत्रगयोः तदा जयत्यधिकविष्कम्भः ।
उभयोरुत्तरगोले विक्षेपेणाधिकस्तदा जयति ।। २२ ।।
उभयोर्दक्षिणगोले तदा जयत्यूनविक्षेपः ।
भिन्नायामाशायामुत्तरगोलस्तदा जयति ।। २३ ।।
(Deva, *KR*, 8. 13-23)

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[1] The conjunction of the planets is known by different names. The conjunction of a planet with Sun is called 'heliacal setting' (*astamaya*); the conjunction of a planet with the Moon is called 'union' (*samāgama*) and the conjunction or any two planets other than the Sun and Moon is called 'encounter' (*yuddha*).

[1] In short, if the greater longitude is A and the lesser B, and the respective motions a and b, (retrograde motion being treated as negative), and t is the time of Conjunction *to be*, $t = (A - B) \div (b - a)$. The longitude at Conjunction $= A + at = B + bt$). For a worked out example see *VK: TSK-KVS*, p. 277.

Computation of conjunction

—Karaṇaratna

A planet is said to be faster or slower (than another planet), according as its velocity is more or less (than that of the other).

If the faster of the two planets has greater longitude, (it should be inferred that) the conjunction of the two planets has aready occurred. If (on the other hand) the slower of the two planets has greater longitude, (it should be understood that) the conjunction of the two planets is to occur. (13)

When one comes to know that the time of conjunction of two planets, in close vicinity, is about to occur one should subtract the planet with smaller longitude from the planet with greater longitude and reduce the difference to minutes. One should then divide that difference (in terms of minutes) by he difference of the daily motions (of the two planets) (in case both the planets are in direct motion), or by the sum of the daily motions (of the two planets) if either of the two planets is in retrograde motion: one should then severally multiply the resulting days, arising from the difference of the two planets, by the daily motions of the two planets and divide (each product) by 60: the result is in terms of minutes. If the two planets are in direct motion, these minutes should be subtracted from or added to the longitudes of the respective planets, according as the conjunction has occurred or is to occur. If either of the two planets is retrograde, subtraction and addition should be reverse for this planet.

This is in brief the method stated by the learned for the equalisation of the longitude of two planets. (14-17).

The Iteration process

Again reduce the difference of the longitudes of the two planets to minutes, as before; (then find the corresponding days); then (severally) multiply those days by the true daily motions of the two planets and divide by 60; and then subtract the (resulting) quotients from or add them to the longitudes of the respective planets, as before. Proceeding in this way, correct the longitudes of the two planets by adding and subtracting the motions of the respective planets. (18-19ab)

The celestial latitude

Subtract the longitude of the planet's ascending node from the instantaneous longitude of the true-mean planet in the case of Mars, Jupiter and Saturn, and from the longitude of the planet's *śīghrocca* in the case of Mercury and Venus; and then multiply the R sine of the *bāhu* of the remainder by the (planet's) greatest latitude and divide by the (planet's *śīghra*) hypotenuse: the result is the (planet's instantaneous) latitude. (19cd-20ac)

Distance between planets in conjunction

To obtain the true distane between (the centres of) the two planets (when they are in conjunction in longitude), one should take the difference or sum of the latitudes of the two planets, according as they are of like or unlike directions. The (resulting) sum or difference divided by 4 gives the distance between (the centres of) the two planets in terms of *aṅgulas*. (20d-21)

When the two planets are together, the planet, with greater diameter is the victor. When both the planets are to the north of the ecliptic, the planet with greater latitude is the victor. (22)

When both the planets are to the south of the ecliptic, the planet with lesser latitude is the victor. When the latitudes of the two planets are of different directions, the planet with north latitude is the victor.[1] (23). (KSS)

—आर्यभटीयार्धरात्रपक्षः

19. 3. 2. कृतयमवसुरसदशकाः पातांशा दशगुणाः कुजादीनाम् ।
नवरविरसार्कमासा विक्षेपकला गुणा दशभिः ।। १ ।।
केन्द्रज्यान्त्यफलज्यागुणिता फलजीवया हृता कर्णः ।
त्रिज्यान्त्यफलज्योना चक्रार्धे संयुता चक्रे ।। २ ।।
भुक्त्यन्तरयुतिभाजितमनुलोमविलोमविवरमाप्तदिनैः ।
अधिकेऽल्पगतावेष्यत्यधिकगतौ ग्रहयुतिरतीता ।। ३ ।।
विवरं स्वभुक्तिगुणितं पृथक् तथैवोद्धृतं क्षयोऽतीते ।
धनमेष्यति समलिप्तो वक्रगतेः क्षयधनं व्यस्तम् ।। ४ ।।
स्वं स्वं विशोध्यं पातं समलिप्तात् सौम्यशुक्रयोः शीघ्रात् ।
जीवा विक्षेपगुणा हृतान्त्यकर्णेन विक्षेपः ।। ५ ।।
एकान्यदिशोरन्तरयोगौ विक्षेपयोर्ग्रहान्तरकम् ।
ग्रहणवदन्यत् साध्यं स्फुटविक्षेपोऽवनत्यैव ।। ६ ।।

(Brahmagupta, *KK*, 1.8. 1-6)

ABh. : Ārdharātrika system

4, 2, 8, 6 and 10, each multiplied by 10, give the degrees in the longitudes of the *pātas* (nodes) of Mars, Mercury, Jupiter, Venus and Saturn, respectively. 9, 12, 6, 12 and 12, each multiplied by 10, give respectively the minutes in the *vikṣepas* of the above planets. (1)

The *jyā* of the *śīghrakendra* of a planet obtained in the fourth operation during the process of finding its true longitude, multiplied by the *jyā* of its maximum *śīghraphala* and divided by the *jyā* of its *śīghraphala* obtained in the fourth operation, gives its *śīghrakarṇa*.

[1] The above rules show that, in general, the planet which lies to the north of the other is the victor. But Venus is always the victor, no matter whether it lies to the north or south of the other planet.

When the *śighrakendra* of a planet is 180°, its *śighrakarṇa* is the *trijyā* less the *jyā* of its maximum *śighraphala*. When the *śighrakendra* is 360°, the *śighrakarṇa* is the sum of *trijyā* and the *jyā* of the maximum *śighraphala*. (2)

Take the difference of the longitudes of the two planets whose conjunction is under consideration. Divide it by the difference of their daily motions, if they are moving in the same direction, or by the sum, if they are moving in the opposite directions. The result is in days, etc. If the slower planet is ahead of the quicker (and if both the planets are moving in the same direction), the conjunction will take place after the time given in the result; if the quicker is ahead of the slower, the conjunction has taken place before the time. (3)

Find the difference between the longitudes of the two planets (whose longitudes are to be made equal). Multiply by it the daily motion of each planet. Divide each of the two products by the difference of the daily motions of the two planets, if they are in the same direction, or by the sum, if they are in opposite directions. Subtract each result from the longitude of the corresponding planet, if the conjunction has taken place, and add, if it is to take place, the planet having a direct motion. If it is retrograde, subtract the result from its longitude if the conjunction is to take place, and add if it has taken place. The two planets will then have equal longitudes. (4)

When two planets have equal longitudes, subtract from each the longitude of the planet's *pāta*. In the case of Mercury and Venus, however, the longitude of the *pāta* should be subtracted from the *śighrocca* of the planet. Find the *jyā* of each of the remaining arcs. Multiply each by the mean *vikṣepa* of the corresponding planet, and divide by its *śighrakarṇa* (*KK*. 1.8.2). The result is the *sphuṭavikṣepa* of the planet. (5)

When two planets are of equal longitudes, the distance between their centres is the difference between the *vikṣepas*, when in the same direction, or is the sum of the *vikṣepas*, if in opposite directions.

All other calculations are the same as those in a solar eclipse. The *vikṣepa* of the lower of the two planets, whose conjunction is being considered, should be corrected by the *avanati*, as in the case of the Moon. (6). (BC)

ब्रह्मगुप्तकृत: शोध:

19. 3. 3. ऊनदिनोदितगुणितादधिकदिनादूनदिनहृताल्लब्धम् ।
अधिकं प्राग्युतिरूनं यद्यधिकदिनोदितात् पश्चात् ॥ १ ॥

अन्तरमाद्यो भूयोऽन्यदिष्टघटिकाफलोनयुतयोश्च ।
प्राक् पश्चाद्वान्तरयोस्तदन्तरेणोद्धृतादाद्यात् ॥ २ ॥

युत्यान्यथेष्टघटिकागुणितात् फलनाडिका यथाद्यवशात् ।
प्राक् समलिप्तिककालात् पश्चाद्वा ग्रहयुतिर्भवति ॥ ३ ॥

स्वदिनघटिकाविभक्तस्तदुदितपरदिवसनाडिकाघात: ।
तुल्य: परोदिताभिर्घटिकाभिर्यदि युतिर्ग्रहयो: ॥ ४ ॥

(Brahmagupta, *KK*, 2.6. 1-4)

Emendation by Brahmagupta

(While considering the conjunction of two planets) multiply the *adhikadina* (the longer day) by the *ūnadinodita-ghaṭikā* (or the *ghaṭikās* passed since the rising of the planet with the shorter day), and divide by the *ūnadina* (the shorter day). If the result is greater than the *adhikadinodita-ghaṭikā* (or the *ghaṭikās* passed since the rising of the planet with the longer day), the conjunction of the planets has taken place. If less, the conjunction will take place. (1)

Find the difference between the above result and the *adhikadinoditaghaṭikā*. Call it *Ādya*.

Now assume some time before or after that, when the planets have equal longitudes, according as the conjunction has taken place or will take place. Find the longitudes of the planets at that time, and hence the lengths of their days, and the number of *ghaṭikās* passed in each day. From these, in the above manner, find the difference, and call it *Anya*. Now if both the *Ādya* and the *Anya* show that the conjunction has taken place, or will take place, find their difference. Multiply the *Ādya* by the assumed time and divide by the difference. The result, in terms of *ghaṭikās* etc., gives the correct time of the conjunction of the planets, either before or after the moment of their having equal longitudes, according as the *Ādya* shows that the conjunction has taken place or will take place.

But if the *Ādya* shows that the conjunction has taken place, and the *Anya* shows that it will take place, or vice versa, then find their sum and proceed as before. (2-3)

Multiply the number of *ghaṭikās* passed in the day of a planet by the number of *ghaṭikās* in the day of the other planet, and divide by the length of the day of the former. If this result is equal to the number of *ghaṭikās* passed in the day of the second planet, there is conjunction of the two planets.[1] (4). (BC)

—भास्कर: १

समकलत्वम्

19. 3. 4. इष्टग्रहान्तरं भाज्यं प्रतिलोमानुलोमगम् ।
भुक्तियोगविशेषेण दिनादिस्तस्य लभ्यते ॥ ४ ॥

[1] For the rationale, see *KK:BC*, I. 155-56.

स्फुटभुक्त्यानुपाताप्तफलेनासन्नयोगिनाम् ।
ग्रहाणां शुद्धिकल्पाभ्यां कुर्यात् समकलावुभौ ।। ५ ।।

पातभागास्ततः शोध्याः शीघ्रोच्चात् सितसौम्ययोः ।
'कृतद्वयष्टर्तुककुभो' द्विगुणास्ते कुजादितः ।। ६ ।।

'नवार्कर्त्वर्करवयो' दशघ्नाः क्षिप्तिलिप्तिकाः ।
पातांशोनभुजामौर्वीसंगुणाः सौम्यदक्षिणाः ।। ७ ।।

विष्कम्भार्धहृतो घातो मन्दशीघ्रोच्चकर्णयोः ।
भूताराग्रहविवरं भागहारः प्रकीर्तितः ।। ८ ।।

विक्षेपलिप्तिका लब्धास्ताभिरन्तरमिष्टयोः ।
एकदिक्त्वे विशिष्टाभिर्युक्ताभिर्भिन्नदिक्कयोः ।। ९ ।।

चतुर्भागाङ्गुला लिप्ता ग्रहयोरन्तरं स्फुटम् ।
वर्णरश्मिप्रभायोगाद्‌ह्यमन्यत् स्वया धिया ।। १० ।।

(Bhāskara I, *LBh.*, 7. 4-10)

—**Bhāskara I**

Common longitude

Divide the difference between the longitudes of the two given planets by the sum or difference of their daily motions according as they are moving in unlike or like directions: then are obtained the days, etc. (elapsed since or to elapse before the time of conjunction of the two planets). The longitude of those two neighbouring planets should then be made equal up to minutes of arc by subtracting from or adding to their longitudes their motions (corresponding to the above days, etc.) obtained by proportion with their true daily motions. (4-5)

Latitudes of the two planets

In the case of Mercury and Venus, subtract the longitude of the ascending node from that of the *śighrocca*: (thus is obtained the longitude of the planet as diminished by the longitude of the ascending node). The longitudes (in terms of degrees) of the ascending nodes of the planets beginning with Mars are respectively 4, 2, 8, 6, and 10, each multiplied by 10.

The greatest latitudes, north or south, in minutes of arc, (of the planets beginning with Mars), are respectively, 9, 12, 6, 12, and 12, each multiplied by 10. (To obtain the R sine of the latitude of a planet), multiply (the greatest latitude of the planet) by the R sine of the longitude of the planet minus the longitude of the ascending node (of the planet) (and divide by the 'divisor' defined below).

The product of the *mandakarṇa* and the *śighrakarṇa* divided by the radius is the distance between the Earth and the planet: this is defined as the 'divisor'.

Thus are obtained the minutes of arc of the latitudes (of the two planets which are in conjunction in longitude). (6-9a)

Distance between the two planets

From those latitudes obtain the distance between those two given planets by taking their difference if they are of like directions or by taking their sum if the are of unlike directions.

The true distance between the two planets, in minutes of arc, being divided by 4 is converted into *aṅgulas*.

Other things should be inferred from the colour and brightness of the rays of the (two) planets or else by the exercise of one's own intellect. (9b-10). (KSS)

—लल्लः

ग्रहसमागमः

19. 3. 5. मन्दकर्णमथ शीघ्रकर्णवत्
संविधाय चलकर्णताडितम् ।
तं भजेत् त्रिभवनज्यया फलं
भूग्रहान्तरमुशन्ति तद्विदः ।। १ ।।

ग्रहाणां समागमकालनिर्णयः

'पञ्च'भि'र्दश'भि'रिन्द्रियेन्दु'भि-
'र्व्योमबाहु'भिरथे'षुलोचनैः' ।
चन्द्रयोजनतनुर्हृताऽऽस्फुजि-
ज्जीवसौम्यशनिभौममूर्तयः ।। २ ।।

व्यासखण्डकृतिघातिता तनु-
र्भूग्रहान्तरहतेन्दुकर्णहृत् ।
योजनत्वमपहाय जायते
सैव तस्य कलिकामयी स्फुटा ।। ३ ।।

योजनानि दशभिर्हृतानि वा
मध्यमाः स्युरथ मानलिप्तिकाः ।
ताडितास्त्रिभवनज्यया पुन-
र्भूग्रहान्तरविभाजिताः स्फुटाः ।। ४ ।।

'नन्द''सूर्य''रस''सूर्यभानवो'
'दिग्गु'णाः शरकलाः कुजादितः ।
'वेद''लोचन''गजा''ङ्क''खेन्दवः'
पातजाः स्युरथ 'दिग्गु'णा लवाः ।। ५ ।।

क्षितिसुतगुरुसूर्यसूनुपाताः
स्वचलफलोनयुता तथा तथैव ।
शशिसुतसितयोस्तु पातभागाः
स्वमृदुफलेन च संस्कृताः स्फुटाः स्युः ।। ६ ।।

ग्रहाणां युतिकालः समकलिकत्वं च

निजगतिविवरेण संविभक्ता
विवरकलास्त्वनुलोमयोर्दिनानि ।
ऋजुगमनविलोमभुक्तियुत्या-
भ्यधिकगतावधिके युतिर्गता तैः ।। ७ ।।

अनधिकगमनेऽधिकेऽथ वक्रि-
ण्यपि चरितान्तरलिप्तिका गतिघ्नाः ।
गतियुतिविवरोद्धृतास्तथैव
स्वफलमृणं स्वमिते गते च योगे ।। ८ ।।

धनमृणमथ तद्विलोमगे स्तः
समकलिकौ समलिप्तिकाद्विपातात् ।
त्रिदशगुरुमहीजसूर्यजानां
भृगुतनयेन्दुजयोस्तथैव शीघ्रात् ।। ९ ।।

निजमध्यमबाणसङ्गुणा
भुजजीवा कुखगान्तरोद्धृता ।
स शरो भवति स्फुटो रवि-
ग्रहणोक्त्या नतिसंस्कृतो विधोः ।। १० ।।

क्षेपयोर्वियुतिरेकगोलयो-
र्या च भिन्नककुभोर्भवेद् युतिः ।
अन्तरं तदुभयोः प्रकीर्तितं
तद्दलैक्यलघुभेदकारकम् ।। ११ ।।

(Lalla, *ŚiDhVṛ.*, 11. 1-11)

—Lalla

Planetary conjunction

Find the *manda* hypotenuse of a planet following the method for the *śighra* hypotenuse. Then multiply it by the *śighra* hypotenuse and divide the product by the radius. The wise astronomers say that the quotient is the distance between the centre of the earth and the planet. (1)

Time of conjunctions

If the diameter of the Moon measured in *yojanas* is divided by 5, 10, 15, 20, and 25, the results are, respectively, the diameters of Venus, Jupiter, Mercury, Saturn and Mars (upon the Moon's orbit, when they are at their mean distances from the Earth). (2)

When the diameter of a planet (expressed in *yojanas*) is multiplied by the square of the radius and divided by the product of the distance between the centre of the Earth and the planet, and the Moon's distance from the Earth, (both in *yojanas*), the result is converted into minutes of the true angular diameter of the planet. (3)

Or, (when the diameter of a planet) in *yojanas* is divided by 10, the quotient is the mean angular diameter in minutes. This again, when multiplied by the radius and divided by the distance between the centre of the Earth and the planet, gives the true angular diameter in minutes. (4)

The latitudes of Mars, (Mercury, Jupiter, Venus and Saturn when at their mean distances from the Earth), are, respectively, 9, 12, 6, 12 and 12 minutes, each multiplied by 10.

4, 2, 8, 6 and 10, each multiplied by 10, are, respectively, the degrees (in the longitudes) of the nodes (of the above planets). (5)

The nodes of Mars, Jupiter and Saturn become more accurate when corrected by adding to their longitudes (given above), or subtracting from time, the respective *śighraphalas* of the planets, according as these are additive or subtractive. But the nodes of Mercury and Venus become more accurate when corrected by adding to the longitudes, or subtracting from them, their respective *mandaphalas*, according as they are additive or sub-subtractive.[1] (6).

Time of conjunction and equality of longitudes

If two planets are moving in the same direction, divide the difference of their longitudes in minutes by the difference of their true daily motions. The quotient gives the days elapsed since the conjunction, if the faster of the two planets is ahead. If the slower is ahead, the conjunction will take place after so many days. (7)

If, again, one planet is direct and the other retrograde, divide the difference of their true longitudes, expressed in minutes, by the sum of their true daily motions. If the retrograde planet has the greater longitude, the conjunction will take place after the days in the quotient; if lesser, the conjunction has taken place. (8)

Multiply the difference between the true longitudes of the two planets severally by the true daily motion of each planet and divide each product by the sum of difference of their true daily motions, as the case may be. Add the quotients to the respective longitudes of the planets if the conjunction is to take place, and subtract if the conjunction has taken place. If the planet is retrograde, add the quotient if the conjunction has taken place and subtract if it is to take place. Then the two planets will have true equal longitudes expressed in minutes.

Now, in the case of Mars, Jupiter and Saturn, subtract the correct longitudes of their respective nodes from their true longitudes when thus made equal. But in the case of Mercury and Venus, subtract the correct longitudes of their nodes from the longitudes of their respective *śighroccas* at the time when the planets themselves have true equal longitudes. (9)

Find the R sine of the remainder in each case. Multiply it by the greatest mean (geocentric) latitude of the planet and divide by its distance from the centre of the Earth. The result is the true (geocentric) latitude

[1] For mathematical notes see *ŚiDhVṛ:BC*, II. pp. 177-79.

of the planet. This again should be corrected by the latitudinal parallax of the planet as in the case of the Moon, following the method given (in the chapter on) Solar eclipse. (10)

The differences of the latitudes (of the two planets of equal longitudes), when of the same denomination, or their sum when of different denominations, is said to be the distance between the centres of the two planets. When this distance is less than the sum of their radii, the conjunction is called *Bhedayuti*.[1] (11). (BC)

ग्रहाणां बिम्बमानम्

19. 3. 6. 'व्यङ्घ्रीषवः' 'सचरणा ऋतवः' 'त्रिभाग-
युक्ताद्रयो' 'नव' च 'सत्रिलवेषव'श्च ।
स्युर्मध्यमास्तनुकलाः क्षितिजादिकानां
त्रिज्याशुकर्णविवरेण पृथग्विनिघ्न्यः ।। १ ।।

त्रिघ्न्या निजान्त्यफलमौर्विकया विभक्ता
लब्धेन युक्तरहिताः क्रमशः पृथक्स्थाः ।
ऊनाधिके त्रिभगुणाच्छ्रवणे स्फुटा स्युः
कल्प्यं खलु त्रिकलमङ्गुलमत्र बिम्बे ।। २ ।।

युतिकालज्ञानम्

दिवौकसोरन्तरलिप्तिकौघाद्
गत्योर्वियोगेन हृताद्यदैकः ।
वक्री जवैक्येन दिनैरवाप्तै-
र्याता तयोः संयुतिरल्पभुक्तौ ।। ३ ।।

वक्रेऽथवा न्यूनतरेऽन्यथैष्या
द्वयोरनृज्वोर्विपरीतमस्मात् ।
दृक्कर्म कृत्वायनमेव भूयः
साध्येति तात्कालिकयोर्युतिर्यत् ।। ४ ।।

एवं कृते दिविचरौ ध्रुवसूत्रसंस्थौ
स्यातां तदा वियति सैव युतिर्निरुक्ता ।
दृक्कर्मणायनभवेन न संस्कृतौ चेत्
सूत्रे तदा त्वपमवृत्तजयाम्यसौम्ये ।। ५ ।।

(Bhāskara II, *SiŚi.*, 1.10. 1-5)

Angular diameters of planetary orbits

The mean diameters of Mars, Mercury, Jupiter, Venus and Saturn are respectively 4′ 45″, 6′ 15″, 7′ 20″, 9′, 5′ 20″. (1)

These estimates being multiplied by the difference of the radius and *śighrakarṇa* and divided by thrice the *śighra-antyaphalajyā* are to be added or subtracted from the mean values above given according as the *śighrakarṇa* is less or greater than the radius to give the rectified values. Three minutes of arc to be constructed as one *aṅgula* in this respect.[1] (2)

Time of conjunction

To obtain the time of the conjunction of two planets, compute the difference of the longitudes of the two planets, and divide by the difference of their daily motions. If one of the planets be retrograde, divide by the sum of the daily motions. The result gives the number of days approximately after the moment of conjunction if the slower planet has a longitude falling short of that of the quicker. (3)

If one of the planets be retrograde, and if its longitude be the lesser, then also the conjunction was past by the number of days computed. In the other cases the conjunction is to take place after the number of days computed. If, however, both the planets be retrograde, then if the slower of them has a longitude less than that of the quicker, then the conjunction is ahead, otherwise past by the number of days. (4a)

Having computed the approximate time of conjunction, obtain the true motions of the planets pertaining to that day, rectifiy them for *āyana-dṛkkarma* and following the process indicated in verse (3) above, again compute the moment of conjunction. (This will be a good approximation). This conjunction will be the one on the polar latitudinal circle. If *āyana-dṛkkarma* be not done, then the conjunction will be on the circle of celestial latitude. (4b-5). (AS)

19. 3. 7. 'पञ्च-र्त्वं-गा-ङ्क-विशिखाः' पृथ'गीश'कर्णा-
योगाहताः 'प्रकृति-भा-न्वरि-सिद्ध-रामैः' ।
भक्ताः फलोनसहिताः श्रवणेऽधिकोने
ते व्युद्धृताः स्युरसृजो वपुरङ्गुलानि ।। १ ।।

युतेर्यातैष्यता

अधिकजवखगेऽधिकेऽल्पभुक्ते-
रथ कुटिलेऽल्पतरेऽनुलोमतो वा ।
अनृजुगखगयोस्तु शीघ्रगेऽल्पे
युतिरनयोः प्रगताऽन्यथा तु गम्या ।। २ ।।

गतगम्यदिवसाः

ऋजुगतिखगयोस्तु वक्रयोर्वा
विवरकला गतिजान्तरेण भक्ताः ।
गतिजयुतिहृता यदैकवक्री
युतिरगता प्रगताप्तवासरैः स्यात् ।। ३ ।।

दक्षिणोत्तरान्तरम्

चाल्यौ खेटौ समौ स्तो
ग्रहयुतिदिवसैश्चन्द्रबाणः स्वनत्या
संस्कार्योऽत्र ग्रहौ स्वेषुदिशि
समदिशोस्त्वल्पबाणोऽपरस्याम् ।

[1] For the rationale, see *ŚiDhVṛ:BC*, II, pp. 179-83.

[1] For comments, see *SiŚi:AS*, pp. 501-02.

एकान्याशौ यदेषू विरहित-
सहितौ खेटमध्येऽन्तरं स्याद्
भेदो मानैक्यखण्डादिह लघुनि
तदाल्पं हि किं लम्बनाद्यम् ।। ४ ।।

(Gaṇeśa, *GL*, 13. 1-4)

Mean diameters of Mars and other planets

Mars 5, Mercury 6, Jupiter 7, Venus 9, Saturn 5 (minutes): Multiply this, in order, by the difference between 11 and the *śighrakarṇa* of the planet. Divide this result respectively by 21, 12, 6, 24, and 3. (Let the result be x.)

If the hypotenuse (*karṇa*) is more than 11, subtract x from the first table; add x if it is otherwise. (Let the result by y). Then y divided by 3 gives the diameter of the planet in *aṅgulas*. (1)

Conjunctions gone and yet to occur

If the planet with quicker daily motion has a greater longitude, then the conjunction has taken place already. If the retrograde planet is lesser in longitude than the other in direct motion, then also conjunction is over. When both the planets are retrograde, if the planet with quicker motion has lesser longitude, then conjunction has already taken place. In the reverse case, conjunction is yet to occur. (2)

Number of days before or after conjunction

When both the planets are in direct or retrograde motion, find the difference of their longitude and divide by the difference of their daily motions. The quotient gives the number of days. Both values should be in *kalās*.

If one is retrograde, divide by the sum of their daily motions. The number of days before or after the conjunction is to be decided as before. (3)

North-south latitudinal difference

Rectify the moment of the conjunction after knowing the number of days (from verse 3). The planets will have the same value is *rāśis*, degrees etc. The latitude of the moon is to be corrected for its parallax.

The direction of the planet and its latitude will be the same. If the two latitudes are in the same direction, then the planet with lesser value is in the direction opposite to that of the other.

If the directions of the latitudes are the same, take their difference; if the direction are different, add. That gives the difference in *aṅgulas* between their discs, (x). If x is less than the sum of radii of the two discs, the two planets intersect. Since the *lambana* is very small, what is the use of these factors here?[1] (4). (VSN)

[1] For explanation and rationale, see *GL:RCP*, II. pp. 114-20.

20. ग्रहनक्षत्रयुतिः – CONJUNCTION OF STARS AND PLANETS

ग्रहनक्षत्रयुतिः विक्षपांशाश्च

20. 1. 1. योगभागसमः सर्वः संयुक्तो लक्ष्यते ग्रहः ।
अधिकोनकलाकालविज्ञानं चानुपाततः ॥ ५ ॥

विक्षपांशाः

उदग्दिशोऽर्कभूतानि याम्ये पञ्च दिशो भवाः ।
उदग् रसास्तया व्योम दक्षिणे मुनयोऽम्बरम् ॥ ६ ॥

उदगर्कास्तथा विश्वे दक्षिणे मुनयोऽश्विनौ ।
सौम्ये रसकृतिः सैका याम्ये सार्धास्त्रयाग्नयः ॥ ७ ॥

अब्धयो वसवः सार्धाः सप्त शैलास्ततः परम् ।
उदक् त्रिंशत् कृतिः षण्णां याम्ये लिप्तास्त्रिषट्ककाः ॥

उदगर्काश्च विश्वे च द्विरभ्यस्ता नभस्तथा ।
विक्षेपांशाः क्रमाद् दृष्टाः पण्डितैर्वाजिभादितः ॥ ९ ॥

(Bhāskara I, *LBh.*, 8. 5-9)

Conjunction of stars and planets

All planets, whose longitudes are equal to the longitude of the junction-star of a *nakṣatra* can be seen in conjunction with that star. (Of a planet and a star) whose longitudes are unequal, the time of conjunction has to be determined by proportion. (5)

Latitudes of the junction-stars

North, ten, twelve, five; south, five, ten, eleven; north, six, zero; south seven, zero; north, twelve, thirteen; south, seven, two; north, thrity-seven; south, one and a half, three, four, eight and a half, seven, seven; north, thirty, thirty-six; south, eighteen minutes of arc; north, twenty-four, twenty-six, and zero—these have been stated by the learned to be the degrees (unless otherwise stated) of the latitudes of the junction stars of the *nakṣatras* beginning with Aśvinī in their serial order. (6-9). (KSS)

ग्रहतारासमागमन्यायः

20. 2. 1. ध्रुवकादूनः पश्चादधिकः प्राग्वक्रितेऽन्यथा योगः ।
अन्यद्ग्रहमेलकवद् ध्रुवक्रान्तेर्भविक्षेपाः ॥ ७ ॥[1]

(Brahmagupta, *KK*, 1. 9. 7.)

[1] *Dhruvaka* of a *yogatārā* is the distance on *krāntimaṇḍala* between the beginning of Meṣa and the point where the *dhruvaprota* passing through the *yogatārā* meets the *krāntimaṇḍala*.
Vikṣepa of a *yogatārā* is the distance of the *yogatarā* from its *dhruvaka* measured on *dhruvaprota*.

Principle of star-planet conjunction

If the longitude of a planet corrected by the *āyanadṛkkarmakalā* is less than the *dhruvaka* of the *yogatārā*, the conjuction is yet to take place; if greater, the conjunction has taken place. This rule is applicable when the planet has direct motion. If the planet is retrograde, the rule is reversed. The rest of the calculation (as regards the time of conjunction, etc.) is the same as in the case of the conjunction of the two planets. (7)

ग्रहताराच्छादनम्

20. 3. 1. छादयति योगतारां मानार्धोनाधिकाद् भविक्षेपात् ।
स्फुटविक्षेपो यस्याधिकोनको भवति स समदिक्स्थः ॥

विक्षेपोंऽशद्वितयादधिको वृषस्य सप्तदशभागे ।
यस्य ग्रहस्य याम्यो भिनत्ति शकटं स रोहिण्याः ॥ १५ ॥

विक्षेपेऽन्त्ये सौम्ये तृतीयतारां भिनत्ति पित्र्यस्य ।
इन्दुर्भिनत्ति पुष्यं पौष्णं वारुणमविक्षिप्तः ॥ १६ ॥

(Brahmagupta, *KK*, 1. 9. 14-16)

Occultation of stars by planets

When a planet is on the same side of the ecliptic as the *yogatārā* of any *nakṣatra*, the planet will occult the *yogatārā*, if its *sphuṭavikṣepa* is either greater than the difference of its semi-diameter and the *vikṣepa* of the *yogatārā* or less than the sum of the two.[1] (14)

When the longitude of a planet, corrected by the *āyanadṛkkarmakalā*, is 1 sign 17° and its *vikṣepa* is greater than 2° S, it occults the Cart of Rohiṇī. (15)

When the Moon has the maximum north *vikṣepa* it occults the third star of Maghā. When it has no *vikṣepa* it occults Puṣya, Revatī and Śatabhiṣaj. (16). (BC)

रोहिणीशकटभेदः

20. 4. 1. वृषे सप्तदशे भागे यस्य याम्योंऽशकद्वयात् ।
विक्षेपोऽभ्यधिको हन्याद् रोहिण्याश्शकटं तु सः ॥ १३ ॥
ग्रहवद् द्युनिशे भानां कुर्याद् दृक्कर्म पूर्ववत् ।
ग्रहमेलनविज्ञेयं ग्रहभुक्त्या दिनादिकम् ॥ १४ ॥

(*SūSi.*, 8. 13-14)

Occultation of Rohiṇī by planets

In Taurus, the seventeenth degree, a planet of which the latitude is a little more than two degrees, south, will

[1] For a demonstration see, *KK:BC*, I. p. 135-36.

split the wain of Rohiṇī (i.e., the V-shaped constellation of Hyades. (13)

Calculate, as in the case of the planets, the day and night of the asterisms, and perform the operation for apparant longitude (*dṛkkarman*), as before: the rest is by the rules for the conjunction (*melaka*) of planets, using the daily motion of the planet as divisor: the same is the case as regards the time. (14). (Burgess)[1]

20. 4. 2. गवि'नगकु'लवे खगोऽस्य चेद्
यमदिगिषु: 'खशरा'ङ्गुलाधिक: ।
कभशकटमसौ भिनत्त्यसृक्
शनिरुडुपो यदि चेज्जनक्षय: ।। ७ ।।
स्वर्भानावदितिभतोऽष्टऋक्षसंस्थे
शीतांशु: कभशकटं सदा भिनत्ति ।
भौमार्क्यो: शकटभिदा युगान्तरे स्यात्
सेदानीं न हि भवतीदृशि स्वपाते ।। ८ ।।
(Gaṇeśa, *GL*, 11. 7-8)

If any planet with 17 degrees in Vṛṣabha has its southern latitude exceeding 50 *aṅgulas*, it cuts the *śakaṭa* of the star Rohiṇī (i.e. the V-shaped constellation of Hyades). If Mars, Saturn and Moon do so, then it brings greater calamity to the people. (7)

If Rāhu stays in any of the eight stars counting from Punarvasu the Moon always cuts Rohiṇī.

This *bheda* with respect to Mars and Saturn will happen in the next aeon (*yuga*). Their *pātas* are such that it will not happen in this *yuga*.[2] (8)

चन्द्रनक्षत्रयोग:

20. 5. 1. यावत्या यद्दिशाक्षिप्त्या यावांस्तारासमागम: ।
तावत्या तद्दिशाक्षिप्त्या तावानिन्दु: समो भवेत् ।। १० ।।
अष्टिर्दशगुणा लिप्ता विक्षेपस्य यदोत्तरे ।
निरुणद्धि तदा व्यक्तं कृत्तिकातारकां शशी ।। ११ ।।
उत्तरां परमां क्षिप्तिं गत्वा शिशिरदीधिति: ।
आवृणोति स्वबिम्बेन मघामध्यस्थतारकाम् ।। १२ ।।
आरोहति शशी षष्टया प्राजेशशकटं स्फुटम् ।
अष्टिवर्गेण याम्यायां योगतारा विलिख्यते ।। १३ ।।
याम्यगं पञ्चहीनेन शतेन त्वाष्ट्रतारकम् ।
मैत्रं शतेन सार्धेन द्विशत्या शक्रतारकम् ।। १४ ।।
सप्ताशीत्या शशी हन्ति तारां सौम्यविशाखयो: ।
याम्यगो दक्षिणाशास्थो व्यक्तं शतभिषग्जिनै: ।। १५ ।।
पुष्यं पौष्णं च पातस्थो निरुणद्धि निशाकर: ।
यष्टियुक्तकलाक्षिप्त्या भेद: स्याद् ग्रहधिष्णयो: ।। १६ ।।
(Bhāskara I, *LBh.*, 8. 10-16)

[1] For notes, see *Sū.Si : Burgess*, pp. 248-49.

[2] For the rationale see *GL: RCP*. II. p. 103-4.

Occultation of stars by the Moon

The Moon is in (absolute) conjunction with a junction-star when its longitude and celestial latitude, both in magnitude and direction, are the same as the longitude and celestial latitude of the star both in magnitude and direction. (10)

Latitudes of the Moon occulting certain stars

When the Moon atains 160 minutes (of arc) of north latitude, it clearly covers the junction-star of the *nakṣtara* Kṛttikā (i.e., the Pleiades). (11)

Having attained her maximum northern latitude, the Moon covers with its disc the central star of the *nakṣatra* Maghā. (12)

With its latitude 60′ (south), the Moon clearly occults the cart of Rohiṇī (i.e., the V-shaped constellation of Hyades); and with latitude 256′ south it covers the junction-star (of Rohiṇī) (i.e., Aldebaran). (13)

With its latitude 95 (minutes of arc) south, (the Moon covers the junction-star of) the *nakṣatra* Citrā (i.e., Spica); with 150 (minutes of arc) south, (the junction-star of) the *nakṣatra* Anurādhā; and with 200 (minutes of arc) (south), (the junctuion-star of) the *nakṣatra* Jyeṣṭhā (i.e., Antares). (14)

With latitude 87 (minutes of arc south), the Moon clearly occults (the brighter of) the two northern stars of the *nakṣatra* Viśākhā; with 24 (minutes of arc) south, (the junction-star of) the *nakṣatra* Śatabhiṣak (i.e., Aquarii). (15)

The Moon, situated at its ascending Node, occults (the junction-star of) Puṣya and Revatī (i.e., Piscium).

The above occultations (*bheda*) of the stars by the planet (Moon) are based on the minutes of latitude determined from actual observation by means of the instrument (called) Yaṣṭi. (16). (KSS)

20. 5. 2. प्राजापत्यदले स्थितस्तु हिमगुर्याम्यै: शरांशैस्त्रिभि-
र्वित्र्यंशै: शकटं भिनत्ति विदलैस्तै: पञ्चभी रोहिणीम् ।
सौम्यै: पञ्चभिरंशकैश्च विदलैस्तारां मघामध्यमां
विक्षेपेण विवर्जितस्तु गुरुभं पौष्णं तथा वारुणम् ।।
(Lalla, *ŚiDhVṛ.*, 11. 11)

The Moon, when in the middle of the asterism of Rohiṇī, pierces the Cart (of Rohiṇī), if its southern latitude is 2° 40′, and occults the principal star, Rohiṇī, when its southern latitude is 4° 30′.

Again, the Moon occults the middle star of the asterism Maghā, when it has a north latitude of 4° 30′. When it has no latitude, it occults Revatī, Puṣya and Śatabhiṣā. (11). (BC)

नक्षत्रग्रहयोगसमयः

20. 6. 1. विधेयमायनं ग्रहे स्वदृष्टिकर्म पूर्ववत् ।
स्फुटश्च खेटसायको ग्रहर्क्षयोगसिद्धये ॥ ९ ॥

युतिकालज्ञानम्

ग्रहध्रुवान्तरे कला नभोगभुक्तिभाजिताः ।
गतागताप्तवासरैर्युतिर्ग्रहेऽधिकोनके ॥ १० ॥

विलोमगे नभश्चरे गतैष्यताविपर्ययः ।
ग्रहर्क्षदक्षिणोत्तरान्तरं नभोगयोगवत् ॥ ११ ॥

(Bhāskara II, *SiŚi*.. 1.11. 9-11.)

Time of star-planet conjunction

The *āyana-dṛkkarma* is to be done (with respect to the planet) and the *sphuṭaśara* is to be computed to know the time of (polar latitudinal) conjunction. (9)

The difference in the longitudes of the planet and the star, divided by the daily motion of the planet, gives the number of days approximately after or before the moment of conjunction.[1] (10)

If the planet be retrograde, the conjunction past or future will be in the reverse, i.e. future or past. (11). (AS)

[1] For the rationale, see *SiŚi:AS*, pp. 512-13.

21. नूतना आविष्काराः – NOVEL INNOVATIONS

राशिगोलस्फुटानीतिः

ग्रहणबीजम्

21. 1. 1. इन्दुमार्गतिरश्चीनं द्विस्पृक्सूत्रं भवेद् यदा ।
तदैव परमासत्तिर्ग्रहणे सोमसूर्ययोः ॥ १ ॥

मध्यकाले तु न तथा द्विस्पृक्सूत्रस्य संस्थितिः ।
परमग्रासकालोऽतो भिद्यते मध्यकालतः ॥ २ ॥

(Acyuta, *Rāśigola.*, 1-2)

True longitude computation on the sphere of zodiac

Essence of an eclipse

In an eclipse,[1] the maximum nearness of the Sun and the Moon occurs when the straight line joining the centres of the two (*dvi-spṛk-sūtra*) is perpendicular to the Moon's orbit. (1)

At the moment of conjunction or opposition (of the Sun and the Moon) (*madhyakāla*[2] or *sphuṭa-parvānta*), the line joining the centres is not so, (i.e., not perpendicular to the Moon's orbit). Consequently, the moment of maximum eclipse is different from the moment of conjunction or opposition. (2). (KVS)

पर्वान्ते पक्षद्वयम्

21. 1. 2. अर्कमार्गतिरश्चीनां रेखां प्राप्नोति चन्द्रमाः ।
यदा, तदा स्फुटैक्यं स्याद्, इति केचन सूरयः ॥ ३ ॥

पाताद् यावतिथे भागे क्रान्तिवृत्ते रविर्भवेत् ।
विक्षेपमण्डले चन्द्रः पातात् तावतिथेऽन्तरे ॥ ४ ॥

यदा भवेत्, तदैव स्यात् पर्वान्त इति केचन ।
अर्कमार्गतिरश्चीनसूत्रात् पातानुसारतः ॥ ५ ॥

प्राग्वा पश्चाच्च भवति वर्त्मसाम्यं, ततोऽपि च ।
पातासन्नप्रदेशे स्यात् सन्निकर्षः परस्तयोः ॥ ६ ॥

व्यहीन्दावोजपदगे पातः पृष्ठगतस्तयोः ।
पुरोगतो युग्मपदे; तस्मान्मध्याख्यकालतः ॥ ७ ॥

परमग्रासकालोऽयं प्राक्पश्चाच्च भवेत् क्रमात् ।
आसत्तिकालः पर्वान्ताद् भिन्नः पक्षद्वयेऽप्यतः ॥ ८ ॥

भेदस्य तारतम्ये हि केवलं कलहो भवेत् ।
विविच्य नोक्तो भेदोऽयं यद्यप्यार्यभटादिभिः ॥ ९ ॥

तथापि युक्तिसिद्धत्वात् स्वीकृतो ह्यच्युतादिभिः ।

(Acyuta, *Rāśigola.*, 3-9)

Two views on conjunction

Some scholars are of opinion that the true longitudes (*sphuṭa*) (of the Sun and the Moon) are equal when the Moon reaches the perpendicular drawn from the Sun's *viz.*, the ecliptic) (to cut the Moon's orbit). (3)

Others, however, say that moment of conjucntion (*parvānta*)[1] occurs only when the Moon in its orbit (*vikṣepa-maṇḍala*) is as distant in degrees (*bhāga*) from the Node) (*pāta*) as the Sun is in its orbit. (4-5a)

The point of equality in distance (*vartma-sāmya*) on the Moon's orbit will fall either to the East or West of the perpendicular drawn to the Sun's path according as to whether the (proximate) Node is to its East or its West. The point of closest proximity lies further from the point of equality to the side where the node lies. (5b-6)

If Moon-*minus*-Node (*vyahīndu*) is in odd quadrant, the Node will be behind the two, and if it be in an even quadrant the Node will be in front of them. And therefore the moment of maximum eclipse will, respectively, be before or after the moment of conjunction. (7-8a)

Thus according to both views, the moment of maximum eclipse differs from the moment of conjunction. A dispute can therefore occur only in the estimation of this difference. (8b-9a)

Although this distinction (between the moments of maximum eclipse and the so-called *madhyakāla* or *parvānta*) has not been expressly distinguished by Āryabhaṭa and others, it is recognized by Acyuta and others, since it is obvious by inference. (9b-10a). (KVS)

[1] The expression 'in an eclipse' (*grahaṇe*) presupposes that the centre of the Sun and the Moon in their respective orbits are very near the Node, so that the small arcs from the Node to the centres may be taken as straight lines. Thus, the Node, the Sun and the Moon may be taken as the vertices of a triangle. Then the least distance of a point to a straight line is the perpendicular from that point to the straight line.

[2] In Indian astronomy by *madhyakāla* is meant the *sphuṭaparvānta* and not, as it would seem from the term, the moment of the middle of the eclipse. Cf., *Sūryasiddhānta*, 4. 16 : *Sphuṭatithyavasāne tu madhyagrahaṇam ādiśet.*

[1] *Parvānta* really applies to both the moments of conjunction and opposition but, for the sake of facility of expression, it is translated by 'conjunction' alone (which is the case in solar eclipses), here as also below.

पर्वान्तनिर्णयः—प्रथमः पक्षः

21. 1. 3. मध्यकाले ह्यर्कबिम्बात् स्फुटक्षेपान्तरे विधुः ।। १० ।।

ततः प्रागथवा पश्चादर्कासत्तिर्भवेदिति ।
ज्ञातुं तत्कालसरणिर्विज्ञेया शीतदीधितेः ।। ११ ।।

विधोस्तत्कालसरणिः कथं ज्ञेयेति चेच्छृणु ।
द्युषष्ट्यंशजभूपृष्ठस्फुटगत्यन्तरं भुजम् ।। १२ ।।

दिनषष्ट्यंशजस्पष्टक्षेपखण्डः शराभिधः ।
तयोर्वर्गैक्यमूलं स्यात् तत्कालसरणिर्विधोः ।। १३ ।।

अर्कमार्गानुसारी स्याद् बाहुस्तद्वयस्तदिक् शरः ।
चन्द्रमार्गात्मकः कर्णः प्रमाणक्षेत्रमीदृशम् ।। १४ ।।

मध्यक्षेपः श्रुतिर्बाहुः परमासत्तिकालजम् ।
बिम्बान्तरं चन्द्रमार्गे तयोरग्रान्तरं शरः ।। १५ ।।

इच्छाक्षेत्रमिदं; तत्र ज्ञातेन श्रवणेन हि ।
अज्ञातौ दोःशरौ नेयावनुपातेन; तद्यथा ।। १६ ।।

प्रमाणक्षेत्रकर्णस्य यद्येतौ बाहुसायकौ ।
तदेच्छाक्षेत्रकर्णस्य कौ स्तो बाहुशराविति ।। १७ ।।

इच्छाक्षेत्रगतौ स्यातां बाहुबाणौ; तयोः शरः ।
गत्यन्तरात्मको ज्ञेयो यतोऽसौ चन्द्रमार्गतः ।। १८ ।।

दिनषष्ट्यंशसम्बन्धिस्फुटगत्यन्तरात्मना ।
प्रमाणक्षेत्रकर्णेन नाडिकैका भवेद् यदि ।
तदेच्छाक्षेत्रबाणे कः काल इत्यनुपाततः ।। १९ ।।

परमग्रासकालस्य भेदः स्यान्मध्यकालतः ।
द्विः कृत्वो हरणं कार्यं कर्णेनैवं यतस्ततः ।। २० ।।

मध्यक्षेपाच्छरहतात् कर्णकृत्याप्तनाडिकाः ।
स्वर्णं कुर्यान्मध्यकाले तयोर्दिग्भेदसाम्यतः ।। २१ ।।

परमग्रासकालोऽयं, मध्यक्षेपाद् भुजाहतात् ।
कर्णाप्तो बिम्बभेदः स्यात् परमग्रासकालजः ।। २२ ।।

तदूनं बिम्बयोगार्धं भवेद् ग्रासप्रमा परा ।
विधोरर्ककलाप्राप्तिः पर्वान्त इति यन्मतम् ।। २३ ।।

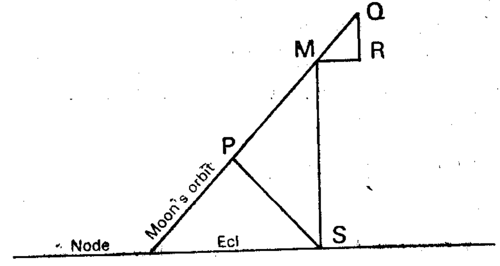
तन्मते प्रोक्तमखिलं युज्यते नात्र संशयः ।

(Acyuta, *Rāśigola.*, 10b-24a)

Determination of Conjunction: First view

At the moment of conjunction, the angular distance of the Moon from the Sun is equal to its latitude corrected for parallax (*sphuṭa-vikṣepa*). In order to know where the point of maximum nearness to the Sun will occur, to the East or West of (the moment of conjunction), it is necessary to scan the path of the Moon at the moment. (10b-11)

If you desire to know the path of the Moon at that moment, listen.



(Let) the (*bhujā*) be one-sixtieth of the difference between the daily motions (of the Sun and the Moon) (*dyu-gatyantara*) as corrected for parallax (*bhūpṛṣṭha-sphuṭa*), (MR); and let one-sixtieth part of the variation in latitude (*kṣepa-khaṇḍa*) per day as corrected for parallax (*spaṣṭa*) be what is called the 'altitude' (*śara*, the other side of the right-angled triangle), (QR). Then the root of the sum of their squares (MQ) will give the Moon's path for that period (*viz.* one *nāḍikā*). (12-13)

The base (of the right-angled triangle so formed) is parallel to the Sun's path, the altitude perpendicular to it and the hypotenuse along the orbit of the Moon. Thus is established the antecedent triangle (*pramāṇa-kṣetra*) (MRQ) (where the values of all the sides are known). (14)

The latitude at the moment of conjunction (*madhyakṣepa*) (SM) is the hypotenuse, the line joining (the centres of) the orbs (of the Sun and the Moon) at the moment of maximum eclipse (SP), the base, and the line along the Moon's orbit joining the ends of the above two lines (PM), altitude: thus is the consequent triangle (*icchā-kṣetra*) (SPM) established. (15-16a)

Here, (in this consequent triangle), since the hypotenuse, (which is the latitude at the moment of conjunction), is known, the base and altitude can be derived by proportion (with the antecedent triangle). This is how it is: (16b)

If such are the base and altitude of the antecedent triangle, what would be the base and altitude of the consequent triangle? (By means of this argument will be derived the base and altitude of the consequent triangle:

$$SP = \frac{SM \cdot MR}{MQ}; \qquad MP = \frac{SM \cdot QR}{MQ} \qquad (17)$$

Of these two, the altitude, since it lies along the Moon's orbit, should be understood to have its value in terms of the relative motions (*gaty-antara-ātmaka*) (per *nāḍikā* projected upon the Moon's orbit). (18)

And if the hypotenuse of the antecedent triangle (MQ), which represents the relative motions (of the

Sun and the Moon) for one-sixtieth of a day, is one *nāḍkā*, what time will the altitude of the consequent triangle represent? This proportion will give the time between the moments of maximum eclipse and conjunction, *i.e.*:

$$\frac{MP}{MQ} = \frac{SM \cdot QR}{MQ \cdot MQ}. \qquad (19\text{-}20a)$$

Since, (in the above calculation), division with the hypotenuse (MQ of the antecedent triangle) is to be done twice, the resultant *nāḍikās* (are) obtained by multiplying the latitude at conjunction by the altitude, and dividing the product by the square of the hypotenuse (of the antecedent triangle, *i.e.*, $\frac{SM \cdot QR}{MQ^2}$.) (20b-21a)

(These *nāḍikās*) are to be added to, or subtracted from, the moment of conjunction, according as to whether the directions (North or South) of the two, (*viz.*, the latitude and altitude), are different or the same, respectively.[1] This is the moment of maximum eclipse. (21b-22a)

Multiply the latitude at conjunction (SM) by the base (MR) and divide by the (antecedent) hypotenuse (MQ); the result will be the distance between (the centres of) the orbs) (of the Sun and the Moon) at the moment of maximum eclipse, *i.e.*, $SP = \frac{SM \cdot MR}{MQ}$. (22b)

This, subtracted from half the sum of the angular diameters (*bimba*) (of the Sun and the Moon), will give the maximum eclipse. (23a)

All that is said here accords, in fact, only with the school that says that the moment of conjunction is when the Moon attains the same (number of) minutes (*kalās*) as the Sun, (both measured along the ecliptic). (23b-24a). (KVS)

पर्वान्ते द्वितीयपक्षखण्डनम्

21. 1. 4. वर्त्मसाम्यं हि पर्वान्त इति पक्षे विधोर्गतिः ।। २४ ।।

क्षेपवृत्तानुसारी स्यात् ततोऽस्य बाहुता कथम् ।
तर्हि गत्यन्तरांशस्य बाहुत्वं नेह कल्प्यते ।। २५ ।।

किन्तु कर्णत्वमिति चेद्, भवत्वेवं शशिग्रहे ।
रविग्रहे तु कर्णत्वमिति तस्या न युज्यते ।। २६ ।।

यतो नतिवशादिन्दोर्वर्त्म भिन्नं प्रतिक्षणम् ।
अनुगत्यन्तरांशस्य बाहुत्वं भास्वतो ग्रहे ।। २७ ।।

बाणत्वं नतिखण्डस्य पक्षेऽस्मिन् यदि कल्प्यते ।
तदा तयोर्वर्गयोगमूलस्यात्रेन्दुमार्गता ।। २८ ।।

इति चेन्न यतो नात्र तयोर्दोःकोटिरूपता ।
तथा हि नतिखण्डः स्याद्वचस्तदिक्कोऽर्कमार्गतः ।। २९ ।।

गत्यन्तरांशकश्चास्मिन् पक्षे स्यादिन्दुमार्गतः ।
अतोऽनयोर्मिथो न स्याद्दोःकोटित्वमिति स्थितम् ।।३०।।

किञ्च लम्बनखण्डेन घटिकाकालजेन च ।
ऊनितो गतिभेदांशो बाहुर्ज्ञेयः पुरोदितः ।। ३१ ।।

तत्र लम्बनखण्डोऽयं रविमार्गगतो भवेत् ।
गत्यन्तरांशकश्चात्र चन्द्रमार्गगतो यतः ।। ३२ ।।

ततोऽनयोर्वियोगोऽपि वस्तुतस्तु न युज्यते ।
एतद्दोषनिरासार्थं यद्यत्र नतिलम्बने ।। ३३ ।।

तयोः खण्डौ च नीयन्ते भास्करोक्तेन[1] मार्गतः ।
तर्हि लम्बनखण्डस्य गत्यन्तरलवेषु हि ।। ३४ ।।

युक्तैव संस्कृतिः किन्तु विक्षेपनतिखण्डयोः ।
मिथो योगो वियोगो वा नोपपन्नो भवेत् तदा ।। ३५ ।।

यतो नतिः क्षेपवृत्ततिरश्चीनेव तन्मते ।
क्रान्तिवृत्ततिरश्चीनो विक्षेपः सर्वसम्मतः ।। ३६ ।।

एवं ह्युभयतःपाशारज्जुरत्रापतिष्यति ।
किञ्च गत्योर्वियोगोऽपि पक्षेऽस्मिन् नैव युक्तिमान् ।।३७

भिन्नमार्गगतत्वस्य तयोरप्यविशेषतः ।
"लग्नोनान्त्येन्दुदोःकोटचो"रित्यादाविन्दुलग्नयोः ।।३८।।

वियोगोऽपि न युक्तोऽत्र यतस्तौ भिन्नवृत्तगौ ।
किञ्चात्र "कृतनत्येन्दोः क्षिप्त्यानीतं स्थितेर्दलम् ।।३९।।

कृत्वा तत्काललम्बं च पर्वान्ते निर्णयस्तयोः"[3] ।
इत्युक्तस्थितिदलस्यानीतिरपि न युज्यते ।। ४० ।।

यतः स्थितिदलक्षेत्रगतिभेदांशरूपयोः ।
इच्छाप्रमाणयोर्नात्र वस्तुतस्तुल्यरूपता ।। ४१ ।।

तस्माद्विधोरन्त्यभुक्तिरन्त्यस्फुट इति द्वयम् ।
क्रान्तिवृत्तानुसार्यैव स्वीकर्तव्यं रविग्रहे ।। ४२ ।।

तथा च सूर्यशशिनोर्गत्योश्च स्फुटयोरपि ।
विश्लेषो युज्यते तद्वद् भुजाद्यानीतिरेव च ।। ४३ ।।

क्रियाक्रमे त्वन्त्यभुक्तिरपि विक्षेपवृत्तगा ।
गृह्यतेऽतो भूजादीनामानीतिर्नातिसुन्दरा ।। ४४ ।।

रशिगोलस्फुटैक्यं हि यदा स्यादर्कचन्द्रयोः ।
स एव कालः पर्वान्तो ग्राह्यः सूर्यग्रहे सदा ।। ४५ ।।

अन्यथा लम्बसंस्कारः पर्वान्ते नैव युज्यते ।
क्रान्तिवृत्तकलारूपा यतो लम्बनलिप्तिकाः ।। ४६ ।।

(Acyuta, *Rāśigola.*, 24-46)

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[1] This follows from the following consideration: When the Moon at conjunction is in advance of the Node, this correction has to be deducted because the maximum-eclipse-position is behind the Moon; under this conditon the *vikṣepa* and the *śara* are both North or both South, *i.e*,. there is sameness of direction, (*dik-sāmya*). Hence the rule: '*Dik-sāmye ṛṇa*', 'Deduction when the directions are the same'. When the Moon at conjunction is behind the Node, the maximum-eclipse-point is in advance of the Moon and hence the correction has to be added. Under this condition, the *vikṣepa* and *śara* are of different directions. one North and the other South. Hence the rule '*Dig-bhededhanam*', 'Addition when the directions are different.

[1] *Mahābhāskarīya*, 5. 27-28.

[2] Acyuta's *Uparāgakriyākārama*, 3. 28ff.

[3] *Ibid.*, 3.41.

Refutation of the second view

But, according to the school which says that the moment of conjunction is equality of distance (from the node), (the calculations as described above are impossible for the several reasons set out below): (24b)

(i) (Firstly), how can this, (*viz.*, the relative motion per *nāḍikā*, MR), be taken as the base in view of the fact that the motion of the Moon is (measured) along its own orbit? (25a)

(ii) It may be argued that the relative motion per *nāḍikā* (*gaty-antara-aṁśa*) is not taken as the base, but only as the hypotenuse. (But it has to be noted that) this might be so in the case of the lunar eclipse; but, in the case of the solar eclipse it cannot be taken as the hypotenuse, since the path of the Moon is changing every moment owing to parallax in latitude (*nati*). (25b-27a)

(iii) If, according to this school and with reference to the solar eclipse, it is argued: Let the relative motion (along the Moon's path) (*anu-gaty-antara-aṁśa*) be taken as the base, and the variation in latitude corrected for parallax (*nati-khaṇḍa*) as the altitude, then the root of the sum of their squares will give the path of the Moon. (27b-28)

(The answer is:) No, because in this case they do not form the base and altitude (of a right-angled triangle); for, the latitudinal variation is perpendicular to the Sun's path and the longitudinal variation is (measured), according to this school, along the Moon's path and hence there cannot be any base-altitude relationship between them. (29-30)

(iv) Further, (if it is said that) the base taken above is the relative variation in longitude (*gati-bheda-aṁśa*) minus the variation in parallax in longitude (*lambana-khaṇḍa*) per *nāḍikā* (*ghaṭikā-kālaja*); (31)

(this is improper); for, here, the variation in parallax in longitude is measured along the Sun's path; but the relative variation in longitude is (measured) along the Moon's path; (so these twoare different); hence the subtraction of one from the other is, in fact, not possible. (32-33a)

(v) If however, in order to remove this objection, the parallaxes in latitude (*nati*) and longitude (*lambana*) and their variations (*khaṇḍas*) be calculated according to the method prescribed by Bhāskara, then, the rectification (*saṁskāra*, addition or subtraction) of the variation in longitude by the variation in parallax in longitude is permissible; but addition or subtraction between variations in latitude (*vikṣepa-khaṇḍa*) and variation in parallax in latitude (*nati-khaṇḍa*) will not be possible, for, according to Bhāskara, the parallax in latitude is (measured) perpendicularly to the Moon's orbit, and the latitude is (measured), according to all, perpendicularly to the ecliptic. (33b-36)

Thus, (according this school), every approach leads us to a dilemma. (37b)

(vi) Again, according to this school, even the subtraction of the velocity of one from the other is not possible because the two are (measured), along different paths. (37b-38a)

(vii) Then again, according to this school, the subtraction between the Moon and the rising point of the ecliptic (*lagna*), as required in the (verse) beginning with, "of the *bhujā* and *koṭi* of Final-Moon-*minus-lagna*", is also impossible, because the two move in different circles. (38b-39a)

(vii) Now there is the rule: "Calculate the half-duration of the eclipse (*sthiter-dala*) from the Moon's latitude as corrected for parallax, (*kṛta-natyā kṣiptyā*); calculate also the parallax in longitude (*lamba*) for the times (*tat-kāla*) obtained, (*viz.* the beginning and end of the eclipse); and thus determine them at conjunction." (39b-40)

The calculation of the half-duration of the eclipse according to this rule is also not possible, since, really, there is no similarity between the fields on which the half-duration and the relative variation in motion are represented, (*viz.*, the ecliptic and the Moon's orbit) which form, respectively, the consequent (*icchā*) and the antecedent (*pramāṇa*). (41)

Therefore in the solar eclipse, the two, the rectified velocity of the Moon (*antya-* or *saṁskṛta-bhukti*) and latitude corrected for parallax, have to be measured only in relation to the ecliptic, (the former along it and the latter perpendicular to it). (42)

Again, it would be proper to take the differences between the velocities of the Sun and the Moon and that between their rectified latitudes and so also in the matter of the determination of the sines etc. (43)

In the *Kriyākrama*, the rectified velocity too is measured along the (planet's) orbit, and hence the derivation of the sines etc. is not quite accurate. (44)

In the solar eclipse, that moment alone is to be taken as the moment of conjunction when the rectified latitudes of the Sun and the Moon on the celestial sphere (of which the ecliptic is a great circle) are identical. (45)

Otherwise, corrections for parallax would not be correct at the moment of conjunction, for the seconds of

parallax (*lambana-liptāḥ*) are in terms of the seconds of the circle of the ecliptic. (46). (KVS)

राशिगोलस्फुटसंस्कारः

21. 1. 5. इन्द्रादीनामपि गतिः क्रान्तिवृत्तानुसारिणी ।
अतो न क्षेपतो भेदः स्फुटस्यापीति यन्मतम् ।। ४७ ।।

समलिप्ताकाल एव पर्वान्तस्तन्मते भवेत् ।
चन्द्रादयः क्षेपवृत्ते भ्रमन्ति सततं यतः ।। ४८ ।।

ततः स्फुटोऽपि तेषां स्यात् स्वतोऽपि क्षेपवृत्तगः ।
इति पक्षे हि पर्वान्तः साम्यकालो न केवलम् ।। ४९ ।।

अस्मिन् पक्षे हि चन्द्रस्य रशिगोलस्फुटाप्तये ।
स्फुटीकरणतः पश्चात् कार्यं यत्नान्तरं यतः ।। ५० ।।

तत्प्रकारश्चाच्युतेन कीर्तितः स्फुटनिर्णये ।
"पातोनस्य विधोस्तु कोटिभुजयोर्जीवे मिथस्ताडये-
दन्त्यक्षेपशराहतं वधममुं विक्षेपकोट्या हरेत् ।
लब्धं व्यासदलोद्धृतं हिमकरे स्वर्णं, विपाते विधौ
युग्मायुग्मपदोपगे; विधुरयं स्पष्टो भगोले भवेत्" ।।

इत्यत्र वासना ज्ञेया कलास्वन्तरवद् बुधैः ।। ५२ ।।

अयमेव हि संस्कारो लघूकृत्य क्रियाक्रमे ।
तेनैव "क्षेपवीरांशं कुर्याद्" इत्यादिनेरितः[1] ।। ५३ ।।

राशिगोलस्फुटानीतिरच्युतेनैवमीरिता ।
क्रान्तिवृत्तगतिश्चैवमानेया गोलवित्तमैः ।। ५४ ।।

(Acyuta, *Rāśigola.*, 47-54)

Correction for Reduction to the Ecliptic

Now, according to the view of those who consider that the true motions of the Moon and the other (planets) are given (by the *śāstrakāras*) along the ecliptic and hence their true longitudes are not vitiated by their orbital motion (*kṣepataḥ* means *kṣepa-vṛtta-gamanataḥ*), the moment of equality in degrees will itself give the moment of conjunction (*parvānta*). (47-48a)

On the other hand, according to the view that since the Moon and other (planets) always move on (their own) orbits *kṣepa-vṛtta*) and hence their true longitudes (*sphuṭa*) are (measured) on their orbits, the moment of conjunction is not equality in degrees (measured on the ecliptic). (49b-50)

It is because, according to this view, a further correction has to be applied to the true longitude (as measured on its orbit) to obtain the true longitude as measured on the ecliptic (*rāśigola-sphuṭa-āptaye*), that a method to this effect has been enunciated by Acyuta in the *Sphuṭanirṇaya*:

"Multiply the tabular cosine (*koṭijyā*) and sine (*bhujājyā*) of Moon-minus-Node and the product by the tabular versine (*śara*) of the maximum latitude (*antya-kṣepa*) of the Moon. Divide this by the tabular cosine of the latitude at the particular moment and the quotient is to be divided again by the tabular radius (*vyāsa-dala*). The result (will give the correction for longitude which) is to be added to, or subtracted from, the Moon's longitude, as Moon-minus-Node is in an even or an odd quadrant, respectively. The True-Moon measured on the ecliptic is thus got."[1]

The proof for this may be understood by wise men learned in the (nuances of astronomical) science (*kalāsu*). (50-52)

This very correction has been enunciated in a simplified form by the same (author) in (his *Uparāga-*) *Kriyākrama* (ch. 1, verse 41) in the words "Apply one twenty-fourth part of the latitude" etc.[2] (53)

Thus has been enunciated the "True-longitude Computation on the Sphere of Zodiac" by Acyuta. True longitudes (of planets) on the ecliptic should be calculated by experts in this manner. (54). (KVS)

स्फुटचन्द्राप्तिः

ग्रन्थोद्देशः

21. 2. 1. अधोऽधः क्रमशोऽतीतचन्द्रतत्तुङ्गसङ्गमात्
प्रत्यहं वाक्यनवकात् 'स्फुटचन्द्राप्ति'रुच्यते ।। २ ।।

श्रुतुमात्रे प्रकारेऽस्मिन् न स्याद् यस्यातिविस्मयः ।
स्वस्यैवानधिकारेण स न गृह्णात्विमां गतिम् ।। ३ ।।

प्रणम्य प्रणये युष्मान् साधवो माधवोऽस्म्यदः ।
भवद्भ्यः प्रणतोन्नत्यै भवद्भ्यः किं न लभ्यते ।। ४ ।।

(Mādhava, *Sphuṭa.*, 2-4)

Computation of True Moon

Aim of the work

(Herein below) is expounded the *Computation of True Moon* by means of placing, daily, one below the other, nine numerical expressions (*vākya-s*) as calculated from (the time of) the (previous) conjunctions of the Moon and its Higher Apsis. (2)

(If ever there be) one who is not delighted on hearing about this method, let him not accept it. To be sure, he will not have the ability (to practise it). (3)

Oh! ye good souls! I, Mādhava, bow before you and beseech you. For, what does not one obtain from you who are bent upon elevating those who bend before you. (4). (KVS)

[1] Acyuta's *Uparāgakriyākrama:*

[1] For elucidation and rationale, see *Rāśigolasphuṭānīti: K. V. Sarma*, Intro., pp. 12-14.

[2] For elucidation and rationale, see, *op. cit.*, Intro., pp. 14-15.

ध्रुवसाधनानि

21. 2. 2. 'दीननम्रानुशास्यो'नं दिनराशिं कलेर्गतम् ।
'शिवदूता'ऽऽहतं हत्वा 'पर्याप्तहृदये'न यत् ॥ ५ ॥

लभ्यते तेन कर्तव्या वक्ष्यमाणविधेर्ध्रुवाः ।
तेनैवाद्यस्तथैकैकरहितेन तदष्टकम् ॥ ६ ॥

वाक्यसंख्याः

शिष्टात् तु 'शिवदूता'प्ता वाक्यसङ्ख्याऽग्रिमा ततः ।
मुहुः प्रक्षिप्त'पर्याप्तहृदया'त् क्रमशः पराः ॥ ७ ॥

ध्रुवकालाः

पृथक् तच्छेषरहित'शिवदुता'त् 'प्रिया'ऽऽहतात् ।
'धृतालय'हृता नाड्यो ध्रुवकाला इमे स्मृताः ॥ ८ ॥

ध्रुवः

लिप्तादि 'सत्त्ववान् रामो' 'मौनकामे'ऽग्रिमे फले ।
स च 'विश्वैकनाथ'श्च तस्मिन् 'धृति'युते ध्रुवः ॥ ९ ॥

ततोऽधिकं तु तत्रांशा 'गो'गुणा 'वि'गुणाः कलाः ।
'सु'गुणा विकलास्तासु तद्'गौरां'शं विशोधयेत् ॥ १० ॥

Basis for zero-corrections

From the elapsed Kali days (for any desired date) deduct (a *khaṇḍa*, a lump number of days, equal to) 15,02,008 (*dīnanamrānuśāsya*). Multiply the remainder (*khaṇḍaśeṣa*) by 6845 (*śivadūta*) and divide by 1,68,611 (*paryāptahṛdaya*).[1] The quotient (got is called *dhruva-sādhana* and) is to be used for deriving the several zero-corrections (*dhruvas*) to be used for calculations which will be enunciated below. The first *dhruva* (is to be calculated) using this quotient itself (*agrimaphala*), while the further eight (*dhruvas*) are to be derived from this *dhruva* reduced increasingly by 1. (5-6)

Moon-sentence numbers

Divide the remainder (in the division in 5) by 6845 (*śivadūta*). The quotient obtained will be the first Moon-sentence-number (*vākyasaṅkhyā*).[2] The further (eight sentence-numbers can be obtained) in the same manner from the division by 6842 (*śivadūta*) of the successive remainders to which 1,88,611 (*paryāptahṛdaya*) has been added. (7)

[1] *Rationale*: *dīnanamrānuśāsya* (15,02,008) is a *khaṇḍa* (Lump of days) at the end of which the Moon and its Higher Apsis are in conjunction at mean sunrise at Ujjain. This Lump, can, therefore be discarded and calculations need be based only on the remaining days (*khaṇḍaśeṣa*). Now, 6845 (*śivadūta*) anomalistic cycles of the Moon are contained in 1,88,611 (*paryāptahṛdaya*) days. Hence the anomalistic cycles completed during the *khaṇḍaśeṣa* is given by the expression:

$$\frac{khaṇḍaśeṣa \times 6845}{188611}$$

[2] *Rationale*: The remainder (of the division in 5) is, in fact, the number of days in the current anomalistic cycle *multiplied* by 6845 (*śivadūta*); hence the division of this remainder by 6845 to get days. It may be noted here that since 188611/6845=27 days, 33 *nā*, 16 *vinā*., the further *vākyasaṅkhyās* will successively increase by 27 or 28.

Zero-moments

Subtract (each of the nine) remainders (obtained above) from 6845 (*śivadūta*). Multiply the (nine) balances by 12 (*priya*) and divide by 1369 (*dhṛtālaya*). The (nine) results obtained will be in *nāḍikās* and are (to be called) Zero-moments (*Dhruvakāla-s*).[1] (8)

Zero-corrections

At the end of 5105 (*mānakāma*) (cycles) of the first (*dhruvāsadhana, viz*., the *agrimaphala* of *vv*. 5-6), the zero-correction, beginning with minutes, is 5^r-24°-27′ (*sattvavān rāmaḥ*). For each increase of 69 (*dhṛti*) (cycles, thereafter), the increase in zero-correction is 7^r-1°-44′ (*viśvaikanātha*).[2] (9)

(To get the *dhruva* for) the remaining cycles, multiply the same by 3 (*go*), getting thereby degrees, by 4 (*vi*), getting thereby minutes, and by 7 (*su*), getting thereby seconds; in the case of seconds, however, reduce them by 1/23 (of the number of the said cycles).[3] (10). (KVS)

परितालिका

21. 2. 3. साधयित्वा ध्रुवांश्चैव ध्रुवकालांस्तथानयेत् ।
आद्यमल्पतमं कृत्वा यथाऽधिकतमोऽन्तिमः ॥ ११ ॥

तैः सार्धं तत्र तत्र स्युः सह तद्वाक्यसंख्यया ।
तद्ध्रुवा स्वार्कमध्याय स्वसाधकफलान्विताः ॥ १२ ॥

अत्राप्यौत्पत्तिकोऽस्त्येव वेण्वारोह इव क्रमः ।
द्युनिशोरविशेषेण द्वयोस्तत्पार्श्वयोरिव ॥ १३ ॥

उपर्युपरि पूर्वस्माद् ध्रुवाः स्युः क्रमशः स्थिताः ।
वाक्यसंख्यास्तथाधोऽधो, व्यत्यये व्यत्ययाद् द्वयम् ॥१४॥

प्रमुष्टसम्प्रदायस्य प्रक्रियेयमुदीरिता ।
तत एवाञ्जसाऽमीषां भवत्यवगमोऽन्यथा ॥ १५ ॥

[1] *Rationale*:

$$\frac{\text{Remainder (in verse 5)}}{6845}\ \text{days} = \text{Remainder} \times \frac{60}{6845}\ nā.$$

$$= \text{Remainder} \times \frac{12}{1369}\ nāḍikās$$

[2] *Rationale*: 5105 (*mānakāma*) anomalistic cycles are chosen in such a manner that their *dhruva* added to the *dhruva* of the *khaṇḍa* 15,02,008, (*dīnanamrānuśāsya*), ends in full minutes and not be carried forward to seconds. 69 (*dhṛti*) is the least number of anomalistic cycles for which too, the *dhruva* will end in full minutes.

It has to be remembered, in this connection, that the *dhruva* 5^r-24°-27′ (*sattvavān rāmah*), includes in it the *dhruva* of the elapsed *khaṇḍa* also. Therefore 5105 (*mānakāma*) cycles should be reduced from the *agrimaphala* only once, even if it is possible to reduce it by 5105 more than once. For further reductions, 69 (*dhṛti*) and its *dhruva* alone should be made use of.

[3] *Rationale*: For *dhṛti* (69) *kendra* cycles, the Moon's *Dhruva*=*dhṛti*+*viśvaikanātha*. Hence,

$$Dhruva\ \text{for 1 cycle} = \frac{dhṛti + viśvaikanātha}{dhṛti} = \frac{69_s\ 7^r\ 1^\circ\ 44'}{69}$$

$$= 1^s\ 0^r\ 3^\circ\ 4'\ 6\ 22/23''$$

$$= 1^s\ 0^r\ 3^\circ\ 4' - (7 - 1/23)''.$$

From this the completed 1 circle can be dropped.

एकस्मिन् ध्रुवकाले या वाक्यसङ्ख्याऽवकल्पते ।
'शश'हीना पुन: सा स्यात् न चेत् स'गुळिको'ऽनयो: ।।
'तीर्थकाङ्गा'त् मृगानीकै:'' 'प्र'गुणात् 'सु'गुणाच्च यत् ।
विभज्य लब्धं भागादि धनर्णं तद् ध्रुवेऽस्तु तत् ।। १७ ।।
ध्रुवकालोऽपि येनैक: साध्यते तदनन्तरम् ।
तत: 'सर्वार्थ'युक्तेन 'दीनदान'युतेन च ।। १८ ।।
अस्मिन्ननन्तरातीते समस्तं तद्विपर्ययात् ।
एकद्वित्र्यन्तरे कार्ये कर्तव्यस्तत्समुच्चय: ।। १९ ।।
नाडीषष्ट्यन्तरेऽप्येवं सैका व्येकाऽथवा भवेत् ।
वाक्यसंख्या ध्रुवो नान्य:, सा गति: सार्वलौकिकी ।।२०।।
(Mādhava, *Sphuṭa.*, 11-20)

The Chart

When all the *dhruvas* have been calculated (as instructed in vv. 9-10) and so also the *dhruva-kālas* (as instructed in v. 8), arrange them in (the ascending) order, so that the smallest (*dhruvakāla*) comes as the first and the largest as the last. (11)

Alongside each (of the *dhruvakālas*), chart the corresponding *dhruvas* with their respective *vākyasaṅkhyās*. Chart also the *phalas* which enabled the calculation (of the above), for use (later) in the computation of the Mean Sun. (12)

Here (in the chart of verses 11-12 above), there will be apparent a natural order, irrespective of (the two) sides of the day, *viz.*, day and night, as in climbing a bamboo tree (wherein the branches will be found equally distributed on its two sides). Thus, the *dhruvas* will be, in order, successively greater than the one before, the *vākyasaṅkhyās* being so in the descending order. If otherwise, the order will be reversed (in both). (13-14)

This method of work has been spelt out for (the benefit) of one who has forgotten the tradition. Otherwise, one would have an easy understanding thereof from tradition itself. (15)

When the *vākyasaṅkhyā* for a (particular) *dhruvakāla* is considered, the *vākyasaṅkyā* next to it will be less than it by 55 (*śaśa*) or greater than it by 193 (*guḷika*). The (*dhruvas*) for these two (*viz.*, 55 and 193 days) will be given by 3176 (*tīrthakāṅga*) divided by 1035 (*mṛgānīka*) multiplied, respectively, by 2 (*pra*) and 7 (*su*). The results, which will be in degrees, are to be added to or subtracted from the previous *dhruvas*.[1] (16-17)

When a *dhruvakāla* has been calculated from a number (*viz.*, 6845 *minus* the remainder, (*vide* verse 8), the subsequent (*dhruvakāla-s*) would have been derived from numbers increased by 747 (*sarvārtha*) or 808 (*dīnadāna*).[2] (18)

In the case of a succeeding (*dhruvakāla*), all (the above said) corrections should be applied, inversely. In the case of those removed by one, two or three (intervening *dhruvakāla-s*), the *sum* of the (relevant) corrections (should be similarly applied). (19)

Again, at 60 *nāḍikās* (after or before) a *dhruvakāla*, the corresponding *vākyasaṅkhyā* will increase or decrease by 1. But the *dhruva* will not change. Indeed, this is a universal rule. (20). (KVS)

ध्रुवकालसंस्कार:

21. 2. 4. इत्थं तथैव वाप्तेषु तत्तद्वाक्यध्रुवै: सह ।
ध्रुवकालेषु कार्योऽन्य: संस्कार:, सोऽभिधीयते ।। २१ ।।

रविमध्यम्

'आदिकूर्मे'ऽग्रिमफले 'कर्कशानेकार्यकृत्' ।
अर्कमध्यं विलिप्तादि विद्यात्, प्रतिफलं पुन: ।। २२ ।।
'दाराधीनसुखं' तद्वत् तत्तद्'धी''घ्न''युगां'शयुक् ।
वाक्यसंख्यावशाद् भूय: तत्काले तद्दिनेऽपि तत् ।। २३ ।।

भुजान्तरसंस्कार:

तत्कालमध्यमार्कस्य स्वोच्चहीनस्य दोर्गुणात् ।
अधऊर्ध्वार्धजात् स्वर्णं 'आतपा'प्ता विनाडिका: ।।२४।।

देशान्तरसंस्कार:

प्राक् पश्चात् समरेखयास्तथा देशान्तरोद्भवा: ।
तद्विदां सम्प्रदायाद्धि तदियत्तावधार्यते ।। २५ ।।
इयत्यो लिप्तिका: स्वर्णमिन्दुमध्य इति स्थितौ ।
व्यत्यस्यर्णधने तास्ता: 'शशिरि'घ्ना'स्तमो'हृता: ।। २६ ।।

चरसंस्कार:

स्फुटीकृत्य पुनर्भानुं सायनस्यास्य दोर्गुणात् ।
'गुणोद्याना'दयो ग्राह्या गुणाश्चरदलाप्तये ।। २७ ।।

गुणोद्यानादि-चरज्या:

गुणोद्यानं मनोलीनं समभिज्ञ: सनातन: ।
तृणासनं लूनधनुर्देवानीक निधिव्यय: ।। २८ ।।
धरालयो वीतभयो मधुमान्यं परार्थकृत् ।
नवोदयं गुणाधिक्यं धर्मनिष्ठा क्षमापर: ।। २९ ।।

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[1] *Rationale*: Now, $\frac{\text{tīrthakāṅga}}{\text{mṛgānīka}} = \frac{3176}{1035} = 3^\circ\, 4'\, 6\frac{22''}{23}$

which is the *dhruva* for one anomalistic cycle (vide *v.* 10). Since one cycle is equal to 27½ days, roughly, (*vide* verse 7), *Dhruva* for

2 cycles or 55 days $= 2 \times \frac{3176}{1035} = pra \times \frac{\textit{tīrthakāṅgo}}{\textit{mṛgānīka}}$

Dhruva for 7 cycles or 193 days $= 7 \times \frac{3176}{1035} = su \times \frac{\textit{tīrthakāṅga}}{\textit{mṛgānīka}}$

[2] This is demonstrated in the example worked out in the Introduction of the edition of this work (*Sphuṭa*: *KVS*) p. 21, where the figures are 460, 1207, 2015, 2762, 3509, 4256, 5064, 5811, and 6558.

शिवरात्रि र्गुरुगिर: काकागरु दिवाध्वर: ।
बन्धुवैरं शिखिशिखा भव: शूर: कृश: स्मर: ।। ३० ।।

छाया वैषुवती यत्र द्व्यङ्गुला तत्र केवलम् ।
गुर्वक्षरात्मकमिदं विद्याच् चरदलम् कृतम् ।। ३१ ।।

ततो न्यूनाधिकायां तु तत् स्यात् तदनुपाततः ।
सायनेऽर्केऽजजूकादौ ज्ञेया तस्य धनर्णता ।। ३२ ।।

तेषामेकविधत्वे स्यादेकीभूतानि तानि स: ।
भेद एकस्य चेत्तस्य चापरैक्यस्य चान्तरम् ।। ३३ ।।

त एते ध्रुवकाला: स्युर्वाक्यकाला: सुसंस्कृता: ।
चरार्धमात्रसंस्काराद् दिनार्धं 'शुक'नाडिका: ।। ३४ ।।

दिनमानाल्लघीयांसस्तेऽह्न्येव, निशीतरे ।

(Mādhava, *Sphuṭa.*, 21-35a)

Correction to the Dhruvakāla-s

To the *dhruvakālas* derived in the above manner, along with their *vākyasaṅkhyās* and *dhruvas* (verses 16-20), or calculated in the manner enunciated before (verses 7-9), another correction has to be applied. That is stated, hereinbelow. (21)

Mean Sun

At the end of 5180 (*ādikūrma*) (anomalistic cycles) contained in the first result (*agrimaphala*), being the first *dhruvasādhana*, vide verses 5-6), the Mean Sun, correct to the seconds, is 11ʳ-11°-5′-11″ (*karkaśānekakāryakṛt*).[1] For each remaining (cycle) the Mean Sun is (to be calculated at the rate of) 27°-9′-28″ (*dārādhīnasukham*) *plus* 9/31″ (*dhī/yuga*)[2] (and added). Again, (is to be calculated and added, the Mean Sun) for (the number of days equal to) the *vākyasaṅkhyā* and for that portion of the day under consideration upto the time (of each *dhruvakāla*).[3] (22-23)

Correction for equation of time due to the equation of centre

The sine of arc of the difference between the Mean Sun and (the Sun's) Higher Apsis (viz., 2ʳ-18°, *duṣṭā strī*) divided by 160 (*ātapa*) would give *vināḍikās*.[4] These should be added (to the *Dhruvakālas*) if Sun *minus* Apsis (*Kendra*) is less than a half-circle (6ʳ) and subtıacted of greater. (24)

Correction for terrestrial longitude

Then again, the corrective (*vināḍikās*), on account of the place (in question)being situated to the east or west of the central (Ujjain) meridian, (has to be calculated). Its measure (for the place) is to be known from the traditional knowledge of the learned. (25)

Therefore, when the correction in minutes to the mean Moon (for the place, as got by tradition) has been found to be additive or subtractive, the minutes are to be multiplied by 255 (*śiśira*) and divided by 56 (*tama*) and applied inversely as *vināḍikās*.[1] (26)

Correction for decl. asc. difference

The True Sun is then computed and the precession added. Its R sine in the Sine Table (below) beginning with 153 (*guṇodyāna*) is then noted in order to compute the declinational difference (*cara-dala*). (27)

Sr. No.	*Gurvakṣaras* (2/5 second)			
(1-4)	153	305	457	607
(5-8)	756	903	1048	1190
(9-12)	1329	1464	1595	1721
(13-16)	1840	1953	2059	2156
(17-20)	2245	2323	2391	2448
(21-24)	2493	2525	2544	2551

(28-30)

It is to be noted that the above (table) gives the half-ascensional differences (*cara-dala*) expressed in terms of *gurvakṣara-s* and pertain to a place where the equinoctial shadow is two fingers' breadh (*aṅgulas*).[2] (31)

(When the equinoctial shadow of the place in question is) less or more than (2 *aṅgulas*), (the half-ascensional difference) will be proportional (to the shadow). Its positive or negative nature is to be understood from the

[1] *Rationale*: 5180 (*ādikūrma*) is a certain number of anomalistic Mean Sun which period when added to the Mean Sun of the *Khaṇḍa* 15,02,008 (*dīnanamrānuśāsya*) gives a result in full seconds, *viz.*, 11ʳ-11°-5ˢ-11″ (*karkāśānekakāryakṛt*).

[2] *Rationale*: The mean motion of the Sun in one anomalistic cycle is $27° \; 9' \; 28 \frac{9}{81} = \textit{dārādhīnasukham} + \frac{\textit{dhi}}{\textit{yuga}}$

[3] The mean motion of the Sun for this calculation, as given in the author's *Veṇvāroha*, (verse 38), is $59'\text{-}8 \frac{8''}{47}$

[4] *Rationale*: The *Dhruvakālas* have been reckoned as from mean sunrise at Ujjain. They should be reckoned from true sunrise of the local place, which depends on: (i) the Sun's equation of the centre, (ii) the reduction to the equator, (iii) the longitude of the place and (iv) the declinational ascensional difference (*caradala*) at the place for that day. Of these item (ii) is neglected by earlier astronomers like Āryabhaṭa, and not given by our author in his work, following Āryabhaṭa, though he must have known its need. Item (i) is given here.

The equation of the centre is taken as 129′ × R sin *Manda-kendra*/3438 (Āryabhaṭa) and the True Sun rises earlier or later as this is negative or positive, at the rate of 1 *prāṇa* of time per minute of arc. Therefore, it is equal to R sin *Mandakendra* × 129/3438 × 6 = R sin *Mandakendra*/160 in *vināḍikās*, and is additive for the first six signs of *Mandrakendra* and subtractive for the next six.

It may be noted that item (iii) is given in verses 25-26 and item (iv) in verses 27-32, below.

[1] *Rationale*: The Sun rises at the rate of 10 *vināḍikās* earlier or later, as the place is 1° east or west of the standard meridian and this is additive or subtractive, respectively, to get the true *dhruvakāla*. Expert astronomers find this time by various means, and usually express it in terms of correction to the Mean Moon, which obviously is negative for the east, and positive for the west. This is transmitted through tradition to succeeding astronomers. This can be re-converted into *vināḍikās* by multiplying the Moon's correction by 255 and dividing by 56, since the mean motion in 255 *vināḍikās* is 56. Since the correction-*vināḍikās* and the Moon's correction are opposite in sign, the sign is asked to be reversed.

[2] This would correspond to a region having a latitude of 9½°, like Central Kerala, from where the author of this work hailed.

sāyana-Sun being in (the six Signs) from Aries (*ajādi*) or from libra (*jūkādi*). (32)

When the sign of all the three is the same, (the total correction) is their sum; when one is different, (the total correction) is the differnce between it and the sum of the other two. The *vākyakāla-s* duly corrected (as above) will be the (correct) *dhruvakāla* (*True dhruvakāla-s*). (33 34a)

Fifteen (*śuka*) *nāḍikās* corrected merely by the half-ascensional difference will give (the length of) the half-day. Those (*vākyakāla-s*) which are less than the length of the (full) day (i.e., twice the half-day as found above) will fall during daytime and the other (*vākyakāla-s*) will fall during night-time. (34b-35a). (KVS)

चन्द्रस्फुटः

21. 2. 5. तेषु स्वध्रुवयुक्तानि वाक्यानि स्फुटशीतगुः ।। ३५ ।।

ततस्तदन्तरालेषु स भवत्यनुपाततः ।
तस्य तत्कालगमनं यतस्तद्‌द्वितयान्तरम् ।। ३६ ।।

इष्टकालस्फुटानयन मार्गान्तरम्

'कठोरं' 'निष्ठुरं' चैके क्षिपन्त्यूर्ध्वं त्यजन्त्यधः ।
यथोक्तवाक्यसंख्यायां 'सुखं' 'दुःख' मतोऽन्यथा ।। ३७ ।।

तदन्तरं निहत्येष्टनाड्या 'नत'हृतं ततः ।
धनर्णं विदधन्त्यूर्ध्वमधश्चादावथोदिते ।। ३८ ।।

इष्टकालरविस्फुटम्

विदधीतैवमेवार्के विदित्वास्य गतिं स्फुटाम् ।
तथा तन्मध्यमे कृत्वा कुर्याद्वा तत्स्फुटक्रियाम् ।। ३९ ।।

स्वर्णं स्वोच्चोनमध्यार्ककर्किनक्रादिषट्कजा ।
कोटिज्या'त्माशय'हृता गतिर्मध्याऽस्य तत्स्फुटा ।। ४० ।।

तत्फलं वा 'जनेना'दि गृहीत्वा स्व'युगां'शयुक् ।
विदधीत विलिप्तासु तद्वदेव धनक्षयौ ।। ४१ ।।

'इष्टाङ्गनासखो नित्यम्' 'निःशेषमदनार्तिनुत्' ।
भागमात्रगतेर्भानोः स्फुटद्वयमिदं विदुः ।। ४२ ।।

'शौरीव नश्शिरो नम्यः' 'शूली शुष्मिनिकेतनः' ।
इमौ तन्मध्यमौ ज्ञेयौ श्रीमदार्यभटोदितौ ।। ४३ ।।

स्वोच्चतुल्यतनोस्तस्य पुनः शिशुतमा गतिः ।
निजनीचसमस्यातः परिपूर्तिं व्रजत्यसौ ।। ४४ ।।

ध्रुवकालोक्तसंस्कारः सध्रुवेषु तथेष्यते ।
सूर्यसंक्रमवाक्येषु सूक्ष्मद्युगतसिद्धये ।। ४५ ।।

अहर्गणेऽप्ययं शक्यः कर्तुमुक्तविपर्ययात् ।
स हि तत्संस्कृतो नित्यं भवत्यर्कोदयाद् गतः ।। ४६ ।।

'कान्तं कर्म'विहीनं सत् प्राक्फलं 'सदना'हृतम् ।
प्रहता'न्मूल'हीनात् स्वात् 'संसारा'प्तसमन्वितम् ।। ४७ ।।

हित्वा लिप्तात्मकं राशिषट्काच्छिष्टं विधुन्तुदः ।
वाक्यसंख्यावशाद् वाक्यकालेष्वेष तदर्कवत् ।। ४८ ।।

(Mādhava, *Sphuṭa.*, 35b-48)

True Moon

The sum of the (relevant) *vākyas* (Moon-sentences) and the (relevant) *dhruvas* will give the True Moon-s (at those *vākyakāla-s*). The True Moon (for times) in between (two *vākyakāla-s*) will have be calculated by interpolation. The Moon's motion during an interval is the difference between the two (relevant) True Moons (and hence the said interpolation). (35b-36)

Another method

(Another method to derive the true Moon at any desired time is now stated. If the desired time is) later (than the *vākyakāla* nearest to it), some add 221 (*kaṭhora*) to the *vākyasaṅkhyā*, and (if the desired time is) earlier, subtract from it 220 (*niṣṭhura*). Or perform the operation with 27 (*sukha*) and 28 (*duḥkha*), applied in the reverse order.[1] (37)

Multiply the difference, i.e., the rate of the daily motion of the *vākya* got, by the desired time, in *nāḍikās*, and divide the product by 60 (*nati*). The resullt should be added to the Moon's longitude (of the relevant *vākyakāla*) if (the desired time) is later and subtracted if earlier. (38)

True Sun at desired time

In the case of the Sun, too, (its true position at any desired time) can be computed using its true motion. Computation of the true Sun can be done also by finding the mean Sun (at the time) using its mean motion. (39)

R cosine of the mean Sun-*minus*-Higher Apsis is to be divided by 1550 (*ātmāśaya*) and the result applied to the Sun's mean motion, positively (when it is) in the six Signs beginning with Cancer (*Karki*) and negatively in the six Signs beginning with Capricorn (*Nakra*).[2] The true (daily) motion of the Sun is got. (40)

Alternatively, take the reading for Sun-*minus*-Higher Apsis in the Sine Table beginning with *janena*.[3] Add

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[1] *Rationale*: Subtraction of 27 from above or addition of 28 from below gives the mid-*vākya*, whose rate is taken as the average for the interval. This rate being for one day or 60 *nāḍikās*, the division by 60 is done. Addition of 221 is the same as subtraction of 27 and subtraction of 220 is the same as adding 28, the total being 248. Either can be chosen according to convenience.

[2] *Rationale*: Since the Sun's equation of centre is proportional to the Sun's Sin *manda-kendra*, the variation in it causing true daily motion is proportionate to the cosine, and, therefore, zero at 90° and 270° of *kendra*.

[3] This is the table of the *mandajyās* of the Sun enunciated in the *Grahacāranibandhana* of Haridatta (ed. K.V. Sarma, K.S.R. Institute Madras, 1954, p. 19):

(1-8) : *janena* (8), *satyena* (17), *mukhena* (25), *liṅginā* (33), *yavena* (41), dhāvena (49), samena (57), vartanam (64)

to it 1/31 (*yugāṁsa* of itself and apply the result, taken as seconds, to the mean motion of the Sun, its being positive or negative being the same as before; (i.e., as stated in previous verse). (The Sun's true motion is got). (41)

The two positions of the True Sun (in its course) correct to seconds, when its daily motion is exactly 1°, are 10^r 27° 3′ 10″ (*iṣṭāṅganāsakho nityam*), and 6^r 8° 56′ 50″ (*niśśeṣamadanārtinut*). (42)

The Mean Sun-s at these positions are, according to *Āryabhaṭa*, 10^r 25° 4′ 25″ (*śaurīva naś śironamyaḥ*) and 6^r 10° 55′ 35″ (*śūlī śuṣminiketanaḥ*). (43)

When the (true) position (*sphuṭa*) (of the Sun) is equal to its Higher Apsis, it will have the slowest motion. And, when it is equal to its Lower Apsis, it will have its fastest motion. (44)

The corrections prescribed for the *dhruvakālas* are to be (computed and) applied also to the mnemonics for the Sun's transits (from one Sign to another) so that correct results might be obtained. (45)

This correction can be applied inversely also to the *Ahargaṇa* (total number of Kali days up to the current day). When corrected in this manner, it will give the True *ahargaṇa* which elapsed at sunrise (on that day). (46)

Node

The first result (*Agrimaphala* of verses 5-6) is reduced by 5161 (*kāntam karma*) and multiplied by 87 (*sadana*). From the result subtract 35 (*mūla*) and add to the remainder its 277th (*saṁsāra*) part. (The result obtained is in) minutes and is to be subtracted from 6^r to get (the position of) the Node. Its position for the (different) *vākyakālas* is to be computed proportionately using the *vākyasaṅkhyā-s* in the same manner as that prescribed for the Sun.[1] (47-48). (KVS)

(9-10): rasena (72), hāsena (78), madena (85), yodhanam (91), sudhenu (97), rājanya (ratnasya) (102), sunīpa (107), rūpakaḥ (112)
(17-24): taṭasya (116), dhānyasya (119), parasya (121), bhadrakaḥ (124), carasya (126), hārasya (128), dharāpa (129) dhārakaḥ (129)

These are the Sun's equation of the centre for every $3\frac{3}{4}°$ of *kendra* beginning from 0° to 90°. These are proportionate to Sin *kendra*. When shifted by 90°, so as to begin from 90° onwards, these will be equal to the cosines, and proportionate to the variations in the equation of the centre causing the true motion. Since the constant variaticn is $1/60 \times (1+1/31)$ of 129′, the instrcution to take it as seconds etc.

[1] *Rationale*: It is taken that the motion of the Node is $(87+87/277)$ minutes per Moon's anomalistic cycle. At 5161 Moon's anomalistic cycles after the *khaṇḍa-dina*, the *kṣepa* for the Node is $(35+35/277)$ minutes, negative. Hence, the subtraction of 35. Since the Node's motion in retrograde, the total result is to be subtracted from the position of the Node at the beginning of Kali, which is 6 *rāśis*. The use of proportion for the days gone during the current cycle is obvious, the motion of the Node being uniform.

समापनोक्तिः

21. 2. 6. वदतैतावदेवेत्थं यन्मया नोक्तमन्तरा ।
सिद्धं कृत्वा समक्षेपि समक्षेऽपि तदस्तु वः ।। ४९ ।।

इति संक्षिप्य सन्देहान् हन्तुं हन्त सतः सताम् ।
केनचित् सुधिया ख्याता सन्मार्गे शशिनो गतिः ।। ५० ।।

'शीलं राज्ञः श्रिये' कृत्वा प्राक् पुनयन निर्मितम् ।
विलिप्तादिकं वाक्यजातं येन तेनेयमारचि ।। ५१ ।।

(Mādhava, *Sphuṭa.*, 49-51)

Conclusion

Stating but this much and that in a succinct manner, possibly certain details might have been left out by me, at places, under the presumption that those (details) are (generally known. May all those (details) be before your (mind's eye). (49)

With a view to dispel the doubts of good men, the motion of the Moon has been set out in a proper manner concisely, by a man of intellect (which I consider myself to be). (50)

By the very same person who composed the set of '*Moon-sentences, beginniug with śīlam rājñaḥ śriye*' (12° 2′ 35″), correct to the second,[1] has this work, too, been composed. (51). (KVS)

लघूपायाः

21. 3. 1. अदृष्टमन्यैरिदमाश्मकीयैः
कर्म ग्रहाणां लघुतन्त्रसिद्धम् ।
सञ्चिन्त्य शास्त्रार्णवमाश्मकीय-
मुद्घाटचते तत्र रहस्यभूतम् ।। २१ ।।

'रुद्रैः' सहस्रहतषट्छकलैश्च हत्वा
वर्षाणि 'रन्ध्रवसुवह्नि'सभानसंख्यैः ।
युक्त्वा सदा प्रविगणय्य 'खराम'भक्ते
मासा भवन्ति दिवसाश्च हृतेऽवशिष्टाः ।। २२ ।।

संहृत्य 'रन्ध्रयमलै' 'रसराम'भागै-
र्भूयोऽ'ग्निवेद'गुणितेषु हरेच्च भागम् ।
'खव्योमखद्विमुनि'भिः प्रलयस्तिथीनां
संयोज्य 'भूतयमरुद्र'हृते दिनानि ।। २३ ।।

तेभ्योऽधिकाह्नान् प्रविशोध्य शेषं
पातादतीतो ह्यवमस्य कालः ।
यदा न शुद्धचेदवमं प्रगृह्य
दत्वा चतुष्षष्टिमतो विशोध्यः ।। २४ ।।

मासाधिमासकगणाद् 'गिरि'भागशेषात्
त्रिशद्गुणादपचयोऽयमुदीर्यतेऽतः ।

[1] This text has been edited as an Appendix to the edition of *Sphuṭacandrāpti*.

'शैला'वशिष्टकलियात'मिषु'प्रणिघ्नं
संयोज्य हीनदिवसेषु 'नगा'वशेषः ।। २५ ।।
एकयुक्तदिवसेषु वर्षपः कीर्तितः सितखगादि तद्विदा ।
हीनरात्रगतयुक्तवासराद् 'वेदवृन्द'विहृतास्तदावमाः ।।
वर्षेषु 'रन्ध्रकृतचन्द्र'समाहृतेषु
षट्सप्तपञ्चविहृतेषु दिनादिलाभः ।
ते योजिता दशहतासु समासु संज्ञां
सम्प्राप्नुवन्ति रविजा इति निश्चयो मे ।। २७ ।।
रविजदिवसयोज्याश्चावमा येऽत्र लब्धाः
सततमधिकमासान् शोधयेत् 'खाग्नि'निघ्नान् ।
भवति यदवशिष्टं शोधनीयं समायां
यदि तदधिकशुद्धं क्षेप्यमेवोपदिष्टम् ।। २८ ।।

(Bhāskara I, *MBh.*, 1. 21-28)

Simplified astronomical procedures

After a careful study of the ocean of the *Āśmakīya-śāstra* (i.e. the system of Āśmakīya or Āryabhaṭa), I reveal, by means of simplified rules, the planetary procedures, the secrets therein, (hitherto) unnoticed by other followers of the Āśmakīya. (21)

Mean lunar days between Caitra and solar year

Having ascertained the number of years (elapsed since the beginning of Kaliyuga), multiply them by 11 and by 389/6000, (separately). Add the two results and divide the sum (thus obtained) by 30. The quotient denotes (the mean intercalary) months, and the remainder (the mean intercalary) days.[1] (22)

Mean lunar days at mean solar year since prev. omitted lunar day

Multiply (the number of years elapsed since the beginning of Kaliyuga) by 29 as divided by 36, (i.e. by 29/36). Again multiply the same (number of years) by 43 and divide by 72,000. The sum of the two quotients gives the (residual mean omitted lunar) days. (Multiply the remainder of the first division by 2000, increase the product by the remainder of the second division, and then) divide (the sum) by 1125; then are obtained (the mean lunar) days (which have elapsed at the beginning of the mean solar year since the occurrence of a mean omitted lunar day).[2] (23)

Mean lunar days at a Mean Caitra since a mean omitted lunar day

From (the mean lunar days elapsed at the beginning of the mean solar year since the occurrence of a mean omitted lunar day), subtract the mean intercalary days (obtained in stanza 22 above): the remainder (obtained) is the time (in terms of mean lunar days) elapsed (at the commencement of mean Caitra) since the fall of a (mean) omitted lunar day. In case the subtraction is not possible, add 64 (to the minuend) and then from the sum perform the subtraction. (24)

Lord of the year

Divide the sum of the months (viz. the solar months obtained by multiplying the years elapsed since the beginning of Kaliyuga by 12) and the (corresponding complete mean) intercalary months (obtained in stanza 22 above) by 7; multiply the remainder by 30. Now, we may say what is to be subtracted from this: Divide the number of years elapsed since the beginning of Kaliyuga by 7 and multiply the remainder (of the division) by 5, add this product to the number of (residual mean) omitted lunar days (obtained in stanza 23) and divide the sum by 7: the remainder (of this division is the quantity of the sum to be subtracted). (Divide the difference of this quantity and the one obtained previously by 7). The remainder increased by one, counted from Friday gives the Lord of the year (i.e. the planet presiding over the first day of Caitra). So has been stated by the learned. (25-26b)[1]

Mean omitted lunar days

Increase the number of (lunar) days (elapsed since the beginning of Caitra) by the number of (mean lunar) days elapsed (at the beginning of Caitra) since the fall of a mean omitted lunar day, and divide that (sum) by 64: the quotient gives the number of (mean) omitted lunar days (which have occurred since the mean omitted lunar day occuring before the beginning of Caitra). (26c-d)

Mean lunar days between mean Caitra and mean solar year

Multiply the number of years (elapsed since the beginning of Kaliyuga) by 149 and then divide by 576: the quotient is in terms of days. Add these days to ten times the number of years (elapsed): thus are obtained the so-called *ravija* days. To the *ravija* days add the (residual mean) omitted lunar days obtained above (verses 23 above). From the sum subtract the (complete mean) intercalary months (obtained in stanza 22) as multiplied by 30. Whatever is obtained as the remainder is the 'subtractive' for the (current) year. When the 'subtractive' is greater, then the difference is prescribed as the ' additive'.[2] (27-28)

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[1] The mean intercalary days obtained from this rule are equal to the number of mean lunar days lying between the beginning of mean Caitra and the beginning of the mean solar year.
The number of mean intercalary days in a mean solar year is qual to $11 \times 389/6000$. Hence the above rule.

[2] The number of mean omitted lunar days in a mean solar year is $5+29/36+43/72{,}000$; and the number of mean lunar days between two successive mean omitted lunar days is approximately 64. Hence the above rule.

[1] For elucidation, see *MBh:KSS*, p. 19.

[2] For the rationale, see *MBh:KSS*, pp. 20-21.

मध्यग्रहः

ग्रहतनुः, तद्द्वारा क्रियाश्च

21. 3. 2. षष्टिशतत्रयनिघ्नो वर्षगणो ग्रहतनुः सदा कथितः ।
तेन समेता विहगा ध्रुवका इति कीर्तिताः सद्भिः ॥

मधुसितदिवसाद्यो हीनहीनो गणोऽह्नां
दिविचरहृतशिष्टो वारमाहाब्दपादिम् ।
अत इदमपि शोध्यं शोधनीयं समायां
पतितसमतिरिक्तो गृह्यते नापरोऽत्र ॥ ३० ॥

सप्तत्या दिवसाद्याः'शरभागा' द्विगुणिता विघटिकाश्च ।
तद्रहितो ग्रहदेहो रविबुधभृगवश्च निर्दिष्टाः ॥ ३१ ॥

कुमुदवनसुबन्धो रन्ध्रवर्गो द्वियुक्तो
ग्रहतनुगुणकारो भागहारः प्रदिष्टः ।
'शरयमयमला'ख्यो भागपूर्वोऽत्र लाभो
ह्यृणमपि'शिव'निघ्ने' 'खेन्द्रिया'प्ता विलिप्ताः ॥

भागाः 'खत्रिघनां'शा'स्त्रिरुद्र'गुणिते विलिप्तिका ज्ञेयाः ।
षड्भिः शतैर्विभक्ते विंशत्यंशो रवेश्च तमः ॥ ३३ ॥

'अचल'हतनवांशा लिप्तिका 'रुद्र'निघ्ने
'गगनरस'विभक्ते लिप्तिकास्ता विपूर्वाः ।
ग्रहतनु'खयमां'शास्तत्पराः शोधनीया
दशलवसमवेतश्चन्द्रतुङ्गः स भानोः ॥ ३४ ॥

'भूभृद्राम'हतां हरेच्छतगुणै 'रन्ध्रै'र्ग्रहाणां तनुं
भागाद्यास्फुजितो विमौरिकगणा भागे शतेनोद्धृते ।
'रामांशेन युतं रवेश्च सकलं द्विघ्नाद् रवेश्शोधयेत्
क्षेपः सोमजसोमयोः कृतगुणः सूर्योऽथ 'विश्वा'हतः ॥

'व्योमशून्यनेत्र'भाजिते फलं
राशयोऽष्टभाजितेऽथ लिप्तिकाः ।
बिन्दुषड्हृते विलप्तिका विदुः
सर्वमेव योज्य गण्यते बुधः ॥ ३६ ॥

अष्टाहते 'शरयमाश्वि'हृते कलाः स्यु-
र्देहे तथा त्रिशतभक्तविलिप्तिकाश्च ।
युक्त्वैतदेवमुभयं शनिरत्र गण्य-
स्त्रिंशल्लवो रविभवो धनमत्र कार्यम् ॥ ३७ ॥

द्विकनिघ्ने ग्रहदेहे स्वविंशभागरहिते तु लिप्ताद्याः ।
पञ्चाशदंशविकलाः क्षेप्या भौमो रवेरर्धे ॥ ३८ ॥

'द्वियम'घ्ने ग्रहदेहे 'शरनगरामो'द्धृते तु लिप्ताद्याः ।
सुरनाथगुरोर्भोगो रविभोगद्वादशांशयुतः ॥ ३९ ॥

राशित्रयं क्षिप निशाकरतुङ्गमध्ये
पातं निपात्य भगणात् क्षिप राशिषट्कम् ।
त्रैराशिकागतदिनेषु च रूपमेकं
व्यावर्णयन्ति गणका भटशास्त्रचित्ताः ॥ ४० ॥

(Bhāskara I, *MBh.*, 1. 29-40)

Mean Planets

Grahatanu and Dhruvaka

The number of years elapsed (since the commencement of Kaliyuga) multiplied by 360 is always called *grahatanu*. The (mean) longitudes (reduced to degrees) of the planets (Sun, Mercury, and Venus) together with the *grahatanu* are called *dhruvaka* by the learned. (29)

The grahatanu for the Sun, Mercury, and Venus

Diminish the (lunar) days elapsed since the beginning of Caitra by the corresponding complete omitted lunar days (obtained in the second half of stanza 26 above) and divide (the difference) by seven: the remainder (of the division) counted with the first day of Caitra is said to give the (current) day. From that, the 'subtractive' for the year (obtained in stanza 27-28) should also be subtracted. (But it must be remembered that) the minuend of this subtraction is the difference of the previous subtraction and not the other (i.e., not the remainder of the division). (The remainder obtained by subtracting the 'subtractive' is the *grahatanu* for the Sun, Mercury, and Venus. It denotes the number of mean civil days elapsed since the beginning of the mean solar year). (30)

Mean longitudes of the Sun, Mercury, and Venus

Divide the *grahadeha* (or *grahatanu*) (for the Sun) by 70: the result is in days, etc. Then multiply one-fifth of the *grahadeha* (*i.e.*, *grahatanu*) by 2: the result is in *vighaṭikās*. These (days and *vighaṭikās*) subtracted from the *grahadeha* are stated to be (the degrees, minutes, etc. of) mean longitudes of the Sun, Mercury, and Venus. (31)

Mean longitude of the Moon

Multiply the *grahatanu* for the Moon by 83 (lit. 9^2+2) and divide by 225; the result is in terms of degrees, etc. From that subtract the seconds obtained by multiplying the *grahatanu* by 11 and dividing by 50. (Then add the remainder to thirteen times the mean longitude of the Sun as prescribed in stanza 35 below: The sum thus obtained is the mean longitude of the Moon). (32)

Mean longitude of the Moon's ascending node

Divide (the *grahatanu*) by 270: these are degrees. Multiply (the *grahatanu*) by 113 and by 600: these: are seconds. These together with one-twentieth part of the (mean) longitude of the Sun (in revolutions, etc.) constitute the (mean) longitude of the Moon's ascending Node. (33)

Mean longitude of the Moon's apogee

Multiply the *grahatanu* by seven and divide by nine: these are minutes. Then multiply the *grahatanu* by 11

and divide by 60: these are seconds. Then divide the *grahatanu* by 20: these are thirds to be subtracted. These together with one-tenth of the Sun's (mean longitude (in revolutions, etc.) constitute the (mean) longitude of the Moon's apogee. (34)

Mean śīghrocca of Venus, and additives for the śīghrocca of Mercury and the Moon

Multiply the *grahatanu* by 37 and divide by 900: these are the degrees, etc., (forming part) of the (mean) longitude of (the *śīghrocca* of) Venus. Then divide the *grahatanu* by 100: these are seconds. Add to these one-third of the Sun's (mean) longitude (in revolutions, etc.). Then subtract the whole of that (sum) from two times the Sun's (mean) longitude. (The difference thus obtained is the mean longitude of the *śīghrocca* of Venus).

To the (mean) longitude of (the *śīghrocca* of) Mercury and the Moon, add four times the Sun's (mean) longitude and thirteen times the Sun's (mean) longitude respectively. (Vide stanzas 32 and 36). (35)

Mean śīghrocca of Mercury

Divide the *grhatanu* by 200: the result is in terms of Signs. Then divide the *grahatanu* by 8: these are minutes. Then divide the *grahatanu* by 60: these are seconds. Adding all these (and also four times the Sun's mean longitude as prescribed in stanza 35) is obtained the (mean) longitude of (the *śīghrocca* of) Mercury. (36)

Mean Saturn

Multiplying the *grahatanu* by 8 and dividing by 225 are obtained minutes; and dividing (the *grahatanu*) by 300 are obtained seconds. Adding these two together and increasing that (sum) by one-thirtieth of the Sun's (mean) longitude is obtained the (mean) longitude of Saturn. (37)

Mean Mars

Multiply the *grahatanu* by two and subtract one-twentieth of itself from that: thse are minutes, etc. Then divide the *grhatanu* by 50: these are seconds. Add these (minutes and seconds) to half the Sun's (mean) longitude (in revolutions, etc.): the sum is the (mean) longitude of Mars. (38)

Mean Jupiter

Multiply the *grahadeha* (i.e., *grahatanu*) by 22 and divide by 375: these are minutes etc. Add them to one-twelfth of the Sun's (mean) longitude (in revolutions, etc.): the result is the (mean) longitude of Jupiter. (39)

Corrections to be applied

Add three Signs to the mean longitude of the Moon's apogee. Subtract the (mean) longitude of the Moon's ascending node from 12 Signs and then add 6 Signs. Also (if necessary) add *one* to the *ahargaṇa* obtained by proportion. So say the astronomers whose hearts are devoted to Āryabhaṭa's system of astronomy (*bhaṭaśāstra*). (40).[1] (KSS).

[1] For rationale, and worked out examples, see *MBh:KSS*, pp. 16-28.

22. गणितयुक्तयः – RATIONALE OF ASTRONOMY

उपपत्तेः प्रयोजनम्

22\. 1. 1. मध्याद्यं द्युसदां यदत्र गणितं तस्योपपत्तिं विना
प्रौढिं प्रौढसभासु नैति गणको निःसंशयो न स्वयम् ।
गोले सा विमला करामलकवत् प्रत्यक्षतो दृश्यते
तस्मादस्म्युपपत्तिबोधविधये गोलप्रबन्धोद्यतः ॥
(Bhāskara II, *SiŚi.*, 2. 1. 2)

Need for rationale

It shall not be possible for an astronomer to assert his views convincingly before an assembly of scholars merely on the basis of the computation of mean planets etc. without a proper exposition of the underlying rationale. I am therefore composing this section (i.e. *Golādhyāya* section of the *Siddhāntaśiromaṇi*) expostulating astronomical rationale by means of astronomical models and diagrams through which they could be clear even as a myrobalan placed on one's palm. (KVS)

सावनदिनादिमानम्

22\. 2. 1. नित्यं रविगतिलिप्तासमासुभिः सावनो भाहः ।
यस्माद्रविभगणदिनान्यतः सरूपाणि भाहाः स्युः ॥ १ ॥
घटिकाष्टविंशत्या दशपलयुतयोनिते सावने मासे ।
रविचन्द्रमसोर्योगो भवति स मासस्तु चान्द्रमसः ॥ २ ॥
चैत्रादिः 'खाग्नि'दिनैः षड्विंशतिनाडिकाधिकैरेकः ।
मासः सहस्ररश्मेर्धृतिविघटीसंयुतैर्भवति ॥ ३ ॥
चान्द्रान्मासात् पातङ्गो मासो यस्मात् सदाभ्यधिकः ।
अधिमासकस्ततः स्यात् 'पञ्चरसैः' शीतगोः पक्षैः ॥४॥
तेन च दिनकरमासाः साधिकमासाः शशाङ्कमासाः स्युः ।
अत एव च चन्द्राहाः सावनरहितास्तिथिप्रलयाः ॥
(Lalla, *ŚiDhVṛ.*, 16. 1-5)

Measure of sidereal day etc.

Since a civil day or *sāvanadina* is equal to a sidereal day or *nākṣatradina plus* the number of *asus* equal to the number of minutes in the Sun's daily motion, the number of sidereal days in a year is equivalent to the number of Sun's revolutions in the year, converted into days and increased by one. (1)

When a civil month is decreased by 28 *ghaṭikās*, and 10 *palas*, the conjunction of the Sun and Moon takes place. This duration is a lunar month or *cāndramāsa.* (2)

A solar month or *sauramāsa* consists of 30 *dinas*. 26 *ghaṭikās* and 18 *vighaṭikās* (in civil units), and begins with Caitra. (3)

As a solar month is always longer than a lunar month, there is an intercalary month or *adhimāsa* in 65 half lunar months or *pakṣas* of the Moon. (4)

So, the number of solar months (during a period) increased by the number of intercalary months is equal to the number of lunar months (in the same period). And, so, the number of lunar days diminished by the number of civil days (during a period) is equal to the number of *avamadinas* or omitted tithis (in the same period).[1] (5)/(B'C.

ग्रहस्फुटयुक्तिः

22\. 3. 1. विदितविहङ्गमवृत्तप्रमिता ग्रहतत्तदुच्चविवरभुजा ।
वेद्यग्रहवलयोद्भवलिप्ताभिर्मीयते, भिदा च तयोः ॥१५॥
कक्ष्यामण्डलमध्यस्वमन्दवृत्तस्थबाहुकोटिभ्याम् ।
श्रुतिवृत्तप्रमिताभ्यां तन्मानेनात्र मीयते कक्ष्या ॥ १६ ॥
स्फुटभुजकोटिभ्यां वा परिधिव्यासार्धतोऽपि तन्मानम् ।
श्रुतिवृत्तमिदं क्षिप्तं ह्यत्रत्यग्रहवशाद्यतः क्षेपः ॥ १७ ॥
तत्रापि स्वप्रमितः क्षेपो, न तु योजनैः समानतया ।
तेन न कर्णेनाप्यः, प्रमितः स हि कक्ष्यया सदा समया ॥
एतत्कक्ष्या शैघ्रे क्षिप्तेऽस्मिन् कोटिमण्डलं च ततः ।
कर्णघ्नत्रिज्याप्तक्षेपभुजामान्दकर्णकोटिकया ॥ १९ ॥
विहगभ्रमवलयांशैरुदितात् परिधेः फलाभ्यां च ।
साध्यः कर्णस्, तेन त्रिज्याघ्नाद् दोःफलात्तु विवरभुजा ॥
एवमिह स्फुटसिद्धिर्द्वाभ्यामेवारमन्दजीवानाम् ।
भूग्रहविवरश्रवणे कोटिः कर्णो भुजोक्तविक्षेपः ॥ २१ ॥
सैव भुजा त्रिज्याघ्ना भूग्रहविवरोद्धृता स्फुटः क्षेपः ।
मान्दश्रुतिनिघ्नोऽतो मान्दः क्षेपो विभज्यतेऽनेन ॥ २२ ॥
विदितग्रहवलयमितात् परिधेर्व्यासार्धतोऽथवा कर्णः ।
तस्य च कोटित्वेऽत्र क्षेपो मन्दो भुजा ततः स्पष्टा ॥२३॥
प्राग्वन्मान्दफलेन स्वमध्यमं स्पष्टमत्र बुधसितयोः ।
तत एव च विक्षेपः पुनः स्वशीघ्रोच्चयोर्विपर्यासः ॥२४॥
अल्पतया कक्ष्यायाः शैघ्राद् वृत्तात्तदाहृत्य ।
मन्दश्रुतिविक्षिप्त्योः कोट्या त्रिज्याहतं स्फुटं तदिह ॥२५
क्षेपो हि बुधैः पठितस्त्रिज्यानिघ्नोऽन्त्यफलभाज्यः ।
भूग्रहविवरार्धमिहाप्यन्त्यो ग्राह्यः स्फुटश्च ततः ॥ २६ ॥
शीघ्रफलेनैव रवेः संस्कृतमिह मध्यमं स्फुटं तु तयोः ।
स्वक्षितिविवरघ्नं तद्योजनमपि केवलान्त्यफलभाज्यम् ॥

[1] For mathematical notes, see *ŚiDhVṛ: BC*, II, pp. 238-39.

बहुकारणगतिकत्वादेवं नियता ग्रहा भगोलगताः ।
कतमेन चिदेवैषां ज्ञेयः कालस्तु तद्गतिकालात् ॥ १८ ॥
(Nīlakaṇṭha, *Golasāra*, 3. 15-28)

Rationale of True planets

The anomaly forming the difference between the planet and its apogee, and measured by the known planet's circle, (whether mean or *śīghra*) is (ultimately) measured by the minutes on the orbit of the knowable (i.e., true) planet. And there is difference between the two. (15)

The orbit is measured by the base and perpendicular on the *manda*-circle centered at the centre of the orbit, and measured by the circle of the hypotenuse. (16)

Or, it is also measured by the radius of the orbit got from the true base and perpendicular. (17a)

It is this hypotenuse circle that is deflected (from the ecliptic), since the deflection (giving the latitude) is that of the planet on this. (17b)

Even there, the deflection is measured, not in *yojanas*, but by itself, (i.e., the angle). Therefore, it is not got by the hypotenuse, since it is measured on a circle always even (i.e., of unchanging radius). (18)

This is on the *śīghra* circle (in the case of Mercury and Venus). When this is projected on the orbit, the resulting perpendicular circle is to be got. The hypotenuse (of the equation of conjugation) is to be got by using the two results, (i.e., R sine and R cosine) of the orbit, formed by the segments of the motion of the planet, of circles varying according to the perpendiculars got from the deflection as base, and multiplying by the hypotenuse, and dividing by the radius. The R sine (of the equation of conjunction) multiplied by the radius, and divided by the above is the R sine forming the result, (to be applied in the last operation). (19-20)

Getting the true planet in this manner is only for the two, (*viz.*, Mercury and Venus). For Mars, Jupiter and Saturn, the perpendicular projected from the hypotenuse forming the distance between the Earth and planet, is the hypotenuse, and the base is the deflection mentioned. (21)

This base, multiplied by the radius and divided by the distance between the Earth and the planet is the true (i.e., geocentric) latitude. Therefore, the latitude got on the *manda* circle, multiplied by the *manda*-hypotenuse is divided by this (*viz.*, the distance between the Earth hand the planet. (22)

Or, the hypotenuse is first to be got, using the radius of the mean planet's orbit. In getting here the perpendicular related to that, the latitude got by the *manda*-circle is the true base. (23)

Here, in the case of Venus and Mercury, their mean is corrected by the equation of the centre. The latitude also is got from this, but this being applied to the apsis of conjunction inversely. (24)

25. But since the orbit is smaller than the *śīghra* circle, it (i.e., the latitude) is multiplied by the perpendicular got from the *manda*-hypotenuse and the latitude, (taken as base), and divided by the radius, to become true. (25)

The deflection listed by the wise (i.e., the authoritative astronomers) is to be multiplied by the radius and divided by the final result. Here, the distance between the Earth and the (true) planet is to be taken as the final result. Therefore, it, (i.e., the latitude), becomes true by that. (26)

The mean Sun corrected by their (i.e., of Mercury and Venus) equation of conjunction, is their true position. Their *yojana* measures are to be multiplied by their true distances and their *manda*-hypotenuse and divided by their distances above. (27)

The (motions of the) planets on the stellar sphere are thus ruled by motions caused multifariously. The motions of these are ruled by those. The inter-dependence is, thus, resolved by resorting to successive approximation. (28-29a). (KVS)

देशान्तरादिवासना

22. 4. 1. पश्चात् पश्चादर्कः प्राक् प्राक् च यतोऽभ्युदेति रेखायाः ।
तद्देशान्तरजातं तेन स्वमृणं ग्रहे क्रियते ॥ ६ ॥
मध्यस्योदयकाले यः स रवेः स्वे चिरेण शीघ्रमृणे ।
तत्फलकलासुभिरतः स्वगतित्रैराशिकं स्वर्णम् ॥ ७ ॥
प्रागपरोन्मण्डलयोरुदयास्तमयौ रवेरिह निबद्धौ ।
याम्योत्तरध्रुववशात् तत्र नतिं चोन्नतिं नीतौ ॥ ८ ॥
उन्मण्डलादधस्तात् क्षितिजमुदग् दक्षिणे तदुपरिष्टात् ।
तेन प्रागुदयोऽस्तमयः स्यात् तेन रविचारेण ॥ ९ ॥
याम्ये गोले रव्युदयश्चिरेण शीघ्रं भवेद् यतोऽस्तमयः ।
तेन तदन्तरभूतं कालफलमृणधने स्वमृणम् ॥ १० ॥
अत एव महान् दिवसस्तनीयसी रात्रिरुत्तरे गोले ।
व्यस्तं याम्ये स्पष्टं विकारणं छेद्यकेऽभिहितम् ॥ ११ ॥
(Lalla, *ŚiDhVṛ.*, 16. 6-11)

Rationale of Deśāntara etc.

As the Sun rises first at a place which is to the east of the prime-meridian line, and then at a place which is to the west of it, the correction for terrestrial longitude is applied positively or negatively, as the case may be, to the mean place of a planet (as found at sunrise at Laṅkā). (6)

Since the time of the mean sunrise (is not the same as the time when the true Sun rises), the latter time being either before or after, (the longitude of a planet should be corrected) by means of proportion using the Sun's *mandaphala* in minutes and the motion of the planet. The correction in minutes, (known as *bhujāntara* correction), is additive or subtractive, as the case may be. (7)

(At the equator), the Sun rises and sets on the eastern and western six o'clock circle, respectively. The elevations and depressions therein are controlled by the north and south celestial poles. (8)

In the northern hemisphere, the horizon is below the six o' clock circle and, in the southern hemisphere, it is above it. (So, when the Sun is in the northern hemisphere), at a place to the north of the equator, it rises earlier and sets later than it does on the equator. (9)

But when the Sun is in the southern hemisphere, it rises later and sets earlier than it does on the equator. So, the result due to the ascensional difference is subtracted (from the mean longitude of a planet at sunrise on the equator in order to obtain its mean longitude at sunrise at the observer's station to the north of the equator, if the Sun is in the northern hemisphere; for sunset), it is added. (When the Sun is in the southern hemisphere,) the contrary is the case. (10)

Hence, when the Sun is in the northern hemisphere, the day is longer and the night is shorter; the reverse is the case when it is in the southern hemisphere, (the place of observation being to the north of the equator). The cause can be explained clearly by means of a diagram.[1] (11). (BC)

चरादीनां वासना

22. 5. 1. लङ्कावृत्ते मध्यस्थिते भुवो यत्कुजं तदुद्वृत्तम् ।
तेन न तत्र चरदलं सदा समत्वं च दिवसनिशोः ।। १२।।

तत्राक्षाभावेऽपि स्वस्वक्रान्त्या स्थितौ तिरश्चीनौ ।
ज्यायस्या मेषवृषौ यतोऽल्पकालोदयौ तेन ।। १३ ।।

मिथुनान्तोऽल्पक्रान्त्या पदान्तत्वादृजुस्थितो यस्मात् ।
तस्माच्चिरोदयोऽसावक्षवशाच्चान्यविषयेषु ।। १४ ।।

आद्यान्त्यचक्रपादौ द्युनिशचतुर्थेन चरदलोनेन ।
उद्गच्छतः पलवशाद् भ्रमाच्च युक्तेन मध्यस्थौ ।। १५ ।।

प्रागायतं कुलीरान्मकरादुदगायतं यतः षट्कम् ।
अक्षे भ्रमवशगत्वादधिकन्यूनोदयं तस्मात् ।। १६ ।।

उदगायतोदया ये यान्त्यस्तं ते परायताः पश्चात् ।
प्रागायतोदया ये गच्छन्त्युदगायतास्तेऽस्तम् ।। १७ ।।

स्वचरार्धेनोनयुता ये दृश्या राशयो भवन्त्यत्र ।
तेन युतोनाः क्रमशस्तेऽक्षवशादस्तमुपयान्ति ।। १८ ।।

यस्य स्वचरार्धसमा निरक्षविषयोदयासवो शशेः ।
दृश्यः स सदा तस्मिन् दृशयोऽदृश्योऽन्यथा भवति ।। १९ ।।

पञ्चभिरधिकाः सप्ततिरंशा यस्मिन् पलस्य विषये स्युः ।
तत्र न वृश्चिककार्मुकमकरघटा दृश्यतां यान्ति ।। २० ।।

ये यत्र न दृश्यन्ते दक्षिणगोलस्थिता ग्रहा विषये ।
तत्रैते समापक्रमा न सौम्येऽस्तमुपयान्ति ।। २१ ।।

मेषान्तमितः सविता 'नगदिग्भि'र्योजनैरुदगुदेति ।
नवधृतिभिर्वृषभान्तं मिथुनान्तं 'नखयमै'रेव ।। २२ ।।
(Lalla, *ŚiDhVṛ.*, 16. 12.-22.)

Rationale of ascensional difference etc.

Along the latitude of Laṅkā which passes through the middle of the Earth, (that is, along the equator), the horizon coincides with the six o'clock circle. So, there is no ascensional difference (at this place). The lengths of days and nights are always the same. (12)

There, though the latitude is 0°, (the times of rising of the Signs of the zodiac above the horizon are not the same). Since Aries and Taurus are more oblique because of their respective larger declinations, they take a shorter time to rise. The end of Gemini is more upright because of its comparatively shorter declination and because of being at the end of the quadrant. Therefore, it takes a longer time to rise. At places other than the equator, (the times of rising differ), because of difference in latitude. (13-14)

(In the northern latitudes) the first and fourth quarters (of the ecliptic) pass the horizon in one fourth of the duration of the day and night (that is, one fourth of 60 *ghaṭikās*) *minus* the ascensional difference. The second and third quarters, however, (pass the horizon) in one fourth of the duration of the day and night *plus* the ascensional difference. This is so because of the latitude of the place and of rotatory motion. (15)

(Since in the northern latitudes) the six Signs from Cancer are inclined towards the south and the six Signs from Capricorn are inclined towards the north, the former take longer and the latter shorter times than those they take at the equator. This is so also because of rotatory motion. (16)

(The Signs of the zodiac) that rise when they are inclined towards the north set when they are inclined towards the west. Again, those which rise when inclined towards the east or west set when they are inclined towards the north. (17)

[1] For notes, see *ŚiDhVṛ*: *BC*, II, p. 240.

If the times of rising of the Signs *at any latitude* are obtained by adding or subtracting their respective ascensional differences (to or from their times of rising at the equator), the times of their setting are obtained by subtracting or adding their respective differences. (18)

If at any latitude, the ascensional difference of a Sign is the same as its time of rising at the equator, expressed in *asus*, that Sign is always visible at that place. At other places it rises and sets (as usual). (19)

Scorpio, Sagittarius, Capricorn and Aquarius are not visible at a place where the latitude is 75°. (20)

When planets in the southern hemisphere are not visible at a particular station, they do not set at that station when in the northern hemisphere and of the same declination. (21)

When the Sun is at the end of Aries, it rises 108 *yojanas* to the north of the east point; when at the end of Taurus, by 189 *yojanas*; and by 220 *yojanas* when at the end of Gemini. (22). (BC)

ग्रहणवासना

22. 6. 1. अथवा नतिः खमध्यान्मध्यज्याग्रं तदुत्थनतभागैः ।
याम्यमुदग् वा यद्वद् ग्रहयोः कक्षान्तरं तद्वत् ।। २८ ।।
जलदवदिन्दुर्दिनपं छादयति समागतो यतः पश्चात् ।
प्रग्रहणमतः पश्चात् प्राग्भागे दिनकरे मोक्षः ।। २९ ।।
आवरणस्य लघुत्वात् तीक्ष्णविषाणोऽर्धखण्डितः सविता ।
भवति स्थितिश्च लघ्वी प्रतिविषयं ग्रासनानात्वम् ।। ३०
प्रविशति यद् भूच्छायावृत्तं त्रैराशिकात् स्वकक्षास्थम् ।
तेन न लम्बनमिन्दोर्नावनतिस्तुल्यकक्षत्वात् ।। ३१ ।।
प्रत्यङ्मुखं व्रजन्त्यां पूर्वाभिमुखो व्रजति शशी यस्मात् ।
तस्मात् प्राक् प्रग्रहणं पश्चन्मोक्षः शिशिररश्मेः ।। ३२ ।।
छादकबिम्बमहत्वाद् विषाणयोः कुण्ठतार्धसंच्छन्ने ।
चन्द्रे स्थितश्च महती भवति न च ग्रासवैचित्र्यम् ।।३३।।
ग्रहणे ग्रहमोक्षदिशो रविशशिनोः खण्डकालवैचित्र्यात् ।
शशिभूच्छाये कारणमकारणं राहुरिति सिद्धम् ।। ३४ ।।
आवरणात् स्वग्रहणे क्षेपेण क्षिप्यते यतश्चन्द्रः ।
ग्राह्यादर्कग्रहणे व्यस्ताव्यस्तौ ततः क्षेपौ ।। ३५ ।।
बिम्बप्राची याम्यां याति यदान्यद्दिशस्तदा सौम्यम् ।
भ्रमत्यसव्यं सव्यं व्यस्तं तेनापरं वलनम् ।। ३६ ।।

(Lalla, *ŚiDhVṛ.*, 16. 28-36)

Rationale of eclipses

The parallax in latitude is (calculated) by means of the degrees in the zenith distance of the meridian ecliptic point. It is the north-south distance of the orbits of the Sun and the Moon at the time of an eclipse. (28)

In a solar eclipse, since the Moon comes from the west like a cloud and obscures the Sun, contact takes place on the west and separation on the east. (29)

(In a solar eclipse) the obscuring body, (that is, the Moon) being smaller in size, the Sun, when half eclipsed, has pointed horns. The duration of the eclipse is short and the obscured portion appears different in different places. (30)

(In a lunar eclipse) the Moon enters the circle of the Earth's shadow cast on its own orbit as shown by calculations. Since the orbit (of the obscuring and the obscured bodies) is the same, there is neither parallax in longitude nor in latitude. (31)

Since, in a lunar eclipse, the Moon moving eastward enters the shadow which is moving westward, contact takes place on the east and separation on the west. (32)

As the disc of the Earth's shadow (which is the obscurring body in a lunar eclipse), is big, the horns of the Moon, when half-eclipsed are blunt. The duration of the eclipse is long and the obscured portion does not appear different from different places. (33)

In a solar and lunar eclipse, the parts of the disc obscured are different; there is a difference in the duration of each; and the directions of the contact and separation vary in each case. The cause (for the eclipse) is the Moon (in a solar eclipse) and the Earth's shadow (in a lunar eclipse). *It is thus established that Rāhu is not the cause of the eclipse.* (34)

The Moon, when eclipsed, is thrown (to the north or south) of its obscuring body, (that is the Earth's shadow), according as its latitude is to the south or north. (So, in the projection of a lunar eclipse), the latitudes (must be marked) in a direction contrary to their own.

But in a solar eclipse, (the Moon is thrown to the north or south of the obscured body, the Sun, according as its latitude is to the north or south). (Hence, in the projection of a solar eclipse), the latitudes of the Moon should be marked in their own direction. (35)

When the east point on the prime vertical moves to the south (with reference to the east point on the ecliptic), the other directions (points) move to the right from the left; when the point moves to the north, the other directions move to the left from the right. Therefore the *valana* (deflection) should be marked in a direction contrary to its own. (36). (BC)

ग्रहगणगणवासना

भूमध्याद् रविशशिनोरन्तरम्

22. 7. 1. 'पञ्चस्विषुरन्ध्रेषुसागरा' योजनश्रुतिः ।
रवेर्, इन्दोर् 'अगाद्रचग्निवेदरामाः' कुमध्यतः ।। २ ।।

अविशेषश्रुतिघ्नौ तौ त्रिज्याभक्तौ स्फुटौ मतौ ।
स्वकक्ष्यामध्यभूमध्याद् यतः स्वान्त्यफलान्तरे ॥ ३ ॥

रविशशिभूव्यासाः

'पङ्क्त्यब्धिसागरा' भानोर्व्यासस्, 'तिथ्यग्नयो'विधोः ।
'खेषुखैकाः' क्षितेः, सर्वे स्वगोलान्तस्थयोजनैः ॥ ४ ॥

स्फुटयोजनकर्णाप्तो व्यासस् त्रिज्याहतो निजः ।
लिप्ताव्यासो रवीन्द्वोः स्यात्, त्रिज्या हि स्यात् कलाश्रुतिः ॥

सर्वेष्वपि च वृत्तेषु व्यासार्धस्य तु लिप्तिकाः ।
त्रिज्यासंख्यास्, ततस्त्रिज्या सर्ववृत्तकलाश्रुतिः ॥ ६ ॥

(Para,, *Grahaṇa-nyāya.*, 2-6)

Rationale of the computation of eclipses

Distances of the Sun and the Moon

The (mean) distance in *yojanas* of the Sun from the centre of the Earth is 4,59,585. Of the Moon, (it is) 34,377. (2)

These two multiplied (respectively) by the (respective) hypotenuses derived by repeated approximation (i.e., the radius vector of the respective orbits) and divided by *Trijyā* (*viz.*, 3438) are known to be the true distances (of the Sun and the Moon) (at the particular time of calculation), for they must be (at some position) between the centre of their orbits, *viz.*, the Earth, and their maximum value. (3)

Diameters of the Sun, the Moon and the Earth

The diameter of the Sun is 4410, that of the Moon 315 and that of the Earth 1050, all in (absolute) *yojanas* within their own spheres. (4)

The respective diameters of the Sun and the Moon divided by the true distances in *yojanas* and multiplied by the *Trijyā* is the diameter of the Sun and the Moon in minutes of arc; for, the *Trijyā* (*viz.*, 3438), the (unit) hypotenuse, is in minutes. (5)

In all circles the minutes contained in the radius is equal to their number in the *Trijya*, *viz.*, 3438; and hence *Trijyā* is the hypotenuse in minutes for all circles. (6) (KVS)

भूच्छाया

22. 7. 2. भानोर्व्यासदलं दीपोन्नतिः शङ्कून्नतिर्भुवः ।
शङ्कुदीपान्तराले भूः स्याद्रवेर्योजनश्रुतिः ॥ ७ ॥

शङ्कुदीपान्तराले भूः शङ्कुघ्ना, शङ्कुदीपयोः ।
भेदेनाप्ता हि तच्छाया, छायादैर्घ्यं क्षितेश्च तत् ॥ ८ ॥

मूले भूमिसमव्यासा साऽग्रे गोपुच्छसम्मिता ।
यतोऽर्करश्मिः पतति क्रमेणाधः समन्ततः ॥ ९ ॥

विधोर्योजनकर्णोनं छायादैर्घ्यं समाहतम् ।
भूव्यासेनोद्धृतं च्छायादैर्घ्येण, स्यान्निशाकृतः ॥ १० ॥

स्थाने छायामितिस् त्रिज्यानिहता सा भवेद् हृता ।
विधोर्योजनकर्णेन तमोव्यासः कलात्मकः ॥ ११ ॥

छायादैर्घ्यान्तरे स्वाग्रात् छायाव्यासो हि भूसमः ।
[इन्दोः स्थाने तमोव्यासः] कियानितीन्दुवत् कलाः ॥१२॥

Para., *Grahaṇa-nyāya.*, 7-12)

The Earth's Shadow

The radius of the Sun is (equivalent to) the height of the lamp, that of the Earth to the height of the gnomon, and the (true) distance of the Sun in *yojanas* to the horizontal distance between the gnomon and the lamp. (7)

The horizontal distance between the gnomon and the lamp multiplied by the gnomon, and divided by the difference (in the heights) between the gnomon and the lamp is the shadow (of the gnomon); this represents the length of the Earth's Shadow. (8)

At the base, the Shadow has the diameter of the Earth and at the tip it is (tapering) like the cow's tail, for the Sun's rays fall (on the Earth) equally on al sides. (9)

The length of the Shadow decreased by the (true) distance in *yojanas* of the Moon (from the Earth) multiplied by the diameter of the Earth and divided by the length of the Shadow will be the measure of (the diameter of) the Shadow at the Moon's position. (10-11a)

That multiplied by *Trijyā* and divided by the (true) distance in *yojanas* of the Moon will be the diameter of the Shadow in minutes. (11 b-d)

(If) the diameter of the Shadow at a distance equal to the length of Shadow from its tip is equal to (the diameter of) the Earth, what will be its diameter at the Moon's position: thus is calculated the diameter of the Shadow. Its (measure in) minutes is derived even as the Moon's. (12). (KVS)

ग्रहणे रविशशिनोः स्थितिः

22. 7. 3. एकदृक्सूत्रगौ यावच्चन्द्रार्कौ ग्रहणं रवेः ।
तावन्, निशाकृतो यावत् तावत् स्यात् तमसि स्थितिः ॥

स्थितिरिन्दो[स्तमो]मध्ये पर्वान्ते स्यात् शशीनयोः ।
युतिः पर्वान्ततः प्राग्वा पश्चाद्वा लम्बनाद् भवेत् ॥ १४ ॥

(Para., *Grahaṇa-nyāya.*, 13-14)

Duration of the eclipses

The Sun's eclipse lasts as long as the Moon and the Sun continue to be in the same eye-line (when viewed from the Earth). Similarly, for the Moon, (the eclipse lasts) as long as it remains in the Shadow (of the Earth). (13)

At full moon, the Moon will be at the middle (of its passage through) the Shadow. The conjunction of

the Moon and the Sun may occur (a little) before or after the time of new moon, on account of parallax. (14) (KVS)

मध्याह्नलग्नं दृक्क्षेपलग्नं च

22. 7. 4. मध्यलग्नं नताल्लङ्कोदयैर्ह्यूनाधिको रविः ।
दृक्क्षेपलग्नं प्राग्लग्नं हीनं राशित्रयेण च ।। १५ ।।

मध्यज्या

मध्यलग्नापमाक्षज्याधनुषोर्योगभेदतः ।
मध्यज्या, मध्यदृज्या सा, तच्छङ्कुर्मध्यसंज्ञितः ।। १६ ।।

दृक्क्षेपशङ्कुः

दृक्क्षेपशङ्कुरपि स एव त्रिज्याहतो भवेत् ।
मध्यलग्नविहीनप्राग्लग्नदोर्ज्याविभाजितः ।। १७ ।।

मध्यशङ्कुर्मध्यलग्नहरिजान्तरजीवया ।
यदि कस्त्रिज्ययेति स्याच् छङ्कुर्दृक्क्षेपसंज्ञितः ।। १८ ।।

तस्य छाया हि दृक्क्षेपजीवा खापममध्यगा ।
छायाकर्णस्य सा कोटिर्, दृग्गतिज्या च तद्भुजा ।। १९ ।।

(Para., *Grahaṇanyāya.*, 15-19)

Meridian ecliptic point and Nonagesimal

The point of the Ecliptic on the Meridian (*Madhya-lagna* or *Madhyāhna-lagna*, M) is the longitude of the Sun (♉ S) reduced or increased by the segment (SM) of the ecliptic got from the *Nata-nāḍikas* (*i.e.*, time from the True noon by using the *Laṅkodayaprāṇas*).[1] (15 ab)

The Nonagesimal (*Dṛkkṣepalagna*) (D) is the Orient Ecliptic point (L) minus three Signs. (15cd)

Sine of the Meridian Ecliptic Point

The Sine of the Point of the Ecliptic on the Meridian (*Madhya-jyā* or *Madhyāhna-jyā*, MZ) is got by the (proper) addition or subtraction of the declination of the *Madhya-lagna* (ME), and the arc of the Sine latitude, *i.e.*, the latitude (ZE).[2] It is actually the *Madhyadṛgjyā* (Sine Zenith distance of the *Madhyalagna*). The (*Mahā-*) *Śaṅku* pertaining to it (*i.e.*, Cos. ZM) is termed the *Madhya-*(*Śaṅku*). (16)

Dṛkkṣepaśaṅku

The *Dṛkkṣepaśaṅku* (*i.e.*, the *Mahāśaṅku* pertaining to the Nonagesimal, *i.e.*, Cos. ZD) is this (*Madhyaśaṅku*) multiplied by *Trijyā* and divided by the Sine of the *Madhyalagna-minus-Prāglana* (Sin. ML).[3] (17)

The *Dṛkkṣepa-*(*śaṅku*) is derived by the proportion: If the *Madhyaśaṅku* is (got) by the Sine of *Madhyalagna*-horizon (*i.e.*, ML), (what will it be) by *Trijyā*. (18)

Its Mahācchāyā is (equal to) the Sine at the Nonagesimal of the arc between the Zenith and the Ecliptic (DZ). (It will be the altitude for the hypotenuse which is the (*Graha-mahā-*)*cchāyā* (*i.e.*, Sin. SZ); Sine *Dṛggati* (*i.e.*, Sine of the intervening segment on the ecliptic, SD) is the base for it. (19) (KVS)

दृग्गतिज्या

22. 7. 5. दृक्क्षेपज्यावर्गहीनच्छायावर्गपदं ततः ।
प्रोच्यते दृग्गतिज्येति दृग्भेदोत्था गतिर्यतः ।। २० ।।

दृग्भेदजो यतः क्षेपो दृक्क्षेपज्येति सोच्यते ।
छायावशाद्धि दृग्भेदो, भवेद् दृग्ज्या ततोऽत्र सा ।। २१ ।।

शून्यं खमध्ये, हरिजे भूव्यासार्धमितो भवेत् ।
दृग्भेदो निजकक्ष्यायाम्, अन्तरालेऽनुपाततः ।। २२ ।।

द्रष्टृभूमध्ययोः खेटस्योन्नती वृत्तयोस्तु ये ।
दृग्भेदोऽपि तयोर्भेदो, द्रष्टृमध्ये स चाप्यधः ।। २३ ।।

लम्बनं यद्यपि छायावशाद् दृष्टं तथापि च ।
सिद्ध्यते दृग्गतिज्यातस्तद्, दृक्क्षेपगुणान्नतिः ।। २४ ।।

(Para , *Grahaṇa-nyāya.*, 20-24)

Sine Dṛggati

The root of the square of the *Dṛgjyā* (*i.e.*, Sine Zenith distance of the planet, *i.e.*, Sin. ZS) minus the square of Sine *Dṛkkṣepa* (Sin. ZD) is called Sine *Dṛggati* (Sin. DS). Since the difference in longitude due to the difference of the position of the observer at the centre and at the surface of the Earth (*dṛgbhedotthā gati*) is got from this, it is called *Dṛggatijyā*.[1] (20)

That (*jyā*) from which (the portion of) the latitude caused by the *Dṛgbheda* (*i.e.*, the difference of view from the surface and the centre of the Earth) is obtained is called the *Jyā* for the *Dṛkkṣepa*. The difference of view is in accordance with the *Chāyā* (Sine of the Zenith distance). Hence, here, the *Dṛkkṣepajyā* is the same as *Dṛgjyā* (Sine Zenith distance) of the Nonagesimal. (21)

The *Dṛgbheda* is zero at the zenith. It is equal to the radius of the Earth at the horizon, in its own orbit (SiŚi). In between, it is (SS′), proportionate (to the Sine of the Zenith distance, ∠ZCS). (22)

In the case of the altitudes of a planet (S and S′) in the two circles with the observer (E) and the centre of the the Earth (C) (respectively, as centres), the difference (of the centres) is the *Dṛgbheda* (SS′). Viewed from the observer as centre, its direction is downwards. (23)

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[1] In the forenoon, taking the position of the Sun as the *Udaya-lagna*, calculate the segment of the ecliptic corresponding to the *Natanāḍikās* at that time using the *Laṅkodayaprāṇas*, (i.e., *Laṅkārāśi-māna*), and subtract it from the longitude of the Sun. In the afternoon, since the *Madhyalagna* is greater than the longitude of the Sun, the corresponding segment is added to it.

[2] That is, the Sine of the angle got from the sum or difference of the declination of the *Madhyalagna* (ME) and the latitude (ZE) is called *Madhyajyā*.

[3] Since the spherical triangle MDZ is right angled.

$$\text{Cos. DZ} = \frac{\text{Cos. MZ}}{\text{Cos. DM}} = \frac{\text{Cos. MZ}}{\text{Sin. }(90-\text{DM})} = \frac{\text{Cos. MZ}}{\text{Sin. ML}}$$

as given in the verse.

[1] The sense is *dṛgbhedotthagati-dāyaka-jyā*, since *paramalambana* x *dṛggatijyā* = actual *lambana* (in longitude).

Though the (composite) *Lambana* (parallax) observed is in accordance with (the difference in) the *Mahācchāyā* (Sine Zenith distance), actually, the Parallax in longitude is (that component of it which is) derived from *Dṛggatijyā* (Sine SD). That (component of it which is) derived from *Dṛkkṣepajyā* (Sine ZD) is the *Nati* (Parallax in latitude). (24) (KVS)

नतिलम्बने

22. 7. 6. अपमानुगतो ह्येव लम्बनांशो,ऽत्र लम्बनम् ।
अपमप्रतिदिक्स्थश्च लम्बनांशो नतिर्भवेत् ॥ २५ ॥

दृक्क्षेपदृग्गतिज्ये द्वे भूव्यासार्धसमासहते ।
[त्रिज्या]भक्ते स्वकक्ष्यायां योजनैर्नतिलम्बने ॥ २६ ॥

दृक्क्षेपदृग्गती त्रिज्यामिते चेन्नतिलम्बने ।
भूव्यासार्धमिते ताभ्यामिष्टाभ्यां ते तु के इति ॥ २७ ॥

स्फुटयोजनकर्णेन भक्ते ते त्रिज्यया हते ।
[त्रिज्यावृत्ते] ग्रहस्य स्वे भवतो नतिलम्बने ॥ २८ ॥

विधोर्नतिश्चार्कनतिहीना स्याद् ग्रहणे नतिः ।
सूर्यस्थानाद् विधोर्यस्माद् गृह्यते नतिलम्बने ॥ २९ ॥

अर्केन्द्वोर्लम्बने तुल्ये योजनैः, स्वनती तथा ।
भिन्ने एव कलाभिस्ते, लिप्ताधिक्यं सदा विधोः ॥ ३० ॥

दृग्गतिज्याथवा भूमिव्यासार्धघ्ना विभाजिता ।
मध्ययोजनकर्णेन विधोः षष्ट्या हता पुनः ॥ ३१ ॥

मध्यभुक्त्या विधोर्भक्ता ग्रहणे लम्बनाडिका ।
दृग्गतिज्या 'त्रिषण्णागै'र्भक्तातो लम्बनाडिका ॥ ३२ ॥

(Para., *Grahaṇanyāya*, 25-32)

Parallaxes in Latitude and Longitude

(For), here, *Lambana* (parallax in longitude) is the component along the Ecliptic (SD′) of the (composite) Parallax (SS′) while the component perpendicular to the Ecliptic (S′D′) of the (composite) Parallax will be the *Nati* (Parallax in latitude). (25)

The two, Sine *Dṛkkṣepa* and Sine *Dṛggati*, multiplied by the radius of the Earth and divided by *Trijyā* will, respectively, be the Parallaxes in latitude and in longitude, in terms of *yojanas*, in the planet's orbit. (26)

(This results from the proportion): If the *Dṛkkṣepa*(*jyā*) and the *Dṛggati*(*-jyā*) are each equal to *Trijyā*, their (corresponding) *Nati* and *Lambana* will be equal to the radius of the Earth, what will each be for the given *Dṛkkṣepa*(*-jyā*) and *Dṛggati*-(*jyā*).[1] (27)

These divided, respectively, by their true distances from the Earth and multiplied by *Trijyā* will give the Parallaxes in latitude and longitude, of the respective planet in the *Trijyā-vṛtta* (Great circle). (28)

The Parallax in latitude of the Moon minus the Parallax in latitude of the Sun is the Parallax in latitude (to be used) in (the computation of) eclipses, since the Parallaxes in latitude and longitude of the Moon are (both) calculated relative to the position of the Sun. (29)

The Parallaxes in longitude are the same for the Sun and the Moon in *yojanas*; as also the Parallaxes in latitude. They, of course, differ, (in their measure) in minutes, the minutes for the Moon being always greater. (30)

Or, if Sine *Dṛggati* is multiplied by the radius of the Earth and divided by the mean distance of the Moon (from the Earth), multiplied again by 60 and divided by the mean daily motion of the Moon, the *Lambana-nāḍikās* (Parallax in longitude in *nāḍikās*) in an eclipse are obtained. (31-32ab).

The *Dṛggatijyā* (Sine SD) divided by 863, therefore, (gives) the *Lambana-nāḍikās*. (32cd). (KVS)

22. 7. 7. मध्यकर्णात्तु यो दोषो मध्यभुक्तिर्हरेद्धि तम् ।
समस्तलम्बनादोषं समस्तेन्दुगतिर्हरेत् ॥ ३३ ॥

दृक्क्षेपे क्रियते कैश्चित् क्षेपो दृक्क्षेपलग्नजः ।
[चन्द्र]क्षेपोऽपरैर्, अन्यैर्लम्बाक्षे नतिदे (?) क्रमात् ॥

दृक्क्षेपज्या 'त्रिषण्णागै'र्भक्ता सूर्यशशाङ्कयोः ।
स्फुटगत्यन्तरक्षुण्णा षष्ट्याप्ता वा स्फुटा नतिः ॥३५॥

पर्वान्ते लम्बनं प्राह्णे पर्वान्तद्युगते त्वृणम् ।
अपराह्णे धनं यस्माच्चन्द्रोऽर्कादवलम्बते ॥ ३६ ॥

अर्केन्द्वोर्योगकालस्य सिद्धये यत्र लम्बनम् ।
क्रियते तत् पश्चिमेऽर्काच्चन्द्रे स्वं शोध्यमन्यथा ॥ ३७ ॥

इह पूर्वाह्णापराह्णशब्दौ दृक्क्षेपलग्नतः ।
पूर्वापराशयोर्भानुसंस्थितेर्वाचकौ मतौ ॥ ३८ ॥

लम्बसंस्कृतपर्वान्ते लम्बनं चापि केवले ।
पर्वान्तद्युगते कुर्यादविशेषान्तमुक्तवत् ॥ ३९ ॥

अविशिष्टलम्बनं हि स्फुटलम्बनमुच्यते ।
यस्माल्लम्बनकालेऽपि भवेत् खेटस्य लम्बनम् ॥ ४० ।

(Para., *Grahaṇa-nyāya.*, 33-40)

Any defect (in the results obtained as above) due to (using) the mean distance (of the Moon) will be rectified (automatically) by the division by (its) mean daily motion. The defect due to (using) the composite Parallax (in the rule of three) is remedied by (the division with) the whole daily motion of the Moon. (33)

The latitude at the Nonagesimal is added to the Zenith distance of the Nonagesimal by some (to get its value on the orbit of the Moon). By others, the latitude of the Moon (is similarly added). And, by still others, the latitude and longitude relating to the Parallax in latitude (?), (respectively, are added). (34)

[1] Thus, *Trijyā*: *radius of Earth* : : *Dṛkkṣepa* : *Nati* (in *yojanas*)
Do. : Do. : : *Dṛggati* : *Lambana*: (in *yojanas*)

Or, the *Dṛkkṣepajyā* divided by 863 and multiplied by the difference between the true daily motions of the Sun and the Moon and divided by 60 is the true Parallax in latitude. (35)

The *Lambana*(*-nāḍikās*) should be subtracted from the time of *Parvānta* (end of the new moon or the time of conjunction of the Sun and the Moon) if it occurs in the forenoon, and added if it occurs in the afternoon, since the Moon is (always) depressed from the Sun. (36)

Where the *Lambana* is calculated to determine the time of (apparent) conjunction of the Sun and the Moon, it is to be additive when the Moon is to the west of the Sun, and subtractive otherwise. (37)

The terms 'forenoon' and 'afternoon' are meant here as stating the positions of the Sun to the east and the west as (reckoned) from the Nonagesimal. (38)

The *Lambana* to be calculated for the time of the conjunction corrected for parallax too should be applied (only) to the time of true conjunction and (continued to be done so) as said (here) till there is no difference (between two successive results). (39)

Only that *Lambana* made difference-less (by successive approximation) is said to be the *True Lambana*, (and not the *Lambana* for the time of True conjunction), for, at the time of the (first) *Lambana* itself there might be a *Lambana* for the planet. (40). (KVS)

ग्रहणविधिः

22. 7. 8. लम्बसंस्कृतपर्वान्ते मध्यग्रहणमुष्णगोः ।
अर्केन्दुबिम्बयोर्मध्यरेखायोगस्तदैव हि ।। ४१ ।।

मध्यकालेन्दुविक्षेपस् तत्कालनतिसंयुतः ।
तुल्यदिक्त्वे,ऽन्यथा हीनः, क्षेप इन्दो रवेर्ग्रहे ।। ४२ ।।

स्वस्थानात् लम्बनं हीन्दोर्विक्षपाग्रात्, ततो नतिः ।
मध्यज्यायास्तु या दिक् सा दृक्षेपस्य नतेरपि ।। ४३ ।।

वेणोः पर्वसु सूत्राणां लम्बितानां यथा भवेत् ।
वेण्वग्रक्षेपवत् क्षेपस्, तथा दृक्क्षेपवन्नतिः ।। ४४ ।।

यदा तु बिम्बयोः स्पर्शस्तदा केन्द्रान्तरं तयोः ।
बिम्बार्धैक्यं ह्यतस्तेन कृतं सम्पर्कमण्डलम् ।। ४५ ।।

सम्पर्कमण्डले भानुमध्ये यावद् विधोः स्थितिः ।
तावत् स्याद् ग्रहणं भानोः, स्पष्टभोगो विधोर्ग्रहे ।। ४६ ।।

सम्पर्कमध्ये कल्प्योऽर्कः सर्वदापि, शशी पुनः ।
मध्यक्षेपान्तरान्मध्ये, परिध्योः स्पर्शमोक्षयोः ।। ४७ ।।

मध्यक्षेपस्य सम्पर्कव्यासार्धस्य च वर्गयोः ।
भेदमूलं गतिक्षेत्रं प्राक्[पश्चात्] चार्कतो विधोः ।। ४८ ।।

(Para., *Grahaṇa-nyāya.*, 41-48)

Computation of the Eclipse

The Mid-eclipse of the Sun occurs at conjunction corrected for Parallax, for, it is then that the lines connecting the centres of the Sun and the Moon (to the observer's eyc) coincide. (41)

The latitude of the Moon (to be made use of) in the solar eclipse is its latitude at the time of Mid-eclipse with the parallax in latitude at that moment added to it when (the two are) in the same direction, and subtracted from it otherwise. (42)

The (composite) parallax of the Moon is (determined) from its position at the end of its latitude and the parallax in latitude from there. Therefore the direction of the *Madhyajyā* (Sine *Madhyalagna*, ZM) is also (the direction) of the *Dṛkkṣepa* and of the parallax in latitude (*i.e.*, the latitudinal component). (43)

As the displacement of the threads suspended from the rings of a bamboo follows the displacement of the bamboo top, like that the parallax in latitude follows the *Dṛkkṣepa*. (44)

When the (eclipsing and the eclipsed) orbs just come into contact, then the distance of their centres is the sum of their semi-diameters. And, hence, the contact-circle (*Samparka-maṇḍala*) is drawn with this sum as its radius. (45)

As long as (the centre of) the Moon is within the contact-circle with the Sun as centre, so long is the Sun eclipsed. In the lunar eclipse the true time of the Moon's passage (from the point of first contact to the point of last contact) is the duration of the eclipse. (46)

The Sun is to be assumed always[1] at the centre of the contact-circle. As for the Moon, at Mid-eclipse it is at a distance (from the centre) equal to its latitude at the middle. At first and last contacts (the Moon) is on the circumference (in either of the two cases of contact-circles). (47)

(The angular) distance moved by the Moon during an eclipse east or west of the Sun, is the square root of the difference of the square of the radius of the contact-circle and the square of the latitude at Mid-eclipse (mentioned in 47). (48). (KVS)

22. 7. 9. क्षेपाग्रे हि स्थितिर्मध्ये विधोः, पूर्वापरा गतिः ।
सम्पर्ककोटिः क्षेपोऽतो, गतिक्षेत्रं च तद्भुजा ।। ४९ ।।

गत्यन्तराप्तं षष्टिघ्नं गतिक्षेत्रं स्थितेर्दलम् ।
तदेवेन्दो, रवेः स्वीयलम्बनांशेन तद्युतम् ।। ५० ।।

[1] Thus, even in the lunar eclipse, the Moon may, with advantage, be taken to be at the centre of the Shadow.

स्थित्यर्धहीनं द्युगतं मध्यजस्पर्शजं स्मृतम् ।
स्पर्शकाले पुनः साध्यं लम्बनं सङ्कटं पतत् ॥ ५१ ॥

स्पर्शजं लम्बनं यद्, यन्मध्यजं, त्वन्तरं तयोः ।
स्थित्यर्धे प्रक्षिपेत् तद्धि स्पर्शस्थित्यर्धमीरितम् ॥ ५२ ॥

स्पर्शनद्युगतं तेन, तस्माच्च स्पर्शलम्बनम् ।
कृत्वा, लम्बनशेषं तु स्थित्यर्धे केवले क्षिपेत् ॥ ५३ ॥

एवं कृत्वाविशिष्टं तु स्थित्यर्धं स्पर्शजं स्फुटम् ।
मोक्षजं च तथा साध्यं स्थित्यर्धं मोक्षकालतः ॥ ५४ ॥

(Para., *Grahaṇa-nyāya.*, 49-54)

At mid-eclipse the Moon is (north or south) at a distance of the latitude. The (angular) distance moved is east-west. Therefore the latitude is the perpendicular relating to the hypotenuse formed by the radius of the contact-circle, and the (angular) distance moved is the base. (49)

The (angular) distance moved multiplied by sixty and divided by the difference of the daily motions (of the Sun and the Moon) gives the half duration (in *nāḍikās*). This (angular distance) itself is (to be used) for the lunar (eclipse). For the solar (eclipse), this, corredted by the parallax in longitude relating to itself, (is to be used). (50)

The time of corrected new moon, less the half-duration (found by 50) is called 'Time of first contact resulting from mid-eclipse' (*Madhyaja-sparśaja*). For this time, the parallax in between is again to be calculated. (51)

The difference between the parallaxes of the middle of the eclipse and the first contact should be added to the half-duration. This is called the First-contact-half-duration. (52)

Using this, the time of the first contact should be done, using which the parallax at this first contact is (again) to be found. Again, the difference of the parallaxes is to be found, and added to the original half-duration. (53)

The half-duration found by successive approximation, by repeating this work, is the True half-duration. In the same manner, the half-duration pertaining to the last contact (*i.e.*, the second half-duration) is to be found. (54). (KVS)

इष्टकालग्रहणम्

22. 7. 10. मध्येष्टयोर्लम्बनयोः ऋणमेकं धनं परम् ।
यदा तदा लम्बनयोर्योगः क्षेप्यः स्थितेर्दले ॥ ५५ ॥

यदन्तरं लम्बनयोस् तद्धि कालद्वयोत्थयोः ।
कालद्वयान्तरालोत्थलम्बनं, भिन्नयोर्युतिः ॥ ५६ ॥

उदये प्राक् पश्चिमेऽस्ते लम्बनादर्कतः शशी ।
गतिर्लम्बनतोऽतोऽधः [ततः] कालान्ततो ग्रहे ॥ ५७ ॥

स्थित्यर्धे लम्बनं यत् स्यात् तस्मिन् काले च लम्बनम् ।
स्याद्, भू[यो]ऽप्यविशेषान्तमतः साध्यं स्थितेर्दलम् ॥५८

तत्तत्कालोत्थविक्षेपो गतिक्षेत्रस्य साधनम् ।
इत्येके,ऽन्ये त्विष्टमध्यकालोत्थौ द्वावपीति तौ ॥ ५९ ॥

इष्टविक्षेपसम्पर्ककर्णकृत्योर्यदन्तरम् ।
मध्येष्टक्षेपयोर्भेदकृतियुक्तं च तुल्ययोः ॥ ६० ॥

विदिशोश्चेद्योगकृतियुक्तं, तस्य पदं तु यत् ।
गतिक्षेत्रं हि तत्स्पष्टमित्याहुर्युक्तिचिन्तकाः ॥ ६१ ॥

क्षेपयोरन्तरं कोटिर्योगो वा, बाहुरिष्टजम् ।
गतिक्षेत्रं हि, तत्कर्णो कृतिक्षेत्रमपि स्फुटम् ॥ ६२ ॥

स्थित्यर्धं लम्बनाभावाद् गतिक्षेत्रभवं स्फुटम् ।
पर्वान्ते मध्यमिन्दोः स्यात् स्फुटक्षेपश्च केवलम् ॥६३॥

(Para, *Grahaṇa-nyāya.*, 55-63)

Eclipse at a desired time

If, of the parallax pertaining to the mid-eclipse and that to the desired time, one is subtractive and the other additive, the *sum* (not the difference) of the two parallaxes is to be added to the half-duration, (to find its correct value). (55)

Verily, the difference between the parallaxes of two moments of time is the parallax resulting during the interval, and, therefore, if (the parallaxes are) different (in sign) they should be added. (56)

At sunrise the Moon is displaced eastward from the Sun, and, at sunset (it is displaced) westward, owing to parallax. Therefore, the (relative) motion is (always) lessened by parallax. And, hence, the duration of the eclipse is increased. (57)

Whatever parallax there is during the half-duration, during the time corresponding to this also there will be a change in parallax. Therefore the half-duration has to be determined by successive approximation. (58)

Some (say) that the (angular) distance moved should be found by using the corrected latitudes of the respective times (alone). Others, (however, say) that the two, *viz.*, the latitudes of mid-eclipse and the desired time should be used (to get these). (59)

Experts in the science have stated: Take the difference between the square of the latitude at the time taken and the square of the sum of the semi-diameters. If this latitude and the latitude at mid-eclipse are of the same direction, add the square of their difference; if of different direction, add the square of their sum. The square root of this is the corrected (angular) distance of half-duration. (60-61)

(Of the triangle giving this corrected angular distance of half duration), the difference or sum of the latitudes (mentioned in 60-61) is the perpendicular, the (angular) distance got (in 48 as a first approximation) is the base and the corrected (angular) distance (of half-duration) is the hypotenuse. (62)

Since there is no parallax for the lunar eclipse, the half-duration got from the (angular) distance (got in 60-61) is the correct (half duration), the mid-eclipse is at opposition, and the true latitude of the Moon itself is the (corrected) latitude. (63). (KVS)

विमर्दः

22. 7. 11. विमर्दवृत्तमिन्दू[नतमसोऽर्धेन] साधितम् ।
गतिक्षेत्रं विमर्दार्धकालश्चात्र च पूर्ववत् ।। ६४ ।।

(Para., *Grahaṇa-nyāya*, 64)

Total Eclipse

The circle of total (lunar) eclipse is drawn with the semi-diameter of the Shadow minus that of the Moon (as radius). Here also, the (angular) distance of half total phase and the time thereof are to be found as before. (64). (KVS)

वलनम्—१ आक्षवलनम्

22. 7. 12. नतोत्क्रमज्यानिहता पलज्या त्रिज्ययोद्धृता ।
आक्षं स्याद् वलनं, सौम्यं प्राह्णे, पश्चात्तु दक्षिणम् ।। ६५

नते तिथ्यधिके तत्तु त्रिंशच्च्छुद्धमिह स्फुटम् ।
तिर्यग्गतिर्न पाताले तदासन्नेऽप्यतो यतः ।। ६६ ।।

आधिक्याद् हरिजे तिर्यग्गतेरुत्क्रमसंग्रहः ।
उदये ह्युदगग्रं स्याद् बिम्बमस्तमयेऽन्यथा ।। ६७ ।।

द्युवृत्त[स्थ]स्य बिम्बस्य ह्युदये त्रिगुणः श्रुतिः ।
तिर्यग्गतिः पलज्या च, न स्यात् सातोऽनुपाततः ।। ६८ ।।

—२ आयनवलनम्

[चन्द्र]कोटयुत्क्रमज्याघ्नस् त्रिज्याप्तः परमापमः ।
आयनं वलनं, दिक् तु वेद्याऽत्राऽयनवत् सदा ।। ६९ ।।

उदङ्मुखो मृगादौ हि खेटो, याम्यायनेऽन्यथा ।
तिर्यग्गतिस्तु गोलान्तेऽत्राधिक्यादुत्क्रमग्रहः ।। ७० ।।

तिर्यग्गतिर्हि बिम्बस्य गोलान्ते परमापमः ।
त्रिज्याकर्णस्त्वयनान्ते, न त्वतः सानुपाततः ।। ७१ ।।

—३ स्फुटवलनम्

तुल्यातुल्यदिशोर्योगभेदाद् वलनचापयोः ।
जीवा [सोमार्क]कर्णघ्ना त्रिज्याप्ता वलनं स्फुटम् ।।७२।।

(Para., *Grahaṇa-nyāya.*, 65-71)

Deviation in Direction

i. Deviation due to Latitude of place

The (tabular) sine of latitude multiplied by the (tabular) versine of (the hour-angle got from) the time before or after midday and divided by *Trijyā* (*i.e.*, by 3438) gives the deviation due to the latitude of the place; this is northward in the forenoon and southward in the afternoon. (65)

If the time is more than 15 (*nāḍikās*) it should be deducted from 30 (*nāḍikās*) and the remainder taken as the correct time to be taken, because at the nadir and its neighbourhood there is no north-south deviation. (66)

The versine is to be used because near the horizon the change of deviation is a maximum. At rising, the orb is turned northward, and at setting (it is turned) in the opposite direction; (and, this is the reason for the northward or southward deviation). (67)

The orb is on its diurnal circle, and, at rising, the sine of three *rāśis* (*i.e*, *Trijyā*) is the hypotenuse, and the sine of the latitude gives the deviation. Therefore the deviation cannot be proportionate (to the sine of the hour-angle).[1] (68)

ii. Deviation due to the Moon's course

The maximum declination, multiplied by the versine of the Moon's *koṭi* and divided by *Trijyā*, gives the deviation due to the Moon's course. Here, the direction is always the direction of the course: (69)

For the planet is in its northward course (when in the six Signs) from Capricorn, and in its southward course, otherwise. The (maximum) deviation is when (it is) at the ends of the (southern and northern) hemispheres, (*i.e.*, at the first points of Aries and Libra). The versine is used sine the maximum variation is at these point.[2] (70)

At the ends of the hemispheres (*i.e.*, at the first points of Aries and Libra) the deviation in the direction of the orb is equal to the maximum declination. But the hypotenuse is equal to *Trijya* at the ends of the courses, (*i.e* at the beginning of Cancer and of Capricorn). Therefore the deviation cannot be proportionate (to the sine). (71).

iii. Total deviation

Find the two arcs of deviation and add them together or take their difference respectively, according as they are of the same or different directions. Find the sine, multiply it by the diameter of the Moon or the Sun (as the case may be) and divide by *Trijyā*. This gives the Total deviation. (72). (KVS)

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[1] The reasoning in 67-68 is shown to be wrong by Bhāskarācārya II (*vide* his *Siddhāntaśiromaṇi*, *Gola.*, *Grahaṇavāsanā*, 36 ff.

[2] Here too the use of the versine is wrong as shown by Bhāskarācārya, *ibid.*

ग्रहणपरिलेख:

22. 7. 13. सम्पर्कमण्डले ग्राह्यगर्भे दिक्सूत्रमण्डिते ।
पूर्वतो वलनाग्रा[त्] स्यात् प्रागाशाऽन्याश्च तद्वशा[त्]॥

स्वक्षेपाग्रे मध्यबिन्दुर्मध्याद् याम्योत्तरे स्थितः ।
याम्योत्तरे नयेत् क्षेपौ मध्यतः स्पर्शमोक्षजौ ॥ ७४ ॥

स्पर्शमोक्षोद्भवौ बिन्दू विक्षेपाग्रान्निजान्निजात् ।
सम्पर्कवृत्तपरिधौ कार्यौ पूर्वापराशयोः ॥ ७५ ॥

स्पर्शबिन्दू रवेः पश्चात्, प्राच्यो बिन्दुस्तु मोक्षजः ।
विधोर्व्यस्तं, ग्राहकत्वाद् ग्राह्यत्वादपि शीतगोः ॥७६॥

क्षेपदिग्विपरीता स्यात् चन्द्रग्रहणलेखने ।
चन्द्रात् क्षेपान्तरे यस्मात् तमोबिम्बं प्रकल्प्यते ॥ ७७ ॥

लिखिते ग्राहके स्वीय[बिम्बान्त]र्ग्रहणस्थितिः ।
मध्ये स्पर्शे च मोक्षे स्या,च्छिष्टे ग्राहकवर्त्मतः ॥७८॥

बिन्दूत्रयस्पृग्वृत्त[स्य] खण्डे ग्राहकवर्त्म [तु] ।
इष्टकालेऽभीष्टभागे वर्त्मनि ग्राहकस्थितिः ॥ ७९ ॥

(Para., *Grahaṇa-nyāya*, 73-79)

Graphical representation of the Eclipse

Draw the contact-circle and draw the eclipsed body concentric with it. Mark the four cardinal directions. On the east, from the point marking the deviation, lies the east point (with reference to the ecliptic). (Mark this on the contact-circle). The other directions are (also to be drawn) in relation to this. (73)

The point of mid-eclipse lies on the north-south line, at a distance equal to the latitude at mid-eclipse. Mark off the latitudes pertaining to the first and last contacts, measuring them from the centre, on the north-south line. (74).

On the eastern and western sides, the points of the first and last contacts are to be marked on the contact-circle, at a distance (from the east-west line) equal to their respective latitudes. (75)

In the case of the Sun, the western is the point of first contact, and the eastern, the point of last contact, while for the Moon, it is the reverse, since the Moon is the eclipser and the eclipsed, (repectively in the two cases). (76)

In representing the lunar eclipse graphically, the point (of first contact etc.) should be taken in the direction opposite to the latitude, because the Shadow-circle is drawn (with its centre) at a distance equal to the latitude from the Moon (taking it as drawn first). (77)

When the eclipsing body is drawn (respectively) at mid-eclipse, first point and last point, the quantity of eclipse is seen in (the form of the cut-off portion of) the eclipsed body itself. At other parts, (the quantity depends) on the path of the eclipser. (78)

As for the path of the eclipser, it is the arc of the circle passing through the three points. At any desired time the eclipser is on this path at the corresponding point. (79) (KVS)

अनादेश्यग्रहणम्

22. 7. 14. अष्टांशेऽर्कस्याष्टिभागेऽ[दृश्यौ] छन्नेऽपि शीतगोः ।
तैक्ष्ण्याद् रवे, विधोः शौक्ल्याच्चासन्नार्कमरीचिभिः ॥

(Para., *Grahaṇa-nyāya*, 80)

Eclipses not to be predicted

When (only) an eighth part of the Sun or a sixteenth part of the Moon is hidden, their eclipses will not be (distinctly) visible, on account, in the case of the Sun, of its brilliance, and in the case of the Moon on account of its being illuminated by the neighbouring rays of the Sun. (Therefore these eclipses should not be predicted.) (80) (KVS)

द्युगते उदयान्तरसंस्कारः

22. 7. 15. लिप्ता लङ्कोदयासूनां चक्रे सायनतीक्ष्णगोः ।
सदान्तरं धनर्णं स्यात् [क]लावद् द्युगतासुषु ॥ ८१ ॥

अर्कभुक्तिभ्रमणजो दिनेशो राशिमानतः ।
समश्चोनाधिको यस्मात् संस्कारश्च [तद]न्तरात् ॥

यद्यप्यनुक्तो बहुभिः संस्कारोऽयं तथापि च ।
कार्यः स्याद् गणकैः . . . . . (?) ॥ ८३ ॥

उपायान्तरमप्यस्ति पर्वलम्बादिभिर्विना ।
'सिद्धान्तदीपिकायां' तल्लिखितं गोलवित्स्मृतम् ॥८४॥

(Para., *Grahaṇa-nyāya*, 81-84)

Correction of the predicted time for Reduction to the Equator

Take the minutes of arc of Right Acension of the *Sāyana* Sun, as also the minutes (of its longitude) on the ecliptic. The difference between them), taken always as *prāṇa-s* (*i.e.*, sixths or *viṇāḍīs*), is to be added to or subtracted from the predicted time according as the minutes of longitude are greater or less. (81)

Since the increment in the longitude of the Sun, during its daily revolution, is mean, less or more, according to the ascensional difference, the corrections also are according to these differences. (82)

Though this correction has not been mentioned by many, it should be done by astronomers. (83)

There is another method (for computing the solar eclipse) without finding the parallax at new moon etc.

This has been explained (by me) in the *Siddhāntadīpikā*,[1] as given by (Mādhava) 'the Golavid' (lit. 'expert in spherics').[2] (84) (KVS)

चन्द्रादीनां सितासितवासना

22. 8. 1. दिनकरकरसङ्गादङ्ग यः शीतरश्मि-
र्व्रजति धवलिमानं मानिनी मानहस्तैः ।
घट इव दिशि भानोरातपस्थोऽन्यभागे
भजति च शितिमानं छाययैवात्ममूर्तेः ।। ३७ ।।

सकलमसितं मासान्ते दलं शशिमण्डले
धवलमखिलं पक्षान्ते स्यान्नृलोचनगोचरम् ।
असितमसिते शुक्ले शुक्लं क्रमादुपचीयते
रविमभि यतः प्रालेयांशोस्तथा च विमुञ्चतः ।।३८।।

दर्पणेऽर्ककिरणा यथा स्थिता नाशयन्ति गृहमध्यगं तमः ।
नैशमन्धतमसं महीकृतं तद्वदेव शशिबिम्बसंस्थिताः ।।३९।।

सर्व एव खचराः सतारका-
श्चन्द्रवत् कुवलयेन संयुताः ।
उज्ज्वला दिशि सहस्रदीधितेः
श्यामलस्तदितरत्र निश्चयः ।। ४० ।।

ऊर्ध्वगस्य नरदृष्टिगोचरं
खेचरर्क्षनिवहस्य यद्दलम् ।
तत् सदार्ककिरणैः समुज्ज्वलं
दृश्यते च तत एव नासितम् ।। ४१ ।।

भार्गवेन्दुसुतयोरधःस्थयोर्दृश्यते यदसितं न चन्द्रवत् ।
तद्रवेर्निकटवर्तिनोस्तयोः सर्वमेव वपुरुज्ज्वलं भवेत् ।।४२।।

(Lalla, *ŚiDhVṛ.*, 16. 37-42)

Rationale for the Brightness of the Moon

Oh friend (*aṅga*)! just as a pot placed in the Sun appears bright on the side facing the Sun and dark on the side away from it on account of its own shadow, the Moon in contact with the rays of the Sun appears bright on the side facing the Sun, white as a woman angry with her lover, and dark on the side away from it. (37)

At the end of a lunar month (*amāvāsyā*) the (lower) half of the disc of the Moon visible to the people (of this Earth) is completely dark, but at the end of the first fortnight (*pūrṇimā*), it is completely white.

Since the Moon approaches the Sun in the dark half of the lunar month, its dark portion gradually increases; in the light half of the month, however, the Moon recedes from the Sun and so its illuminated portion gradually increases. (38)

Just as the Sun's rays reflected by a mirror dispel the darkness in a room, the Sun's rays reflected by the Moon dispel the blinding darkness of the night caused on the Earth. (39)

All the spheres of planets, together with the stars including the sphere of Earth, are like the Moon, bright on the side facing the Sun and dark on the side away from it. (40)

That half of the disc of every planet or star having its orbit above that of the Sun, which is visible to the people (of the Earth), is always seen illuminated by the rays of the Sun. Hence it is never dark. (41)

Venus and Mercury though (moving in orbits) beneath (that of the Sun, do not appear dark like the Moon; this is so because they are nearer the Sun and thus their whole discs are illuminated. (42). (BC)

ग्रहबिम्बदर्शनवासना

22. 9. 1. यत्र लग्नमुदितं शशी तथा यत्र चास्तमुदयं व्रजत्यहः ।
अन्तरालमनयोर्भुजस्ततः कोटिरिन्द्वभिमुखो यतो नरः ।।

तत्र भास्करमशीतदीधितिं कृछ्रदृश्यमनुरु प्रपश्यति ।
यत्र रश्मिनिकरेण सर्वतः सूर्यबिम्बपरिधिः पिधीयते ।।४४

कर्णिकेव कमलस्य केसरै-
स्तेन लाघवमुपैति मध्यगः ।
व्याप्नुवन्ति सकलं नभस्तलं
रश्मयो गगनमध्यगे रवौ ।। ४५ ।।

तत्कदम्बपरिवेष्टितो नरो
भास्करं दिवसनाथमीक्षते ।
दूरजः क्षितिजमण्डलोपगो
भूमिरुद्धकिरणश्च तिग्मगुः ।। ४६ ।।

यत्सुखं समवलोक्यते तथा
भात्युरुर्विकिरणश्च सोऽरुणः ।
यद् विमण्डलवतोऽपमण्डलं
क्षेपसंस्कृतिमुपैति तेन तत् ।। ४७ ।।

(Lalla, *ŚiDhVṛ.*, 16. 43-47)

Visibility of the planetary orbs

The interval between the *lagna* and the rising or the setting Moon is the *bhuja* and that from its end directed towards the Moon is the perpendicular or *koṭi*. (43)

Man (on the Earth) sees the Sun having hot rays, difficult to look at and as small in size (in the zenith of the sky) where the whole disc is fringed on all sides with innumerable rays. (44)

[1] The *Siddhāntadīpika* is the author's sub-commentary on the *Bhāṣya* of Govindasvāmin on the *Mahābhāskarīya* of Bhāskara I (Ed. T.S. Kuppanna Sastri, Madras, 1957). The method referred to is given on pp. 314-17 of this edition, on *Mbh.* 5. 68-71.

[2] 'Golavid' Mādhava is one of the teachers of Parameśvara on astronomy, and author of *Sphuṭacandrāpti*, (ed. K. V. Sarma, Vishveshvaranand Institute), and other works.

When the Sun is in the middle of the sky, it appears small (covered with rays) just like the pericarp of a lotus covered with filaments. Again, when it is in the middle of the sky, its rays pervade through the entire space. Thus man sees the Sun, the Lord of the day, surrounded by its rays. (45-46a)

But when it (the Sun) is on the horizon, it is at a distance and its rays are obstructed by the earth. But it can seen without discomfort, and looks big, red and and less hot. (46b-47a)

Since (the Moon and the five planets) move in their respective orbits, (which deviate from that of the Sun), their declinations have to be corrected by their latitudes. (47b). (BC)

दृक्कर्मवासना

22. 10. 1. द्रष्टा समकलकाले
भूतलमध्यस्थित: सूर्यम् ।
पश्यति शशिना पिहितं
न तदा भगोलपृष्ठस्थ: ।। २३ ।।

भूपृष्ठगतो द्रष्टा
पूर्वनतं पूर्वमेव तिथ्यन्तात् ।
पश्यति समुच्छ्रितत्वा-
च्छशिना रविमण्डलम् पिहितम् ।। २४ ।।

पश्यति समकलकालात्
परतोऽन्तरधीयते गतं नीचम् ।
तेन प्राक्पश्चिमयो:
कुदलकलालम्बनमृणं स्वम् ।। २५ ।।

भूतलमध्यस्थस्य
दष्टुर्भूपृष्ठगस्य वा दृष्टि: ।
स्वाभिमुखं याति समं
न लम्बनं तेन मध्याह्ने ।। २६ ।।

पूर्वापरे कुवृत्ते
लम्बनलिप्तोपपत्तिरुक्ता या ।
याम्योदक्क्षितिजवशात्
सा ज्ञेयाऽवनतिलिप्तानाम् ।। २७ ।।

(Lalla, *ŚiDhVr.*, 16. 23-27)

Visibility correction

While an observer at the centre of the Earth can see the Sun obscured by the Moon at the time of conjunction, an observer on the surface of the Earth does not see it as such. (23)

The observer on the surface of the Earth sees the disc of the Sun obscured by the Moon even before (the calculated time for conjunction), as he is elevated above the centre. (This is so, if the Sun is) in the eastern hemisphere.

But if the Sun is in the western hemisphere, he sees it after the calculated time, when the Sun has set, that is, has disappeared below the horizon.

So, the parallax in longitude due to the radius of the Earth is subtracted (from the calculated time of conjunction) if the eclipse takes place in the eastern hemisphere; but added if it takes place in the western hemisphere. (24-25)

The line joining the observer at the centre of the Earth and the zenith coincides with the line joining the observer on the surface of the Earth to the zenith. Thus, there is no parallax at midday (when the Sun is at the zenith). (26)

Whatever reasoning has been given (by the astronomers) for the parallax in longitude in minutes due to the eastern and western horizons, similar reasoning is to be understood for the parallax in latitude in minutes due to the northern and southern horizons. (27). (BC)

दृक्कर्मसंस्कारवासना

22. 10. 2. आयनाख्यमिह दृष्टिकर्म तत्
तद्वशेन पलजं तथा बुधै: ।
दृष्टिकर्म कुजयोर्विधीयते
खेचरे चरदलोपपत्तिवत् ।। ४८ ।।

क्षेपयो: समदिशो: समानयो:
खेचरौ चरत एकवर्त्मना ।
अगच्छतोरिह दिशोर्यतो यथा
लघ्विषुर्भवति तेन सोऽन्यथा ।। ४९ ।।

(Lalla, *ŚiDhVr.*, 16. 48-49)

Correction to visibility correction

The wise men have laid down the rule that to a planet, both on the eastern and western horizons, visibility correction due to deviation of the ecliptic or *āyanadṛkkarmāsu* and visibility correction due to the terrestrial latitude or *ākṣadṛkkarmāsu* should be applied. The reason is somewhat similar to that for the application of ascensional difference. (48).

When the latitudes (of two planets) are equal in magnitude and direction, the planets have the same day-circle. But when one planet has a latitude smaller than that of the other planet, it appears in a direction opposite to that of the latitude (with regard to the centre of the other planet) even if the latitudes are of the same denomination. (49). (BC)

भुजा कोटिश्च

22. 11. 1. पदं राशित्रयं, तत्र भुजाकोटी गतागते ।। १ ।।
ओज,युग्मे क्रमाज्ज्ञेयं कोटिबाहू इति स्थिति: ।

(Bhāskara I, *LBh.*, 9. 1b-2a)

Sine and cosine

(In a circle), three Signs make a quadrant. In the odd quadrants, the arc traversed is called *bhujā* (or *bāhu*) and that to be traversed is called *koṭi*; in the even quadrants, the two are, respectively, called the *koṭi* and *bāhu* (or *bhujā*). This is the convention. (1b-2a). (KVS)

ज्यानयनयुक्तिः (परिलेखद्वारा)

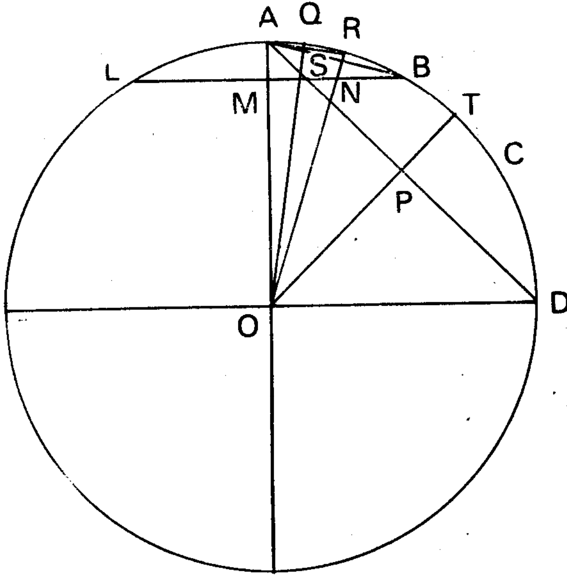
22. 12. 1. समवृत्तपरिधिपादं छिन्द्यात् त्रिभुजाच्चतुर्भुजाच्चैव ।
समचापज्यार्धानि तु विष्कम्भार्धे यथेष्टानि ॥ ११ ॥
(Āryabhaṭa I, *ABh.*, 2.11)

R sine table (geometrically)

Divide the quadrant of the circumference of a circle (into as many parts as desired). Then, from (right) triangles and quadrilaterals, one can find as may R sines of equal arcs as one likes, for any given angle (and thus prepare the R sine table).[1] (11). (KSS)

Demonstration

Find twelve R sines at intervals of 7° 30′ in the circle of radius R (=3438′).



Let the fig. represent a circle of radius R (3438′). Divide the quadrant into two at T (45°) each; Trisect TA into TB, BR, RA (15° each), RA into two (RQ, QA, $7\frac{1}{2}$ degrees each). Mark off AL (30°). Join LB. This is equal to R and denotes chord 60°. Half of this is R sin 30°. Thus,

R sin 30°=R/2=1719′.

[1] Cf. sections 7.18 and 19 above. The construction of the table envisaged might be illustrated as given under 'Demonstration' following the instructions given in the Bhāṣya of Bhāskara I on this verse (cf. edn., pp. 77-83).

This is the fourth Rsine, in the $7\frac{1}{2}°$ table to be computed.

Now, from the right-angled triangle OMB,

$$OM=\sqrt{R^2-(R/2)^2}=\frac{\sqrt{3}}{2}R=2978'.$$

This is R sin 60°, *i.e.*, the eighth R sine.

Now, from the right-angled triangle AMB,

$$AB=\sqrt{(\text{Rsin } 30°)^2+((\text{Rvers } 30°)^2}$$
$$=\sqrt{(1719')^2+(460')^2}=1780'.$$

This is chord 30°. Half of this, *i.e.*, AN, is Rsin 15°.

Thus, Rsin 15°=890′.

This is the second Rsine.

Now from the right-angled triangle ANO

$$ON=\sqrt{(AO)^2+(AN)^2}=\sqrt{R^2-(\text{Rsin } 15°)^2}=3321'.$$

This is Rsin 75°, *i.e.*, the tenth Rsine.

Now, from the right-angled triangle ANR, where R is the mid-point of the arc AB, we have

$$AR=\sqrt{(AN)^2+(NR)^2}=\sqrt{(\text{Rsin } 15°)^2+(\text{Rvers } 15°)^2}$$
$$=\sqrt{(890')^2+(117')^2}=898'.$$

This is chord 15°. Half of this (*i.e.*, AS) is Rsin 7° 30′.

Thus, Rsin 7° 30′=449′.

This is the first Rsine.

Now, from the right-angled triangle ASO,

$$OS=\sqrt{R^2-(\text{Rsin } 7° 30')^2}=3409'.$$

This is Rsin (82° 30′), *i.e.*, the eleventh Rsine.

Now, Rvers 75°=R−Rsin 15°, so that

$$\text{chord } 75°=\sqrt{(\text{Rsin } 75°)^2+(\text{Rvers } 75°)^2}=4186'.$$

Half of this is Rsin 37° 30′. This is the fifth Rsine.

Now, $\text{Rsin } 52° 30'=\sqrt{R^2-(\text{Rsin } 37° 30')^2}=2728'$.

This is the seventh Rsine,

Thus, seven Rsines have been obtained by using triangles.

Now, we make use of the semisquare AOD. Its side OA and OD are each equal to R. Therefore,

$AD=\sqrt{2}\,R=4862'$.

This is chord 90°. Half of this, *i.e.*, AP, is Rsin 45°. Thus, Rsin 45°=2431′. This is the sixth Rsine.

Now, from the right-angled triangle APT,

$$AT=\sqrt{(\text{Rsin } 45°)^2+(\text{Rvers } 45°)^2}=2630'.$$

This is chord 45°. Half of this is Rsin 22° 30′. This is the third Rsine.

Hence, as before,

$$\text{Rsin } 67° 30'=\sqrt{R^2-(\text{Rsin } 22° 30')^2}=3177'.$$

This is the ninth Rsine.

Thus, we get all the twelve Rsines, which might be set out as follows:

Rsin 7° 30′=449′	Rsin 37° 30′=2093′
Rsin 15°=890′	Rsin 45°=2431′
Rsin 22° 30=1315′	Rsin 52° 30′=2728′
Rsin 30°=1719′	Rsin 60°=2978′

Rsin 67° 30′=3177′
Rsin 75°=3321′
Rsin 82° 30′=3409′
Rsin 90°=3438′

Analysis. Stanza *ABh.* 9 (c-d) gives the fourth Rsine. This fourth Rsine yields the eighth and the second Rsines. The eighth Rsine does not yield any new Rsine. The second Rsine yields the tenth and the first Rsines. The first Rsine yields the eleventh Rsine, and the tenth Rsine yields the fifth and the seventh Rsines. These Rsines do not yield any new Rsines. So this process ends here.

Again, the radius is the twelfth Rsine. This yields the sixth Rsine, and the sixth Rsine yields the third and the ninth Rsines. These do not yield any further Rsines. So the process ends here.

Thus, from the fourth and the twelfth Rsines one gets all the twleve desired Rsines. (KSS)

रविचन्द्रयोः स्फुटवासना

22. 13. 1. 'त्रिभि' 'र्नगैः' संगुणिते 'खकुञ्जरैः'
स्वकेन्द्रजीवे विभजेत् फलं कलाः ।
तदूनयुक्तौ स्फुटतामिनोडुपा-
वितः स्वकेन्द्रेऽजतुलादिके क्रमात् ॥ १४ ॥
(Lalla, *ŚiDhVṛ.*, 2. 14)

True Sun and Moon

Multiply the R sines of the mean anomalies (of the Sun and the Moon) by 3 and 7, respectively, and divide each by 80. The results (called *mandaphala* or corrections to be applied to the mean longitudes) are in minutes. Add or subtract the results (to or from the mean longitudes of the Sun and Moon) according as their mean anomalies are within the six Signs beginning from Libra or within the six Signs beginning from Aries (i.e., greater or less than 180°). Thus are obtained their longitudes. (14). (BC)

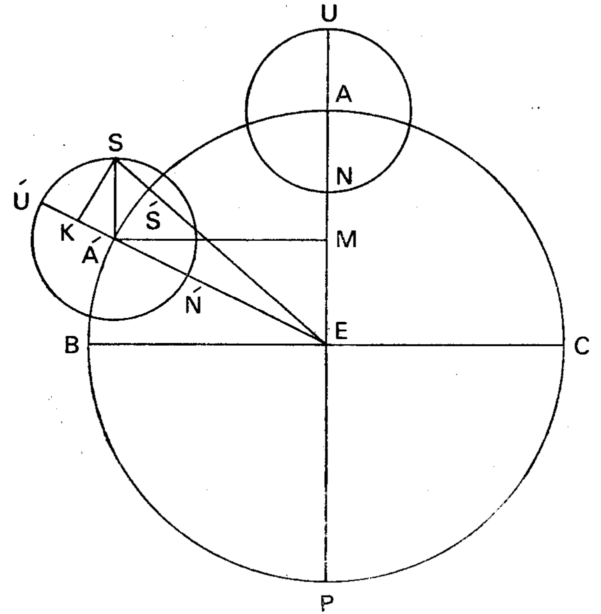
Diagrammatic rationale

Here is given the formulae for determining the true longitudes of the Sun and the Moon. This can be achieved either by the epicyclic method or by the eccentric method.

Epicyclic method

First the epicyclic method or *nīcoccavṛttabhaṅgi*. According to this method, the body is supposed to describe, with uniform motion, the circumference of a circle called *nīcoccavṛtta* or epicycle, the centre of which is supposed to move along the *kakṣāvṛtta* or deferent or circular orbit of the body, with a motion equal to the mean motion of the body, but in a reverse direction. The time the body takes to make one revolution about the centre of the epicycle is the same as the time the epicycle takes to revolve once round the orbit.

This method is illustrated below:



Let ABPC be the deferent of the Sun, with centre E, the centre of Earth. Let AEP be the apse line. With centre A and radius equal to *mandāntyaphalayjā* or radius of the Sun's epicycle describe a circle. Let the apse line cut it at U and N, which are the *mandocca* or apogee and *mandanīca* or perigee of the Sun on the epicycle, respectively. When the centre of the epicycle is at A, the Sun is at U. Let A move with the mean motion of the Sun up to A′ along the orbit and let the Sun move from U′ to S along the epicycle so that arc U′S=arc A′A. Join SE cutting the orbit at S′. Then A′ is the *madhyamasūrya* or mean place of the Sun and S′ is the *sphuṭasūrya* or true place of the Sun in the orbit. Thus the correction to be given to the mean longitude of the Sun to get its true longitude is the value of the arc A′S′. This value is valled *mandaphala* or equation of the centre of the Sun and the process is called *mandakarma*.

Now to find the arc A′S′. Join SA′. Draw SK and A′M perpendiculars to U′A′E and UAE, respectively.

The angle A′EA, that is the angle between the apogee A and the mean Sun A′, is called *mandakendra* or mean anomaly.

A′M is the *mandakendrajyā* or R sine mean anomaly. Now, since arc U′S=arc A′A, angle U′A′S=angle A′EA. So, SA′ is parallel to AE.

Then, from the similar triangles SA′K and A′EM,

$$SK = \frac{A'M \times SA'}{A'E}$$

$$= \frac{\text{R sin mean anomaly} \times \text{radius of epicycle}}{\text{R}}$$

$$= \frac{\text{R sin mean anomaly} \times \text{circumference of epicycle}}{360}$$

$$= \frac{\text{R sin mean anomaly} \times 13\frac{1}{2}}{360}$$

$$= \frac{\text{R sin mean anomaly} \times 3}{80} \qquad (1)$$

3 is called *guṇaka* (multiplier). SR is *doḥphala*.

The arc corresponding to (1) as R sine is approximately taken to be the value of arc A′S′ or *mandaphala*; which is

$$\frac{\text{R sin mean anomaly} \times 3}{80} \text{ minutes, nearly,}$$

from $225 : 225' : : \frac{\text{R sin mean anomaly} \times 3}{80} : ?$.

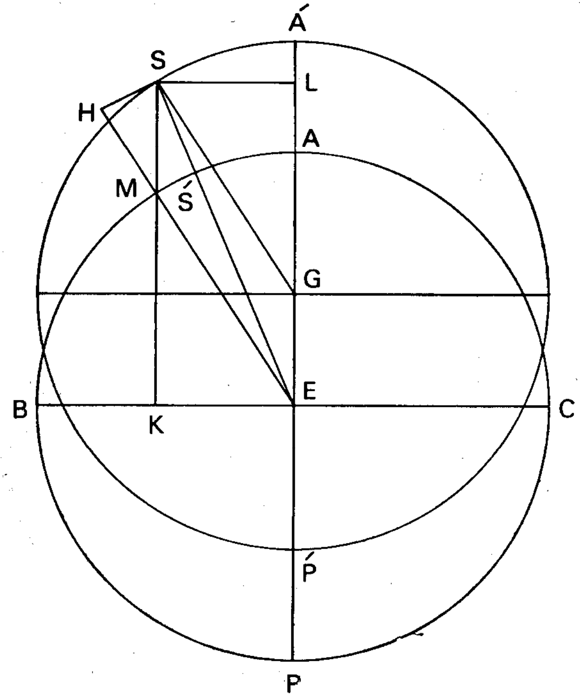
The *mandaphala* is subtracted from the Sun's mean longitude if its mean place is in advance of its true place, that is, when the mean anomaly is in the first or second quadrant; and added, if its true place is in advance of its mean place, that is, in the third or fourth quadrant.

SE is called *mandakarṇa* or *manda* hypotenuse.

Eccentric method

The second method is called *prativṛttabhaṅgi* or method of concentric and eccentric. This method supposes that the mean Sun or Moon moves with a uniform velocity along its circular orbit or concentric and the true Sun moves with the same velocity and in the same direction along the circumference of an equal circle called *prativṛtta* or eccentric, the centre of which is situated on the line joining the apogee to the centre of the Earth, at a distance equal to *mandāntyaphalajyā* or radius of the epicycle of the body, from the latter point.

The method is illustrated below.



Let ABPC be the Sun's circular orbit or concentric with centre E, the centre of the Earth. Let AP be the apse line, so that A is the apogee and P the perigee on the orbit. Let GE be equal to the radius of the Sun's epicycle. With G as centre, describe a circle equal to the orbit. This is the *mandaprativṛtta* or eccentric. Let AP cut it at A′ and P′, which are, respectively the apogee and perigee on the eccentric. Let the mean Sun and true Sun start from A and A′, respectively, and move along the concentric and the eccentric with the same velocity and in the same direction. Let their new places be M and S. Join SE cutting the concentric at S′. Then M is the mean Sun and S′ is the true Sun. So the correction to be given to the mean longitude of the Sun to get its true longitude is the value of the arc MS′, which is the *mandaphala*.

Now to find this value. Join GS and EM. Draw SH perpendicular to EM produced. Join SM and produce it to meet BC at K. Draw SL perpendicular to A′P.

Now, since arc A′S=arc AM, angle A′GS=angle AEM. So GS is parallel to EM and GS is also equal to EM. Thus, SM is parallel and equal to GE or radius of the epicycle.

Now, angle A′GS=angle AEM, the angle between the apogee and the mean Sun or mean anomaly. Therefore SL=EK=R sin mean anomaly.

Now, from similar triangles SHM and EKM,

$$SH=\frac{KE\times SM}{EM}$$

$$=\frac{\text{R sine mean anomaly}\times\text{radius of epicycle}}{R}$$

$$=\frac{\text{R sine mean anomaly}\times\text{circumfrence of epicycle}}{360}$$

$$=\frac{\text{R sine mean anomaly}\times 13\tfrac{1}{2}}{360}$$

$$=\frac{\text{R sine mean anomaly}\times 3}{80}$$

Hence the value of the *mandaphala*, which is the same as before.

Thus both methods give the same result.

The same methods are followed to find the true longitude of the Moon.

This *mandaphala* is the arc corresponding to

$$\frac{\text{R sine mean anomaly}\times 31\tfrac{1}{2}}{360}\text{ as R sine,}$$

$$\text{or to }\frac{\text{R sine mean anomaly}\times 7}{80}\text{ as R sine,}$$

$$\text{or to }\frac{\text{R sine mean anomaly}\times 7\text{ minutes, nearly}}{80}$$

7 is called *guṇaka*.

As has already been pointed ourt, Lalla follows Ārya-bhaṭa when he takes the circumferences of the epicycles of the Sun and Moon as $13\frac{1}{2}$ and $31\frac{1}{3}$, respectively. Again, following him he gives the circumferences divided by $4\frac{1}{2}$.

Even from the time of Bhāskara I, if not earlier, the Indian astronomers were aware of the fact that the circumferences of the epicycles of the Sun, Moon and the planets as tabulated, are their mean circumferences and so the true positions calculated from these are not really the true positions. They, therefore, prescribed the method of successive approximations to obtain more correct results.

Lalla gives the method for the Sun and Moon (ch. 3 17), which is as follows.

Consider the diagram above for the epicyelic method. SK or *doḥphala* is already obtained. From the same similar triangles, KA′ or *koṭiphala*

$$=\frac{\text{R cos mean anomaly}\times 3}{80}$$

the Sun.

Then, $KE=KA'+R$ is known, and hence

$$ES=\sqrt{SK^2+CE^2}$$

= *mandakarṇa* or *manda* hypotenuse.

Then $\frac{3\times ES}{R}$ gives the more correct *guṇaka* or circumference of the epicycle. Hence the *dohphala*, *koṭiphala* and *karṇa* should again be calculated. Then again the *guṇaka*. The process should be repeated till the *karṇa* or the distance of the Sun is fixed and hence the correct epicycle.

Similarly, for the Moon, by using 7 as the *guṇaka*, the correct distance will be found.

The same procedure should be followed in the case of concentric and eccentirc methods. (BC)

रविचन्द्रयोः स्फुटभुक्तिः

22. 14. 1. स्वभोग्यखण्डं 'क्षितिखेन्दु'भिर्हृतं
रवेर्विधो'र्दिग्'गुणितं 'सुरो'द्धृतम् ।
तदूनयुक्ते भवतः स्फुटे गती
क्रमात् स्वकेन्द्रे मृगकर्कटादिके ॥ १५ ॥

(Lalla, *ŚiDhVṛ.*, 2. 15)

True motion of Sun and the Moon

Divide the *bhogyakhaṇḍa* of the Sun by 101 and multiply that of the Moon by 10 and divide by 33. (The results, called *mandagatiphalas* or corrections to be applied to the mean motions) should be applied to their respective mean motions negatively if the respective mean anomalies are within six Signs beginning with Capricorn, and positively if they are within six Signs beginning with Cancer; (i.e., negatively, positively, positively or negatively, according as their mean anomalies are in the first, second, third or fourth quadrant.) Thus are obtained their true motion. (15)

In the figure, let O be the centre of the Sun's eccentric circle. Let the arc AB denote the Sun's *mandakendra* or mean anomaly for any day, and the arc AC that for the next day. Then the arc BC is the motion of the mean anomaly in 1 day or *mandakendragati*. Measure off arc BD=225′. Draw BE, CF and DG as perpendiculars to AO. Draw BL perpendicular to DG cutting CF in K. Now BE is R sine of the arc AB and CF that of arc AC. Thus CK is their difference. DL is called *bhogyakhaṇḍa* by Lalla.

Now, from the proportion, arc BD: DL=arc BC: CK

$$CK=\frac{\text{arc BC}\times \text{DL}}{\text{arc BD}}$$

$$=\frac{\text{motion of anomaly}\times \textit{bhogyakhaṇḍa}}{225}$$

From above, the Sun's *mandaphala* on any day

$$=\frac{\text{R sine mean anomaly on that day}\times 3}{80}$$

and its *mandaphala* on the next day

$$=\frac{\text{R sine mean anomaly on the next day}\times 3}{80}$$

Thus, the difference between the *mandaphalas* on two consecutive days or *mandaphala* or correction to be given to the mean motion

$=\frac{3}{80}$ (R sine mean anomaly for one day $\sim$ R sine mean anomaly for the next day)

$=\frac{3}{80}\times$ CK

$$=\frac{3}{80}\,\frac{\text{motion of anomaly}\times \textit{bhogyakhaṇḍa}}{225}$$

$=\frac{3}{80}\times\frac{\textit{bhogyakhaṇḍa}}{225}\times$(Sun's motion—motion of its apogee)

$$=\frac{8}{84}\times\frac{\textit{bhogyakhaṇḍa}\times \text{Sun's motion}}{225}$$

(since Sun's apogee is supposed to have no motion)

$$=\frac{3}{80}\times\frac{\textit{bhogyakhaṇḍa}}{225}\times 59'\ 8''$$

$=\frac{\textit{bhogyakhaṇḍa}}{101}$ minutes, nearly.

Similarly, the Moon's *mandagatiphala*

$$=\frac{7}{80}\times\frac{\textit{bhogyakhaṇḍa}}{225}\times(790'\ 35''-6'\ 41'')$$

$=\frac{10}{33}\times$ *bhogyakhaṇḍa* minutes, nearly.

The *mandagatiphalsa* added to or subtracted from the mean motion of the Sun or Moon, gives its true motion subtracted when the mean anomaly is in first fourth quadrant; otherwise it is added. (BC)

ग्रहस्फुटयुक्तिः

22. 15. 1. मन्दोच्चभागरहितग्रहबाहुमौर्व्या
संसाध्य बाहुफलमस्य धनुर्दलेन ।
संस्कृत्य मध्यममृणं स्वमवेत्य केन्द्रात्
संशोधयेच्च तदनष्टमतश्चलोच्चात् ।। ४ ।।

शेषं भवेत्तु चलकेन्द्रमतो भुजज्यां
कोटयाह्वयां च विदधीत तयोः फले च ।
कोटीफलेन रहिता सहिता त्रिभज्या
कार्या कुलीरमकरादिगते स्वकेन्द्रे ।। ५ ।।

तद्वर्गबाहुफलवर्गसमासमूलं
कर्णो भवेद् भुजफलं गुणितं त्रिमौर्व्या ।
कर्णोद्धृतं कृतधनुःफलमाशुसंज्ञं
स्यात्तद्दलं स्वमथवार्णमनष्टसंज्ञे ।। ६ ।।

कार्यं क्रियाद्यथ तुलाद्यवगम्य केन्द्रं
प्राग्वत्ततो मृदुफलं सकलं विधेयम् ।
मध्ये पुनश्चलफलेन ततोऽखिलेन
प्राग्वत् सुसंस्कृततनुः स्फुटतामुपैति ।। ७ ।।

शीघ्रोद्भवेन दलितेन फलेन पूर्वं
संस्कृत्य वा ग्रहमतो विदधीत मान्दम् ।
तेनाखिलेन सकलेन च शीघ्रजेन
प्राग्वत् स्फुटो भवति संस्कृतभाग् ग्रहः सः ।। ८ ।।

(Lalla, *ŚiDhVṛ.*, 3. 4-8)

Computation of true planets

Subtract the longitude in degrees of the apogee of a planet from its mean longitude. (The remainder is the mean anomaly.) It should be reduced to the) first quadrant. Find its R sine and hence the *doḥphala*. Find corresponding arc, (which is the *mandaphala*). Add/or subtract half of it to or from the mean longitude of the planet, according as the mean anomaly is (greater or less than 180°). (The result is the longitude of the planet after the first correction) called *anaṣṭa*. (4)

Subtract (*anaṣṭa*) from the longitude of the planet's *śīghrocca*. The remainder is called *śīghrakendra*. Find its R sine and R cosine and hence the *doḥpala* and *koṭiphala*. Add or subtract the *koṭiphala* to or from the radius, according as the *śīghrakendra* is in the the first and the fourth or second and third quadrants. (The result is called *sphuṭakoṭi*). The hypotenuse or *karṇa* is the square root of the sum of the squares of the *doḥphala* and *sphuṭakoṭi*. (5)

Multiply the *doḥphala* by the radius and divide by the hypotenuse. The arc corresponding to the result as R sine is called *śīghraphala*. Add or subtract half of the *śīghraphala* to or from the once-corrected longitude of the planet (*anaṣṭa*), according as the *śīghrakendra* is

in the first and second quadrants or in the third and the fourth. (The result is the longitude of the planet after the second correction.) (6)

From the twice-corrected longitude calculate the *mandaphala* as before and apply the whole of it to the mean longitude. (From the longitude of the planet thus corrected) calculate the *śīghraphala* as before, and apply the whole of it to the (thrice-corrected longitude of) the planet. The result is the true longitude of the planet. (7)

Or, (first calculate the *śīghraphala* from the planet), and apply half of it to the longitude of the planet. Then (calculate the *mandaphala* from the corrected longitude) and apply half of it to the corrected longitude. Then, as before, calculate and apply the whole of the *mandaphala* and *śīghraphala*. Thus corrected, the planet's true longitude is obtained. (8)

Diagrammatic Rationale

There above verses give the method to determine true longitude of planets. In the case of planets revolving round the Sun, two corrections are given: (1) *mandaphala* correction, and (2) *sighraphala* correction. The first is equaivalent to the zodiacal inequality, and the second to the solar inequality of Ptolemy. The *sighraphala* roughly represents the elongation in the case of an infereior planet and the annual parallax in the case of a superior planet. Both these corrections are calculated by the epicyclic or cecentric method.

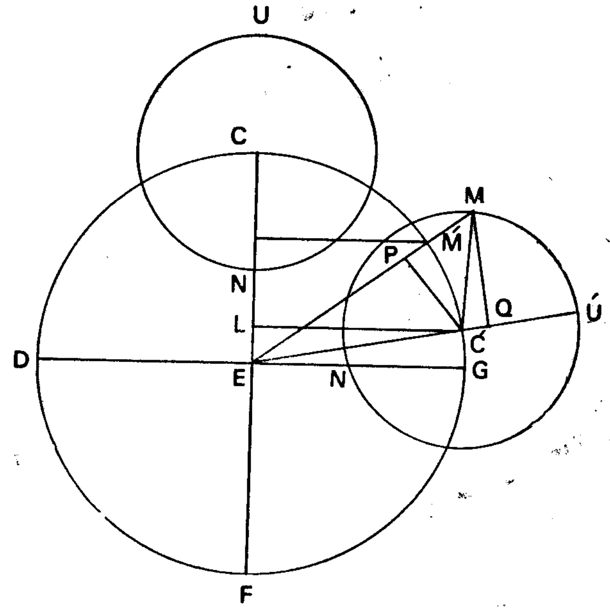
The epicyclic method. In the case of the Sun and the Moon, the tabulated epicycle are the same in the odd and even quadrants. In the case of the planets, both the *manda* epicycles and the *śīghra* epicycle vary, and for every anomaly, *manda* or *śīghra*, the variation is calculated and the correct epicycle or epicycle $\div 4\frac{1}{2}$ or *sphuṭamandaguṇaka*, as it is called, is obtained.

The method of calculation of the *mandaphala* is the same as in the case of the Sun. From *ŚiDhVṛ.*, 2. 14,

$$\textit{doḥphala} = \frac{\text{R sin mean anomaly} \times \textit{sphuṭamandaguṇaka}}{80}$$

The *mandaphala* will therefore be the corresponding arc. As in the case of the Sun, this is added to or subtracted from the mean longitude of the planet. The corrected planet is called *mandasphuṭagraha* or *mandaspaṣṭgraha* or true mean planet.

Now the second correction or *śīghra* correction using this corrected planet.



Let CDFG be the deferent or *kakṣāvṛtta* of the planet, with centre, E, centre of the Earth. With centre C and radius equal to *śīghrāntyaphalajyā* of radius of the *śīghra* epicycle of the planet, describe a circle. This is the *śīghranīcoccavṛtta*. Let CEF cut it at U and N, which are respectively *śīghrocca* or *śīghra* apogee and *śīghranīca* or *śīghra* perigee on the epicycle. Let C move with the velocity of the corrected planet or *mandaspaṣṭagraha* along the deferent and let the planet move from U′ to M along *śīghranīcoccavṛtta* so that arc U′ M is equal to arc C′C. Join EM cutting the deferent at M′. Then C′ is the *mandaspaṣṭagraha* and M′ is true planet or *sphuṭagraha*. So the correction to be given to the longitude of the true mean planet to find its true longitude is the value of arc C′M′. This value is clled *śīghraphala* and the process is callcd *śīghrakarma*.

Now to find arc C′M′. Join MC′. Draw C′L, C′P and MQ respectively perpendiculars to CE, EM and U′E.

Now angle C′EC, which is the angle between *śīghrocca* and *mandaspaṣṭagrha*, is *śīghrakendra* or *śīghra* anomaly.

Therefore C′L is R sine *śīghra* anomaly and EL is R cosine *śīghra* anomaly.

Now, since arc U′M=arc C′C, angle U′C′M=angle C′EC. So, triangles M′CA and C′EL are similar.

$$\text{So, MQ} = \frac{\text{C′L} \times \text{MC′}}{\text{C′E}}$$

$$= \frac{\text{R sine } \textit{śīghra} \text{ anomaly} \times \text{radius } \textit{śīghra} \text{ epicycle}}{\text{R}}$$

$$= \frac{\text{R} \sin \textit{śīghra} \text{ anomaly} \times \text{circumference of } \textit{śīghra} \text{ epicycle}}{360}$$

$$= \frac{\text{R} \sin \textit{śīghra} \text{ anomaly} \times \textit{sphuṭaśīghraguṇaka}}{80}$$

= *dohphala*.

Again, from the same triangles,

$$\text{C}'\text{Q} = \frac{\text{EL} \times \text{MC}'}{\text{C}'\text{E}}$$

$$= \frac{\text{R} \cos \textit{śīghra} \text{ anomaly} \times \textit{sphuṭaśīghraguṇaka}}{80}$$

= *koṭiphala*.

Now, EQ, *sphuṭakoṭi* $= \text{C}'\text{E} + \text{C}'\text{Q}$
$= \text{R} +$ *koṭiphala*.

In the first and fourth quadrants, *sphuṭakoṭi* is equal to the sum of R and *koṭiphala*, but in the second and third quadrants it is equal to their difference.

EM, *śīghrakarṇa* $= \sqrt{\text{EQ}^2 + \text{MQ}^2}$

or *śīghra* hypotenuse $= \sqrt{(\text{C}'\text{E} + \text{C}'\text{Q})^2 = \text{MQ}^2}$

Now, from similar triangles EC′P and EMQ,

$$\frac{\text{C}'\text{P}}{\text{EC}'} = \frac{\text{MQ}}{\text{EM}}$$

$$\text{So, CP}' = \frac{\text{MQ} \times \text{EC}'}{\text{EM}} = \frac{\textit{dohphala} \times \text{R}}{\textit{śīghra} \text{ hypotenuse}} .$$

Arc C′M′, *śīghraphala*, is the arc corresponding to C′P as R sine. The *śīghraphala* is added to the corrected longitude, when *śīghrakendra* is in the first and second quadrants, and subtracted when it is in the third and fourth quadrants. The *śīghraphala* can also be calculated in the same way by the eccentric method.

As regards their application to the mean longitude of a planet, the first operation should be to apply the amount of the first inequality to the mean longitude, getting thereby, in the case of a superior planet, its heliocentric longitude, and in the case of an inferior planet, the centre of its circular orbit; the second operation should be to apply the amount of the second inequality to the corrected mean longitude, which inequality is the annual parallax in the case of a superior planet and the elongation in the case of an inferior planet.

But Indian astronomers have given various methods of application, perhaps to synchronize calculation with observation. Lalla's rules are as follows:

1. Calculate the mean longitude of the planet at the observer's station and hence *mandaphala*. Apply half of it to the mean longitude. The result is the planet corrected once. Now calculate *śīghraphala* and apply half of it to the once-corrected planet. From this again calculate *mandaphala*. Apply the whole of it to the mean longitude. The result is *mandasphuṭa* planet. Calculate *śīghraphala* from it and apply the whole of it to the *mandasphuṭa* planet. The result is the true longitude of the planet.

2. According to the second method as given in verse 8, first the *śīghraphala* is calculated and half of it is applied and then the *mandaphala* is calculated and half of it applied. The third and fourth operations remain the same.

Here again, the tabulated circumferences of the epicycles, both *manda* and *śīghra*, give only the mean circumferences and, as in the case of the Sun and the Moon, the method of successive approximation should be followed. (BC)

मन्दशीघ्रभुक्तिवासना

22. 16. 1. ज्याखण्डकेन गुणिता मृदुकेन्द्रजेन
भुक्तिर्ग्रहस्य शरयुग्मयमैर्विभक्ता ।
क्षुण्णा स्फुटन गुणकेन हृता 'खनागै'-
लिप्ता गते: फलमृणं धनमुक्तवच्च ।। ११ ।।

तद्वर्जिता स्वचलतुङ्गगति: स्वभोग्य-
खण्डाहता 'शरयमाक्षि'हृता हता च ।
स्वेन स्फुटेन गुणकेन 'खनाग'भक्ता
त्रिज्याहता श्रुतिहृताऽशुफलं गते: स्यात् ।। १२ ।।

मन्दस्फुटा ग्रहगति: स्फुटतामुपैति
युक्तोनिता विरहिता सहितामुना च ।
शीघ्राभिधानंनिजकेन्द्रपदक्रमेण
वक्रा गतिर्भवति चेदृणतो विशुद्धा ।। १३ ।।

'बाणाब्धिभि:' 'शशिगुणै:' 'खयमै:' 'खबाणै:'-
'रङ्गै'र्लवैस्त्रिगृहमाद्यपदं युतं स्यात् ।
ऊनं तृतीयमिति केन्द्रपदोक्तलक्ष्म
बुध्वा गतौ चलफलं स्वमृणं विधेयम् ।। १४ ।।

(Lalla, *ŚiDhVṛ*., 3. 11-14)

Manda and Śīghra motion

The mean motion of a planet multiplied by the *bhogyakhaṇḍa* resulting from its mean anomaly and also by the corrected *mandaguṇaka* and divided by 225 and 80 gives in minutes the correction to the motion or *mandagatiphala*. It is to be applied to the mean motion as explained before. (The result is the motion corrected once.) (11)

Subtract it (viz., the corrected motion) from the motion of the *śīghrocca* of the planet. The remainder multiplied by the *bhogyakhaṇḍa* resulting from the *śīghrakendra*, the radius and the corrected *śīghraguṇaka* and

divided by 225, 80, and the hypotenuse gives the second correction of *śīghragatiphala*. (12)

When it is applied to the once-corrected motion, (i) positively, (ii) negatively, (iii) negatively, and (iv) positively, according as the *śīghrakendra* is in the first, second or third or fourth *pāda* (see next verse), the result is the true motion.

When the result is negative, the motion is said to be retrograde. (13)

The first *pāda* extends from 0° to 90° *plus* 45°, 31°, 20°, 50° and 6° for Mars, Mercury, Jupiter, Venus and Saturn, respectively. The third *pāda* extends from 180° to 270° *minus* 45°, 31°, 20°, 50° and 6°, respectively, for these planetes. This definition of *pada* should be kept in view while appliying the *śīghragatiphala* positively or negatively to the motion. (14)

Diagrammatic rationale

The verses of Lalla quoted above give formulae for *mandagatiphala* and *śīghragatiphalas* or two corrections to the mean daily motion of a planet to find its true motion on any day.

For the formula for *mandagatiphala* see above Section 22.14.1, pp. 303-4.

Now, the second formula. According to Lalla, R sine *śīghraphala* of a planet on any day.

$$= \frac{\textit{dohphala} \times R}{\textit{karna}\ (\textit{śīghra hypotenuse})}$$

$$= \frac{R \sin \textit{śīghra} \text{ anomaly on that day} \times \text{correct } \textit{śīghraguṇaka} \times R.}{80 \times \textit{śīghra} \text{ hypotenuse}}$$

Again, R sine *śīghraphala* for the next day

$$= \frac{R \text{ sine } \textit{śīghra} \text{ anomaly on the next day} \times \text{correct } \textit{śīghraguṇaka} \times R}{80 \times \textit{śīghra} \text{ hypotenuse}}$$

So *śīghragatiphala* or *śīghra* correction to motion

$$= \frac{R}{\text{hypotenuse}} \times \frac{\text{correct } \textit{śīghraguṇaka}}{80} \times \text{difference of}$$

the two R sines, approximately.

$$= \frac{R}{\text{hypotenuse}} \times \frac{\text{correct } \textit{śīghraguṇaka}}{80} \times \frac{\textit{bhogyakhaṇḍa}}{225} \times$$

motion of *śīghra* anomaly in minutes.

Now, motion of *śīghra* anomaly=motion of *śīghrocca* of the planet *minus* motion of the planet corrected by the *mandagatiphala*, found in the first part of the rule.

As regards application, according to the text, the first correction or *mandagatipala* should be applied positively or negatively, as the case may be, to the mean motion. The result is called *mandasphuṭagati*. Then to this corrected motion, the second correction, *śīghragatiphala*, should be applied positively or negatively, as the case may be. The result is the true motion. The *śīghrakendras* have been defined in verse 3. 14.

While commenting on these verses Mallikārjuna has stated that the application of correction to motion should follow the same pattern as that of a planet.

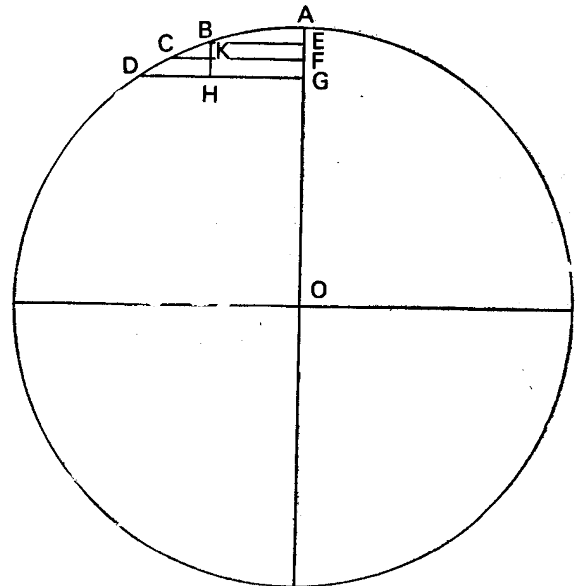
Alternate method for true motion

Lalla gives another method to find the true motion of planets.[1] Here, the first correction follows from the proportion,

mandakarṇa or *manda* hypotenuse : mean motion :: R : ?.

The result is the motion of the planet corrected once. It is called *mandasphuṭagati*. The *mandakarṇa* is found as in verse 3. 17.

From this corrected motion the second correction or *śīghragatiphala* is calculated in the following manner:



Let O be the centre of a circle. Mark off arc AB equal to the *śīghraphala* of any planet, and arc BC as *śīghrakendragati* or motion of *śīghra* anomaly. Let BD= 225′. Draw BE, CF, DG perpendiculars to AO from

[1] *ŚDhVṛ*: *BC* I. 49, II. 55-56.

B, C and D, respectively. Draw BH perpendicular to DG meeting CF at K. Then, DH is *bhogyakhaṇḍa* of *śīghraphala*.

Then, from the proportion, arc BD : DH : : arc BC : CK,

$$CK = \frac{\text{arc BC} \times \text{DH}}{\text{arc BD}}$$

$$= \frac{\text{motion of } \textit{śīghra} \text{ anomaly} \times \textit{bhogyakhaṇḍa of śīghraphala.}}{225}$$

Now, CK is the difference between R sine *śīghraphala* and R sine of *śīghraphala* increased by *śīghrakendragati*. Then this difference is reduced to the *kakṣāvṛtta* or orbit of the planet by the proportion,

śīghrakarṇa or *śīghra* hypotenuse : CK : : R : ?.

$$\text{The reduced result} = \frac{CK \times R}{\textit{śīghra} \text{ hypotenuse}}$$

This is considered by Lalla as approximately the difference between the R sines of *spaṣṭakendras* (*śīghrakendra—śīghraphala*) of two consecutive days. The angle MEC is *spaṣṭakendra* in diagram in Section 22.15, above.

Then, from the proportion,

$$225 : 225' : : \frac{CK \times R}{\textit{śīghra} \text{ hypotenuse}} : ?,$$

spaṣṭakendragati or motion of *spaṣṭakendra*

$$= \frac{CK \times R}{\textit{śīghra} \text{ hypotenuse}} \text{ minutes, nearly ;}$$

$$= \frac{\text{motion of } \textit{śīghrakendra} \times \textit{bhogyakhanda} \text{ of } \textit{śīghraphala} \times R}{225 \times \textit{śīghra} \text{ hypotenuse}}$$

(*śīghrakendragati*= motion of *śīghrocca—mandasphuṭagati* or motion corrected once as calculated above).

It is evident from the diagram in Sn. 22.15.1, that *śīghrocca* on any day—*spaṣṭakendra* on that day=*sphuṭagraha* or true planet on that day, and *śīghrocca* on the next day—*spaṣṭakendra* on the next day =*sphuṭagraha*, or true planet on the next day. So, subtracting one from the other, *śīghroccagati* or motion of *śīghrocca—spaṣṭakendragati*=*sphuṭagrahagati* or true motion of the planet.

While commenting on these verses Mallikārjuna says that the method given here gives a more correct result.

(Brahmagupta gives the same method in *BrSpSi*, ii. 43-44 and *Śrīpati* in *ŚSe*, iii. 42-43.

Bhāskara II has criticized Lalla's formulae for correction to motion given in 3.11-13 (see *SiŚi*, I. ii. 40), as they give only rough results. His formulae are as follows:

First correction or *mandagatiphala*

$$= \frac{\textit{koṭiphala} \times \text{motion of mean anomaly}}{R}$$

and second correction or *śīghragatiphala*

=motion of *śīghra* anomaly—motion of *spaṣṭa* anomaly

=motion of *śīghra* anomaly

$$- \frac{\text{motion of } \textit{śīghra} \text{ anomaly} \times R \cos \textit{śīghraphala}}{\text{hypotenuse}}$$

(*SiŚi*, 1. 2. 36-39).

But in his commentary on the above verses of Lalla he says nothing. The above rules of Bhāskara are based on instantaneous velocities of planets or *tātkālikagatī*. This subject was also dealt with by Muñjāla and Āryabhaṭa II.

In the case of the Moon, which has a great velocity, the above formula for the first correction will give better results. Bhāskara points this out in *SiŚi*, 1. 2. 38. (BC)

APPENDICES

APPENDIX I

TABLE OF INDIAN ERAS

ERA	Zero-year of Era	Year of era current in 1954 A.D. (latterpart)	Date of commencement in 1954 A.D.	Year-beginning: Solar	Year-beginning: Luni-Solar	Pūrṇimānta or Amānta (Lunar months)	Provenance	Remarks
	A.D.							
Kali yuga	—3101	5055	April	Meṣādi (Ver. equi.)	Caitra S 1	—	—	Extrapolated
Saptarṣi	—3176	—	—	—	Caitra S 1	Pūrṇimānta	Kashmir	—
Yudhiṣṭhira	—2448	—	—	—	—	—	—	—
Laukika	724 (?)	—	—	—	—	—	Multan & Kashmir	Adopted by Kalhaṇa
Buddha-nirvāṇa	— 544	2498	May 17	—	Vaiśākha S 15	—	Ceylon	—
Mahāvīra-nirvāṇa	— 527	2481	—	—	Kārtika S 1	—	—	—
Vikrama (I)	— 57	2011	April 4	Ver. equi.	Caitra S 1	Pūrṇimānta	N. India except Bengal	Earlier known as Kṛta
,, (II)	— 57	2011	Oct. 27	—	Kārtika S 1	Amānta	Gujarat	or Mālavagaṇa
,, (III)	— 57	2011	July 1	—	Āṣāḍha S 1	Amānta	Kathiawar	
Christian	0	1954	Jan. 1	Jan. 1	—	—	World	—
Śaka	**78**	**1876**	**April**	**Mesādi** (Ver. equi.)	**Caitra S 1**	**P (N. India) A (S. India)**	**All India**	**Astronomers' era**
Chedi (Kālācuri)	248	—	—	—	Āśvina S 1	Pūrṇimānta	Western & Central India	—
Valabhi	318	—	—	—	Kārtika S 1	Both P. & A.	Kathiawar & Saurashtra	From Gupta era
Gupta	319	—	—	—	Caitra S 1	Pūrṇimānta	Gupta empire (Cen. I. & Nep.)	—
Harṣa	606	—	—	—	—	—	Mathura & Kanauj	—
Hejirā	622	1374	Aug. 31	—	Muharram(Lun.)	—	—	Lunar reckoning
Bengali San	—	1361	April 14	Meṣādi	—	—	Bengal	963+Solar years since 1556
Vilāyati	—	1362	Sept. 16	Kanyādi	—	—	Bengal & Orissa	—
Āmli	—	1362	Sept. 10	—	Bhādra S 12	—	Orissa	—
Fasli (I)	—	1362	Sept. 13	—	Bhādra K 1	Pūrṇimānta	Bengal	992+Solar years since 1584
,, (II)	—	1364	July 1	July 1	—	—	Deccan	—
,, (III)	—	1364	June 8	Sun enters Mṛga. nakṣ.	—	—	Bombay	—
Magi	638	—	—	Meṣādi	—	—	Arakan, Chittagong	Similar to Bengali San
Gaṅgā	—	—	—	—	—	—	Eastern Deccan	—
Kollam (I)	824	1130	Sept. 17	Kanyādi	—	—	North Malabar	—
,, (II)	824	1130	Aug. 17	Siṁhādi	—	—	South Malabar	—
Newar	879	—	—	—	Kārtika S 1	Amānta	Nepal	In vogue till 1768 A.D., suppressed by Gurkhas
Cālukya Vikrama	1075	—	—	—	—	—	Western Deccan	Current only for 100 years
Lakṣmaṇa Sena	1104-1118	—	—	—	Kārtika S 1	—	Mithilā	—
Siṁha	1113	—	—	—	Āṣāḍha S 1	Amānta	Gujarat	Started by Siddharāja Jayasiṁha
Tārikh Ilāhi	1555	—	—	Ver. equi.	—	—	Akbar's empire	Introduced by Akbar (963 Hejirā)
Rāja Śaka	1673	—	—	—	Jyeṣṭha S 13	Amānta	Maharashtra	From the coronation of Śivāji

(Source: *Report of the Calendar Reform Committee*, p. 258)

SELECT BIBLIOGRAPHY OF INDIAN ASTRONOMY

Note: This Bibliography seeks to present, in chronological order, about 300 texts on Indian astronomy, mainly in Sanskrit, including original texts, commentaries, digests and tables, arranged under the authors of the several works.

The dates in Christian era are given in the margins. While pre-Christ dates are suffixed with 'B.C', no special indication is given to A.D. dates.

Texts from which passages have been extracted are marked by an asterisk and the details of the editions and translations used are also indicated. See also App. VI.

Against the works have been given relevant references to three source-books which contain detailed information on the manuscripts, editions, translations and studies on the works. The said three source-books are:

1. CESS — *Census of the Exact Sciences in Sanskrit*, by David Pingree, American Philosophical Society, Philadelphia, Series A, vols. 1 to 4, 1970-81.
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3. Kerala — *A Bibliography of Kerala and Kerala-based Astronomy and Astrology*, by K.V Sarma, Vishveshvaranand Institute, Hoshiarpur, 1972.

The references are to page numbers in the respective source-books.

Abbreviations used: *b* born. *c* circa. *d* died. *fl.* flourished. C commentary.

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(current text redated *c*. 400 B.C.)

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—*Sūryaprajñapti*. CESS 4. 383-84; INSA 215
—*Candraprajñapti*. CESS 4. 387-88; INSA 48.

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—*Vṛddhagargasaṃhitā*. CESS 2. 116-17; INSA 251

Do. Garga
—*Gargasaṃhitā*. CESS 2.117-18; 4.78

80 **Paitāmahasiddhānta I*, summarised mainly in ch. 12 of the *Pañcasiddhāntikā* of Varāhamihira.

(?) Vṛddha-Vasiṣṭha
—*Vṛddha-Vāsiṣṭhasaṃhitā* or *Viśvaprakāśa*. INSA 252

3rd cent. Vasiṣṭha
—* *Vāsiṣṭhasaṃhitā*, summarised in the *Pañcasiddhāntikā* of Varāhimahira

Do. —**Saurasiddhānta*, summarised in the *Pañcasiddhāntikā* of Varāhamihira.

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—**Brāhmasphuṭasiddhānta*. CESS 4. 254-55; INSA 40-42. Ed. by Sudhakara Dvivedi, Varanasi, 1901-2.
—**Khaṇḍakhādyaka*. CESS 4. 256-57; INSA 42-43.
Ed. with the C. of Bhaṭṭotpala and Translation by Bina Chatterjee, Calcutta, 2 vols., 1970.

After 600 **Sūryasiddhānta*. INSA 216-23.
Ed. with the C. of Parameśvara, by K.S. Shukla, Lucknow, 1957; Translated with Notes by Ebenezer Burgess, 1880. Indian rep. Varanasi-Delhi, 1977.

fl. 629 Bhāskara I
—**Mahābhāskaraīya*. CESS 4. 297-98; INSA 19-20; Kerala 70.
Ed. with Translation by K.S. Shukla, Lucknow, 1960.
—**Laghubhāskarīya*. CESS 4. 298-99; INSA 19; Kerala 80. Ed. with Translation, by K. S. Shukla, Lucknow, 1963.
—C. *Bhāṣya* on the *Āryabhaṭīya* of Āryabhaṭa I. CESS 4.297; INSA 19.

650-700 Haridatta
—*Grahacāranibandhana*. Kerala 29, 98.
—*Mahāmārganibandhana*. Kerala 70, 98.

fl. 689 Deva (Devācārya)
—**Karaṇaratna*. CESS 3. 121.
Ed. with Translation by K.S. Shukla, Lucknow, 1979.

c. 768 Lalla, son of Bhaṭṭa Trivikrama
—**Siṣyadhīvṛddhida*. INSA 125
Ed. with Translation by Bina Chatterjee, INSA, New Delhi, 2 vols., 1981.

c. 800 Govindasvāmin
—C. *Bhāṣya* on the *Mahābhāskarīya* of Bhāskara I. CESS 2. 143-44; 4.86; INSA 78; Kerala 28, 70.
—*Govindakṛti*. CESS 2.143; Kerala 28.
—*Govindapaddhati*. CESS 2.143; Kerala 28.

After 821 Śākalya or Śākapūṇi
—*Brahmasiddhānta* or *Śākalyasaṃhitā*, in 6 chs., a dialogue between Nārada and Brahmā. CESS 4. 259-60; INSA 188-89.

(?) *Brahmasiddhānta* or *Brahmasaṃhitā*, in 23 chs., a dialogue between Prabrahman and Brahman. CESS 4.260.

c. 825-900 Śaṅkaranārāyaṇa
—*C. *Vivaraṇa* on the *Laghubhāskarīya* of Bhāskara I. Kerala 80, 91.
Ed. with text, Kerala Univ., Trivandrum, 1949.

fl. 850 Mahāvīra, of the Digambara sect
—**Gaṇitasārasaṅgraha*. CESS 4.388-89; INSA 132.

fl. 864 Pṛthūdakasvāmi, Caturveda
—C. *Vāsanābhāṣya* on *Brāhmasphuṭasiddhānta* of Brahmagupta. CESS 4.221-22; INSA 173.
—C. *Vivaraṇa* on the *Khaṇḍakhādyaka* of Brahmagupta. CESS 4.222; INSA 173.

fl. 904 Vaṭeśvara
—**Vaṭeśvarasiddhānta*. INSA 239.
Cr. ed. with Translation and Notes by K.S. Shukla, INSA, New Delhi.

fl. 932 Muñjāla or Mañjula
—**Laghumānasa*. CESS 4.435-36; INSA 141-42; Kerala 80-81.

c. 950-1100 Āryabhaṭa II
—**Mahāsiddhānta*. CESS 1.53-54; 2.15-16; 4.28; INSA 10-11.
Cr. ed. with Translation by S. R. Sarma, Pūrva-Gaṇita Sn.: 2 Pts., Marburg, 1966.

fl. 958 Prakāśadhara of Kashmir
—C. on the *Laghumānasa* of Muñjāla. CESS 4. 227-28; INSA 172.

fl. 966 Bhaṭṭotpala, of Kashmir
—C. *Cintāmaṇi* on the *Khaṇḍakhādyaka* of Brahmagupta. CESS 4. 282.
—C. *Vivṛti* on the *Bṛhatsaṃhitā* of Varāhamihira. CESS 4.270-72; INSA 34.

fl. 975 Nemicandra, pupil of Abhayanandin
—*Trilokasāra*. CESS 3. 207-8.

fl. 999 Śrīpati, son of Nāgadeva
—* *Siddhāntaśekhara*. INSA 206.
Ed. by Babuaji Misra, Calcutta, 1932.
—* *Dhikoṭikaraṇa*. INSA 206.
Ed. with Translation by K.S. Shukla, Lucknow, 1969.

11th cent. Someśvara
—C. on the *Āryabhaṭīya* of Āryabhaṭa I. INSA 202.
—C. on the *Khaṇḍakhādyaka* of Brahmagupta, INSA 202.

fl. 1055 Daśabala son of Vairocana
—*Cintāmaṇisāraṇikā*. CESS 3. 96; INSA 52.

fl. 1073 Udayadivākara
—* C. *Sundarī* on *Laghubhāskarīya* of Bhāskara I. CESS 1.56-57; INSA 280; Kerala 13,80.

fl. 1092 Brahmadeva, son of Candrabudha
—*Karaṇaprakāśa*. CESS 4. 257-58; INSA 39.

fl. 1099 Śatānanda
—*Bhāsvatī-karaṇa*. INSA 193

b. 1114 Bhāskara II, son of Maheśvara
—* *Siddhāntaśiromaṇi*. CESS 4. 311-19; INSA 27-31.
Ed. with *Vāsanābhāṣya* and *Vārttitka* of Nṛsiṃha Daivajña, Varanasi, 1981; Eng. Translation of the Gaṇita section, by Arka Somayāji, Kendriya Vidyapeetha, Tirupati, 1980.
—*Karaṇakutūhala*. CESS 4.322-26; INSA 31-32.
—C. *Vivaraṇa* on *Śiṣyadhīvṛddhida* of Lalla. CESS 4. 326.
—C. * *Mitākṣarā* or *Vāsanābhāṣya* on his own *Siddhāntaśiromaṇi*. CESS 4. 319-22; INSA 27-32.

fl. 1132 Āśādhara, son of Rihluka
—*Grahajñāna* or *Brahmatulyānayana*. CESS 1.54; INSA 12.

fl. 1185 Caṇḍeśvara of Mithila
—C. on *Sūryasiddhānta*. CESS 3.40-41; INSA 47.

fl. 1150 Malayagiri, Jain monk from Gujarat
—C. *Vṛtti* on *Candraprajñapti*. CESS 4.360.
—C. on *Jyotiṣakaraṇḍaka*. CESS 4.361; INSA 137
—C. on the *Sūryaprajñapti*. CESS 4. 362; INSA 138.

fl. 1178 Mallikārjuna Sūri
—C. on the *Sūryasiddhānta*. CESS 4. 368; INSA 140.

b. 1191 Sūryadevayajvan
—* C. on the *Āryabhaṭīya* of Āryabhaṭa I. INSA 214; Kerala 10, 97.
Cr. ed. by K.V. Sarma, INSA, New Delhi, 1976.
—* C. on the *Laghumānasa* of Muñjāla. INSA 214; Kerala 80, 97.

fl. 1200 Āmarāja, son of Mahādeva
—*C. Vāsanābhāṣya* on the *Khaṇḍakhādyaka* of Brahmagupta. CESS 1.50; 2.15; INSA 3.

(?) —*Somasiddhānta* in 10 chs. INSA 201.

1235 Bhāskara Yogi, son of Kumāra
—C. *Tantrapradīpa* on the *Sūryasiddhānta*. INSA 119, 257.

fl. 1258 Mahādeva, son of Bandhuka
—C. on *Cintāmaṇisāraṇikā* of Daśabala. CESS 4. 372-73.

fl. 13th cent. Keśavārka, son of Rāṇaga
—*Karaṇakaṇṭhīrava*. CESS 2.77.

c. 1300 —* *Vākyakaraṇa*. Kerala 84.
Cr. ed. with the C. of Sundararāja by T.S.K. Sastri and K.V. Sarma, KSR Inst., Madras, 1964.

c. 1300 *Kujādipañcagrahavākya* or *Samudravākya*. Kerala 19.

fl. 1307 Kuvera Śarman of Kañjivihāra
—C. on *Bhāsvatī* of Śatānanda. CESS 2.47.

fl. 1315 Ṭhakkura Pheru
—* *Gaṇitasāra*. CESS 3. 78.

fl. 1316 Mahādeva, son of Paraśurama

—*Mahādevī-sāraṇī* CESS 4.374-76; INSA 131.

fl. 1325 Abhayacandra or °nandin

—C. on *Trilokasāra*. CESS 1.45.

c.1340-1425 Mādhava of Saṅgamagrāma in Kerala

—**Sphuṭacandrāpti*. CESS 4.414.

Ed. K.V. Sarma with Translation and Appendices, Vishveshvaranand Inst., Hoshiarpur, 1973.

—*Veṇvāroha*. CESS 4.414; Kerala 71, 87.

—*Agaṇitagrahacāra*. CESS 4.414-15; Kerala 1, 71.

—*Candravākyāni* or *Viliptādi-candravākyāni*. CESS 4.415; Kerala 71, 91.

14th cent. Somākara (Śeṣa or Śeṣanāga)

—C. on the *Vedāṅga-Jyotiṣa* of Lagadha. INSA 200-1.

fl. 1370 Mahendra Sūri, pupil of Madana Sūri

—*Yantrarāja*. CESS 4.393-95; INSA 133-34.

fl. 1370 Ekanātha, son of Śāraṅga

—C. *Brahmatulyabhāṣya* on the *Karaṇakutūhala* of Bhāskara II. CESS 1.60; 2.18; INSA 61.

Before 1370 Viddaṇācārya

—*Vārṣikatantra*. INSA 242.

fl. 1374 Madanapāla of the Ṭaka royal line

—C. *Vāsanārṇava* on the *Sūryasiddhānta*. INSA 128.

fl. 1375 Īśvara

—*Karaṇakaṇṭhīrava* or *Karaṇakesarī*. CESS 1.55.

fl. 1377 Makkibhaṭṭa

—C. *Gaṇitabhūṣaṇa* on the *Siddhantaśekhara* of Śrīpati. CESS 4.343; INSA 137.

—C. *Gaṇitavilāsa* on the *Mahābhāskarīya* of Bhāskara I. CESS 4.343.

(?) Bhūtiviṣṇu, son of Devarāja

—C. *Bhaṭapradīpa* on the *Āryabhaṭīya* of Āryabhaṭa I. CESS 4.331; INSA 37.

—C. *Gurukaṭākaṣa* on the *Sūryasiddhānta*. CESS 4.331.

fl. 1377 Malayendu Sūri, pupil of Mahendra Sūri

—C. on the *Yantrarāja* of Mahendra Sūri. CESS 4.363-64; INSA 138.

—*Yantrarājaracanā*. INSA 138.

1380-1460 Parameśvara, a Nampūtiri of Vaṭaśśeri in Kerala

—**Goladīpikā I*. CESS 4. 188-89; INSA 168; Kerala 58-59.

Cr. ed. with the author's own com. and Translation by K.V. Sarma, Adyar Library, Madras, 1957.

—*Goladīpikā II*, different from the above. CESS 4.191; INSA 168; Kerala 26, 58-59.

—**Grahaṇanyāyadīpikā*, CESS 4.188; Kerala 31, 58-59.

Cr. ed. by K.V. Sarma, with Translation, Vishveshvaranand Inst., Hoshiarpur, 1966.

—*Grahaṇamaṇḍanam*. CESS 4.188; Kerala 32, 58-59.

Cr. ed. with Translation by K.V. Sarma, Vishveshvaranand Inst., Hoshiarpur, 1965.

—*Grahaṇāṣṭaka*. CESS 4. 189; INSA 169; Kerala 32, 58-59.

—*Candracchāyāgaṇita*. CESS 4. 189; Kerala 35, 68-69.

—**Dṛggaṇita*. CESS 4.188; INSA 168; Kerala 48, 58-59.

Cr. ed. by K.V. Sarma, Vishveshvaranand Inst., Hoshiarpur, 1963.

—*Vākyakaraṇa*. CESS 4.189; Kerala 58-59, 84.

—C. *Bhaṭadīpikā* on the *Āryabhaṭīya* of Āryabhaṭa I. CESS 4.189-90; INSA 167-68; Kerala 10, 58-59.

—C. *Karmadīpikā* on the *Mahābhāskarīya* of Bhāskara I. CESS 4. 190; INSA 169; Kerala 58-59, 70.

—**Siddhāntadīpikā* on the *Mahābhāskarīya-bhāṣya* of Govindasvāmin. CESS 4.188; INSA 169; Kerala 58-69, 70.

Cr. ed. by T.S.K. Sastri, Madras, 1957.

—C. *Pārameśvarī* on the *Laghubhāskarīya* of Bhāskara I. CESS 4.187; INSA 169; Kerala 58-59, 80.

—C. on the *Vyatīpātāṣṭaka*. CESS 4.191; Kerala 58-59, 88.

—*C. *Vivaraṇa* on the *Sūryasiddhānta*. CESS 4. 190; INSA 169; Kerala 58-59, 97.

—C. *Pārameśvarī* on the *Laghumānasa* of Muñjāla. CESS 4.188; INSA 169; Kerala 58-59.

1387-1477 Sūrya, son of Bālāditya

—*Gaṇakānanda*. INSA 212.

fl. 1400 Padmanābha, son of Narmada

—*Yantraratnāvalī* or *Yantrakiraṇāvalī*. CESS 4. 170-73; INSA 162-63.

—C. on the above. CESS 4.170-72; INSA 162-63.

fl. 1417 Dāmodara, son of Padmanābha

—*Bhaṭatulya*. CESS 3.100-1.

—*Sūryatulya*. CESS 3.101.

fl. 1434 Gaṅgādhara, son of Candrabhaṭṭa

—*Cāndramāna*, based on the *Sūryasiddhānta*. CESS 2.82; INSA 70.

fl. 1438 Makaranda

—*Makarandasāraṇī*. INSA 135-46.

b. 1444 Nīlakaṇṭha Somayāji or °Somasutvan, of Kerala

—**Golasāra*. CESS 3.175; INSA 155; Kerala 27, 52.

Cr. ed. with Translation by K.V. Sarma, Hoshiarpur, 1970.

—*Candracchāyāgaṇita*. CESS 3.176; 4.142; INSA 155; Kerala 37, 52.

—C. *Bhāṣya* on the *Āryabhaṭīya* of Āryabhaṭa I. CESS 3.177; 4.142; INSA 155; Kerala 10, 52.

—**Jyotirmīmāṃsā*. CESS 4. 142.

Cr. ed. by K.V. Sarma, Hoshiarpur, 1977.

—**Tantrasaṅgraha*. CESS 3.176-77; 4.142; INSA 156; Kerala 46, 52.

Cr. ed. with two commentaries by K.V. Sarma, Hoshiarpur, 1977.

—**Siddhāntadarpaṇa*. CESS 3.175-76; 4.142; INSA 155-56; Kerala 52,95.

Cr. ed. by K.V. Sarma, Hoshiarpur, 1965.

—C. on the *Siddhāntadarpaṇa* .CESS 3.175-76; Kerala 52, 95.

15th cent. Cakradhara

—*Yantracintāmaṇi* or *Sad-yantracintāmaṇi*. CESS 3. 36-37; 4.88; INSA 46.

—C. on his own *Yantracintāmaṇi*. CESS 3. 36-37; 4.88; INSA 46.

15th cent. Keśava

—*Siddhāntalaghukṣamaṇikā*, based on the *Sūryasiddhānta*. CESS 2.64; INSA 110.

15th cent. Perā-Jyosyalu of Ākhaṇḍala

—*Grahacandrikāgaṇita*. CESS 2.14.

c. 1450 Muñjāditya

—*Bālabodha* or *Jyotiṣasārasaṅgraha*. CESS 4.31-35; INSA 146.

(?) Colasūri or Colaviścit

—C. *Gaṇakopakāriṇi* on the *Sūryasiddhānta*. CESS 3.52-53; 4. 94; INSA 50.

b. 1463 Aniruddha, son of Mahāsarman

—C. on *Bhāsvatīkaraṇa* of Śatānanda. CESS 1.43.

1472 Yallaya, son of Śrīdharācārya

—C. *Kalpataru* on the *Laghumānasa* of Muñjāla. INSA 255.

—C. *Kalpavallī* on the *Sūryasiddhānta*. INSA 255.

—C. Supp. to the com of Sūryadevayajvan on the *Āryabhaṭīya* of Āryabhaṭa I. INSA 254.

—*Jyotiṣadarpaṇa*. INSA 255.

fl. 1491 Appaya, son of Perubhaṭṭa

—C. (in Telugu) on the *Grahacandrikāgaṇita* of Perā-Jyosyulu of Ākhaṇḍala. CESS 1.44; 2.13; INSA 7.

b. 1495 Balabhadra, son of Vasanta

—C. *Ṭīkā* on the *Bhāsvatīkaraṇa* of Śatānanda CESS 4. 233-34; INSA 14.

fl. 1496 Keśava, son of Kamalākara of Nandigrāma

—*Grahakautuka*. CESS 2.65-66; INSA 109.

fl. 1500 Sundararāja, son of Anantanārāyaṇa

—C. *Laghuprakāśikā* on the *Vākyakaraṇa*. INSA 211.

fl. Jyeṣṭhadeva of Kerala

1500-75 —*Yuktibhāṣā* (in Malayalam). CESS 3. 76-77. 4.100; Kerala 44, 76.

—**Dṛkkaraṇam* (in Malayalam). Kerala 44, 48.

c. 1501 Lakṣmīdāsa, son of Vācaspati Miśra

—*Gaṇitatattvacintāmaṇi*. INSA 123.

c. 1500-60 Śaṅkara Vāriyar of Kerala, pupil of Jyeṣṭhadeva

—*Karaṇasāra*. Kerala 17.

—C. *Laghuvivṛti* on the *Tantrasaṅgraha* of Nīlakaṇṭha Somayāji. Kerala 46, 89.

—C. *Yuktidīpikā* on the *Tantrasaṅgraha* of Nīlakaṇṭha Somayāji. Kerala 46.

—C. on *Pañcabodha* IV.

fl. 1503 Jñānarāja

—*Siddhāntasundara* or *Sundarasiddhānta*. CESS 3. 75-76; INSA 93-94.

fl. 1505-34 Acyuta Bhaṭṭa, son of Sāgara Bhaṭṭa

—C. *Ratnadīpikā* or *Ratnamālā* on the *Bhāsvatīkaraṇa* of Śatānanda. CESS 1.36; INSA 2.

b. 1507. Gaṇeśa Daivajña, son of Lakṣmī and Keśava

—**Grahalāghava*. CESS 2.94-100; 3.72-74; INSA 64.

—*Pātasāraṇī*. CESS 2.100; INSA 67.

—*Cābukayantra*. CESS 2. 106.

—*Pratodayantra*. CESS 2.106.

—*Tithicintāmaṇi* or *Pañcāṅgatithicintāmaṇi*. CESS 2.100-3; 4.74-76; INSA 66.

—*Bṛhat-Tithicintāmaṇi* CESS 2. 104; INSA 62.

—C. on *Pratodayantra*. CESS 2.106; 4.75; INSA 67.

—*Sudhīrañjanayantra*. CESS 2.106; INSA 68.

1480-1550 Gaṇapati Bhaṭṭa, father of Govindānanda

—*Jyotiṣmatī*. CESS 2.89.

fl. 1510 Govindānanda Kavikaṅkaṇa, son of Gaṇapati Bhaṭṭa

—C. *Artharatnaprabhā* on *Jātakārṇava*. CESS 2.144; 3.35.

fl. 1525 Nārāyaṇa of Kerala

—*Uparāgakriyākrama*. CESS 3.150; 4.137; INSA 150; Kerala, 13, 50.

fl. 1525 Mādhava, son of Kandarpa

—C. on the *Bhāsvatīkaraṇa* of Śatānanda. CESS 4. 415-17; INSA 129.

fl. 1530 Citrabhānu of Kerala

—*Karaṇāmṛta*. CESS 3.47; 4.93.

fl. 1540 Gopīrāja Paṇḍita

—*Grahagaṇitakalpataru*. CESS 2.133; INSA 76.

—C. on the above. CESS 2.133; INSA 76.

—*Siddhāntakaustubha*. CESS 2.133; 4.83.

fl. 1540 Gopīrāja of Dadhigrāma

—C. *Vilāsavatī* on the *Yantrarāja* of Mahendra Sūri. CESS 2.133; INSA 77.

b. 1548 Nṛsiṃha, son of Rāma of Nandigrāma

—*Grahakaumudī*. CESS 3. 202-3; INSA 160.

—*Kheṭamuktāvalī*. CESS 3.203; INSA 160.

16th cent. Ḍhuṇḍhirāja, son of Nṛsiṃha

—*Ayanatattva* or *Sāyanatattva*. INSA 56.

—*Grahamaṇi*. INSA 50.

fl. 1550 Kṛṣṇa Cakravartin

—*Jyotiṣkaṇikā* or *Jyotiṣkalikā*. CESS 2.52-53.

c. 1550-1621 Acyuta Piṣāraṭi

—*Uparāgakriyākrama*. CESS 1.37; 4.12; Kerala 6, 13.

—*Karaṇottama*. CESS 1.37; 2.11; 4.12; INSA 1; Kerala 6, 18.

—**Rāśigolasphuṭānīti*. CESS 1.38; 4.13; Kerala 6, 78.

—C. (in Malayalam) on the *Veṇvāroha* of Mādhava. CESS 1.38; Kerala 6, 87.

—**Sphuṭanirṇaya*. CESS 1.38; 4.12-13; Kerala 6, 98.

—C. on *Sphuṭanirṇaya*. CESS 4.13; Kerala 6, 98.

—C. *Vivaraṇa* on his own *Karaṇottama*. CESS 1.37; Kerala 6, 38.

—*Chāyāṣṭaka*. CESS 1.38; 4. 13; Kerala 6, 38.

Before 1567 Ahobilanātha

—*Grahatantra*. INSA 2-3.

fl. 1572 Bhūdhara, son of Devadatta

—C. *Vivaraṇa* on *Sūryasiddhānta*. CESS 4. 331-32; INSA 35-36.

fl. 1576 Pītāmbara Siddhāntavāgīśa of Assam

—*Grahaṇakaumudī*. CESS 4.204-5.

—*Saṅkrāntikaumudī*. CESS 4.204.

. 1578 Viśvanātha Daivajña, son of Divākara

—C. *Udāharaṇa* on the *Sūryasiddhānta*. INSA. 246.

—C. *Udāharaṇa* on the *Grahalāghava* of Gaṇeśa Daivajña. INSA 246-48.

—C. *Udāharaṇa* on the *Rāmavinoda* of Rāma Daivajña. INSA 249.

—C. *Udāharaṇa* on the *Tithicintāmaṇi* of Gaṇeśa Daivajña. INSA 250.

fl. 1578 Dinakara, son of Rāmeśvara

—*Candrārkī*. CESS 3.102; 4.109; INSA 57.

—*Tithisāraṇī* or *Dinakarasāraṇī*. CESS 3. 104.

—*Kheṭasiddhi*. INSA 57.

b. 1586 *Nṛsiṃha*, son of Kṛṣṇa

—C. *Bhāṣya* on the *Sūryasiddhānta*. CESS 3. 204-5; INSA 159.

—C. *Vāsanāvārttika* on the *Siddhāntaśiromaṇi* of Bhāskara II. CESS 3.205-6; INSA 159.

fl. 1586 Gaṅgādhara, son of Nārāyaṇa

—C. *Manoramā* on the *Grahalāghava* of Gaṇeśa CESS 2.82; INSA 69.

fl. 1587 Nīlakaṇṭha Jyotirvid, son of Ananta

—*Grahakautuka*. INSA 154

—C. on *Makarandasāraṇī*. INSA 154.

fl. 1598 Bhāva-Sadāśiva Bhaṭṭa

—*Laghukaraṇa*. CESS 4.296; INSA 188

fl. 1599 Tamma Yajvan, son of Malla Yajvan

—C. *Kāmadogdhrī* on the *Sūryasiddhānta*. CESS 3. 85-86; INSA 223.

fl. 1599 Rāghavānanda Cakravartin

—*Dinacandrikā*. INSA 175.

—C. *Rahasya* on *Sūryasiddhānta*. INSA 175.

fl. 1600 Gaṇeśa, great-grandson of Gaṇeśa Daivajña (*b.* 1507)

—C. *Prakāśa* on the *Siddhāntaśiromaṇi* of Bhāskara II. CESS 2.106-7; 4.76; INSA 57.

b. 1603 Munīśvara Viśvarūpa, son of Raṅganātha

—*Siddhāntasārvabhauma*. CESS 4. 438-39; INSA 145-46.

—C. *Āśrayaprakāśinī* on the above. CESS 4. 439-40.

—*Gaṇitaprakāśa*. CESS 4.440.

—*Ekanāthamukhabhañjana*, a refutation of Ekanātha on *krāntipātārthatraya* in the *Siddhāntaśiromaṇi* of Bhāskara II. CESS 4.440.

—C. *Marīcī* on the *Siddhāntaśiromaṇi* of Bhāskara II. CESS 4.439-40; INSA 145.

fl. 1603 Raṅganātha, son of Ballāla

—C. on the *Sūryasiddhānta*. INSA 184.

fl. 1608 Viṣṇu Daivañja, son of Divākara

—*Sūryapakṣa-śaraṇa-karaṇa*. INSA 244-45.

—C. *Subhodinī* on *Bṛhat-Tithicintāmaṇī* of Gaṇeśa Daivajña. INSA 244.

b. c. 1610 Kamalākara, son of Nṛsiṃha

—* *Siddhāntatattvaviveka*. CESS 2.21-22; 4.33; INSA 102.

—C. *Udāharaṇa* on the above. CESS 2.23.

—C. *Śeṣavāsanā*. CESS 2.23; 4.33; INSA 101.

—C. *Sauravāsanā* on *Sūryasiddhānta*. CESS 2.23; 4.33; INSA 101.

fl. 1609 Mathurānātha Śarman Cakravartin

—*Ravisiddhāntamañjarī* or *Sūryasiddhāntamañjarī*. CESS 4.349; INSA 143.

fl. 1615 Rāmadaivajña, son of Madhusūdana

—C. *Yantradīpikā* on the *Yantracintāmaṇi* of Cakradhara. INSA 180.

fl. 1619 Nāgeśa Daivajña, son of Śiva Daivajña

—*Grahaprabodha*. INSA 140.

—C. on the above. INSA 146.

Before 1621 Ekanātha, son of Caṇḍika

—*Gaṇaprakāśa*. CESS 1.59-60; INSA 60.

b. 1624 Acalajit, son of Rāmeśvara

—*Candrārkī*. CESS 4.12.

fl. 1627 Kṛpāśaṅkara, son of Chājūrāma

—*Jyoiṣkedāra*. CESS 2.49-50.

fl. 1628 Nityānanda, son of Devadatta

—*Siddhāntabindu*. CESS 3.173; 4.141; INSA 159.

fl. 1629 Balabhadra, son of Dāmodara

—*Hāyanaratna*. CESS 4.234-36; INSA 14.

fl. 1635 Nārāyaṇa, son of Govinda, of Vidarbha

—C. *Udāhṛti* on the *Grahalāghava* of Gaṇeśa Daivajña. CESS 3.165; INSA 151.

fl. 1643 Raṅganātha, son of Nṛsiṃha Daivajña

—*Bhaṅgīvibhaṅgīkaraṇa*. INSA 184.

—*Lohagolakhaṇḍana*. INSA 184.

—*Siddhāntacūḍāmaṇi*. INSA 185.

fl. 1643 (Mālajit) Vedāṅgarāya, son of Tigalā Bhaṭṭa

—*Giridharānanda*. CESS 4.421-22.

—* *Pārasīprakāśa*. CESS 4.421; INSA 239-40.

fl. 1649 Kalyāṇa

—*Khecaradīpikā*. CESS 2.25.

fl. 1650 Gadādhara, son of Mahādeva
—*Lohagolasamarthana*. CESS 2.115.

fl. 1653 Kṛṣṇa, son of Mahādeva
—*Karaṇakaustubha*. CESS 2.55-56; INSA 116.

c. 1660-1740 Putumana Somayāji of Kerala
—**Karaṇapaddhati*. CESS 4.206-7; INSA 104-5; Kerala 16-17, 60-61.
Ed. by K. Sambasiva Sastri, Trivandrum, 1937.
—*Nyāyaratna*. CESS 4. 208-9; Kerala 53, 60-61.
—*Pañcabodha* III. CESS 4.202; Kerala 54-55, 60-61.
—*Veṇvārohāṣṭaka*. CESS 4.209; Kerala 60-61, 87.

fl. 1685 Kuvera Miśra
—C. on *Bhāsvatīkaraṇa* of Śatānanda. CESS 2.47; INSA 119.

fl. 1685 Gaṅgādhara, son of Vidhicandra
—C. *Udāharaṇa* on the *Bhāsvatīkaraṇa* of Śatānanda. CESS 2.85; INSA 70.

fl. 1674 Ānandamuni
—*Gaṇitasāroddhāra*. CESS 1.49; INSA 4.

b. 1686 Sawāi Jayasiṃha
—*Jayavinodasāraṇī*. CESS 3.63; 4.97.
—*Yantrarājaracanā*. CESS 3.63-64; 4.97; INSA 92.

fl. 1687 Rāmakṛṣṇa, son of Lakṣmaṇa
—C. on the *Siddhāntaśiromaṇi* of Bhāskara II. INSA 181.

fl. 1689 Puruṣottama of Kerala
—*Pañcabodhaśataka*. CESS 4.211; Kerala 56-57, 61.

fl. 1713 Trivikrama, son of Kṛṣṇajit
—*Grahasiddhi* or *Grahaśīghrasiddhi*. CESS 3. 92-93
—*Bhramasāraṇī*. CESS 3.93.

fl. 1719 Dādābhaṭṭa *alias* Dādābhāi
—*Turīyayantrotpatti*. CESS 3.97.
—C. *Kiraṇāvalī* on the *Sūryasiddhānta*. CESS 3.97; 4.107; INSA 51.

fl. 1720 Jagannātha Sāmraṭ
—*Samrāṭsiddhānta*. CESS 3.57; INSA 90.
—*Siddhāntasārakaustubha*. CESS 3.57; INSA 90.

fl. 1728-36 Tulajārāja of Tanjore
—*Inakularājatejonidhi*. CESS 3.87-88.
—*Vākyāmṛta*. CESS 3.88; INSA 230.

fl. 1728-62 Kevalarāma Pañcānana
—*Gaṇitarāja*. CESS 2.63; 4.63; INSA 110.
—*Grahacarita*. CESS 2.63; INSA 111.
—*Grahacāra*. CESS 2.64; INSA 111.
—*Tithisāraṇī*. CESS 2.63.
—*Rekhāpradīpa*. CESS 2.63.

fl. 1730 Nayanasukhopādhyāya
—*Ukara*. CESS 3.132; 4.122; INSA 153.

fl. 1740 Lakṣmīpati
—*Dhruvabhramaṇa*. INSA 124.
—*Samrāṭyantra*. INSA 124.

fl. 1740 Kamalanārāyaṇa
—C. *Udāharaṇa* on the *Bhasvatīkaraṇa* of Śatānanda. CESS 2.20; INSA 103

fl. 1750 Rājacandra
—*Siddhāntaratnāvalī*. INSA 177.

fl. 1750 (?) Āzhvāñceri Tamprākkal
—*Gaṇitasārasaṅgraha* (in Malayalam). CESS 2.11.
—*Jyotiśśāstrasaṅgraha*. CESS 4.25; Kerala 11-12, 44.
—*Saṅgrahasādhanakriyā*. Kerala 12, 93.

fl. 1750 Bhāradvājadvija of Kerala
—*Karaṇadarpaṇa*. CESS 4.294; Kerala 16, 67.
—**Gaṇitayuktayaḥ*. CESS 4.249; Kerala 25, 67. Cr. ed. by K. V. Sarma, Vishveshvaranand Inst., Hoshiarpur, Pt. I, 1979.

fl. 1751 Mallāri, son of Divākara
—C. *Siddhāntarahasya* on the *Grahalāghava* of Gaṇeśa Daivajña. INSA 139.
—*Grahasāraṇī*. INSA 139.

fl. 1753 Hemāṅgada Ṭhakkura
—*Grahaṇamālā*. INSA 89.

1756-1812 Kṛṣṇadāsa *alias* Koccu-Kṛṣṇan Āśān of Kerala
—*Pañcabodha* VIII. Kerala 20, 57.
—C. on the *Āryabhaṭīya* of Āryabhaṭa I. Kerala 11, 57.

fl. 1763 Nandarāma Miśra
—*Goladarpaṇa*. CESS 3.129.
—*Grahaṇapaddhati*. CESS 3.128-29; INSA 148.
—*Yantrasāra*. CESS 3.130; INSA 148.

fl. 1766 Śaṅkara, son of Śukadeva Bhaṭṭa
—*Karaṇavaiṣṇava*. INSA 190.

fl. 1782 Mathurānātha Śukla
—*Jyotissiddhānta*. CESS 4. 349-50; INSA 143.
—*Yantrarājakalpa* or *Yantrarājaghaṭanā*. CESS 4.349; INSA 143.

fl. 1791 Cintāmaṇi, son of Vināyaka Somayājin
—*Golānanda*. INSA 50

fl. 1792 Kṛpārāma Miśra, son of Lakṣmīnārāyaṇa
—C. on *Yantracintāmaṇi*. INSA 113-14.

1796-1830 Bālakṛṣṇa Vedavṛkṣa, son of Jyotiḥsvarūpa
—*Śiddhāntarāja*. CESS 4.245-46.

c. 1800 *Ghaṭīgopa*
—C. on the *Āryabhaṭīya* of Āryabhaṭa I. CESS 2.147; 3.36; 4.87; Kerala 10, 35.

c. 1830 Śaṅkaravarman, king of Kaḍattanāḍ, Kerala
—*Sadratnamālā*. INSA 191; Kerala 91, 93.

fl. 1807 Abbaya Kavi
—*Gaṇitāmṛta*. CESS 1.44-45; INSA 1.

fl. 1810 Kulānanda, son of Viśvarūpa
—*Mihiraprakāśa*. CESS 2.47.

fl. 1812 Dinakara, son of Ananta
—*Grahavijñānasāraṇī*. CESS 3.105.
—*Candrodayāṅkajāla*. CESS 3.105.

fl. 1813 Kāśīnātha, son of Nṛhari
—*Grahaprakāśa*. CESS 2.44.

fl. 1833 Nīlāmbara Jhā of Mithila
—C. on different sections of the *Siddhānta-śiromaṇi* of Bhāskara II. CESS 3.193-95; INSA 157-58.

fl. 1832 Jyotirāja of Nepal
—*Jyotirājakaraṇa*. CESS 3.77.

fl. 1835 Candraśekharasiṃha of Orissa
—*Siddhāntadarpaṇa*. CESS 3.45; INSA 48.

fl. 1854 Kodaṇḍarāma, son of Veṅkaṭakṛṣṇa Śāstrin
—C. *Āryabhaṭatantragaṇita* on the *Āryabhaṭīya* of Āryabhaṭa I. CESS 2.77-78; INSA 113.

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GLOSSARY OF TECHNICAL TERMS

(*Note*: The terms have been arranged according to the Devanagari alphabet.)

Aṁśa (*Bhāga*) (1) Degree. (2) Part

Akṣa Latitude (The term *Akṣa* is an abbreviation of the complete term *Akṣonnati*, meaning 'the inclination of the (Earth's) axis (to the plane of the celestial horizon)', i.e., the latitude of the place.

Akṣa-jīvā R sine of latitude

Akṣajyā R sine of latitude

Agrā Amplitude at rising, or the R sine thereof, i.e., the arc of the celestial horizon lying between the east point where the heavenly body concerned rises; or the R sine thereof, which is equal to the distance between the east-west line and the rising-setting of the heavenly body concerned.

Aṅgula Unit of length, 1/24th of a cubit

Adhimāsa(ka) Intercalary month. The intercalary months denote the excess the lunar (synodic) months over the solar months. Thus intercalary months in a *yuga*=lunar months in a *yuga minus* solar months in a *yuga*.

Anādeśyagrahaṇa Eclipse not to be predicted

Anuloma Direct or anticlockwise

Antya-jyā The current R sine-difference, i.e. the R sine difference corresponding to the elementary arc occupied by a planet. (In Hindu trigonometry a quadrant of a circle is divided into 24 equal parts, called elementary arcs.)

Apakrama (1) Greatest declination. (2) Declination

Apama Declination

Apamaṇḍala (*Apakramamaṇḍala*) Ecliptic

Ayana Northward or southward motion of a planet

Arkāgrā Sun's amplitude at rising, or the R sine thereof

Ardhajyā (jyā) R sine

Avanati Moon's true latitude as corrected for parallax

Avamarātra Omitted lunar days or omitted *tithis*

Aviśeṣa-kalākarṇa The distance (lit. hypotenuse) of a planet, in minutes, obtained by the method of successive approximations

Aśvayuk Month of Āśvina

Aśvi(*n*) Two

Aṣṭi Sixteen

Asita Asita-pakṣa, i.e. the dark half of a lunar (synodic) month. (2). The measure of the unilluminated part of the Moon

Asu A unit of time equal to four sidereal seconds

Asta The setting of a heavenly body. (2) *Asta-lagna*, i.e. the setting point of the ecliptic

Astamaya Setting, diurnal or heliacal

Astamayōdayasūtra R sine setting line

Ahargaṇa The number of mean civil days elapsed since the beginning of Kaliyuga (or any other epoch)

Ahorātra (1) A day and night, a nychthemeron. (2) The day radius, i.e. the radius of the diurnal circle

Ahorātrārdha-viṣkambha Semi-diameter of the diurnal circle (of a heavenly body, particularly the Sun), i.e., the day radius.

Ahorātrāsu The number of *asus* in a day and night, i.e., 21600

Indupāta Ascending node of the Moon

Indūcca The Moon's apogee, i.e. the remotest point of the Moon's orbit

Iṣu Five

Iṣṭa Given, desired or chosen at pleasure. (2) *Iṣṭa-graha*, i.e. desired or given planet

Ucca The *Ucca* of a planet is of two kinds : (1) *Mandocca* (apex of slowest motion), and (2) *Śīghrocca* (apex of fastest motion)

Uccanīcaparivarta Anomalistic or synodic revolutions

Uccanīcavṛtta Epicycle

Utkramaṇa (*Utkramajyā*) R versed sine

Udagayana (*Uttarāyaṇa*) Sun's northward journey from winter solstice to summer solstice

Udaggola Northern hemisphere

Udaya The rising of a planet on the eastern horizon. (2) Heliacal rising of a planet. (3) *Udaya-lagna,* i.e. the rising point of the ecliptic. (4) Addition as in *kṣayodayau* (Subtraction and addition)

Udayajīvā (*Udayajyā*) R sine amplitude of the rising point of the ecliptic

Udayāstamaya Heliacal rising and setting

Unmaṇḍala Equatorial horizon

Kakṣyā Orbit

Kakṣyāmaṇḍala Mean orbit, deferent of concentric

Karaṇa (1) The name of one of the five principal elements of the Hindu calendar. (2) An astronomical manual

Karṇa Hypotenuse, lateral side

Kalā Minute of arc

Kalārdhajyā The 24 R sine-differences in terms of minutes

Kālpa A period of 1000 *yugas*

Kāha A day of Brahmā known as *Kalpa*

Ku Earth

Kuja Mars

Kuvāyu Terrestrial wind

Kṛta Four

Kṛti Square

Kṛttikā The *nakṣatra* Kṛttikā

Kendra (1) Anomaly. The *Kendra* is of two kinds: *Manda-kendra* and *Śīghra-kendra*. The *manda-kendra* of a planet is equal to the 'longitude of the planet *minus* the longitude of the planet's *mandocca* (apogee)' and the *śīghra-kendra* of a planet is equal to the longitude of the planet's *śīghrocca* *minus* the longitude of the planet. (2) Centre.

Koṭi (*Koṭī*) (1) Vertical side of a right-angled triangle; (2) Complement of the *bhuja*

Koṭiphala The result obtained by multiplying the R sine of *koṭi* due to the planet's *kendra* by the tabulated epicycle and dividing the product by 80.

Krama Serial order

Kramajyā Same as *Jyā*

Krānti Declination

Kriya The Sign Aries

Kvāvarta Rotations of the Earth

Kṣiticchāyā Earth's shadow

Kṣitija (1) Mars, (2) Horizon

Kṣitijā (*Kṣtijyā*) Earthsine

Kṣitijyā Earthsine. The distance between the rising-setting line and the line joining the points of intersection of the diurnal circle and the six o'clock circle

Kṣipti Celestial latitude

Kṣipti-liptikāḥ The minutes of celestial latitude

Kṣepa (1) Additive quantity. (2) Celestial latitude, see under *Vikṣepa*

Kha Sky

Khagola Sphere of the sky

Khaṇḍagrahaṇa Partial eclipse

Khamadhya Middle of the sky

Gata Traversed, elapsed, past, preceding

Gati Motion. Generally used in the sense of 'daily motion' of a planet etc.

Gatyantara Motion-difference

Gantavya To be traversed; to come, succeeding

Gurvakṣara Long syllable

Gurvabda Jovian year

Gola (1) Sphere. (2) Celestial sphere. (3) Hemisphere; northern or southern hemisphere

Gola-yantra Automatic sphere model of the *Bhagola*

Graha Planet

Grahaṇa Eclipse

Grahaṇamadhya Middle of the eclipse

Grāsa (1) Measure of eclipse. (2) Erosion by overlapping

Grāhaka The eclipsing body the eclipser

Grāhakārdha Half the diameter of the eclipsing body

Grāhya The eclipsed body

Grāhya-bimba the disc of the eclipsed body

Grāhya-maṇḍala The circle of the eclipsed body

Ghaṭikā Same as *ghaṭī*

Ghaṭī A unit of time equivalent to 24 minutes

Ghanagola Solid sphere

Ghanabhūmadhya Earth's centre

Ghāta Product; multiplication

Cakra (1) Circle. (2) twelve Signs or 360°

Cakraliptā The number of minutes of arc in a circle, i.e. 21600

Cakrārdha Half of a circle, i.e., 180°

Candrocca Moon's apogee

Cara Ascensional difference. It is defined by the arc of the celestial equator lying between the six o'clock circle and the hour circle of a heavenly body at rising

Carajīvārdha The R sine of the ascensional difference

Caradala Ascensional difference

Caraprāṇa Same as *Carāsu*

Carāsu The *asus* of ascensional difference

Cala-kendra (*Śīghra-kendra*) See *Kendra*

Cala-kendra-phala Śīghraphala

Calocca Śīghrocca

Cāndramāsa Lunar month

Cāpajyārdha (*jyā*) R sine

Cāpa-bhāga An element of arc of elementary arc (i.e., one of the twentyfour equal divisions of a quadrant, (the R sine differences for which have been tabulated by Āryabhaṭa I)

Cāpita Converted into (or reduced to) the corresponding arc

Caitra The name of the first month of the year

Chāyā (1) Shadow. (2) The R sine of the zenith distance

Jina Twenty-four

Jīvā R sine

Jīvābhukti True daily motion derived with the help of the table of R sine differences

Jūka Sign Libra

Jyā R sine (Radius × sine). The R sine differences corresponding to the twentyfour equal divisions of a quadrant

Jyārdha (*Jyā*) R sine

Tama(*s*) Section of Earth's shadow cone at the Moon's distance

Tamomūrti The Moon's ascending node

Tamoviṣkambha Diameter of Shadow, i.e. diameter of the Earth's shadow cone at Moon's distance

Tārāgraha Star planets, i.e. the planets Mars, Mercury, Jupiter, Venus and Saturn

Tithi Lunar day

Tulā Sign Libra

Trijyā Radius or 3438′, Literally the R sine of three Signs

Tribhuja Triangle

Dakṣiṇāyana Sun's southward motion from summer solstice to winter solstice

Darśana-saṃskāra (usually called *Dṛkkarma*). Visibility corrections. There are three visibility corrections: 1. *Akṣa-dṛkkarma* which is the measure of the arc of the ecliptic lying between the hour circle and the circle of position of the planet concerned, 2. *Ayana-dṛkkarma* which is measured by the arc of the ecliptic lying between the cirle of celestial longitude and the hour circle of planet concerned and 3. a correction of 48′ to be subtracted from the Moon's longitude or added to it according as the Moon is in the eastern or western hemispheres (see above 17.5.8). These corrections having been applied to the true longitude of a planet, we obtain the longitude of that point of the ecliptic which rises on the local horizon simultaneously with the actual planet.

Dasra Two

Dinagaṇa Same as *Ahargaṇa*

Dināntodayalagna The rising point of the ecliptic at sunset

Dṛkkarma See *Darśanasaṃskāra*

Dṛkkṣepa Ecliptic zenith distance or its R sine. Thus, the *dṛkkṣepa* is the zenith distance of that point of a planet's orbit which is at the shortest distance from the zenith. This term is sometimes also used for the R sine of that zenith distance.

Dṛkkṣepajyā The R sine of the *dṛkkṣepa*. See *Dṛkkṣepa*

Dṛkkṣepamaṇḍala Vertical circle through the central ecliptic point

Dṛggati Arc of the ecliptic between the Sun or Moon and the central ecliptic point or its R sine

Dṛggatijyā R sine of *dṛggati*

Dṛggola Visible celestial sphere

Dṛkchāyā Parallax

Dṛṅmaṇḍala Vertical circle

Deśāntara The longitude of a place. It is either the distance of the place from the prime meridian or the difference between the local and standard times.

Deśāntara-ghaṭī *Deśāntara*, in *ghaṭīs*, i.e. the *ghaṭīs* of the difference between the local and standard times.

Dyugaṇa Same as *Ahargaṇa*

Dhanu Arc

Dhanurbhāga The element of arc or elementary arc (i.e. one of the twenty-four equal divisions of a quadrant the R sine differences of which have been tabulated by Āryabhaṭa I)

Dhṛti Eighteen

Natajyā R sine of zenith distance

Natabhāga Meridian zenith distance.

Natabhāgajyā (*Natajyā*) R sine of zenith distance

Nati (1) Meridian zenith distance or the R sine of that. (2) Difference between the parallaxes in latitude of the Sun and the Moon.

Nabha Zero

Nākṣatradivasa Sidereal day

Nirakṣajāḥ (*asavaḥ*) Asus of right ascension or the time in the *asus* of rising at the equator

Pakṣa Lunar fortnight, i.e. the period from new moon to full moon or from full moon to new moon. The period from new moon to full moon is called the light fortnight (of the light half of a lunar month) and that from full moon to new is called the dark fortnight (or the dark half of a lunar month)

Paṅkti Ten

Parama-krānti Greatest declination of the Sun, i.e. the obliquity of the ecliptic

Parama-kṣipti Greatest celestial latitude of the Moon, i.e. inclination of the Moon's orbit

Paramāpakrama Greatest declination; obliquity of the ecliptic

Paramāpakrama-guṇaḥ The R sine of the Sun's greatest declination

Paramāpakramajīvā R sine of the greatest declination

Paraśaṅku (*Paramaśaṅku*) R sine of the greatest altitude, i.e. R sine of meridian altitude

Pariṇāha Periphery, Circumference

Paridhi Circumference

Parivarta Revolution

Parva (1) Time of conjunction or opposition of the Sun and the Moon. (2) Full moon or new moon *tithi*. (3) An eclipse of the Sun or Moon.

Parvanāḍī The *nāḍīs* of the full moon or new moon *tithi* (also called *parva*) which are to elapse at sunrise on that day. Or, in other words, the time in *nāḍīs* which is to elapse at sunrise before the time of conjunction or opposition of the Sun and the Moon

Pala Latitude

Palajyā The R sine of the latitude

Paścārdha The western half

Pāta The ascending node of a planet's orbit (on the ecliptic).

Pātabhāga The degrees of the longitude of the ascending node

Pāda Ascending node

Puṣkara Three

Pūrvāparāyata Directed east to west

Pauṣṇa The *nakṣatra* Revatī which is presided over by Pūṣā.

Prakṛti Eight

Prakriyā Process

Pragrahaṇa First contact in an eclipse

Pragrāsa The beginning of an eclipse, i.e. the first contact

Pratipad The first *tithi* of either half of a lunar month is called Pratipad

Pratimaṇḍala Eccentiric circle of a planet

Pratiloma Retrograde. A planet is said to be *pratiloma* when its motion is retrograde

Prabhā The shadow of a gnomon

Pravahavāyu Provector wind

Prāk-kapāla The eastern hemisphere

Prāglagna (*Lagna*) Rising point of the ecliptic

Prāgvilagna The rising point of the ecliptic

Prāṇa A unit of time equal to four sidereal seconds or one-sixth of a *vināḍikā*

Bava The name of the first movable *karaṇa*, the *karaṇa* being one of the five important elements of the Hindu calendar

Bāhu (1) The base of a right angled triangle. (2) The *bāhu* (or *bhuja*) corresponding to a planet's anomaly (or to any arc or angle)

Bāhuphala Correction due to the *mandocca* or *śīghrocca* of a planet.

Bimba Disc or orb of a planet

Brahmadivasa A day of Brahmā, a *kalpa*

Bha (1) Asterism. (2) Sign

Bhagaṇa The revolution number of a planet, i.e., the number of revolutions that a planet performs around the earth in a certain period

Bhagola Sphere of asterisms, with its centre at the Earth's centre

Bhapañjara (*Bhacakra*) Circle of the asterisms

Bhapariṇāha Circumference of the circle of the asterisms

Bhāga Degree

Bhinna-dikka Unlike direction

Bhukti Motion or daily motion

Bhukti-yoga Sum of daily motions

Bhukti-viśeṣa Motion difference

Bhujā (*bhuja*) Lateral side of a right angled triangle

Bhujājyā The R sine of *Bhuja* (*Bhujā* or *Bāhu*)

Bhujā-phala Same as *Bāhu-phala*

Bhūgola Sphere of the Earth

Bhūgolaviṣkambha Diameter of the Earth

Bhūcchāyā Earth's shadow

Bhūjyā Same as *Kṣitijyā*

Bhū-tārāgraha-vivara The distance between the Earth and a star-planet

Bhūdina Civil days

Bhūdivasa Terrestrial day or civil day

Bheda Occultation of a star

Bhoga Motion

Makara Capricorn

Maṇḍala Circle, Revolution

Madhya (1) Centre, middle. (2) Mean. (3) Middle term in a series

Madhyagraha Mean planet

Madhya-cchāyā The midday shadow (of the gnomon)

Madhya-jivā The R sine of the zenith distance of the meridian ecliptic point

Madhyajyā Meridian sine, i.e. R sine of the zenith distance of the meridian ecliptic point

Madhyamā bhuktiḥ Mean (daily) motion

Madhya-lagna Meridian ecliptic point

Madhyasphuṭa (*Sphuṭamadhya*) True mean position of a planet

Manu (1) A period of time equal to 72 yugas. (2) Fourteen

Manda Slow, apex of slow motion

Mandakarṇa Hypotenuse associated with *mandocca*

Mandavṛtta Manda epicycle

Mandāṁśa The longitudes of the apogees of the planets in terms of degrees

Mandocca Apogee or aphelion of a planet

Mandocca-karṇa Same as *Manda-karṇa*

Mandocca-phala Correction due to a planet's *mandocca*

Māsa Month

Mīna Sign Pisces

Mṛga Sign Capricorn

Meṣa Sign Aries

Maitra The *nakṣatra* Anurādhā which is presided over by Mitra

Mokṣa The seperation of the eclipsed body after an eclipse, the last contact, or the end of eclipse

Yāmya (1) The south direction which is presided over by Yama. (2) The southern hemisphere (*yāmyagola*). (3) The *nakṣatra* Bharaṇī, which is presided over by Yama

Yāmyottara The local meridian

Yuga A period of 43,20,000 years

Yuti (1) Union. (2) Junction

Yoga (1) Conjunction in longitudes of two heavenly bodies. (2) Addition

Yoga-tārā Junction-stars, being the prominent stars of the twenty-seven nakṣatras used by the Hindu astronomers for the study of the conjunction of the planets, especially of the Moon with them

Yoga-bhāga The degrees of longitudes of the junction-stars

Yojana A unit of distance. 8000×4 cubits. The length of a *yojana* has differed at different and at different times. The *yojana* of Āryabhaṭa I and Bhāskara I is roughly equivalent to $7\frac{1}{2}$ miles

Yojana-karṇa The distance of a planet in terms of *yojanas*

Yojana-vyāsa The diameter in terms of *yojanas*

Randhra Nine

Ravi (1) The Sun. (2) Twelve

Ravimāsa Solar month

Ravivarṣa Solar year

Rasa Six

Rāma Three

Rāśi Sign

Rudra Eleven

Rūpa One

Ṛtu (1) Season. (2) Six

Lagna The rising point of the ecliptic

Laṅkā A hypothetical place on the equator where the meridian of Ujjain intersects it.

Laṅkōdaya Times of rising of the Signs at Laṅkā, i.e. right ascensions of the Signs

Lambaka R cosine of latitude

Lambaka-guṇa The R sine of the colatitude

Lambana Parallax in longitude; or, in particular, the difference between the parallaxes in longitude of the Sun and the Moon.

Liptā-śeṣa The residue of the minute

Vakra Retrograde motion

Valana (lit. deflection)

Valana relates to an eclipsed body. It is the angle subtended at the body by the arc joining the north point of the celestial horizon and the north pole of the ecliptic (i.e. the angle between the circle of position and the circle of celestial longitude of the eclipsed body). *Valana* is generally divided into two components, (1) *Akṣa-valana* and (2) *Ayana-valana*. The *Akṣa-valana* is the angle subtended at the body by the arc joining the north point of the celestial horizon and the north pole of the celestial equator (i.e., the angle between the circle of position and the hour circle of the eclipsed body). The *Ayana-valana* is the angle subtended at the body by the arc joining the north poles of the equator and the ecliptic (i.e., the angle between the hour circle and the circle of celestial longitudes of the eclipsed body). The *Valana* is also defined as follows: The Great circle of which the eclipsed body is the pole is called the horizon of the eclipsed body. Suppose that the prime vertical, equator and the ecliptic intersect the horizon of the eclipsed body at the points A, B and C, respectively, towards the east of the eclipsed body. Then the arc AB is called the *Akṣa valana*, arc BC is called *Ayana-valana*, and the arc AC is called *valana*. *Valana* is also called *Spaṣṭa-valana*

Vasu Eight

Vahni Three

Vāra Day

Vāsava The *nakṣatra* Dhaniṣṭhā which is presided over by Vasu

Vikṣipti Celestial latitude

Vikṣepa Celestial latitude

Vikṣepa-jyā The R sine of celestial latitude

Vikṣepa-liptikā The minutes of celestial latitude.

Vikṣepāṁśa The degrees of celestial latitude

Vidikka Contrary direction

Vināḍikā A unit of time, being one sixth of a *ghaṭikā*, equivalent to 24 seconds

Vimardārdha Half of the duration of totality of an eclipse

Viyat Zero

Viliptā Second of arc

Viliptikā Same as *Viliptā*

Viloma Retrograde

Vilomavivara Difference of two planets, one direct and the other retrograde

Vivara Difference

Viśākha The *nakṣatra* Visākhā

Viśva Thirteen

Viṣuvajjīva (*Akṣajyā*) R sine of latitude

Viṣuvajyā The R sine of the latitude

Viṣuvat Equator

Viṣuvaddina The day of the equinox

Viṣuvaddina-madhyāhna-cchāyā The equinoctial midday shadow

Viṣkambha Diameter

Viṣkambha-dala Semi-diameter, radius

Viṣkambhārdha Semi-diameter, radius

Vistṛti Radius

Vṛtta (1) A circle or its circumference. (2) Epicycle

Vṛttapariṇāha Circumference of a circle

Vṛttaparidhi Circumference of a circle

Vega Velocity

Veda Four

Vaidhṛta An astronomical phenomen for which cf. above 11. 15. 1-3

Vaiśva The *nakṣatra* Uttarāṣāḍha which is presided over by Viśve Devāḥ

Vaiṣṇava The *nakṣatra* Śravaṇa, which is presided over by Viṣṇu

Vyatipāta An astronomical phenomenon; see above, 11. 15. 1-3

Vyāsa (1) Diameter. (2) (sometimes) Radius

Vyāsa-dala Semi-diameter, radius

Vyāsa-yojana Diameter in terms of *yojanas*

Vyāsārdha Semi-diameter, radius

Vyoma Zero

Śakābda The year of the Śaka era

Śakra Fourteen

Śakra-tārakam The *nakṣatra* Jyeṣṭhā which is presided over by Indra (Śakra)

Śaṅku (1) Gnomon. (2) The R sine of altitude of a heavenly body

Śaṅkvagra The distance of the projection of a heavenly body on the plane of the celestial horizon from the rising setting line of the heavenly body

Śara (1) Arrow. (2) R versed sine. (3) Five

Śaśi (1) The Moon. (2) One

Śaśidivasa Lunar day

Śaśimāsa Lunar month

Śikhi Three

Śīghra *Śīghrocca*, *Śīghra* epicycle

Śīghra-kendra The *śīghra* anomaly. See *Kendra*

Śīghravṛtta *Śīghra epicycle*

Śīghrocca Apex of fastest motion. See *Ucca*

Śīghrocca-karṇa (*Śīghra-karṇa*) It is equal to $[(R + \text{or minus } R \sin k)^2 + (R \sin b)^2]^{\frac{1}{2}}$ where $R = 3438'$, $k = $ *koṭi* due to *Śīghrakendra*, and $b = $ *bhuja* due to *Sighra-kendra*

Śṛṅgonnati The elevation of the Moon's horns (or cusps)

Śaila Seven

Saṃskṛta Corrected

Sakṛt By the application of the rule only once (i.e., without the application of the method of successive approximations)

Samapūrvaparaḥ Śaṅkuḥ The R sine of the prime vertical altitude (of the Sun)

Samakala Two planets are said to be *samakala* when they are either in conjunction or opposition in longitude

Samamaṇḍala The prime vertical

Samarekhā The meridian

Samaliptendu The longitude of the Moon for the time of opposition or conjunction of the Sun and the Moon

Samparka (1) The sum of the diameters of two bodies in contact. (2) Used in the sense of "the sum of the diameters of the eclipsed and eclipsing bodies".

Samparka-dala Same as *Samparkārdha*

Samparkārdha Half the sum of the diameters of the eclipsed and eclipsing bodies

Sarvagrāsa Total eclipse

Savya Clockwise

Sāgara Four

Sāyaka Five

Sārpamastaka Name of an astronomical phenomenon. One of the *Vyatipātas*. *cf.* 11. 15. 1-3, above.

Sita (1) The measure of the illuminated part of the Moon's disc; the phase of the Moon. (2) The light half of a lunar month (*sita-pakṣa*). (3) Venus.

Sita-pakṣa The light or bright half of a lunar month

Sitamāna The measure of the illuminated part of the Moon's disc

Saumya (1) North. The northern (hemisphere). (2) Mercury

Sauri Saturn

Sthityardha Half the duration (of an eclipse)

Sthityardha-nāḍīkā Half the duration (of an eclipse) in terms of *nāḍīs*

Sthūla Gross, approximate

Sparśa First contact of an eclipse

Sphuṭa True, corrected

Sphuṭa-graha True planet

Sputa-bhukti True (daily) motion.

Sphuṭa-bhoga True motion

Sphuṭa-madhya (1) True mean. (2) The true-mean planet. (3) The true-mean longitude of a planet

Sphuṭa-yojana-karṇa The true distance of a planet in terms of *yojanas*

Sphuṭa-vṛtta True or corrected epicycle

Svadeśa-bhūmi-vṛtta The local circumference of the Earth, i.e., circumference of the local circle of latitude

Svadeśa-bhodaya Times of rising of the Signs at the local place, or oblique ascensions of the Signs

Svadeśākṣa The latitude of the local place.

Svadeśodaya Same as *Svadeśabhodaya*

Svara Seven

Harija Horizon

Hasta Cubit, measure of length

BHŪTASAṄKHYĀ—WORD NUMERALS

Used in Indian Mathematical texts

0 ananta, antarikṣa, abhra, ambara, ākāśa, kha, gagana, jaladharapatha, nabha, pūrṇa, bindu, randhra, viyat, viṣṇupada, vyoma, śūnya; all synonyms of 'Sky'.

1 abja, ādi, indu, ilā, urvarā, kalādhara, ku, kṣapākara, kṣiti, kṣmā, go, candra, jagati, tanu, dharaṇi, dharā, nāyaka, pitāmaha, pṛthvī, prāleyāṃśu, bhū, mahī, mṛgāṅka, rajanīkara, rūpa, vasudhā, vasundharā, vidhu, śaśadhara, śaśāṅka, śaśi, śītakara, śītaraśmi, śītāṃśu, śveta, sudhāṃśu, soma, himakara, himagu, himāṃśu; all synonyms of 'Earth' and 'Moon'.

2 akṣi, ambaka, ayana, aśvin, īkṣaṇa, oṣṭha, kara, karṇa, kuca, kuṭumba, gulpha, cakṣu, jaṅghā, jānu, dasra, dṛṣṭi, dvanda, dvaya, naya, nayana, nāsatya, netra, pakṣa, bāhu, bhuja, yama, yamala, yugala, yugma, ravicandrau, raviputra, locana; all synonyms of 'Eye' and 'Hand'.

3 agni, anala, kāla, kṛśānu, guṇa, gṛha, jvalana, tapana, trikāla, trigata, triguṇa, trijagat, trinetra, dahana, pāvaka, pura, bhuvana, ratna, rāma, loka, vaiśvānara, vahni, sahodarāḥ, śikhin, haranetra, hutabhuk, hutabhuj, hutāśa, hutāśana, hotṛ; all synonyms of 'Fire' and 'Worlds'.

4 abdhi, ambudhi, ambhodha, ambhodhi, ambhonidhi, arṇava, āya, āśrama, udadhi, kaṣāya, kṛta, kendra, koṣṭha, gati, ghana, caraṇa, jala, jaladhi, jalanidhi, turya, diś, payodhi, payonidhi, praṇimnageśa, bandhu, yuga, lavaṇoda, varṇa, vāridhi, viṣanidhi, veda, śruti, samudra, salilākara, sāgara, sukha; all synonyms of 'Ocean'.

5 akṣa, artha, indriya, iṣu, karaṇīya, tattva, parva, pavana, pāṇḍava, prāṇa, bāṇa, bhāva, bhūta, mahābhūta, rāga, ratna, viṣaya, vrata, śara, śastra, sāyaka; all synonyms of 'Arrow'.

6 aṅga, ari, ṛtu, kāya, kāraka, kumāravadana, khara, tarka, darśana, dravya, māsārdha, rasa, rāga, lekhya, ṣaṇmukha, śāstra.

7 aga, acala, atri, adri, aśva, ṛṣi, kalatra, giri, graha, chandaḥ, tattva, turaga, dvipa, dhātu, dhī, naga, pannaga, parvata, bhaya, bhūbhṛt, mātṛka, muni, yati, vāji, vāra, vyasana, śaila, svara, haya; all synonyms of 'Horse' and 'Mountain'.

8 anīka, anuṣṭubha, ahi, ibha, karman, kuñjara, gaja, takṣa, tanu, danti, dik, diggaja, durita, dvīpa, dvirada, dhī, nāga, puṣkarin, bhūti, maṅgala, mada, mātaṅga, mati, vasu, sarpa, siddhi, sindhura, hastin; all synonyms of 'Elephant' and 'Serpent'.

9 aṅka, anilāhva, upendra, keśava, gīr, go, graha, chidra, tārkṣyadhvaj, durgā, dvāra, nanda, nidhi, padārtha, randhra, labdha, labdhi

10 avatāra, aṅgulī, āśā, kakubh, karman, dik, diś, diśā, paṅkti, rāvaṇaśira

11 akṣauhiṇī, īśa, īśvara, bharga, bhava, mahādeva, mṛḍa, rudra, śaṅkara, śiva, śūlin, śvargeśa, hara; all synonyms of god 'Śiva'.

12 arka, āditya, ina, tīkṣṇāṃśu, dinanātha, dinapa, divākara, dyumaṇi, bhānu, bhāskara, maṇḍala, mārtaṇḍa, māsa, ravi, rāśi, vyaya, sūrya; all synonyms of 'Sun'.

13 aghoṣa, atijagatī, karaṇa, kāma, viśva, viśvedevāḥ

14 indra, manu, loka, vidyā, śakra, śarva; all synonyms of 'Indra'.

15 ahan, ghasra, tithi, dina, pakṣa

16 aṣṭi, kalā, nṛpa, bhūpa, bhūpati

17 atyaṣṭi

18 dhṛti, purāṇa, vidyā

19 atidhṛti

20 kṛti, nakha

21 utkṛti, prakṛti, mūrchanā, svarga

22 kṛti, jāti

23 vikṛti

24 arhat, gāyatrī, jina, siddha

25 tattva

26 utkṛti

27 uḍu, nakṣatra, bha; all synonyms of 'Star' and 'Asterism'.

32 danta, rada; all synonyms of 'Teeth'.

33 amara, tridaśa, deva, sura, surādhipa; all synonyms of 'Gods'.

40 naraka

48 jagati

49 tāna

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Dṛkkaraṇa of Jyeṣṭhadeva

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Siddhāntaśekhara-vyākhyā of Makkibhaṭṭa

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Śiṣyadhīvṛddhida of Lalla

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Tantravārttika of Kumārila Bhaṭṭa

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